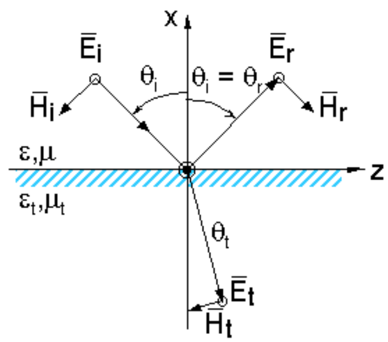
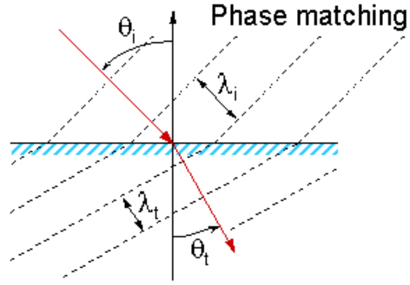
PLANE WAVES AT PLANAR BOUNDARIES

Amplitudes and phases of reflection and transmission:

First consider TE waves



Phase matching



$$\bar{E}_i = \hat{y}\ E_o e^{+jk_x x - jk_z z}$$

$$\bar{E}_r = \hat{y}\underline{\Gamma}\ E_o e^{-jk_x x - jk_z z}$$

$$\bar{E}_t = \hat{y}\underline{T}\ E_o e^{+jk_x x - jk_z z}$$

where

$$k_{x_i} = k_i \cos\theta_i$$

$$k_i = \omega\sqrt{\mu\varepsilon}\ ,\ k_t = \omega\sqrt{\mu_t \varepsilon_t}$$

Apply boundary conditions at $x \triangleq 0$

$\bar{E}_{//}$ continuous: $E_o + \underline{\Gamma} E_o = \underline{T}E$

$$1 + \underline{\Gamma} = \underline{T}$$

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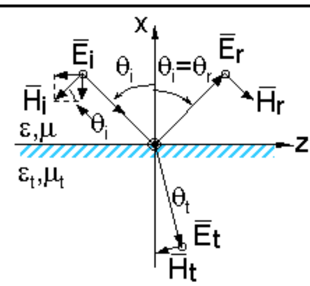
PLANE WAVES AT PLANAR BOUNDARIES (2)

(1) $\bar{E}_{//}$ continuous: $1 + \underline{\Gamma} = \underline{T}$



(2) $\bar{H}_{//}$ continuous: $-\cos\theta_i (E_o/\eta_o) + \cos\theta_i \underline{\Gamma}(E_o/\eta_o) = -\cos\theta_t \underline{T}(E_o/\eta_t)$

$$1 - \underline{\Gamma} = \frac{\cos\theta_t}{\cos\theta_i}\frac{\eta_o}{\eta_t}\underline{T} \quad \text{(Note: } \eta \neq n\text{)}$$

(1) & (2) $\Rightarrow$ $\underline{T} = 2/\left[1 + (\eta_o \cos\theta_t / \eta_t \cos\theta_i)\right] \to 0$ as $\theta_i \to \frac{\pi}{2}$

(1) $\Rightarrow$ $\underline{\Gamma} = \underline{T} - 1 \quad \to -1$ as $\theta_i \to \frac{\pi}{2}$, and $\to \begin{Bmatrix} 1 \text{ for } \eta_o \to \ll \eta_t \\ -1 \text{ for } \eta_o \gg \eta_t \end{Bmatrix}$ for $\theta_i = 0$

where $\cos\theta_t = \sqrt{1 - \sin^2\theta_t}$; $\sin\theta_t = (c_t/c_i)\sin\theta_i$ (Snell's Law)

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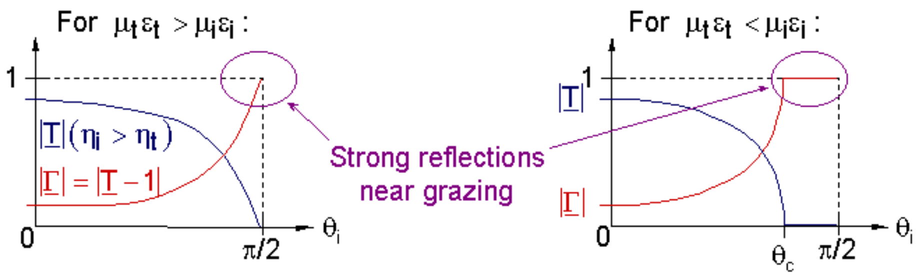
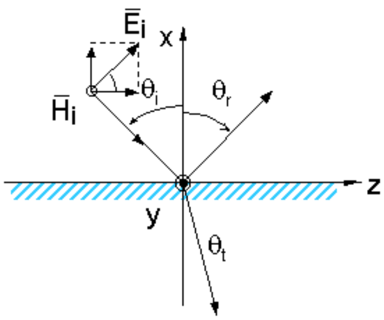
PLANE WAVES AT PLANAR BOUNDARIES (3)



Consider TM Waves



$$\underline{\overline{H}}_i = \hat{y} H_o e^{+jk_{x_i}x - jk_z z}$$

$$\underline{\overline{E}}_i = (\hat{x}\sin\theta_i + \hat{z}\cos\theta_i) H_o \eta_i e^{+jk_{x_i}x - jk_z z}$$

Can use same method as for TE waves (see above) or "Duality"

L8-3

METHOD OF DUALITY

Note: for $\rho = \bar{J} \equiv 0$ we have:

$\nabla \times \underline{\overline{E}} = -j\omega\mu\underline{\overline{H}}$	$\nabla \times \underline{\overline{H}} = j\omega\varepsilon\underline{\overline{E}}$
$\nabla \times \underline{\overline{H}} = j\omega\varepsilon\underline{\overline{E}}$	$\nabla \times \underline{\overline{E}} = -j\omega\mu\underline{\overline{H}}$
$\nabla \bullet \mu\underline{\overline{H}} = 0$	$\nabla \bullet \varepsilon\underline{\overline{E}} = 0$
$\nabla \bullet \varepsilon\underline{\overline{E}} = 0$	$\nabla \bullet \mu\underline{\overline{H}} = 0$

Claim: if given valid solution in zone where $\rho = \bar{J} \equiv 0$

and we write $\underline{\overline{E}} \to \pm\underline{\overline{H}}$, $\underline{\overline{H}} \to \mp\underline{\overline{E}}$

$\mu \to \varepsilon$, $\varepsilon \to \mu$

This "dual solution" is valid (if the new fields satisfy the new boundary conditions)

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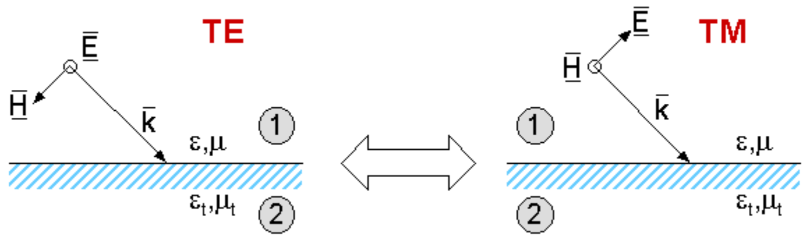
EXAMPLE OF DUALITY



A) If TE is solution in (1), then TM is too if $\varepsilon \to \mu$, $\bar{E} \to \bar{H}$
$\mu \to \varepsilon$, $\bar{H} \to -\bar{E}$

B) In region (2), use same transformation to find TM solution, always

C) TM solutions in (1), (2) match boundary conditions (B.C.) too
only if B.C. are also dual.

Here: B.C. are $E_{/\!/}, H_{/\!/}$ continuous; they satisfy duality $(E_{/\!/} \leftrightarrow \pm H_{/\!/})$
Therefore duality yields TM solution immediately when B.C. satisfy duality

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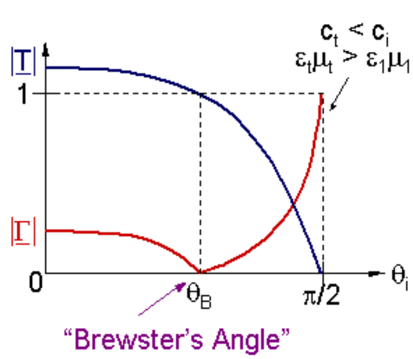
EXAMPLE OF DUALITY (2)

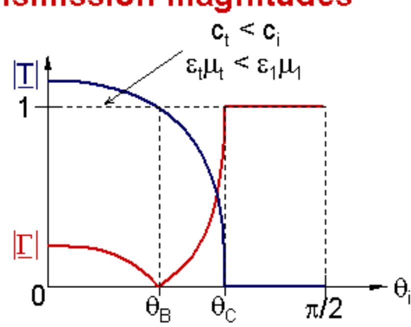
TE Solution: **Dual TM Solution:**

$$\underline{T}_{TE} = 2 \Big/ \left(1 + \frac{\cos\theta_t \sqrt{\mu_o/\varepsilon_o}}{\cos\theta_i \sqrt{\mu_t/\varepsilon_t}}\right) \rightarrow \underline{T}_{TM} = 2 \Big/ \left(1 + \frac{\cos\theta_t}{\cos\theta_i} \frac{\sqrt{\varepsilon_o/\mu_o}}{\sqrt{\varepsilon_t/\mu_t}}\right)$$

$$\underline{\Gamma}_{TE} = \underline{T}_{TE} - 1 \rightarrow \underline{\Gamma}_{TM} = \underline{T}_{TM} - 1$$

Example: TM reflection and transmission magnitudes





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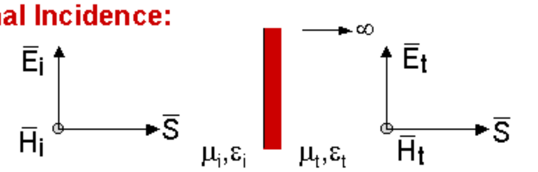
PHYSICAL INTERPRETATIONS OF BREWSTER'S ANGLE

Normal Incidence:



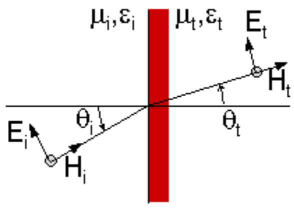
$|\bar{E}_i|/|\bar{H}_i| = \eta_i = \sqrt{\mu_i/\varepsilon_i}$

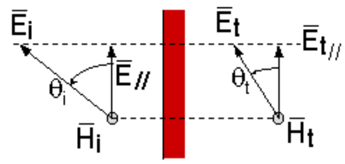
$|\bar{E}_t|/|\bar{H}_t| = \eta_t \neq \eta_i$

For NO reflection $\bar{E}_{//}, \bar{H}_{//}$ must be continuous.

For normal incidence this is possible only IF $\eta_i = \eta_t$

Angular Incidence:





If $|\bar{H}_t| = |\bar{H}_i|$, and $|\bar{E}_t| < |\bar{E}_i|$, then there is an incidence angle where impedances match.

e.g. If: $\mu_i = \mu_t$ and $\varepsilon_t > \varepsilon_i$

Then: $\eta_t < \eta_i = \sqrt{\mu_i/\varepsilon_i}$, $|\bar{E}_t| < |\bar{E}_i|$

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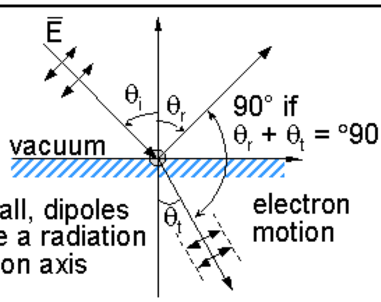
ALTERNATIVE INTERPRETATION OF θ_B



Recall, dipoles have a radiation null on axis

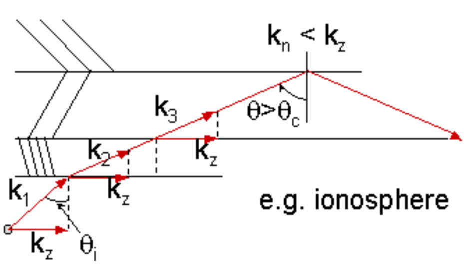
If $\mu_t = \mu_i$; $\varepsilon_i \neq \varepsilon_t \Rightarrow$ null for TM wave when $\theta_r + \theta_t = 90°$ (then $\theta_i = \theta_B$) .

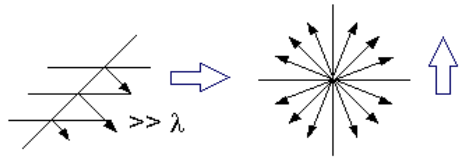
What happens when $\varepsilon_i = \varepsilon_t$, $\mu_t \neq \mu_i$?

Magnetic dipoles are included, and null for TE wave when $\theta_i + \theta_t = 90°$ (then $\theta_i = \theta_B$)

Multilayer refraction:

$$\left|\sum_m \Gamma_m E_o\right|^2 \cong 0$$





reflection phases cancel

$\Rightarrow$ no losses, 100% transmission

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