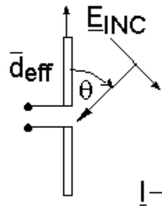
RECEIVING ANTENNAS

Recall: $A_e(\theta,\phi) = \left(\lambda^2/4\pi\right)G(\theta,\phi)$

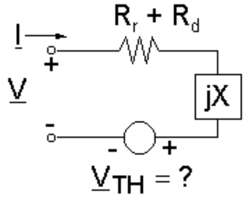
We never proved it; sometimes untrue (when?)

Proof for Short Dipole Antenna:



If d << λ/2π, quasistatics applies

We seek V_{Th} in equivalent circuit:



Assume Normally Incident Uniform Plane Wave:

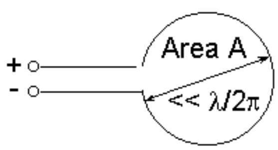
Case 1: small loop antenna (D << λ/2π)

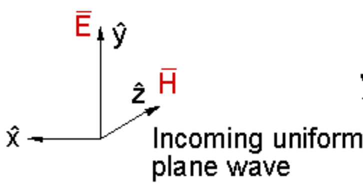
Case 2: short dipole antenna (d << λ/2π)

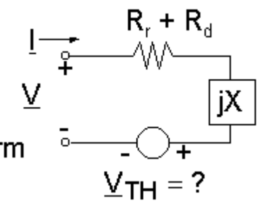
L9-1

SMALL LOOP ANTENNA: OPEN CIRCUIT VOLTAGE

Quasistatic Limit:







Faraday's Law:

$$\nabla \times \bar{E} = -\partial \bar{B}/\partial t \Rightarrow \int_A \left(\partial \bar{B}/\partial t\right) \bullet da = \int_c \bar{E} \bullet d\bar{s} = V_{Th}$$

Open Circuit Voltage:

$$V_{Th} = A\mu_o\left(\partial[\bar{H} \bullet \hat{z}]/\partial t\right)$$

UPW: Power $P\left[W/m^2\right] = \eta_o |\bar{H}_o|^2/2$

Where: $\bar{H} = \hat{z}H_o \cos\omega t$ at $z = 0$, and

$$\left(\partial[\bar{H} \bullet \hat{z}]/\partial t\right) = -\omega H_o \sin\omega t$$

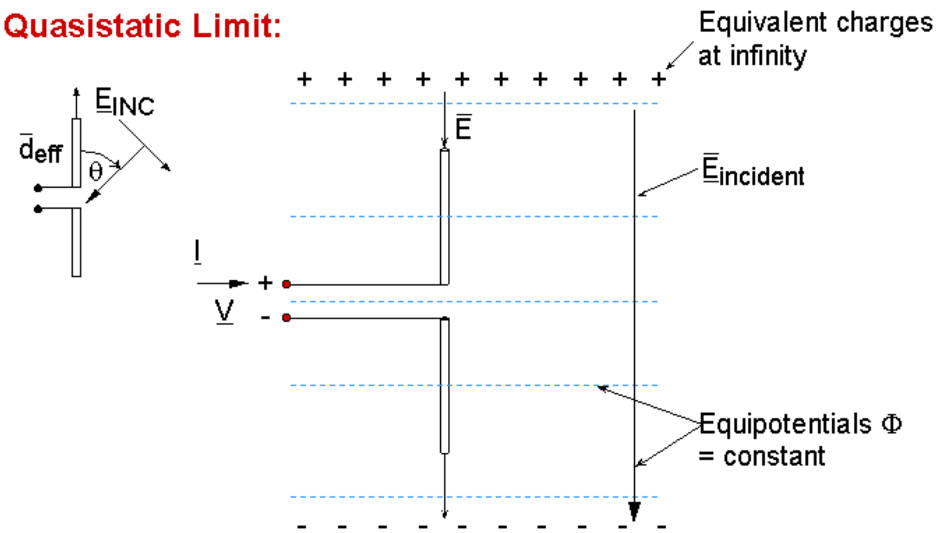
Therefore: $\left(\partial[\bar{H} \bullet \hat{z}]/\partial t\right) = -\omega(2P/\eta_o)^{0.5} \sin\omega t$ and

$$V_{Th} = -A\mu_o\omega(2P/\eta_o)^{0.5} \sin\omega t$$

L9-2

SHORT DIPOLE ANTENNA: OPEN CIRCUIT VOLTAGE

Quasistatic Limit:

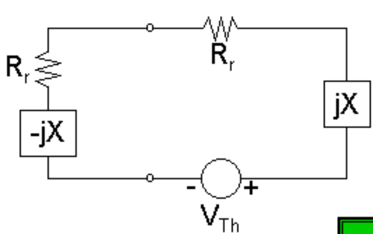


$$\underline{V}_{Th} = -\bar{\underline{E}}_{INC} \bullet \bar{d}_{eff} = -\bar{\underline{E}}_{INC} d_{eff} \sin\theta$$

L9-3

MAXIMUM POWER EXTRACTABLE FROM A SHORT DIPOLE ANTENNA

Antenna Equivalent Circuit plus Matched Load:



$$P_{rec.max} = \frac{1}{2}\left|\underline{V}_{Th}/2\right|^2 \Big/ R_r$$

$$= \left|\bar{\underline{E}}_{INC}\right|^2 \frac{d_{eff}^2}{8R_r} \sin^2\theta$$

where $R_r = \eta_o (k d_{eff})^2 / 6\pi$

$$\text{Define } A_{eff}(\theta,\phi) \triangleq \frac{P_{rec}}{\left|\underline{E}_{inc}\right|^2 / 2\eta_o} \quad \text{"Effective Area"}$$

For short dipole: $A_{eff}(\theta) = \dfrac{2\eta_o P_{rec}}{\left|\underline{E}_{inc}\right|^2} = \dfrac{\eta_o d_{eff}^2 \sin^2\theta}{4R_r} = \dfrac{\lambda^2 3}{8\pi} \sin^2\theta$

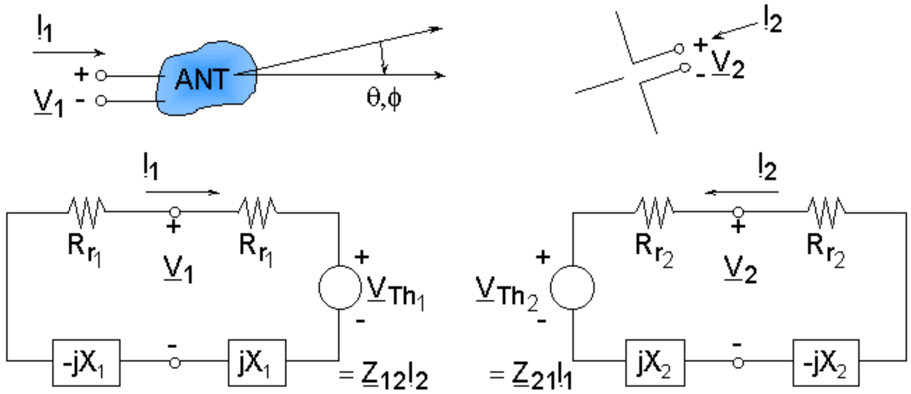
Therefore: $A_e(\theta,\phi) = \dfrac{\lambda^2}{4\pi} G(\theta,\phi) (m^2)$ [true for almost all antennas]

Recall, for short dipole: $G(\theta) = 1.5 \sin^2\theta \;\; [\neq f(\omega)]$

L9-4

PROOF THAT A = Gλ²/4π FOR MOST ANTENNAS

Test Range = Unknown plus Short Dipole Antenna:



$$P_{rec_1} = \frac{|\underline{Z}_{12}\underline{I}_2|^2}{8R_{r_1}} = P_{t_2}\frac{G_2}{4\pi r^2}A_1 \qquad P_{rec_2} = \frac{|\underline{Z}_{21}\underline{I}_1|^2}{8R_{r_2}} = P_{t_1}\underbrace{\frac{G_1}{4\pi r^2}}_{}A_2$$

(Power transmitted by antenna #1)

Wm^{-2} at antenna 2

L9-5

PROOF THAT A = Gλ²/4π FOR MOST ANTENNAS (2)

$$P_{rec_1} = \frac{|\underline{Z}_{12}\underline{I}_2|^2}{8R_{r_1}} = P_{t_2}\frac{G_2}{4\pi r^2}A_1 \qquad P_{rec_2} = \frac{|\underline{Z}_{21}\underline{I}_1|^2}{8R_{r_2}} = P_{t_1}\frac{G_1}{4\pi r^2}A_2$$

$$\therefore \quad \frac{P_{rec_2}}{P_{rec_1}} = \frac{G_1 A_2 P_{t_1}}{G_2 A_1 P_{t_2}} \Rightarrow \frac{A_1}{G_1} = \frac{A_2}{G_2}\left[\frac{P_{t_1}}{P_{t_2}} \bullet \frac{P_{rec_1}}{P_{rec_2}}\right]$$

$$\text{But} \quad \frac{P_{rec_1}}{P_{rec_2}} = \frac{|\underline{Z}_{12}\underline{I}_2|^2}{|\underline{Z}_{21}\underline{I}_1|^2} \bullet \frac{R_{r_2}}{R_{r_1}} = \frac{|\underline{Z}_{12}|^2}{|\underline{Z}_{21}|^2} \bullet \frac{P_{t_2}}{P_{t_1}}$$

Therefore, if $|\underline{Z}_{12}|^2 = |\underline{Z}_{21}|^2$, then $\boxed{\frac{A_1}{G_1} = \frac{A_2}{G_2} = \frac{\lambda^2}{4\pi} \quad \text{Q.E.D.}}$

L9-6

RECIPROCITY AND NON-RECIPROCAL DEVICES

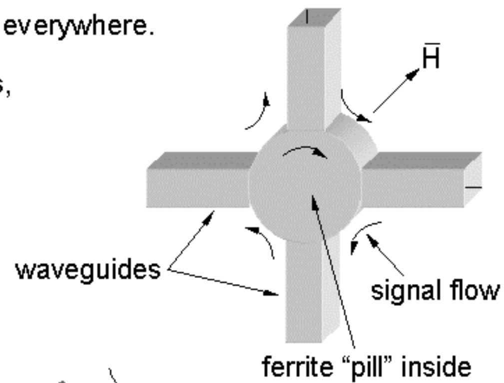
Reciprocity:

$|\underline{Z}_{12}|^2 = |\underline{Z}_{21}|^2$ if $\bar{\bar{\varepsilon}} = \bar{\bar{\varepsilon}}^t$, $\bar{\bar{\mu}} = \bar{\bar{\mu}}^t$ everywhere.

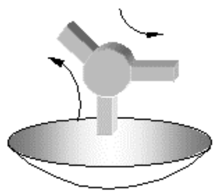
Exceptions: magnetized plasmas,
magnetized ferrites

Non-reciprocal Devices:

4-Port Circulators



Non-reciprocal Antennas:

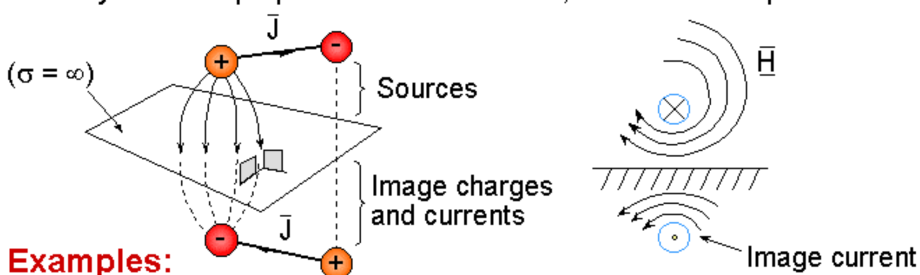


L9-7

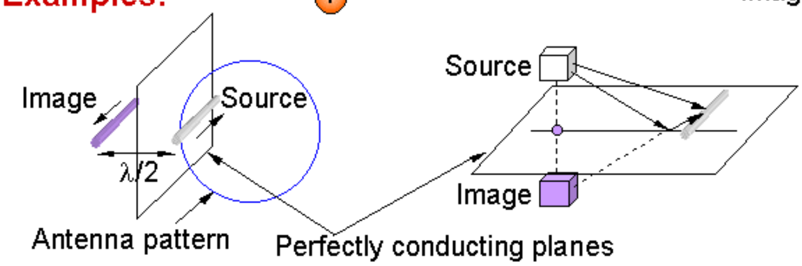
MIRROR IMAGES

Mirror Images: Infinite flat conducting surfaces are "mirrors"

Consider the charges and currents shown, and how $\bar{E}$ must everywhere be perpendicular to the mirror, and $\bar{H}$ must be parallel



Examples:



L9-8