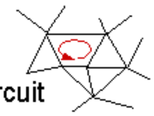
KIRCHOFF'S VOLTAGE LAW

Kirchoff's Voltage Law: (KVL)

Around any loop: $\sum_i V_i = 0$

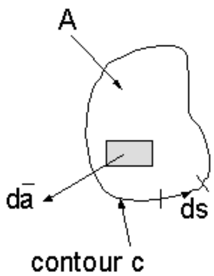


Faraday's Law:

$\nabla \times \bar{E} = -\partial\bar{B}/\partial t$ [differential form]

Integral form is:

$\int_c \bar{E} \bullet d\bar{s} = -\int_A (\partial\bar{B}/\partial t) \bullet d\bar{a}$



[Recall Stoke's Theorem: $\int_A (\nabla \times \bar{G}) \bullet \hat{n} da = \int_c \bar{G} \bullet d\bar{s}$]

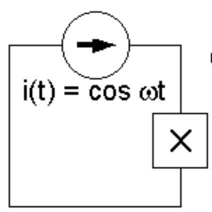
If $\partial\bar{B}/\partial t = 0$ in A, then Kirchoff's voltage law must be satisfied

More generally: **KVL assumes all magnetic energy is stored inside circuit elements**

L10-1

KIRCHOFF'S VOLTAGE LAW (2)

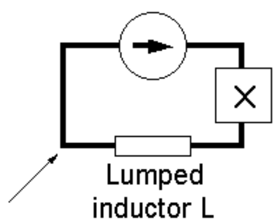
Undefined Circuit:



Current in loop varies, so $(\partial\bar{B}/\partial t) \neq 0 \Rightarrow$ voltage!

Defined Circuit:

We assume voltage drops occur only across elements, and ignore H fields generated by currents through them.



We use small loops, thick wires, and high-impedance lumped elements

L10-2

KIRCHOFF'S CURRENT LAW (KCL)

Kirchoff's Current Law: (KCL) $\sum_i I_i = 0$ = Total current into any node

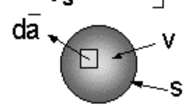
Ampere's Law: $\nabla \times \bar{H} = \bar{J} + \partial \bar{D}/\partial t$ [differential form]

$$\nabla \bullet (\nabla \times \bar{H}) = \nabla \bullet (\bar{J} + \partial \bar{D}/\partial t) = 0$$
$$= \nabla \bullet \bar{J} + \partial (\nabla \bullet \bar{D})/\partial t$$
$$= \nabla \bullet \bar{J} + \partial \rho/\partial t$$

Conservation of Charge: $\nabla \bullet \bar{J} = -\partial \rho/\partial t$ = 0 at each node

[Recall Gauss's Divergence Theorem: $\int_V (\nabla \bullet \bar{G}) dv = \int_S \bar{G} \bullet d\bar{a}$]

Current Law follows from: $\int_S \bar{J} \bullet d\bar{a} = \sum_i I_i = 0$



More generally: KCL assumes nodes store no charge, or that all electric energy is stored inside circuit elements

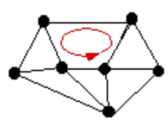
L10-3

SOLVING CIRCUITS

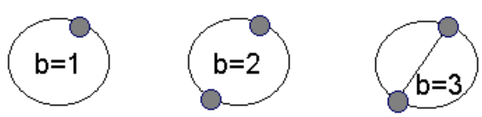
Generic Circuit Topology:

Assume b branches, n nodes, and p unique loops

b=12, p=6, n=7



We can show b = n + p –1.



n = p = 1 n = 2 n = p = 2

Every time we add a branch, the number of nodes or loops (meshes) increases by one.

Voltage sources are branches characterized by v = V_o
Current sources are branches characterized by i = I_o

L10-4

SOLVING CIRCUITS (2)

Number of Unknowns: Number of unknowns is 2b (v,i for each branch)

Number of Equations: We have one device equation for each branch relating or specifying v and i

We also have n – 1 independent node equations (KCL) and p loop equations (KVL), for a total of:

$$b + (n - 1) + p = 2b \text{ equations} = \text{number of unknowns}$$

Given initial conditions and all sources we can solve the equations analytically for simple circuits, or by simulation for any circuit.

(Non-unique exceptions: indeterminate flip-flop or chaotic circuits)

L10-5

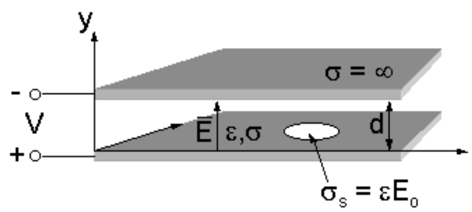
SIMPLE CIRCUIT ELEMENTS, R AND C

Electrostatics:

In general:

Everywhere	$\nabla \times \bar{E} = 0, \quad \nabla \bullet \bar{E} = \rho/\varepsilon$
At conductor	$\hat{n} \bullet \bar{E} = \sigma_s/\varepsilon, \quad \bar{E}_{//} = 0$
Between conductors,	$\nabla \bullet \bar{E} = \rho/\varepsilon = 0$

Parallel-Plate Devices:



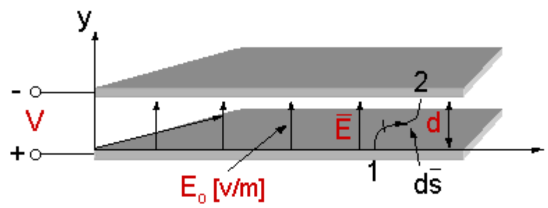
Field Solution:

$\bar{E} = \hat{y}E_o$ for rectangular geometry here

$\sigma_s = \varepsilon E_o$

L10-6

SIMPLE CAPACITOR



Relating Fields to Potentials:

Since: $\nabla \times \bar{E} = 0$

We define $\bar{E} = -\nabla\Phi$ $\quad \Phi = \int_1^2 \bar{E} \bullet d\bar{s}$

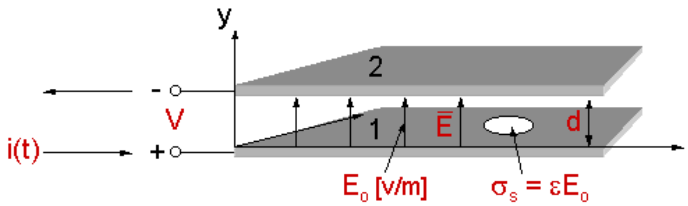
Where Φ is the electrostatic potential relative to [2]

Therefore here: $E_o d = V$, $E_o = V/d$

In general, absolute potential $\Phi = 0$ at infinity, by definition

L10-7

SIMPLE CAPACITOR (2)



Capacitor Charge Q:

We define: $Q = \int_A \sigma_s da = A\varepsilon E_o = A\varepsilon V/d$

We also define: **Q =CV**

Capacitance C:

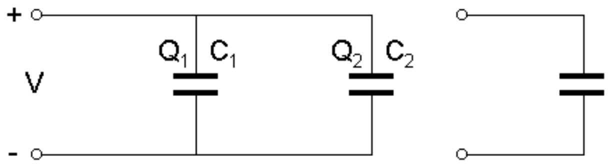
Therefore: **C = Aε/d**

We know: $q(t) = \int_{-\infty}^{t} i(t)dt = Cv(t)$

Therefore: $v(t) = (1/C)\int_{-\infty}^{t} i(t)dt$ Also, i = C dv/dt

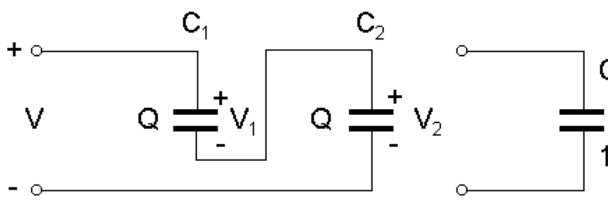
L10-8

CAPACITORS IN SERIES AND PARALLEL



$C_{eq} = Q_{eq}/V =$
$(Q_1 + Q_2)/V = C_1 + C_2$

Capacitors in Parallel: $C_{eq} = C_1 + C_2$



C_{eq}

$1/C_{eq} = V/Q = (V_1 + V_2)/Q$

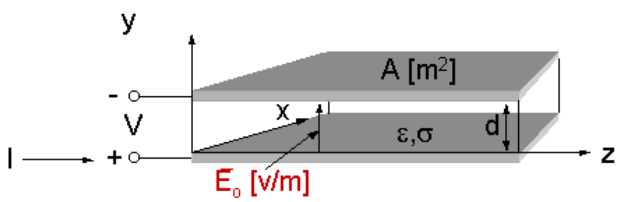
Capacitor in Series: $1/C_{eq} = 1/C_1 + 1/C_2$

L10-9

SIMPLE RESISTORS

Conductance σ:

$\bar{J} = \sigma\bar{E} \left[am^{-2}\right]$
$I = AJ = A\sigma E_o = A\sigma V/d$



Resistance R: $R = V/I = d/A\sigma$

Resistors in Series: $R_{eq} = R_1 + R_2$ $\qquad R_{eq} = (V_1 + V_2)/I$

Resistors in Parallel: $1/R_{eq} = 1/R_1 + 1/R_2$ $\qquad 1/R_{eq} = (I_1 + I_2)/V$

L10-10

CHARGE RELAXATION

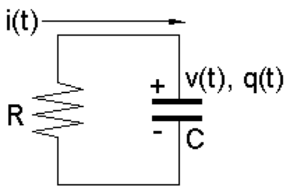
RC Circuits:

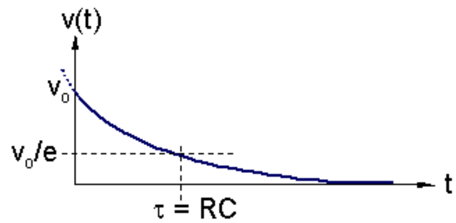
$q(t) = Cv(t) = \int i(t)dt + q_o$

$v(t) = -i(t)R = -RC\,dv(t)/dt$

Try: $v(t) = v_o e^{-t/RC}$ [v_o is initial condition]

It works: $v_o e^{-t/RC} = (-RC)(-1/RC)\,v_o e^{-t/RC}$





Dielectric Relaxation:

$R = d/A\sigma$, $C = \epsilon A/d$

$\tau = RC = \varepsilon/\sigma$ seconds "Relaxation time constant" — independent of geometry

L10-11