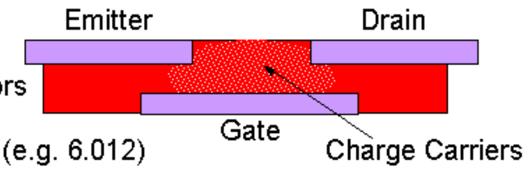
LIMITS TO COMPUTATION SPEED

Devices:

e.g.: Field-effect transistors



Beyond scope of subject (e.g. 6.012)

Interconnect:

Speed of light = 3×10^8 meters/sec
Say CPU and memory separated by 10 cm
$\Rightarrow 2 \times 0.1/(3 \times 10^8) = 0.7$ nsec round-trip delay
$\Rightarrow$<1.5 Gops without pipelining, but matters are worse

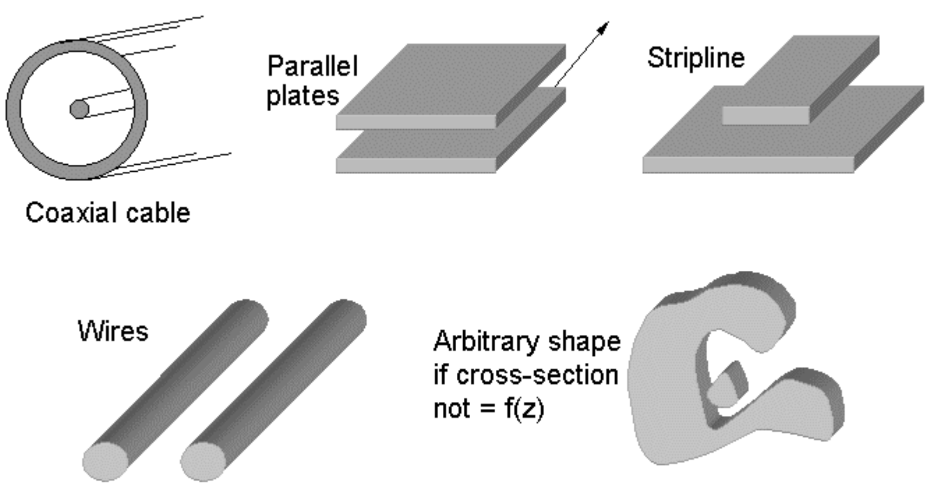
1) $c = (\varepsilon\mu)^{0.5}$ where ε might be $\sim 2\varepsilon_o$
2) Reflections may occur at changes in wire dimensions and at device junctions
3) Wire resistance can slow speeds, too

L12-1

SIMPLE INTERCONNECTIONS

Transverse EM Transmission Lines:

TEM: $\bar{E}_z = \bar{H}_z = 0$



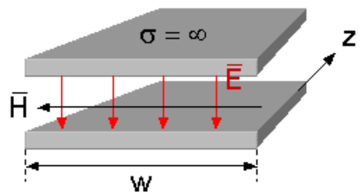
L12-2

PARALLEL-PLATE TRANSMISSION LINE

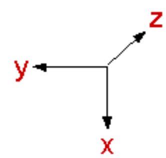
Boundary Conditions:

$\bar{E}_{//} = \bar{H}_{\perp} = 0$ at perfect conductors

Uniform plane wave along z satisfies boundary conditions



Wave Equation Solution:



Recall: $\bar{E} = \hat{x}E_o \cos(\omega t - kz) = \hat{x}E_o \cos\omega(t - z/c)$ for an x-polarized wave at ω radians/sec propagating in the +z direction in free space $(k = \omega/c = 2\pi/\lambda)$

$\bar{H} = \hat{y}(E_o/\eta_o)\cos(\omega t - kz)$ for the same wave

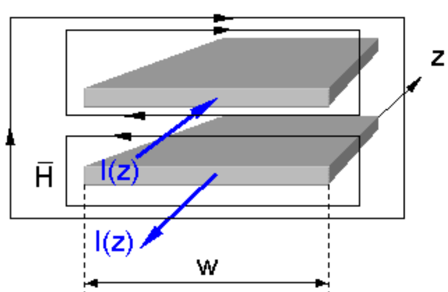
L12-3

PARALLEL-PLATE TRANSMISSION LINE (2)

Currents in Plates:

$$\int_c \bar{H} \bullet d\bar{s} = \int_A \bar{J} \bullet d\bar{a} = I(z)$$

= Hw independent of path



Surface Currents K_s(a m^{-1}):

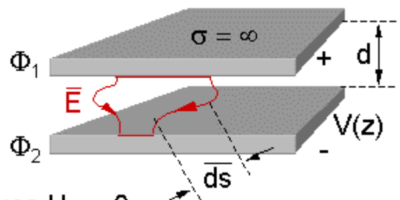
Boundary conditions: $\bar{K}_s(z) = \hat{n} \times \bar{H}(z)$ amperes/meter

[since $K_s = I/W = H$ (from above); $\bar{H}_{AC} = 0$ in conductor]

L12-4

PARALLEL-PLATE TRANSMISSION LINE (3)

Voltages Across Plate:



$$\int_1^2 \overline{E} \bullet d\overline{s} = \Phi_1 - \Phi_2 = V(z)$$

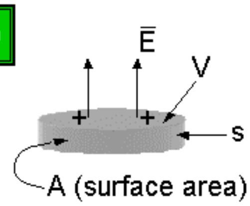
Since all $\int_c \overline{E} \bullet d\overline{s} = 0$ at fixed z because $H_z = 0$,

Therefore **All $\int_1^2 \overline{E} \bullet d\overline{s} = V(z)$ and V(z) is uniquely defined**

Surface Charges (Coulombs/m²):

Boundary conditions: $\hat{n} \bullet \varepsilon \overline{E}(z) = \sigma_s(z)$

since $\nabla \bullet \overline{D} = \rho$ and $\int_s \varepsilon \overline{E} \bullet \hat{n} da = \int_V \rho dv = A\sigma_s$ for

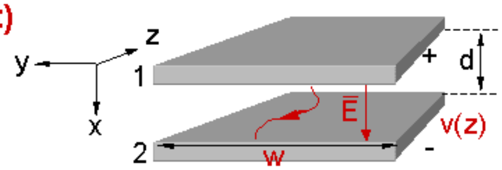


L12-5

VOLTAGE AND CURRENT ON TEM LINES

Integrate $\overline{E}, \overline{H}$ to find v(z, t), i(z, t)

Voltage v(z) on TEM Lines:



Recall:

$\overline{E} = \hat{x} E_o \cos(\omega t - kz)$ for an x-polarized wave

at ω radians/sec propagating in the +z direction

$v(z) = \int_1^2 \overline{E} \bullet d\overline{s} = E_o d \cos(\omega t - kz)$ for our example

Currents I(z) on TEM Lines:

Recall: $H = \hat{y}(E_o/\eta_o)\cos(\omega t - kz)$ for the same wave

$i(z) = \int_c \overline{H} \bullet d\overline{s} = (E_o w/\eta_o)\cos(\omega t - kz)$

Note: v(z) violates KVL, and i(z) violates KCL, why?

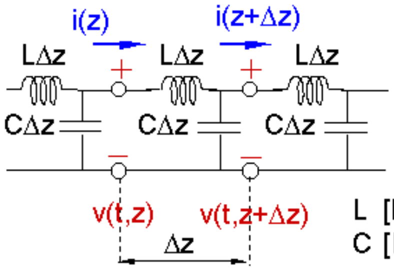
[Note $\partial \overline{B}/\partial t$ through $\overline{E}$ loop, and $\partial \overline{D}/\partial t$ into plates]

Note: $v(z,t)/i(z,t) = \eta_o d/w$ ohms for +z wave alone

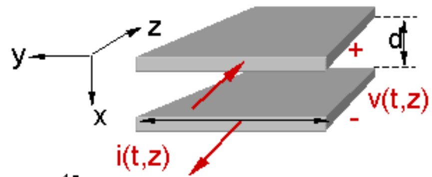
L12-6

TELEGRAPHER'S EQUATIONS

Equivalent Circuit:



L [Henries m^{-1}]
C [Farads m^{-1}]



$\bar{E} \bullet \hat{z} = \bar{H} \bullet \hat{z} = 0$

Difference Equations:

$v(z + \Delta z) - v(z) = -L\Delta z\, \partial i(z)/\partial t$
$i(z + \Delta z) - i(z) = -C\Delta z\, \partial v(z)/\partial t$ [Q = CV]

Limit as $\Delta z \to 0$:
$dv(z)/dz = -L\, di(z)/dt$
$di(z)/dz = -C\, dv(z)/dt$ $\Rightarrow$

Wave Equation
$d^2v/dz^2 = LC\, d^2v/dt^2$

$d^2i/dz^2 = LC\, d^2i/dt^2$

L12-7

SOLUTION: TELEGRAPHER'S EQUATIONS

Wave Equation: $d^2v/dz^2 = LC\, d^2v/dt^2$

Solution: $v(z,t) = f_+(t - z/c) + f_-(t + z/c)$

f_+ and f_- are arbitrary functions

Substituting into Wave Equation:

$(1/c^2)[f_+''(t - z/c) + f_-''(t + z/c)] = LC[f_+''(t - z/c) + f_-''(t + z/c)]$

Therefore $c = 1/\sqrt{LC} = 1/\sqrt{\mu\varepsilon}$

Current I(z,t)

Recall: $di(z)/dz = -C\, dv(z)/dt = -C[f_+'(t - z/c) + f_-'(t + z/c)]$

Therefore: $i(z,t) = cC[f_+(t - z/c) - f_-(t + z/c)]$ where

$cC = C/\sqrt{LC} = \sqrt{C/L} = Y_o = 1/Z_o$ "characteristic admittance"

And therefore: $i(z,t) = Y_o[f_+(t - z/c) - f_-(t + z/c)]$

L12-8