GUIDED WAVES

Applications:

Low-loss transport: Microwave and optical signals
-within and between systems

In devices: Resonators, amplifiers, isolators, optical switches etc.

Standing Waves:

Free space: Waves at any ω, angle, and polarization

Single planar reflector: Same, plus reflected wave
-forms periodic parallel null planes

Parallel-plate waveguide: Same, but the spatial structure depends on ω
-only fields with properly positioned null planes propagate

Today:

Restricted to TE, TM reflected waves

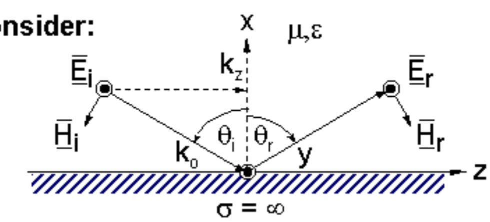
Yields discrete modes, each propagating a continuum of ω

L17-1

TE MODES IN PARALLEL-PLATE GUIDES

Single Reflected Wave:

Consider:



$$\bar{E}_i = \hat{y}E_o e^{jk_x x - jk_z z}$$

$$\bar{E}_r = \underbrace{-\hat{y}E_o}e^{-jk_x x - jk_z z}$$

$\mapsto$ Note: $\bar{E}_i + \bar{E}_r = 0$ @ $x = 0$ is our Boundary Condition "B.C."
B.C. also requires k_z be the same, and therefore k_x

$$\theta_i = \theta_r = \sin^{-1}(k_z/k_o) \text{ where } k_x^2 + k_z^2 = k_o^2 = \omega^2\mu\varepsilon$$

Waveguide Solution:

Recall: $\sin\alpha \equiv \dfrac{e^{j\alpha} - e^{-j\alpha}}{2j}$, since: $\bar{E} = \bar{E}_i + \bar{E}_r$,

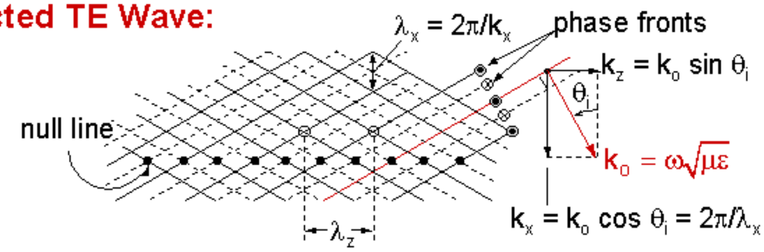
Therefore: $\bar{E} = \hat{y}E_o\left[2j\sin k_x x e^{-jk_z z}\right]$ Since: $\bar{H} = \dfrac{\nabla\times\bar{E}}{-j\omega\mu}$,

Therefore: $\bar{H} = \dfrac{1}{-j\omega\mu}[\hat{z}k_x\cos k_x x + \hat{x}jk_z\sin k_x x]\bullet 2jE_o e^{-jk_z z}$

L17-2

NULL PATTERNS FOR STANDING WAVES

Reflected TE Wave:



Waveguide Wavelength: $\lambda_z = 2\pi/k_z \triangleq \lambda_g$

Since: $d = m(\lambda_x/2) = m\pi/k_x$ where d is waveguide plate separation

Therefore: $k_x = m\pi/d$

TE Modes:

TE_m : TE_1, TE_2, …TE_m…($TM_o \equiv TEM$)

TE_o does not exist because all $\bar{E} = \bar{H} = 0$ to match boundary conditions

L17-3

FIELD SKETCHES FOR TE MODES

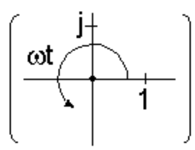
Time-Domain Field Expressions: $\bar{E}(t) = Re\{\underline{\bar{E}}e^{j\omega t}\}$; $\bar{H}(t) = Re\{\underline{\bar{H}}e^{j\omega t}\}$

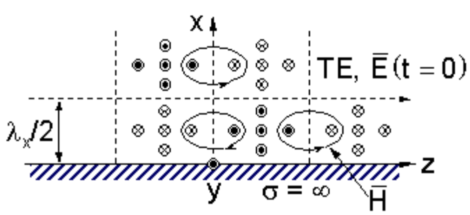
Where: $\underline{\bar{E}} = \hat{y}E_o\left[2j\sin k_x x e^{-jk_z z}\right]$ [from L17-2]

$$\underline{\bar{H}} = \frac{1}{-j\omega\mu}\left[\hat{z}k_x \cos k_x x + \hat{x}jk_z \sin k_x x\right] \bullet 2jE_o e^{-jk_z z}$$

Electric Field Sketches:

Therefore: $\bar{E}(t) = -2\hat{y}E_o \sin k_x x \bullet \sin(\omega t - k_z z)$





Field Motion, Propagation:

For a fixed argument: $\frac{d}{dt}(\omega t - k_z z) = 0 \Rightarrow \frac{dz}{dt} = \omega/k_z$

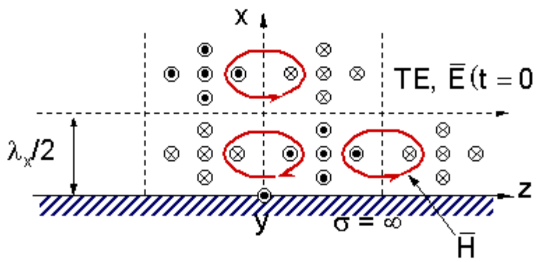
Phase velocity: $v_p = \omega/k_z = \omega/(k_o \sin\theta_i) > c$

L17-4

FIELD SKETCHES FOR TE MODES (2)

Magnetic Field Expressions:

$$\bar{H}(t) = \hat{z}2E_o\left(\frac{-k_x}{\omega\mu}\right)\cos k_x x \bullet \cos(\omega t - k_z z) + \hat{x}2E_o\left(\frac{k_z}{\omega\mu}\right)\sin k_x x \bullet \sin(\omega t - k_z z)$$



Test Boundary Conditions:

$E_{//}, \bar{H}_{\perp} = 0$ at $x = 0$, $k_x x = m\pi$ $(m = 0,1,2,...)$

Therefore we can put waveguide walls at x = 0, $d = m\pi/k_x = m\lambda_x/2$

L17-5

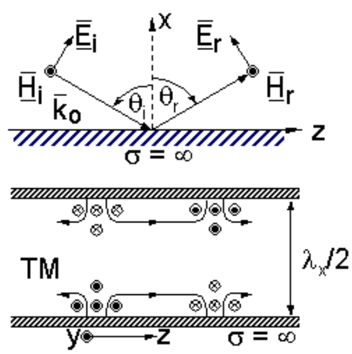
TM WAVEGUIDE MODES

Magnetic Fields:

$\underline{\bar{H}}_i = \hat{y}H_o e^{jk_x x - jk_z z}$

$\underline{\bar{H}}_r = \hat{y}H_o e^{-jk_x x - jk_z z}$

$\underline{\bar{H}} = \underline{\bar{H}}_i + \underline{\bar{H}}_r = \hat{y}H_o 2\cos k_x x e^{-jk_z z}$



Electric Field Expressions:

$\underline{\bar{E}} = (\bar{\nabla} \times \underline{\bar{H}})/j\omega\varepsilon$

$= 2\eta H_o\left(\hat{x}\frac{k_z}{k_o}\cos k_x x + \hat{z}\frac{jk_x}{k_o}\sin k_x x\right)e^{-jk_z z}$

TM Modes:

$TM_0, TM_1, \ldots k_x x = m\pi (m = 0, 1, 2\ldots)$

$[TM_0 \cong TEM]$

L17-6

MODAL CUT-OFF FREQUENCIES

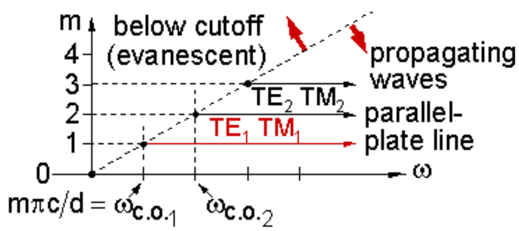
Cut-Off Frequencies:

$$k_{zm} = \sqrt{k_o^2 - k_x^2} = \sqrt{\underbrace{k_o^2}_{(\omega/c)^2} - \underbrace{\left(\frac{m\pi}{d}\right)^2}_{\text{discrete}}} = \underbrace{\pm j\alpha \ \text{ if } \ \frac{m\pi}{d} > k_o = \frac{\omega}{c}}_{\text{"evanescent wave"}} = \omega\sqrt{\mu\varepsilon}$$

discrete "evanescent wave" $\leftrightarrow \omega < m\pi c/d = \omega_{c.o._m}$

$k_z = 0$ at $\omega_{c.o.}$ "cut-off frequency"

Note: discrete modes, each at a continuum of possible ω's



Phase Velocity v_p Above Cut-Off:

$$v_{p_m} = \omega/k_{zm} = \omega\Big/\sqrt{(\omega/c)^2 - (m\pi/d)^2}$$

$$v_{p_m} \rightarrow \begin{cases} c, \omega \rightarrow \infty \\ \infty, \omega \rightarrow c.o._m \end{cases}$$

L17-7

MODAL CUT-OFF FREQUENCIES (2)

Group Velocity v_g Above Cut-Off:

$$k_{zm} = \sqrt{k_o^2 - k_x^2} = \sqrt{\underbrace{k_o^2}_{(\omega/c)^2} - \left(\frac{m\pi}{d}\right)^2}$$

$$v_{g_m} = (\partial k_{zm}/\partial\omega)^{-1} = \left\{\frac{\partial}{\partial\omega}\left[\left(\frac{\omega}{c}\right)^2 - \left(\frac{m\pi}{d}\right)^2\right]^{0.5}\right\}^{-1} = c\left[1 - \left(\frac{m\pi c}{\omega d}\right)^2\right]^{1/2}$$

$$v_{g_m} \rightarrow \begin{cases} c, & \omega \rightarrow \infty \\ 0, & \omega \rightarrow \omega_{c.o._m} \end{cases}$$

Here, $v_p v_g = c^2$

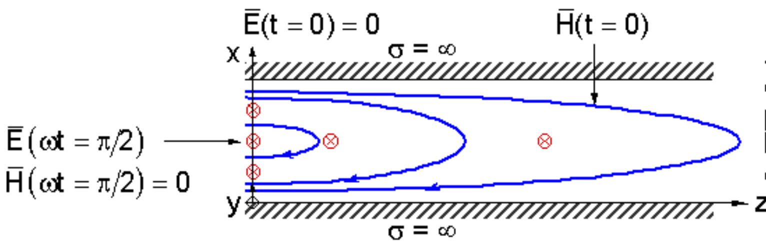
L17-8

EVANESCENT TE MODES

Evanescent TE_1 Mode Fields:

Recall: $\underline{\bar{E}} = \hat{y}2jE_o \sin k_x x e^{-jk_z z}$ where $\begin{cases} k_z = \sqrt{k_o^2 - (\pi/d)^2} = \pm j\alpha \\ e^{-j(\pm j\alpha)z} = e^{-\alpha z} \Rightarrow \text{"}-j\alpha\text{"} = k_z \end{cases}$

Therefore: $\bar{E}(t) = \text{Re}\{\underline{\bar{E}}e^{j\omega t}\} = \hat{y}2E_o \sin k_x x(-\sin\omega t)e^{-\alpha z}$



For $\omega < \omega_{c.o.}$: Stationary oscillating field pattern vs. rigid body motion for $\omega > \omega_{c.o.}$

Evanescent TE_1 Magnetic Fields:

Similarly: $\underline{\bar{H}} = \dfrac{\nabla \times \underline{\bar{E}}}{-j\omega\mu} = \dfrac{j}{\omega\mu}[\hat{z}k_x \cos k_x x + \hat{x}\alpha \sin k_x x] \bullet 2jE_o e^{-\alpha z}$

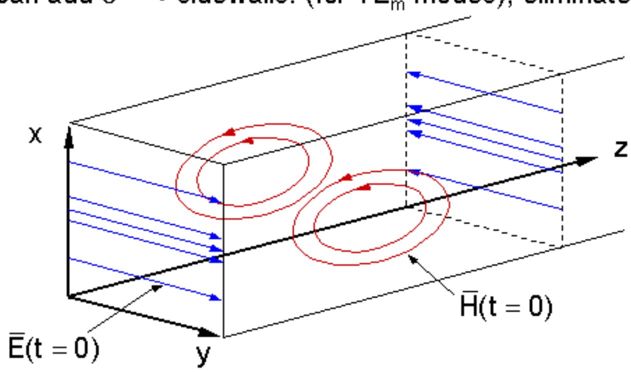
$$\bar{H}(t) = -\frac{2E_o}{\omega\mu}[\hat{z}k_x \cos k_x x + \hat{x}\alpha \sin k_x x]\cos\omega t\, e^{-\alpha z}$$

L17-9

TE_{10} MODE IN RECTANGULAR WAVEGUIDE

Add Sidewalls to TE_1 Waveguide:

Note: can add $\sigma = \infty$ sidewalls! (for TE_m modes), eliminates fringing fields



Surface Charge: Use $\nabla \bullet \underline{\bar{D}} = \underline{\rho}_s$

Surface Current: Use $\underline{\bar{J}}_s = \hat{n} \times \underline{\bar{H}}$

Note:

$\bar{E} \perp (\sigma = \infty)$, $\bar{H} // (\sigma = \infty)$

$\bar{E} \times \bar{H}$ in $+\hat{z}(\pm\hat{x})$ direction

Evanescent waves for $\omega < \omega_{c.o.}$

L17-10