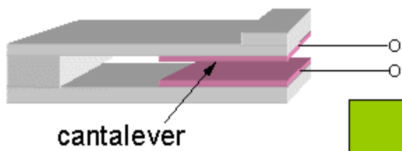
# MICRO-ELECTROMECHANICAL SYSTEMS (MEMS)

## Micro-, Meso-, and Mega-Electromechanical Systems:

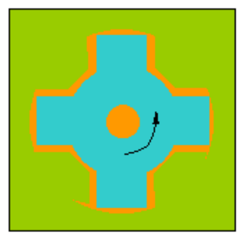
Micro: Micro-fabrication in IC fabs; micro-molds possible (<1 micron)
Possible integration with circuits on same chip
Mass production and low cost even for complex options

Meso: Meso-molds, laser cutting, meso-machine fabrication, miniature parts

## Examples of MEMS:




- Microphones
- Accelerometers
- Video projectors
- Microfluidics
- Motors (electrostatic)
- Chemical, thermal, pressure sensors
- Mechanical "computers" (hot environments)
- Mechanical memories (hot environments; static)



Rotary electrostatic motor

L20-1

# LORENTZ FORCES

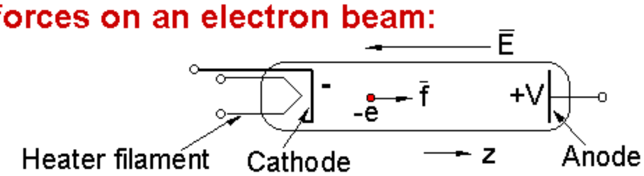
## Lorentz Force Law:

$\bar{f} = q(\bar{E} + \bar{v} \times \mu_o \bar{H})$ Newtons

q = electric charge (Coulombs)
$\bar{v}$ = velocity vector (m $s^{-1}$)

## Example: electric forces on an electron beam:

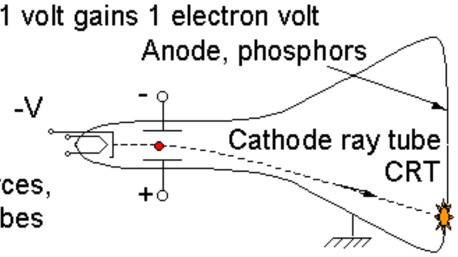
Longitudinal forces:




$\bar{f} = m\bar{a}$ , $\bar{v} = \bar{a}t$ , $z = vt = at^2/2$
Kinetic energy = $mv^2/2$ = eV Joules
So an electron moving through 1 volt gains 1 electron volt
= "e" = $1.6 \times 10^{-19}$ Joules

Lateral forces:

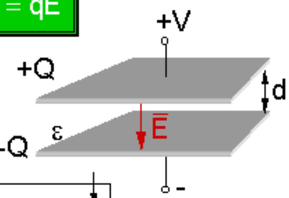
Same results as longitudinal forces, but laterally; e.g. cathode ray tubes




L20-2

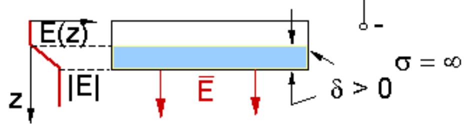
## ELECTRIC FORCES ON CAPACITOR PLATES

**Electric forces calculated from Lorentz equation:** $\bar{f} = q\bar{E}$




Capacitor equations: $Q = CV$
$C = \varepsilon A/d$
$E = V/d$ [V m$^{-1}$]
$w_e = CV^2/2$ [J]

Forces attraction capacitor plates:
Average |E| acting on q is E/2
$E = \sigma_s/\varepsilon_o = Q/A\varepsilon_o$



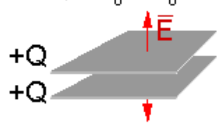

$f = QE/2 = QV/2d = CV^2/2d = \varepsilon A(V/d)^2/2 = f = A\varepsilon E^2/2$ Newtons
Force density $= \varepsilon E^2/2$ [N m$^{-2}$][J] Maximize by maximizing E, limited by breakdowr
For $d \cong$ atomic mean-free-path between collisions < ~1 micron; $V/d \rightarrow E_{max} \cong 10^9$ '
Maximum MEMS electric force density [N m$^{-2}$] $\cong \varepsilon_o E_{max}{}^2/2 \cong 4.4 \times 10^6$ [J m$^{-3}$]

Alternatively, $f = QE/2 = Q^2/2A\varepsilon_o$ Newtons $\Rightarrow$ Force density $= Q^2/2\varepsilon_o A^2$ [N m$^{-2}$]

**Repulsive forces between parallel metal plates:** $f = Q^2/2\varepsilon_o = \varepsilon_o E^2/2$ [J]

$\bar{E}$ inside = 0

Forces arise from charges at infinity




L20-3

## COMPUTING FORCES FROM ENERGY DERIVATIVES

**Force, work, and energy:**

Work is:


Box increases in kinetic energy

Force on box [Newtons] × Distance [m] = Increase in box energy dw [J]

$dw = f_{on\ box} dz$ (dw is negative if the velocity and force are opposite)

Therefore: $f_{on\ box} = dw/dz$

**Work required to separate charged capacitor plates:**

Example: Two plates with ±Q, open-circuit. Therefore
$w_e = CV^2/2 = Q^2/2C$ [J]
(We want expresssion in terms of Q, which is contant)
$C = \varepsilon_o A/d$, so
$w_e = Q^2 d/2\varepsilon_o A$ ($\rightarrow \infty$ as $d \rightarrow \infty$), therefore
$f = \partial w_e/\partial z = Q^2/2\varepsilon_o A$ [Newtons]
(same answer as on L20-3; z = d here)

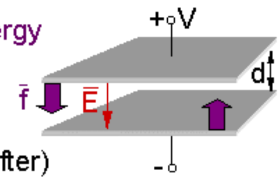
Note that the operand $[Q^2 d/2\varepsilon_o A]$ for $\partial/\partial z$ has not elements (like V,E) that vary with d.
If we connect C to a circuit, we must compute $\partial/\partial z$ for the total system energy, not just for C.

L20-4

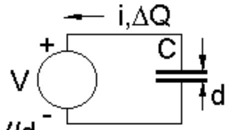
# ELECTROSTATIC ENERGY GENERATORS

**Mechanical work separating charged plates increases electric energy:**

Reversiblity: Mechanical energy ⟺ Electrical energy




**Battery charger:**

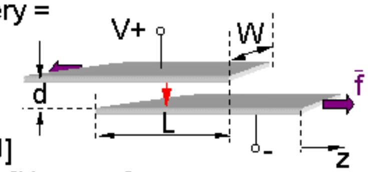



(d$_1$ before, d$_2$ after)

Since $Q = CV = \varepsilon A V/d$,
Separating plates (increasing d) reduces Q on plates and puts i into battery

Generated energy stored in rechargeable battery =
$V\,\Delta Q_{\text{into battery}} = \varepsilon A V^2(d_1^{-1} - d_2^{-1})$
Mechanical work on capacitor plates using battery =
$V\,\Delta Q_{\text{from battery}} = \varepsilon A V^2(d_1^{-1} - d_2^{-1})$

**Lateral mechanical forces:**

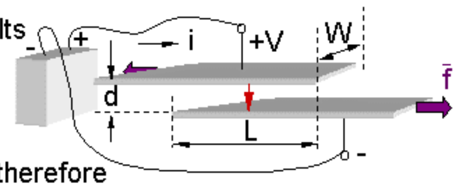



Electrical energy: $w_e = Q^2 d/2\varepsilon_o A = Q^2 d/2\varepsilon_o WL$ [J]
Lateral force: $f = \partial w_e/\partial z = (-\partial/\partial L)(Q^2/2\varepsilon_o WL)$ [Newtons]
(on plate) $= Q^2/2\varepsilon_o WL^2$ ($\to \infty$ as $L \to 0$)
(We assumed Q = constant; d << W,L)

L20-5

# LATERAL FORCES—CONSTANT VOLTAGE

**Parallel plates plus battery:**




$w_e = CV^2/2$ [J] $= \varepsilon_o WLV^2/2d$
$f = -\partial w_e/\partial L = -\varepsilon_o WV^2/2d + V\,\partial Q/\partial L$ where
$Q = CV = \varepsilon_o WLV/d$, so $\partial Q/\partial L = \varepsilon_o WV/d$, therefore
$f = -\varepsilon_o WV^2/2d + \varepsilon_o WV^2/d = \varepsilon_o WV^2/2d$ [Newtons]
(Note: f is independent of L, as is $\partial w_e/\partial L$)

$\bar{f}$ is force exerted on plates by environment

**Example:**

Let: W = 10 cm, d = 10 μm, V = 10 volts (10 kV/cm)
Then: $f = \varepsilon_o WV^2/2d = 8.854\times10^{-12}\times0.1\times10^2/2\times10^{-5} =$
$4.4\times10^{-6}$ Newtons (~1μpound)
Try: d = 1μm, V = 100 volts (1MV/cm) ⇒ $4.4\times10^{-3}$ Newtons (~1 millipound)

**Multi-segment actuator:**

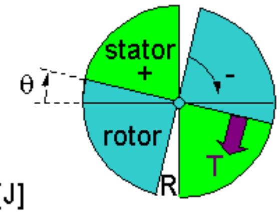
N teeth in length L:


Force is proportional to W, so N edges boost force × N
(N is limited by d << L/N)

L20-6

# ELECTROSTATIC ROTARY MOTOR

## Example, ideal 4-segment motor:

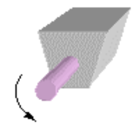



Radius R, plate separation d, $\varepsilon_o$, $E_{max} = 10^{10}$ [V m$^{-1}$]
Plate overlap = A = Rθ [m$^2$]
Δstored electrical energy (from L20-6)

$dw_e = dCV^2/2 - VdQ$ [J] $= dA\, \varepsilon_o V^2/2d - V^2dC$
$= 2Rd\theta\, \varepsilon_o V^2/2d - V^2 2Rd\theta\, \varepsilon_o/d = -Rd\theta\, \varepsilon_o V^2/d$ [J]

## Lateral torque T [Newton meters] on rotor:

$T = -dw_e/d\theta = R\, \varepsilon_o V^2/d$ [N m]



Cadillac Engine
~year 2040
300 hp × 746W/160W = 1400 motors
⇒ ~0.22 cm$^3$! P = VI, I = 746A. Heat

## Motor power P [Watts] at ω radians/sec: P = Tω [W]

## Example for R = 1 mm, d = 1 micron, V = 300 volts, $v_{max}$ = ~200 ms$^{-1}$:

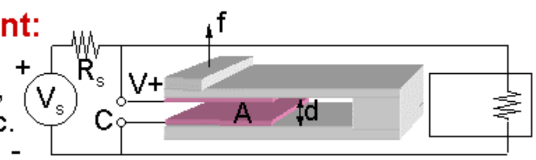
$\omega = v/R = 200/10^{-6} = 2 \times 10^8$ radians/sec (~530,000 rpm)
$P = \omega T = \omega R\, \varepsilon_o V^2/2 \cong 2\times10^8 \times 10^{-6} \times 8.854\times10^{-12} \times 10^5/10^{-6} \cong 160$ Watts!
(could lift 70-kg person [f = ma = 70 × 9.8 = 686 N] at ~23 cm/sec [P = Fv])
Back-off mode: 10 volts ⇒ 180 mW. At 4 mm$^2$, 10-micron thick ⇒ 6250 motors/cm$^3$



# MEMS SENSORS

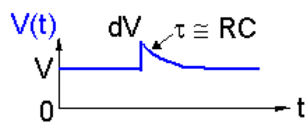
## Measuring displacement:

Microphone, accelerometer, barometer, strain gauge, etc.




Cantilevered capacitor C, stressed at V volts

## Circuit response to displacement δ:




Assume $R_s >> R$; impulse increase in separation d is $\delta << d$
Mechanical energy fδ first increases $w_e$ in C as V →V+dV
Then voltage relaxes back to $V = R/(R + R_s)$ as current flows mostly through R releasing dw[J]
Recall $C = \varepsilon A/d$
$dw \cong (C - C')V^2/2 = V^2\varepsilon_o A(d^{-1} - [d+\delta]^{-1})/2 = (V^2\varepsilon_o A/2d)(1 - [1+\delta/d]^{-1}) \cong V^2\varepsilon_o A\delta/2d^2$ [J]

## Example:

Let: $dw = 4\times10^{-20}$ Joules (~energy needed to convey one bit of information to sensor)
Plate separation d = 1 micron, A = 1-mm square ($10^{-6}$), V = 300; solve for minimum δ
Then: $\delta = dw\, 2d^2/V^2\varepsilon_o A = 4\bullet10^{-20} \times 2 \times 10^{-12}/(300^2 \times 8.8\bullet10^{-12} \times 10^{-6}) = 10^{-19}$ meters $= 10^{-9}$ Å
or: $d = 4\bullet10^{-16} \times 2 \times 10^{-12}/(10^2 \times 8.8\bullet10^{-12} \times 10^{-8}) \cong 10^{-10}$ meters = 1 Å