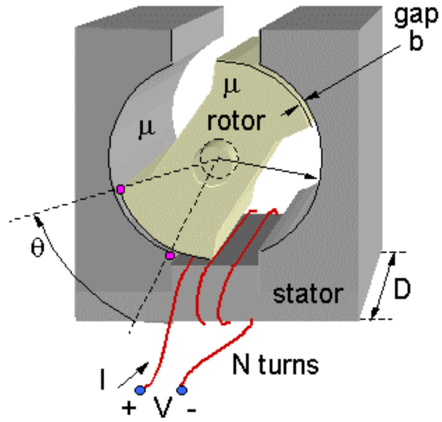
VARIABLE RELUCTANCE MOTORS

Basic 2-Pole Reluctance Motor:

What is the torque on the rotor?



First find B and H:

Since $\nabla \bullet \bar{B} = 0$, then $\bar{B}_{stator} \cong \bar{B}_{gap}$

and $\bar{H}_{stator} \cong (\mu_o/\mu)\bar{H}_{gap} << \bar{H}_{gap}$

$(\mu \cong 10^4 \mu_o)$

$\nabla \times \bar{H} = \bar{J} \Rightarrow$

$NI = \int_c (\bar{H}_{gap} + \bar{H}_{stator}) \bullet d\bar{s} \cong 2bH_{gap}$

Therefore $H_{gap} \cong NI/2b$

(independent of rotor cross-sectional area)

L22-1

VARIABLE RELUCTANCE MOTORS (2)

Find Torque T[Nm] by Differentiating Energy Storage:

Since: $W_m \propto \mu|H|^2/2 \left[Jm^{-3}\right]$ and $\bar{H}_{stator} \cong (\mu_o/\mu)\bar{H}_{gap}$

Therefore: $W_{gap} \cong (\mu/\mu_o) W_{stator} \left[Jm^{-3}\right]$

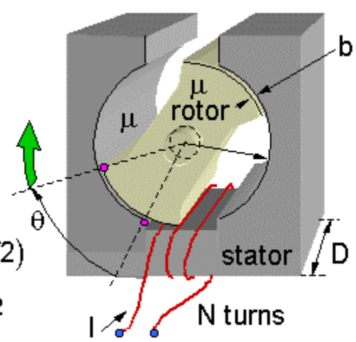
So if: $b/2R << \mu_o/\mu$

Then: $w_{gap} >> w_{stator} [J]$ where

$w_{gap} \cong 2bR\theta D\mu_o |H_{gap}|^2/2$

Where: Gap area $= R\theta D\left[m^2\right]$ (for $0 < \theta < \pi/2$)

Therefore(?): $T = -\partial w_{gap}/\partial\theta = -bRD\mu_o |H_{gap}|^2$



WRONG SIGN!

To find force we must always differentiate total energy, including the energy in the power supply!

To Find Torque Correctly:

Can include the changing source energy, but easier to reformulate the energy expression

L22-2

FLUX LINKAGE Λ AND INDUCTANCE L

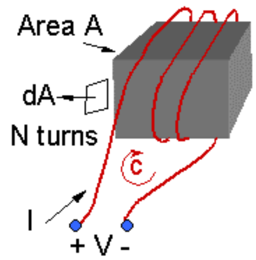
Definition of Flux Linkage Λ:

Flux Linkage $\Lambda = N\int_A \bar{B} \bullet d\bar{a}$

$$-\nabla \times \bar{E} = \partial\bar{B}/\partial t = N\int_A (d\bar{B}/dt) \bullet d\bar{a} = d\Lambda/dt$$

$$-\int_{c\ coil} \bar{E} \bullet d\bar{s} = V = \frac{d\Lambda}{dt} = L\,di/dt$$

Therefore: $L = \Lambda/i$



Reluctance-Motor Flux Linkage Λ:

$V = d\Lambda/dt = 0$ if motor short-circuited; therefore

$\Lambda = \text{constant} \neq f(\theta)$ (key step)

$\Lambda = N\mu_o H_{gap} A_{gap} = N^2\mu_o I A_{gap}/2b$, where $H_{gap} = NI/2b$ [see L22-1]

$A_{gap} = RD\theta$ here [L22-2]

$L = \Lambda/I = N^2\mu_o A_{gap}/2b$

L22-3

RELUCTANCE MOTOR TORQUE

Energy Storage and Torque with the Source Short-Circuited:

$w_m = LI^2/2 = \Lambda^2/2L \quad (I = \Lambda/L)$

$T = -\partial w_m/\partial\theta = -\Lambda^2\partial(1/2L)/\partial\theta = -\Lambda^2\partial(b/N^2\mu_o A_{gap})/\partial\theta \quad [A_{gap} = R\theta D]$

$= -(\Lambda^2 b/N^2\mu_o RD)\partial\theta^{-1}/\partial\theta = \Lambda^2 b/N^2\mu_o RD\theta^2$

Recall $\Lambda = N^2\mu_o I A_{gap}/2b$ [see L22-3]

$T = (N^2\mu_o IR\theta D/2b)^2 b/N^2\mu_o RD\theta^2$

Therefore:

$T = N^2\mu_o I^2 RD/4b$ [Nm] $= (\mu_o H_{gap}{}^2/2)(2bRD) \neq f(\theta)$ (e.g. for $0 < \theta < \pi/2$)

$T = W_{mgap}(dV_{olume}/d\theta)$ [Nm]

~Same as for electric motors!
T is limited by maximum energy density

Maximize T: maximize $(NI)^2$ and RD (~weight), minimize b (tolerances)

Motor Drive Circuit:

I is turned on when A_{gap} is minimum; torque then increases A_{gap} as the rotor is pulled by the stator. When A_{gap} = maximum, I is set to zero and the rotor coasts until A_{gap} = minimum, and the cycle repeats. (<50% duty cycle here)

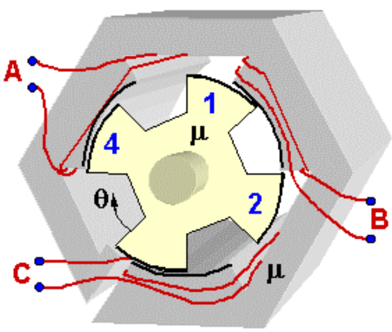
L22-4

¾-POLE VARIABLE RELUCTANCE MOTOR

Winding Excitation Plan:

Start by exciting windings A,B; this pulls pole 1 into pole B; for winding A, the pole area is temporarily constant. When $\Delta\theta = \pi/3$, the currents are switched to B,C; when $\Delta\theta = 2\pi/3$ we excite C,A. Repeating this cycle results in nearly constant clockwise torque.

To go counter-clockwise, excite BC, then AB, then CA.



Torque:

Only one pole is being pulled in; the other winding has either one rotor pole fully in, or one entering and one leaving that cancel. Many pole combinations are used (more poles, more torque).

L22-5

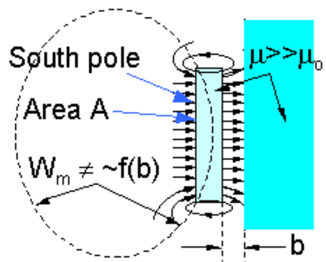
PERMANENT MAGNET SYSTEMS

Refrigerator Magnets:

Force: $f = -dw_m/db$

Variable energy: $w_m \cong bA\mu_o H_{gap}^2/2$ [J] $= bAB_{gap}^2/2\mu_o$

Force density: $f/A = B_{gap}^2/2\mu_o$ [Nm^{-2}]



Example: Let B = 1 Tesla (10,000 gauss), A = 10 cm^2

Then $f = 0.001 \times 1^2/2\times4\pi10^{-7} \cong 400$ N $\cong$ 100-lb force

Permanent Magnet Properties:

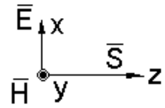
Strength: Typical strong magnets ~0.2T; can → 1Tesla

Temperature : Above ~200C they fail; some fail at low temperatures

L22-6

FORCES FROM ELECTROMAGNETIC WAVES

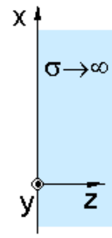
Waves Impacting Conductors:



$\bar{E}_x(z=0) = 0 = \left[\bar{E}_+ \cos(\omega t - kz) + \bar{E}_- \cos(\omega t + kz)\right]$

$H(z=0) = \hat{y}(2E_+/\eta_o)\cos\omega t \Rightarrow \bar{J}_s = \hat{x}(2E_+/\eta_o)\cos\omega t$

Forces on Conductor:



$\bar{f} = q(\bar{E} + \bar{v} \times \mu_o \bar{H}) \Rightarrow$ quasistatic pressure $P\left[Nm^{-2}\right]$;

$F\left[Nm^{-3}\right] = nqv\mu_o H = J\mu_o H$ where $n = \#/m^3$ charge carriers

$\bar{P} = \int_0^\infty \bar{J}(z) \times \mu_o \bar{H}(z) dz$ where $\bar{J}(z) = \nabla \times \bar{H}(z)$ for $\sigma \to \infty$

$$\bar{J}(z) \cong \nabla \times \bar{H} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x \to 0 & \partial/\partial y \to 0 & \partial/\partial z \\ H_x \to 0 & H_y & H_z \to 0 \end{vmatrix} = -\hat{x}\partial H_y/\partial z$$

$$\bar{P}(t) = -\hat{z}\mu_o \int_0^\infty (\partial H_y/\partial z) H_y dz = -\hat{z}\mu_o \int_{H_y(z=0)=H_{y_o}}^{H_y=0} H_y dH_y = \hat{z}\mu_o H_{y_o}{}^2(t)/2$$

[same as L22-6]

$$\langle \bar{P}(t) \rangle = \hat{z}\mu_o \langle H_{y0}{}^2(t) \rangle /4 = 2\langle \bar{S}(t) \rangle /c$$

where $H_{y0} = 2H_+$ and

$\langle S(t) \rangle = \eta_o H_+{}^2/2 = \mu_o c H_+{}^2/2$

L22-7

PHOTON PRESSURE

Photon Energy: hf [J] = "m"c^2

h = Planck's constant = 6.625×10^{-34} [Js], f = frequency [Hz]

Photon Momentum: "m"c = hf/c [Nms^{-1}]

Transferred to mirror upon reflection: Δ momentum = 2hf/c

Photon Pressure P [Nm^{-2}]:

Pressure P is change of momentum $s^{-1}m^{-2}$ [Mechanics: f = d(mv)/dt]

$$\langle P \rangle = n2hf/c = 2\langle S(t) \rangle /c \quad \left[Nm^{-2}\right]$$

where n = # photons reflected $s^{-1}m^{-2}$

If a photon is absorbed, then Δ momentum = hf/c, and $P = \langle S(t) \rangle /c$

In general, if the incident and reflected fluxes are S_1 and S_2 [Wm^{-2}],

$$\langle P \rangle = \langle S_1 - S_2 \rangle /c \quad \left[Nm^{-2}\right]$$

Solar Sailing Across Solar System:

Say 1 km^2 at 1.4 $kW/m^2 \Rightarrow A\langle P \rangle = A2\langle S \rangle /c = 10^6 \times 2800/3 \times 10^8 = 9N$

(say 10^{-6} thick, $\rho = 1 g/cm^3$)

$v = at = (f/m)t \cong (9/1000)3\times10^7 \cong 3\times10^5$ ms^{-1} in 1 year = $10^{-3}c$

L22-8