6.014 Lecture 23:
Power Transmission and Switching

A. Overview

Electric power has transformed society to such a degree it is difficult to imagine its loss. Key to its economic success has been the economies of scale achieved by concentration of generating resources in a few hundred major sites from which power is transmitted over continental distances. Environmental impact has also been reduced by such concentration, although future technical developments may make dispersal of generation more competitive.

We can quickly estimate the total power requirements for the USA by noting that the average home uses roughly the equivalent of one kilowatt continuously, so ~100 million homes need ~100GW average. Government, commerce, and industry require comparable amounts of power, consistent with our present national capacity of ~250GW. Local daily and annual peaks due to air conditioning and the work day can triple baseline requirements, so the use of long power lines to shift this capacity across the continent and time zones is an important tool to minimize power rationing during periods of peak demand.

Two issues are addressed here: 1) what limits power transmission?, and 2) what are the key issues in switching such high powers? First we see from Maxwell's equations that both voltage and current are limited largely by economics, and that both electrical and magnetic effects pose problems that impact transmission design. Switching problems are somewhat different when large power is involved, and so we revisit the behavior of transients in TEM lines and circuits.

B. Limits to Line Voltage

Voltage on long-distance power transmission lines is limited largely by electrical breakdown. The dominant physical mechanism for breakdown begins with mobile electrons being accelerated in an ambient electrical field. If these electrons gain sufficient kinetic energy prior to some atomic collision, then that energy may be sufficient to dislodge another electron. Then the pair can be accelerated to produce a chain reaction and a large current that shorts out the system. Such breakdowns can occur in solids, liquids, or gases. This avalanche of electrons can also heat and destroy the medium, although in air the resulting arc discharge usually fades quickly or is convected away. This model is roughly consistent with the breakdown electric field strengths for various materials such as ceramics, plastics, and air, which are ~4-40, ~10-130, and ~1-3 kV/mm, respectively. The mean free path before a free electron loses momentum in a collision varies among materials, but we might associate shorter paths with solids and much longer paths with gases. For ceramic, plastics, and air we might therefore explain the observed breakdown field strengths cited above by noting they are consistent with ~5 volts over a 20-nm path, ~1 volt over a 10-nm path, and ~0.2 volt over a 70-nm path, respectively. One immediate insight from these numbers is that electrical systems with air gaps on the order of a few microns or less can utilize substantially larger electrical field strengths because the distances are too short to permit any avalanche. Instead the electron might dislodge a positive ion (which requires more energy), that can then accelerate in the opposite direction to dislodge the next electron; this process requires much higher field strengths before breakdown occurs. Another insight is that gases that are more easily ionized will have lower breakdown field strengths. Even a small amount of humidity in air can have this effect. Also, gases at lower pressures have longer mean-free paths, and so lower field strengths can initiate breakdowns such as those occurring in fluorescent light bulbs or neon lights.

These considerations suggest that the maximum voltage we can use on a power cable will be limited in part by the breakdown strength of the surrounding air, 1-3 kV/mm. Figure L23-2 roughly sketches the electric field lines that would accompany a two-wire TEM line located over a ground plane. Near the wires the electric fields are nearly radial, and if the wires are far from any ground plane (relative to wire diameter) the fields increasingly resemble those for a coaxial cable having inner and outer radii of a and b, respectively. In that case:

$$\overline{E} = \hat{r}E_o/r \tag{1}$$

consistent with Gauss's law (see the earlier discussions of radial fields near cylinders). The voltage V between the inner and outer conductor is:

$$V = \int_a^b \overline{E} \bullet \hat{r}dr = \int_a^b \left(\hat{r}E_o/r\right) \bullet \hat{r}dr = E_o \ln\left(b/a\right) \tag{2}$$

Because this voltage is dominated by the radial electric field within several radii of the wire (by virtue of the ln(b/a) factor), (1) is also approximately correct for a wire of radius a that is a distance b from a ground plane. More precise estimates for larger wires can be obtained using contour mapping techniques such as those suggested in Figure L23-2b.

The relation between the maximum electric field (E_{max}) and V can be found from (1); i.e., $E_{max} = E(r=a) = E_o/a$. Therefore $E_o = aE_{max}$, and (2) yields:

$$V_{max} = aE_{max}\ln(b/a) \tag{3}$$

Using reasonable values for a (1 cm), b (10m) and E_{max} (3×10^6), (3) yields $V_{max} \cong 210$ kV. This is less than the maximum used in practice.

High voltage lines operate up to 750 kV, which can produce corona discharges around the wire and down its full length, particularly in humid air. Because the avalanche ionization process increases the conductivity of the air significantly, the effective radius of the wire is correspondingly increased many centimeters. The radius a of the corona is approximately that given by (2) where $E_o = aE_{max}$. At night the corona surrounding such high-tension wires emits an eerie glow. Other effects also become important at these high voltages. For example, rain falling on the wires will drip, particularly from the lowest points where the wires sag. Each drop is at the potential of the wire and is strongly charged the same. Thus as the drop breaks away there is a strong repulsive electrostatic force that rapidly accelerates it. The result is a sound resembling a small machine gun—an acoustic hazard for the neighbors.

C. Limits to Line Current and Power

There are three main physical effects that limit the maximum current a power line can readily convey: 1) heating and thermal expansion that causes the wire to sag, 2) power losses (revenue) due to line resistance, and 3) attractive magnetic forces between adjacent wires.

Wires must be designed for worst-case conditions, i.e., a hot day in the sun. The problem is that the towers are often so far apart that the wires can sag ~10 meters, leaving them ~20 meters above the ground. As they sag further they may hit trees or tall shrubs. Mowing and clearing helps, but must be repeated regularly. If we assume that under these hot conditions the maximum allowable heat flux F from the wires is $\sim 1 W/cm^2$ (the hotter the wire, the greater the flux), then the maximum D.C. current I_{max} can be found by equating the heat lost per meter to the power dissipated. The wire resistance per meter R is:

$$R = 1/\sigma\pi a^2 \ \ [\text{ohms/m}] \tag{4}$$

and the heat generated and dissipated per meter is:

$$P = I^2 R = F2\pi a \tag{5}$$

Therefore this nominal thermal sag limit I_{max} (for DC power and σ for copper) is:

$$I_{max} = (F2\pi a/R)^{0.5} = (F2\pi^2 a^3 \sigma)^{0.5} \cong (1\times 2\pi^2(10^{-2})^3 \times 5\times 10^7)^{0.5} = 1000\pi \text{ amperes} \tag{6}$$

The voltage drop $I_{max}R$ $[Vm^{-1}]$ becomes ~2 mv, which implies this wire will lose only 10 percent of its power after ~10,000 km. Running long distances at lower currents can reduce this loss substantially. In practice aluminum wires are used with conductivity 3.5×10^7 Siemens m^{-1}, about two-thirds that of copper but with one-third the density. The net weight advantage is about a factor of two. The two prices can differ by more, depending on the market.

If two wires are carrying similar currents in the same direction one meter apart they will experience an attractive magnetic force f given by the Lorentz force law:

$$\overline{f} = \Sigma_i q_i(\overline{E} + \overline{v} \times \mu_o \overline{H}) = \overline{I} \times \mu_o \overline{H} \ \ [Nm^{-1}] \tag{7}$$

$$|\overline{H}| = I/2\pi r \tag{8}$$

$$f = \mu_o I^2/2\pi r \cong 1.26\bullet 10^{-6}(1000\pi)^2/2\pi\bullet 1 \cong 2 \text{ [Nm}^{-1}\text{]} \quad (9)$$

Typical high-voltage lines have groups of four parallel wires carrying the same currents, where the four are arranged in a one-meter square. For D.C. lines a few such groups might run in each direction, whereas AC lines generally have multiples of three sets of four wires, where each set of four corresponds to one of three possible electrical phases (0, 120, 240 degrees) at 60 Hz (50Hz in Europe and many other places). A ground wire may be suspended overhead to intercept lightening strikes and carry ground currents. The forces of (9) increase inversely with r, so if wind should sway the wires these forces could increase many times, creating an unstable situation that would inevitably lead to adjacent wires sticking together in a stable fashion. The solution to this attraction is to insert line spacers periodically that hold the lines apart in any wind. These spacers can also cross link the currents in case one wire frays.

Another concern that limits current is the fact that the current does not flow uniformly over the cross-section of the wire. Instead it is generally confined within a nominal "skin depth" at the surface. Earlier we have seen that electromagnetic waves in highly conducting materials decay exponentially with depth, e.g. $|\overline{E}| \propto e^{-z/\delta}$, where the skin depth:

$$\delta = (2/\omega\mu\sigma)^{0.5} \text{ meters for } \sigma >> \omega\varepsilon \quad (10)$$

as derived below. It can be shown (see text) that the average power dissipated P_d [Wm^{-2}] $= |\underline{J}_s|^2/2\sigma\delta = |\underline{H}_{//}|^2/2\sigma\delta$, as if the surface current J_s [Am^{-1}] were distributed uniformly over the depth δ. Using the conductivities σ for copper, aluminum, iron, and sea water of ~5.8×10^7, 3.5×10^7, 10^7, and 3-5 Sm^{-1}, we find the skin depths δ are 9, 11, 21, and ~3×10^4 mm, respectively. Thus standard aluminum power cables one inch in diameter dissipate extra power because the currents are slightly confined near the surface. This tendancy is reduced somewhat by composing the cable of many strands of aluminum wire a few mm in diameter so that the fields can penetrate slightly more deeply.

Skin depth follows from Ampere's law:

$$\nabla \times \overline{H} = \overline{J} + j\omega\varepsilon\, \overline{E} = (\sigma + j\omega\varepsilon)\, \overline{E} = j\omega\varepsilon_{eff}\, \overline{E} \quad (11)$$

where $\varepsilon_{eff} = \varepsilon(1 - j\sigma/\omega\varepsilon)$. But $\overline{E} = \overline{E}_o e^{-jkz}$ where:

$$k = \omega(\mu\varepsilon_{eff})^{0.5} = \omega(\mu\varepsilon)^{0.5}(1 - j\sigma/\omega\varepsilon)^{0.5} \cong \omega(\mu\varepsilon)^{0.5}(- j\sigma/\omega\varepsilon)^{0.5} \quad (12)$$

$$= (\omega\mu\sigma)^{0.5}(-j)^{0.5} = (\omega\mu\sigma/2)^{0.5}(1 - j) = k' - jk'' \quad (13)$$

Therefore:

$$e^{-jkz} = e^{-jk'z}\, e^{-k''z} \quad \text{where } e^{-k''z} = e^{-z/\delta} \quad (14)$$

where the expression for skin depth (10) follows from (13) and (14).

Using the nominal limits to V and I on power lines we can estimate the maximum power P_{max} that can be transported on a single line pair.

$$P_{max} = VI \; < \; V_{max}I_{max} \cong 300\text{kV} \times 1000\pi \cong 1 \text{ GW per line pair} \tag{15}$$

One question that sometimes arises is whether the power in such lines is transferred inside the wires or outside. If we calculate Poynting's vector $\overline{S}$, it is zero inside (because $|\overline{E}|$ inside a good conductor is zero) and non-zero outside. Integrating $\overline{S}$ over any flat surface normal to the wire and outside it yields the correct power, however. Thus the power flows outside the wire. On the other hand, nothing visible is happening outside the wire whereas all the moving electrons are inside. One law case involved interstate commerce and the question of whether taxes were owed because the power did (or did not) flow across state lines. The primary of a power transformer was on one side of the line, and the secondary on the other. Since no wires (power?) crossed the state line, taxes might be avoided.

D. Transients on Power Lines

We recall that the TEM line impedance $Z_o = (L/C)^{0.5} = 1/cC$ where $c = (LC)^{-0.5}$, and the structure of power lines is essentially TEM. We can quickly approximate the capacitance per meter C for a power line of radius a located distance b above the ground plane by:

$$C[\text{Fm}^{-1}] = Q[\text{Cm}^{-1}]/V = 2\pi a\rho_s/V \tag{16}$$

where $\rho_s \cong \varepsilon_o E_{max}$ $[\text{Cm}^{-2}]$ and

$$V = \int_a^b \overline{E} \bullet \hat{r}dr \cong aE_{max} \ln(b/a) \quad [\text{see (2)}] \tag{17}$$

Therefore,

$$C \cong 2\pi a\varepsilon_o E_{max}/aE_{max}\ln(b/a) = 2\pi\varepsilon_o/\ln(b/a) \tag{18}$$

Consider, for example, the case where b = 20m and a = 1 cm; then:

$$C \cong 2\pi 8.85\times10^{-12}/\ln(20/0.01) \cong 7.3\times10^{-12} \text{ [F], and}$$
$$Z_o = 1/cC \cong 1/(3^\bullet 10^8 \times 7.3^\bullet 10^{-12}) \cong 457 \text{ ohms}$$

It is interesting to note that the ratio of maximum practical voltage to maximum practical current is approximately $V_{max}/I_{max} =\cong 750\text{kV}/1000\pi[\text{A}] \cong 320$ ohms, so the impedances are well matched. That is, a nominal power line can have Z_o near the load impedance. However this is of little practical importance because the wavelength at 60 Hz (50 Hz in Europe and elsewhere) is $\lambda = c/f = 3\times10^8/60 = 5000$ km, and typical lines are 250 km or $\sim\lambda/20$ long, so it is more important to match the source to load at D.C. than to worry about the small alterations introduced by line mismatch. That is, rotation of 36^o (λ/20) on a Smith chart does not alter impedance greatly unless the line is strongly mismatched.

Nonetheless, even though the lines are short compared to the 60-Hz wavelength, switching transients can occur on shorter time scales. For example, suppose a switch B is used to engage a load Z_o at the load, as suggested in Figure L23-8. If the source were in fact V_{Th} in series with Z_o, then the voltage everywhere on the open-circuited line would be V_{Th} and the current I(z) would be zero, corresponding to equal forward and backward propagating signals $v_+(t - z/c)$ and $v_-(t + z/c)$. That is,

$$v_+(t–z/c) + v_-(t+z/c) = V_{Th} \text{ at } t = 0 \tag{19}$$

and

$$I(z,t) = Y_o[v_+(t–z/c) - v_-(t+z/c)] = 0 \tag{20}$$

Solving (19) and (20), we obtain:

$$v_+(t-z/c) = v_-(t+z/c) = V_{Th}/2 \text{ at } t < 0 \text{ for all } z \tag{21}$$

Once the switch B closes, the load is matched and the subsequent $v_-(t)$ at $z = 0$ is zero. The result is plotted in Figure L23-8 where for positions $z < -ct$ the total voltage is V_{Th} and for $z > ct$ the line voltage is v_+ only, or $V_{Th}/2$. Since it does not make sense to double line voltage specifications just to permit switches to be thrown, this configuration is not used. One might then suppose that switch A at the input end might be used. Again, if the line is matched, then there is no v_- transient, and the maximum line voltage is the operating voltage. This configuration is also not used. For example, consider what would happen if the load open circuits. A v_- transient would be generated, again requiring line voltage specifications to be double the operating level, as illustrated.

In practice, the source is not a voltage source in series with a resistor. It is generally a multipole magnetic generator driven by a steam turbine. It has perhaps 50-percent mechanical/electrical conversion efficiency so that the source impedance is partly mechanical in origin. As the load draws more current, more force is required to rotate the generator and the steam pressure (or torque) is deliberately increased. If a major load suddenly vanishes because a major power line or circuit breaker opens, the problem can be severe because typical steam reservoirs driving turbines are substantial. Absent an electrical load, there is no back-torque to restrain the motor drive shaft and it could therefore accelerate to produce unacceptable voltages $\left(V \propto \frac{\partial B}{\partial t}\right)$. Typically in an emergency like this the steam pressure in the boiler would be instantly released up the stack, making a terrible noise, and the generator would then free wheel, undriven and unloaded, producing a gradually declining voltage as it slows down.

Another source of transients on power lines is lightning strikes. Although these generally strike the ground wire suspended above the power lines, the power lines themselves can be shorted to ground by the column of ionized air that results. Figure L23-9 illustrates what would happen if a 200-ohm line carrying 2000 amperes at 500kV to a 250-ohm load were short-circuited suddenly. For $t < 0$, since $v_+ + v_- = 500$ kV, and $v_+ - v_- = Z_oI = 200\times2000 = 400$ kV, it follows that $v_+ = 450$ kV and $v_- = V - v_+ = 50$ kV. When the line is short-circuited in the middle, as illustrated, the reflection coefficient Γ becomes -1 for waves approaching the short from either direction. As a result the zero-voltage condition (indicating a short) propagates both right and left at velocity c, while the current to the left of the short is roughly doubled to more than 4000 amperes.

In general, because transients can be induced on lines in so many ways, it is common practice to design them to handle transients that are roughly double the highest steady-state levels expected.

Sometimes switching power introduces other problems, as illustrated in Figure L23-10, which treats the case where two capacitors at different voltages are suddenly connected, and the case where an inductor is suddenly disconnected. The problem is easily seen if two identical capacitors C have voltages V and zero, and a switch is closed at t = 0, connecting them. Since the final total charge $Q_1' + Q_2'$ must equal the initial total charge Q_1, we see:

$$Q_1 = CV = Q_1' + Q_2' = 2CV' \qquad (22)$$

where the 'prime' indicates $t > 0$. Therefore $V' = V/2$. The initial (w_o) and final (w_f) energies in the pair of capacitors are thus:

$$w_o = CV^2/2 \text{ [J]} \qquad (23)$$

$$w_f = 2CV'^2/2 = CV^2/4 = w_o/2 \qquad (24)$$

Thus half the initial energy is lost. Since the two capacitors are lossless, where did the missing energy go?

The answer is that the problem is incompletely specified. When the switch is thrown the current flow would become infinite were it not impeded by inductance L in the wires that could support oscillations at the $\omega_o = (LC)^{-0.5}$ resonant frequency; these oscillations would slowly attenuate due to losses. Also, a significant voltage drop might occur across any arc that formed as the switch contacts closed, dissipating the energy resistively and thermally. Similar loss mechanisms would account for the loss in magnetic energy stored in an inductor that is suddenly open-circuited. The voltages across the inductor could theoretically rise toward infinity, except that residual capacitance is always present that again could support oscillations. Also an arc might form in the switch as it opens that could absorb power by heating and ionizing the gas there. Such arcs can be damaging, so special switches are sometimes used that can blow the arc away. Good power management avoids all such transients by avoiding sharp changes in any circuits storing or carrying significant power.