POWER TRANSMISSION, SWITCHING

How Much Power Do We Need?

Homes: ~10^8 in USA using ~ 1kw⇒100 GW average (Power plants are ~ 1GW)
Rest: Comparable; U.S. generating capacity ≅ 250 GW
Total: Daily and annual peaks (e.g. air conditioning) can triple requirements

What Limits Maximum Power Transmission?

Voltage: Corona, arcing, E_{max} is weather dependent, < ~3×10^6 vm^{-1}
Current: Power loss ∝ I^2R, heating, magnetic field forces, cost

What are the Major Concerns When Switching?

Voltage: Transients on TEM lines can nearly double normal voltage
Transient V's when L's are open-circuited can → ∞
Current: Transients on TEM lines can nearly double normal currents
Transient I's when C's are short-circuited can → ∞

What Lines Deliver the Most Short-Circuited Current?

Instantly: For fixed TEM Z_o, the highest voltages (Thevenin)
Later: ~600V is worst; it overcomes arc and has $Z_{source} \cong Z_{Th}N^2$ (N << 1)

*Accidents worst @ ~600V because arc impedance overcome + N^2 ⇒ low Z_o, ~10^5 A!

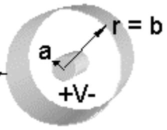
LIMITS TO TRANSMISSION VOLTAGES

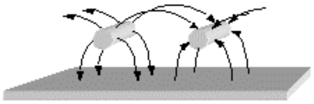
Field Breakdown:

~mean free path

Breakdown voltage: ~4kV – 40kV/mm for ceramics ⇒ ~5 volt/200Å
~10kV – 130kV/mm for plastics ⇒ ~1 volt/100Å
~3kV/mm for air = E_{max} ⇒ ~0.2 volt/700Å

Maximum $\bar{E}$ Field Around Wire:



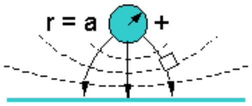


Coaxial cable: $\bar{E} = \hat{r}E_o/r$

Cable voltage: $V = \int_a^b \bar{E} \bullet \hat{r}dr$

$$= \int_a^b (\hat{r}E_o/r) \bullet \hat{r}dr = E_o \ln(b/a)$$

Maximum field at r = a: $E_{max} = E_o/a \Rightarrow E_o = aE_{max}$



Maximum voltage V_{max}: $V_{max} \cong aE_{max}\ln(b/a)$

$$\cong 10^{-2}\, 3 \times 10^6 \ln(10/10^{-2})$$

$$\cong 210 \text{ kV in air}$$

Corona (glow discharge): Bare 2-cm wires produce corona at 750 kv, and are noisy in rain
Corona radius has $|\bar{E}| \cong E_{max}$, varies with humidity

Use insulation? No, it eventually breaks down, concentrating fields and leakage

LIMITS TO TRANSMISSION CURRENTS

Maximum Current (DC case):

Thermal limit: $F[\mathrm{W/m^2}] = I^2R/2\pi a$ $(R[\mathrm{ohms/m}] = 1/\sigma\pi a^2)$

Let $F_{max} = 10^4$ W/m^2 heat flux, $a = 1$cm

(wire sag) $\Rightarrow I_{max} = (2\sigma\pi^2a^3F)^{0.5} \cong (2 \times 5 \bullet 10^7\pi^2 10^{-6} \times 10^4)^{0.5}$

$= 1000\pi$ amperes

Voltage drop: $\Delta V[\mathrm{Vm^{-1}}] = IR = (1000\pi)(1/5{\bullet}10^7\pi^2 10^{-2})$

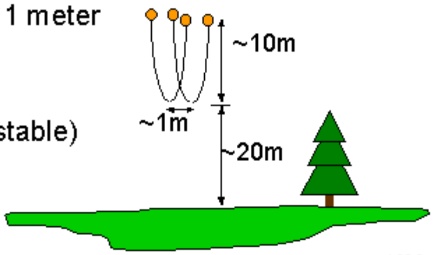
$= 2 \times 10^{-3} \Rightarrow \sim$10,000 km @ 10%drop

Force limit: $\text{Force } f = \sum_i q_i\left(\bar{E} + \bar{v} \times \mu_o\bar{H}\right) = \bar{I} \times \mu_o\bar{H} = \mu_o I^2 \big/ 2\pi r$

$= 1.26 \bullet 10^{-6}\,(1000\pi)^2 \big/ 2\pi \bullet 1$

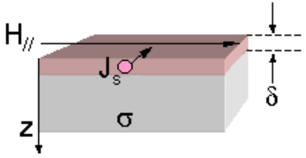
$\cong 2$ Newtons/m at 1 meter

Attractive force for parallel currents
Mandates spacers on long runs (unstable)



SKIN DEPTH

Maximum Current (AC case) is Skin Depth Limited:



Skin depth δ: Waves in conductors decay exponentially with depth, $e^{-z/\delta}$
Assuming uniform current flowing in δ yields correct power dissipation P_d Wm^{-2}

$$\boxed{P_d\,[\mathrm{Wm^{-2}}] = J_s^{\,2}R = H_{//}^{\,2}/\sigma\delta}$$

J_s [Am^{-1}], R[Ωm^{-1}], $H_{//}$ [Am^{-1}], σ[Sm^{-1}], δ[m]

DC limit: DC axial H can coexist throughout conducting wire with $\sigma \to \infty, \neq \infty$ $(\delta \to \infty)$

AC limit: Perfectly conducting wire has longitudinal surface currents J_s $= H_{//}$ [Am^{-1}] that match boundary conditions H inside $= 0$ $(\delta = 0)$; finite σ, $f \Rightarrow$ finite δ

SKIN DEPTH (2)

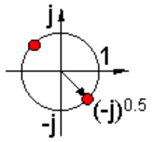
Waves in Conducting Media:

$\nabla \times \overline{H} = \overline{J} + j\omega\varepsilon\overline{E} = (\sigma + j\omega\varepsilon)\overline{E} = j\omega\varepsilon_{eff}\overline{E}$ where $\varepsilon_{eff} = \varepsilon(1 - j\sigma/\omega\varepsilon)$

$\overline{E} = \overline{E}_o e^{-jkz}$ where $k = \omega(\mu\varepsilon_{eff})^{0.5} = \omega(\mu\varepsilon)^{0.5}(1 - j\sigma/\omega\varepsilon)^{0.5}$ If $\sigma >> \omega\varepsilon$, then

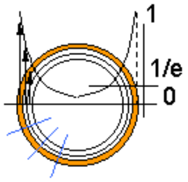
$k \cong \omega(\mu\varepsilon)^{0.5}(-j\sigma/\omega\varepsilon)^{0.5} = (\omega\mu\sigma)^{0.5}(-j)^{0.5} = (\omega\mu\sigma/2)^{0.5}(1 - j) = k' - jk''$

$e^{-jkz} = e^{-jk'z}e^{-k''z}$ where $e^{-k''z} = e^{-z/\delta}$ and:



Skin depth $\delta = (2/\omega\mu\sigma)^{0.5}$ meters for $\sigma >> \omega\varepsilon$

At 60 Hz: Copper $\sigma \cong 5.80 \times 10^7$ Sm^{-1} ⇒ δ = 9 mm
Aluminum $\sigma \cong 3.54 \times 10^7$ Sm^{-1} ⇒ δ = 11 mm
Iron $\sigma \cong 1 \times 10^7$ Sm^{-1} ⇒ δ = 21 mm
Sea water $\sigma \cong$ 3-5 Sm^{-1} ⇒ δ = 37-29 m



Screening Reduced by Multiple Conductors:

Must allow field penetration
Generally use braided cable with
strand diameter d << δ

LIMITS TO POWER TRANSMISSION

Limits Posed by V_{max} and I_{max}:

Power: $P = VI < \sim V_{max}I_{max}$ = ~300kV × 1000π ≅ 1 GW per line pair
Assuming two 2-cm diameter braided copper wires, thermal limit

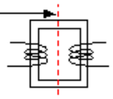
Least cost: "Dollars per pound" ⇒ minimize mass, weight (e.g. Al widely used
Maintainability, redundancy

Where Does Power Flow—Inside or Outside Wires?

Outside: $\overline{E} \times \overline{H} \neq 0$ only outside; inside $\sigma = \infty$ and $\overline{E} = 0$, therefore $\overline{S}(\text{power}) = 0$

Inside: Cut the wires and power stops; no electrons, etc. outside (only fields)

Debates: Edison Co. vs. academic; interstate commerce case at transformer state line



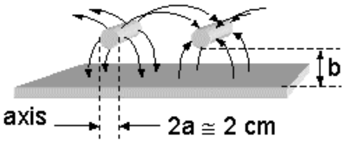
Limits Posed by Switching:

Transients: Can stress specifications—overvoltage, overcurrent, model lines as TEM

Arcing: Arcs are sometimes unavoidable circuit elements
When two capacitors at different voltages are connected
When two inductors with different currents are connected
When currents are interrupted

TRANSIENTS ON POWER LINES

TEM Model for Power Lines:



Can have one or more wires over ground plane
TEM geometry results—uniform cross-section along z axis

Characteristic Impedance Z_o of Nominal Power Line:

Impedance: $Z_o = (L/C)^{0.5} = 1/cC$ where $c = (LC)^{-0.5}$

Capacitance: Say TEM line is 2-cm cylinder above plane b = 20 m away ($\sigma = \infty$)

$C[Fm^{-1}] = Q[Cm^{-1}]/V = 2\pi a\rho_s/V$; $\rho_s = \varepsilon_o E_{max}$ $[Cm^{-2}]$

Recall: $V = \int_a^b \bar{E} \bullet \hat{r}dr \cong aE_{max} \ln(b/a)$ [see L23-2]

Therefore: $C \cong 2\pi\varepsilon_o E_{max}/aE_{max}\ln(b/a) = 2\pi\varepsilon_o/\ln(b/a)$

$\cong 2\pi\ 8.85 \cdot 10^{-12}/\ln(20/0.01) = 7.3 \cdot 10^{-12}$ $[Fm^{-1}]$

Therefore: $Z_o = 1/cC \cong 1/(3 \cdot 10^8 \times 7.3 \cdot 10^{-12}) \cong 457$ ohms

Relation Between Z_o and Maximum Power Possible:

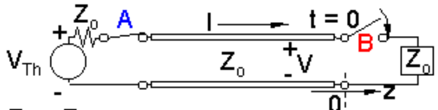
Note: $V_{max}/I_{max} \cong 750kv/1000\pi \cong 320$ ohms, so matching is feasible (useful)

Wavelength: $\lambda = c/f = 3 \cdot 10^8/60 = 5000$ km ⇒ ~DC behavior for lines < ~500 km

Lines > ~500 km often use DC, not 3 phases

SWITCHING TRANSIENTS ON POWER LINES

Making contact to dead load:



Initial Conditions: $I = 0$, $V = V_{max}$, assume $Z_L = Z_o$

Initial V_+ and V_-: $V_+(t-z/c) + V_-(t+z/c) = V_{Th}$ at $t = 0$; $I \propto [V_+(t-z/c) - V_-(t+z/c)] = 0$

Therefore: $V_+(t-z/c) = V_-(t+z/c) = V_{Th}/2$ at $t = 0$ for all z

For $t \geq 0$: $V_+(t-z/c)$ unchanged, $V_-(t+z/c) = 0$ for $z > -ct$ when B is closed

Problem (unreal): Line voltage too high at rest

Solution: Use switch A instead of B

Reality: $V \cong$ constant, current source varies

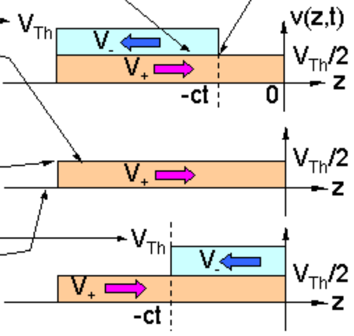
Breaking contact with load:

Initial Conditions: Assume $V/I = Z_o$, so $V_- = 0$

When B opens: $\Gamma \rightarrow +1$, $V_- = V_+$

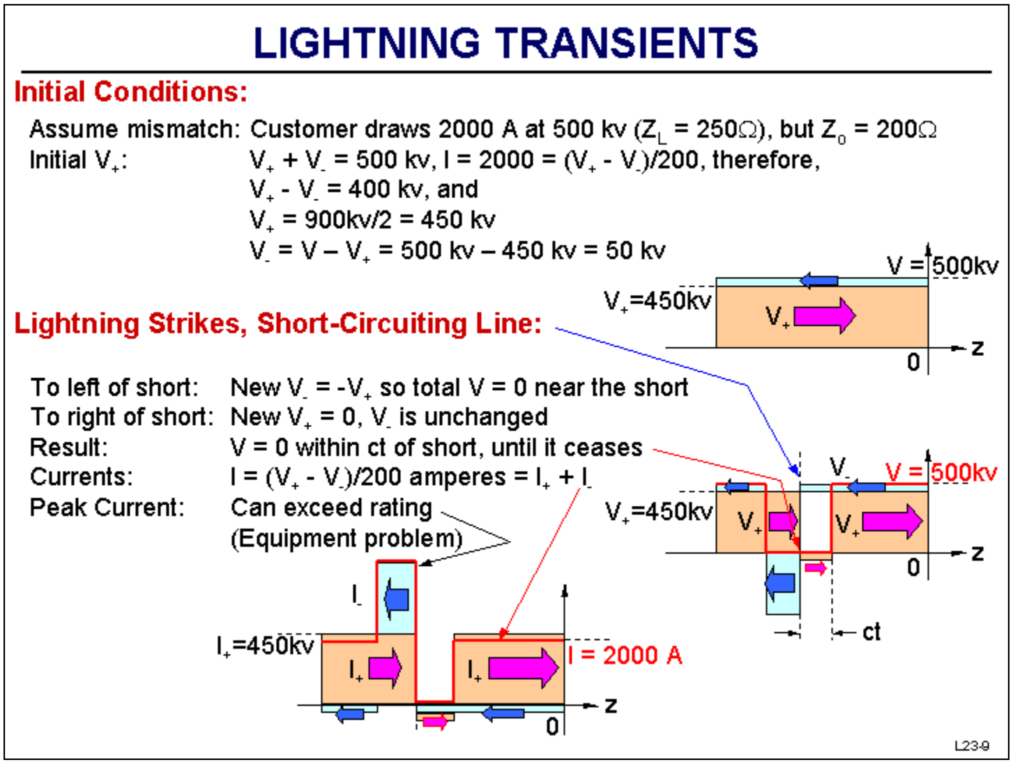
Total voltage: $V_- + V_+ = V_{Th}$, line voltage doubles

When A opens: V_+ slowly disappears, $V \rightarrow 0$



R_{source} is mechanical, inside generator:

If load open circuits, generator steam is released up stack; generator spins unloaded



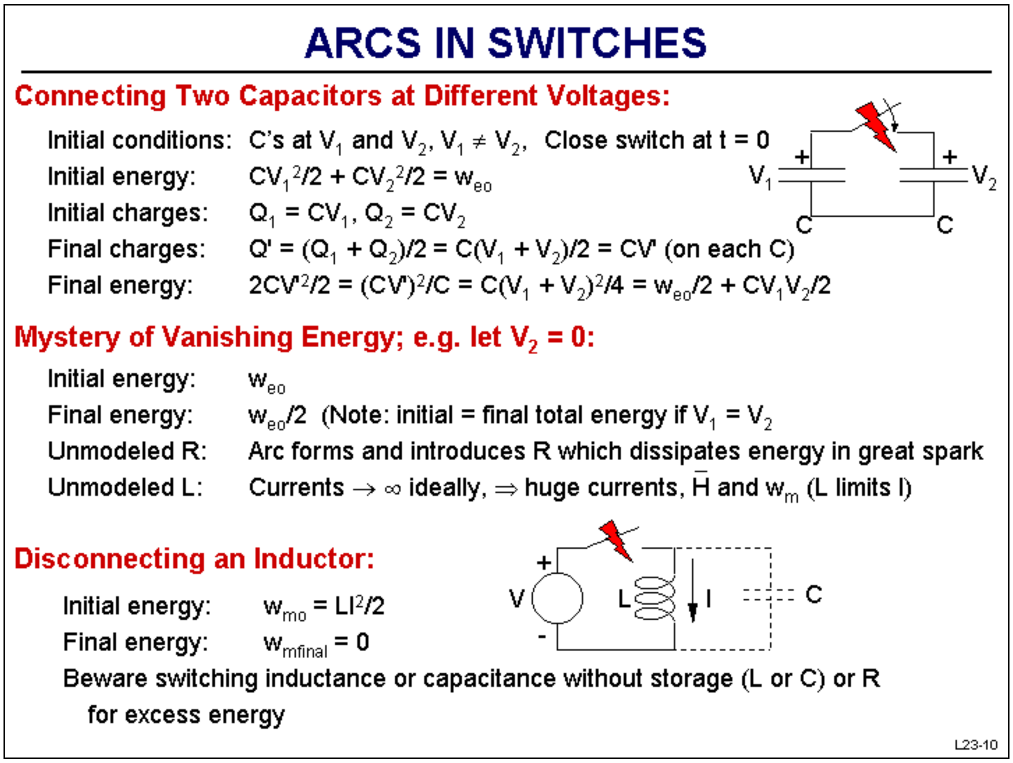
LIGHTNING TRANSIENTS

Initial Conditions:

Assume mismatch:	Customer draws 2000 A at 500 kv ($Z_L = 250\Omega$), but $Z_0 = 200\Omega$
Initial V_+:	$V_+ + V_- = 500$ kv, $I = 2000 = (V_+ - V_-)/200$, therefore, $V_+ - V_- = 400$ kv, and $V_+ = 900\text{kv}/2 = 450$ kv $V_- = V - V_+ = 500\text{ kv} - 450\text{ kv} = 50$ kv

Lightning Strikes, Short-Circuiting Line:

To left of short:	New $V_- = -V_+$ so total V = 0 near the short
To right of short:	New $V_+ = 0$, V_- is unchanged
Result:	V = 0 within ct of short, until it ceases
Currents:	$I = (V_+ - V_-)/200$ amperes $= I_+ + I_-$
Peak Current:	Can exceed rating (Equipment problem)



ARCS IN SWITCHES

Connecting Two Capacitors at Different Voltages:

Initial conditions:	C's at V_1 and V_2, $V_1 \neq V_2$, Close switch at t = 0
Initial energy:	$CV_1^2/2 + CV_2^2/2 = w_{eo}$
Initial charges:	$Q_1 = CV_1$, $Q_2 = CV_2$
Final charges:	$Q' = (Q_1 + Q_2)/2 = C(V_1 + V_2)/2 = CV'$ (on each C)
Final energy:	$2CV'^2/2 = (CV')^2/C = C(V_1 + V_2)^2/4 = w_{eo}/2 + CV_1V_2/2$

Mystery of Vanishing Energy; e.g. let $V_2 = 0$:

Initial energy:	w_{eo}
Final energy:	$w_{eo}/2$ (Note: initial = final total energy if $V_1 = V_2$)
Unmodeled R:	Arc forms and introduces R which dissipates energy in great spark
Unmodeled L:	Currents → ∞ ideally, ⇒ huge currents, $\bar{H}$ and w_m (L limits I)

Disconnecting an Inductor:

Initial energy:	$w_{mo} = LI^2/2$
Final energy:	$w_{mfinal} = 0$

Beware switching inductance or capacitance without storage (L or C) or R for excess energy