6.014 COURSE HISTORY AND PHILOSOPHY

History

50 years ago EE(CS) taught quasistatics, motors, and waves (3 subjects)
~20 years ago motors merged into quasistatics (6.013)
Next fall waves (6.014) and quasistatics (6.013) will merge becoming:

6.013 (New)

Final 6.014 Lecture for all Time is Today

Philosophy

From Maxwell's Equations to applications, plus review of basics
Both physical and mathematical thinking
Exposure to key modern technologies

L24-1

ACOUSTICS (REVIEW OF 6.014 WAVE CONCEPTS)

Basic Variables:

$\bar{E}, \bar{H}$ are both vectors	$\bar{u}$(velocity ms^{-1}), p(pressure Nm^{-2}) are not.
Physics is linear	Non-linear; $\bar{u}$ and p are perturbations
Vectors are $\perp$ to power $\bar{S}$	$\bar{u}$ is // to power $\bar{S}$, ρ is gas density $\left(kg\ m^{-3}\right)$
	$P_o = \langle \text{pressure} \rangle, \rho_o = \langle \text{density} \rangle$,
	γ = adiabatic constant

Basic Equations (vacuum):

$\nabla \times \bar{E} = -\mu_o \partial \bar{H}/\partial t$

$\nabla \times \bar{H} = \varepsilon_o \partial \bar{E}/\partial t$

$\nabla p = -\rho_o \partial \bar{u}/\partial t$ [force = mass $\times$ accel.]

$\nabla \bullet \bar{u} = -(\gamma P_o)^{-1} \partial \rho/\partial t$ [mass conservation]

$\nabla \times \underline{\bar{E}} = -j\omega\mu_o \underline{\bar{H}}$

$\nabla \times \underline{\bar{H}} = j\omega\varepsilon_o \underline{\bar{E}}$

$\left(\nabla^2 + \omega^2 \mu_o \varepsilon_o\right) \underline{\bar{E}} = 0$

$\nabla \underline{p} = -j\omega\rho_o \underline{\bar{u}}$

$\nabla \bullet \underline{\bar{u}} = -j\omega(\gamma P_o)^{-1} \underline{p}$

Acoustic differential equations

$\left(\nabla^2 + \omega^2 (\rho_o/\gamma P_o)\right) \underline{p} = 0 \rightarrow$ Wave Equation

L24-2

ACOUSTIC WAVE EQUATION SOLUTION

Basic Solution:

$\underline{p}(\bar{r}) = \underline{p}_o e^{-j\bar{k}\bullet\bar{r}}$ satisfies the Wave Equation $\left(\nabla^2 + \omega^2(\rho_o/\gamma P_o)\right)\underline{p} = 0$ if:

$k = \omega(\rho_o/\gamma P_o)^{0.5}$ Acoustic dispersion relation $\Rightarrow$

$v_p = \omega/k = (\gamma P_o/\rho_o)^{0.5} = c_s$ and $v_g = (dk/d\omega)^{-1} = (\gamma P_o/\rho_o)^{0.5} = c_s$

Example: 0°C air, surface pressure, $\gamma = 1.4$, $\rho_o = 1.29$ kg m^{-3} $\Rightarrow c_s = 330$ ms^{-1}

Solids: $c_s = (K/\rho_o)^{0.5} \cong 1500$ ms^{-1} in H_2O, 1500-13,000 ms^{-1} in solids

"bulk modulus" ~ spring constant (analogous to γP_o)

z-Directed Sinusoidal Acoustic Waves:

$\underline{p}(z) = \underline{P}_+ e^{-jkz} + \underline{P}_- e^{+jkz}$ [Nm^{-2}] $\quad \underline{\bar{u}}_z(z) = \eta_s^{-1}\left(\underline{P}_+ e^{-jkz} - \underline{P}_- e^{+jkz}\right)$[ms^{-1}]

$\eta_s = \underline{p}_+(z)/\underline{u}_+(z) = \omega\rho_o/k = \rho_o c_s = (\rho_o\gamma P_o)^{0.5}$ [Nsm^{-3}] $\cong$ 425 for air at 20°C

L24-3

ACOUSTIC POWER AND ENERGY

Acoustic Power Density (Intensity):

Power density $(Wm^{-2}) = p\bar{u}$ $\quad$ [Nm^{-2} ms^{-1}] = [Nms^{-1} m^{-2}] in time domain

$\langle I_s \rangle [Wm^{-2}] = 0.5 R_e\{\underline{p}\underline{\bar{u}}^*\} = \hat{z}|\underline{p}|^2/2\eta_s = \hat{z}\eta_s|\underline{\bar{u}}|^2/2$

Example: 1Wm^{-2} at sea level $\Rightarrow |\underline{p}| = (1 \bullet 2\eta_s)^{0.5} = (850)^{0.5} \cong 30$ Nm^{-2}

$|\underline{\bar{u}}| = |\underline{p}|/\eta_s = 0.07$ ms^{-1} and

Distance moved = $\delta z \cong u/\omega \cong 1\mu$ at 10 kHz
(~1 nm at hearing threshold)

Poynting Theorem (lossless):

W_k[Jm^{-3}] kinetic energy density

W_p[Jm^{-3}] potential energy density

$$\nabla \bullet (\underline{p}\underline{\bar{u}}^*)/2 = -2j\omega\left(\rho_o|\underline{\bar{u}}|^2/4 - |\underline{p}|^2/4\gamma P_o\right)[Wm^3]$$

L24-4

SNELL'S LAW AND EVANESCENT WAVES

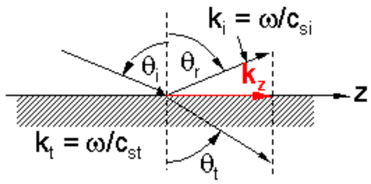
Boundary Conditions: p and $u_\perp$ continuous across boundaries

Incident Wave:

$$\underline{p}_i(\bar{r}) = \underline{p}_o e^{-j\bar{k}\cdot\bar{r}} = \underline{p}_o e^{-jk_i \sin\theta_i z + jk_i \cos\theta_i x}$$

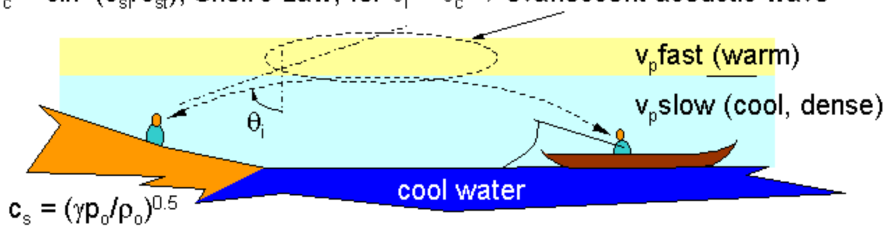
Matching phases at boundary $\Rightarrow$

$\theta_i = \theta_r$ and $\sin\theta_i/\sin\theta_t = c_{si}/c_{st}$



Critical Angle (when $\theta_{si} < \theta_{st}$):

$\theta_c = \sin^{-1}(c_{si}/c_{st})$, Snell's Law; for $\theta_i > \theta_c \Rightarrow$ evanescent acoustic wave



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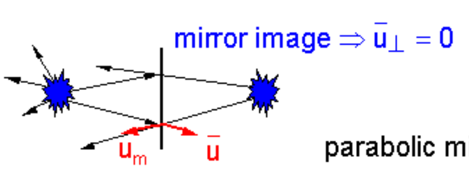
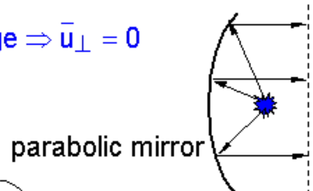
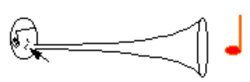
ACOUSTIC ANTENNAS

Boundary Conditions: p and $u_\perp$ continuous across boundaries

Velocity $\bar{u}_\perp$ must be zero at rigid body $\Rightarrow$ mirror images work, reflectors

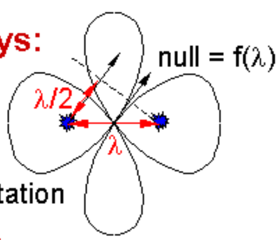




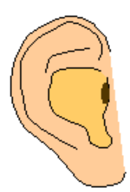


exponential horn matches imepedances

Phased Arrays:



In-phase excitation



ear has nulls that are frequency dependent

Human Ears:

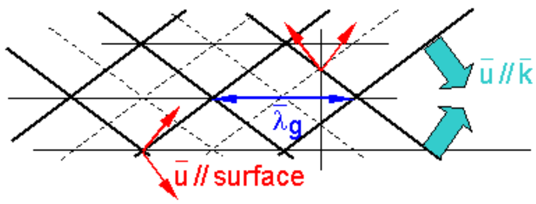
Have horizontal and vertical directionality
And can sense distances close to the head

L24-6

ACOUSTIC WAVEGUIDES AND RESONATORS

Boundary Conditions: p and $u_\perp$ continuous across boundaries

Velocity $\bar{u}_\perp$ must be zero at rigid body



Modes have m,n, $\lambda_i/2$'s in x,y

$\Rightarrow A_{m,n}$ mode

Resonators have m,n,p

$\lambda_i/2$'s in x,y,z directions

$\Rightarrow A_{mnp}$ resonator modes

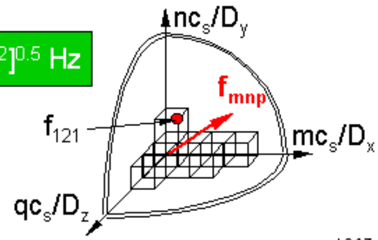
Resonant Frequencies of a Box:

$$(\omega/c_s)^2 = k_o^2 = \Sigma_i k_i^2 = \Sigma_i (2\pi/\lambda_i)^2 = (2\pi)^2[(m/D_x)^2 + (n/D_y)^2 + (q/D_z)^2]$$

where:

$$f_{mnp} = c_s[(m/D_x)^2 + (n/D_y)^2 + (q/D_z)^2]^{0.5} \text{ Hz}$$

$$c_s = (\gamma p_o/\rho_o)^{0.5}$$



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