MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

	Issued:	February 12, 2002
Problem Set 2	Due in Recitation:	February 20, 2002

Suggested Reading: Text: Sections 1.4-1.6, 2.1-2.3, and 9.2<p414.
In general, each homework and related reading assignment covers the material presented in the preceding week's lectures and recitations (e.g. February 12-15).

Problem 2.1

a) For the uniform plane wave in vacuum $\underline{\overline{E}} = 2\hat{x}e^{jy}$, please give the direction of propagation, the wavelength [m], frequency [Hz], power [W/m^2], and sketch the polarization ellipse (e.g. see Fig. 1.4 in text).

b) Sketch the polarization ellipse for the wave $\underline{\overline{E}} = (3j\hat{x} - \hat{y})e^{-j2\pi z}$.

Problem 2.2

Any monochromatic wave $\underline{\overline{E}}$ can be represented as the linear combination of two opposite circularly polarized waves such as $\underline{\overline{E}}_R = (\hat{x} - j\hat{y})e^{-jkz}$ and $\underline{\overline{E}}_L = (\hat{x} + j\hat{y})e^{-jkz}$, where the subscripts R and L indicate right- and left-circularly polarized waves.

a) If we have the elliptically polarized wave $\underline{\overline{E}} = \underline{a}\,\underline{\overline{E}}_R + \underline{b}\,\underline{\overline{E}}_L = 2\hat{x} + (1 - 5j)\hat{y}$ at z = 0, then what are the complex constants $\underline{a}$ and $\underline{b}$ associated with the equivalent two superimposed circular waves?

b) If the left-circular wave is then delayed a quarter wavelength relative to the right-circularly polarized wave as they propagate together through a certain magnetized plasma, what is the polarization of the emerging wave?

Problem 2.3

A uniform x-polarized wave is propagating in vacuum in the +z direction with a wavelength of 1 micron and a flux density of 10^{12} W/m^2 (e.g., a 100-watt CW laser focused on a square spot 10 microns on a side). The electric and magnetic fields are proportional to $\cos(\omega t - kz)$.

a) What are $\overline{E}(\overline{r}, t)$ and $\overline{H}(\overline{r}, t)$ for this wave? Evaluate all constants numerically. Note how high the field strengths get in such high-power laser beams, and that some lasers can create brief Terawatt pulses with fields that are 10^5 greater.

b) Two such beams are made to propagate in opposite directions so that $\overline{E}$ is proportional to $\cos(\omega t - kz) + \cos(\omega t + kz)$. Please evaluate quantitatively the electric and magnetic energy densities (J/m^3), i.e., $W_e(z)$ and $W_m(z)$, for t = 0, and also $W_e(t)$ and $W_m(t)$ for z = 0.

c) For the wave of (a) evaluate Poynting's vector $\overline{S}(z = 0, t)(W/m^2)$, and $\overline{S}(z, t = 0)$.

d) For the waves of parts (a) and (b), evaluate the complex Poynting vector $\underline{\overline{S}}$.

Problem 2.4

a) What charge Q centered inside a sphere 1 mm in radius would produce an electric field strength at its surface that equals the nominal air breakdown voltage of 3×10^7 V/m?

b) What is the corresponding electrostatic potential Φ (volts) at this surface?

Problem 2.5

A certain computer drives a printer using commands through a single wire that for a portion of its length is isolated in space and carries digital signals consisting of 2-mA pulses. For the purposes of this problem these pulses can be modeled as a sine wave at 1 MHz with a 2-mA peak-to-peak amplitude.

a) What is the total power radiated by this current assuming it is uniform along its 10-cm length? Use the Hertzian dipole approximation.

b) If we want to monitor this printer at a distance and need then to intercept 10^{-19} J/pulse, or $\sim10^{-13}$ watts, what is the maximum range r we can do so if our receiving antenna has gain G = 2?