

6.014 Electrodynamics

Problem Set 3 Issued: February 20, 2002
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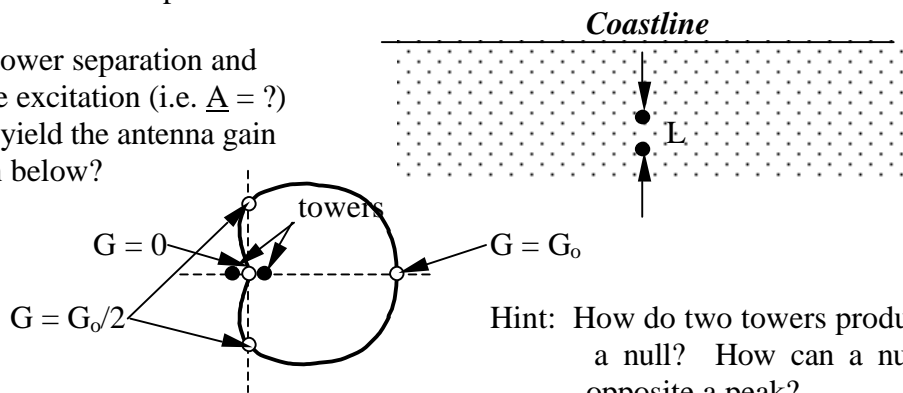
Suggested Reading: Text: Sections 2.4, 3.1-3.6<p.103.

Problem 3.1

A popular coastal AM radio station operating at 1 MHz transmits using two towers L meters apart.

- a) In terms of L and λ what is the smallest angular spacing between nulls of the resulting pattern for the case where $L \gg \lambda$?
- b) Sketch the angular pattern (top view) for the case where $L = 1.5\lambda$ and the currents in the two towers are equal and in phase. Quantitatively determine the angular locations of the nulls and peaks.
- c) To best serve an urban population settled along a straight low coastline, the tall towers are placed 75 meters apart 10 miles from the coast on a line perpendicular to the coast, as shown; they are excited equally and in phase. Sketch the shape of the area served by this station, and determine the relative magnitudes and angles for the maxima and minima of the antenna pattern.

- d) What tower separation and relative excitation (i.e. $\underline{A} = ?$) would yield the antenna gain pattern below?



Hint: How do two towers produce a null? How can a null be opposite a peak?

Problem 3.2

Two perfect Polaroid filters are positioned parallel and polarized at right angles so as to pass no light (but would pass 50 percent of incident unpolarized light if polarized the same).

- a) If a third such filter is placed between them polarized at 45 degrees to each, what percentage of the incident light passes through the trio? (Hint: it's non-zero; explain).
- b) In the limit of an infinite number of such zero-loss filters in series, each rotated a vanishingly small angle relative to its neighbors, what is the limiting transmission percentage for the ensemble as the polarization rotates 90 degrees? Explain briefly.
- c) If a quarter-wave plate is placed between these same two orthogonal Polaroid filters, what is the maximum percentage power that can be transmitted then? How should the quarter-wave plate "fast axis" be positioned relative to the axes of the two polaroids?

Problem 3.3

A distant civilization wishes to transmit messages to Earth from across the galaxy and wants to know the lowest radio frequency that will propagate. The average electron density in intra-galactic space is ~ 0.03 electrons per cubic centimeter.

- a) What is this frequency? Explain briefly how you determined your answer.
- b) How much faster than the speed of light is the phase velocity for a 1-MHz wave in intragalactic space?

Problem 3.4

A 900-MHz wireless phone user wants to know how deep within a human head the radiated power might propagate, and assumes the answer is the same as for meat having $\epsilon = 50(1 - 0.3j)\epsilon_0$, and $\mu = \mu_0$. What is this power penetration depth Δ in a user's head, where power decays as $e^{-z/\Delta}$?