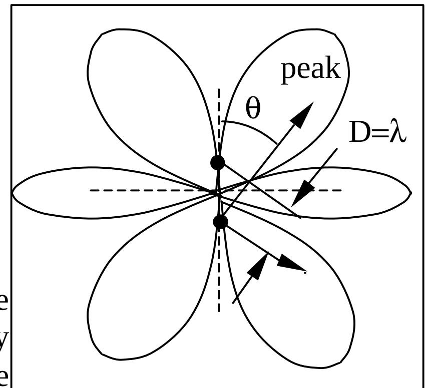
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

Problem Set 3 Solutions Available in Tutorials: February 28, 2002

Problem 3.1

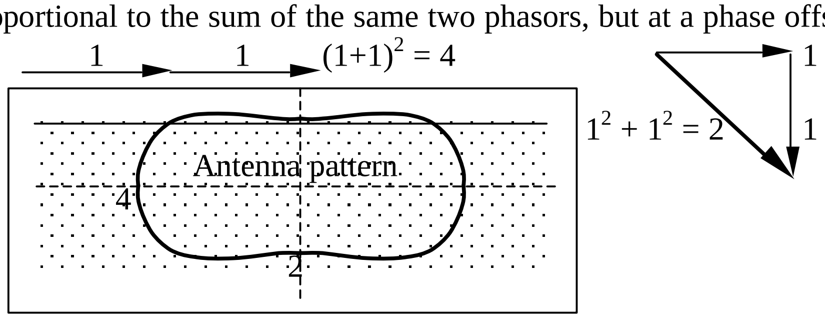


a) When L >> λ, $\cos f_{null} = \frac{nl}{L}$.

$\therefore \Delta f_{null} \approx \frac{l}{L}$.

b) Because the two currents are in phase, we have peaks in the gain when the phase offset in any particular direction is 0 or λ (2λ is not possible here.) So peaks occur when $\boxed{\theta = \cos^{-1}(\pm 0.6667)}$ and $\boxed{\theta = \pm 90^0}$. Nulls occur when this phase offset is λ/2 or 3λ/2, which happens when $\boxed{\theta = 0^o, 180^o}$, and $\boxed{\cos^{-1}(\pm 0.3333)}$. $\mathrm{Cos}^{-1}(\pm 0.6667) = 48.2^o$, 131.8^o, 228.2^o, and 311.8^o. $\mathrm{Cos}^{-1}(\pm 0.3333) = 70.5^o$, 109.5^o, 250.5^o, and 289.5^o.

c) Maxima occur when the two signals add in phase, which occurs when $\theta = \pm 90^o$, and minima occur when the phase difference is maximum, which occurs when $\theta = 0^o$ or 180^o. The power at maximum is proportional to $(1 + 1)^2$, while at the minima it is proportional to the sum of the same two phasors, but at a phase offset of $\lambda/4 \Rightarrow 90^o$.



d) For two towers to produce a null the two phasors must have equal magnitudes, and for them to add to produce a null in one direction, and a peak in the opposite direction, they must be 90^o out of phase in both space and time. Alternatively, any combination of phase offsets in time and space that add to 180^o also works, although with slightly different gain patterns. Here we assume the 90^o option works, so the two towers are separated by 75 meters (λ/4) and are driven 90^o out of phase. The tower positioned toward the peak gain lags the other, i.e. $\boxed{\underline{A} = -j.}$

Problem 3.2

a) The first Polaroid absorbs half the light power and passes half ($E_1 = 2^{-0.5}E_o$, where E_o and E_1 are the magnitudes of the input and output electric vectors, respectively). The electric vector input to the second Polaroid $\overline{E}_1$ can be resolved into two perpendicular components, one parallel and one perpendicular to the axes of the second Polaroid, both having magnitudes of $E_2 = 2^{-0.5}E_1 = E_0/2$. Emerging from the second Polaroid is $\overline{E}_2$, which again can be decomposed into two equal orthogonal components of magnitude $E_3 = 2^{-0.5}E_2 = E_1/2 = 2^{-1.5}E_o$. $\overline{E}_3$ passes through the third Polaroid unattenuated. These vectors each split into two equal components because of the 45^o angles. The fraction of the incident power which then passes through the three polaroids is thus $(E_3/E_o)^2 = 2^{-3} = \boxed{1/8,}$ rather than zero when the middle Polaroid is absent.

b) The fraction of power passing each Polaroid after the first (which absorbs half the power) is $\cos^2\theta$, where θ is the angle between two consecutive Polaroid axes. If N polaroids are in series, then $(N-1)\theta = 90^o$. The question is, does $\cos^{2N}[90/(N-1)])$ approach unity as $N\rightarrow\infty$? Yes, is the answer. For example, 97.5 percent of the power is preserved after 100 polaroids, or 48.8 percent, counting the loss in the first Polaroid. The final answer is $\boxed{\text{half,}}$ this being lost in the first step.

c) Half is lost in the first Polaroid, as usual, then the emerging light can all be converted to circular polarization by $\boxed{\text{placing the fast axis at 45 degrees}}$ with respect to the incident polarization angle. Half of this light can then pass the second Polaroid, or $\boxed{\text{one quarter of the initial light power.}}$ If the fast axis is at any other angle, then less is transmitted. Zero is transmitted if the fast axis is either parallel or perpendicular to the incident polarization. See Example 3.4.3 in the text.

Problem 3.3

a) Radio waves can not propagate below the plasma frequency $\omega_p/2\pi$ Hz, which is $(n_e e^2/\varepsilon_o m_e)^{0.5}/2\pi = (3\times10^4\times[1.60\times10^{-19}]^2/8.854\times10^{-12}\times9.11\times10^{-31})^{0.5}/2\pi$
$= \boxed{1.6\text{ kHz.}}$ (See (3.6.6) in the text.)

b) $v_p = (\mu_o\varepsilon)^{-0.5} = c(\varepsilon_o/\varepsilon)^{0.5} = c(1 - f_p^{\,2}/f^2)^{-0.5} = c\,(1-[1.6\times10^3/1\times10^6]^2)^{-0.5} = \boxed{c(1 + 10^{-6})}$
(See (3.6.5) in the text.)

Problem 3.4

Electric fields propagate as $e^{-jkz} = e^{-jk'z - k''z}$, and power decays as $e^{-2k''z}$, where $k = \omega(\mu_o\varepsilon)^{0.5}$ and ε is given for meat; $\varepsilon = 50(1 - 0.3j)\varepsilon_o$.

Here $\Delta = 1/2k'' = 1/2I_m\{\omega(\mu_o[50(1 - 0.3j)\varepsilon_o])^{0.5}\} = (c/7.07\omega)/2I_m\{(1 - 0.3j)^{0.5}) = (3\times10^8/[7.07\times2\pi9\times10^8])/0.30 = \boxed{2.5\text{ cm}}$. Actually the head also includes fat, bone, and discontinuities that complicate the true answer. So $e^{-j\underline{k}z} = e^{-j134.8-19.8}$.