MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

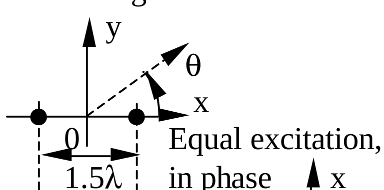
	Issued:	February 27, 2002
Problem Set 4	Due in Recitation:	March 6, 2002

Suggested Reading: Lecture 6 supplementary notes
Text: Examples 9.4.1, 9.4.2;
Sections 4.1-4.2 < p.131 (skip force calculations in Example 4.1.1); 4.3 < p.151

Problem 4.1

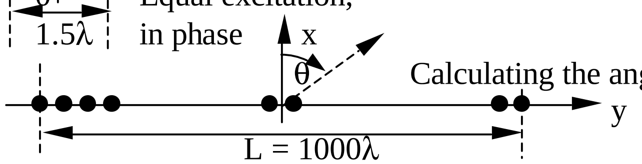
Sketch the antenna patterns in the x-y plane for the following arrays of vertical (z-oriented) dipole antennas. Include quantitative expressions for the angles and relative amplitudes for gain maxima and minima.

a)



b) Same as (a), but excitation = 1 for left antenna, and 2j for right antenna

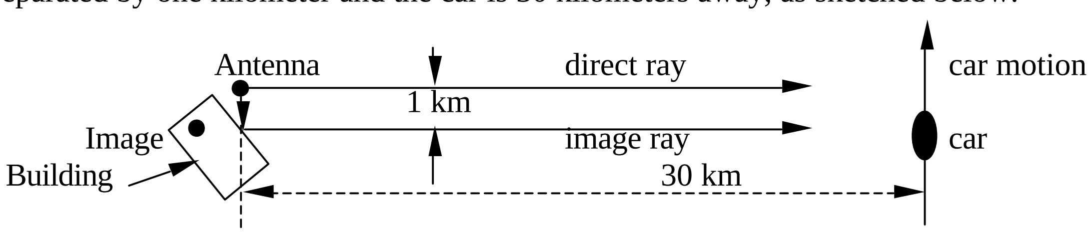
c)



Calculating the angles of the first two nulls is sufficient.

Problem 4.2

A car drives along the highway listening to a 100-MHz FM station and hears small quick bursts of noise as the car passes through successive nulls caused by multipath, as suggested in the figure below, where a perfect reflection (image ray) from a large smooth flat metal building interferes with the direct ray perfectly. The antenna and its image are separated by one kilometer and the car is 30 kilometers away, as sketched below.



a) If a perfect null occurs at 100 MHz, what is the nearest frequency at which another null occurs?

b) When the car is moving perfectly broadside to the direct ray, approximately how far apart [wavelengths and meters] do these noise bursts occur? Briefly explain your method.

c) At the moment the car is exactly broadside to the direct ray and traveling at velocity 40 ms^{-1}, what is the approximate Doppler shift (Hz) associated with the reflected ray?

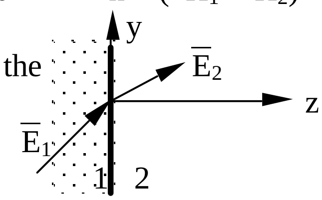
Problem 4.3

The four basic boundary conditions are:

$$\overline{n} \bullet (\overline{D}_1 - \overline{D}_2) = \sigma_s \qquad \overline{n} \bullet (\overline{B}_1 - \overline{B}_2) = 0$$

$$\overline{n} \times (\overline{E}_1 - \overline{E}_2) = 0 \qquad \overline{n} \times (\overline{H}_1 - \overline{H}_2) = \overline{J}_s$$

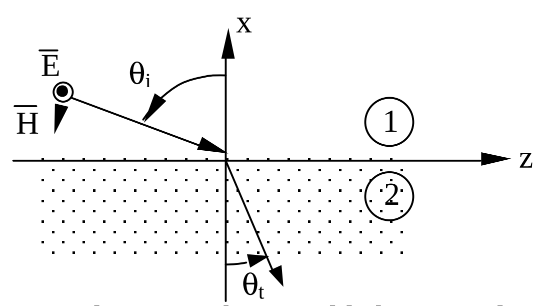
a) If $\sigma = 0$, $\varepsilon_2 = 2\varepsilon_1$, and $\overline{E}_1 = \overline{y} + \overline{z}$, what is $\overline{E}_2$ near the boundary z = 0 that separates medium 1 and 2?



b) If $\sigma_2 = \infty$, $\varepsilon = \varepsilon_o$, and $\mu = \mu_o$, and $\overline{E}$ incident = $\overline{x}E_o\, e^{-jk(\overline{y} + \overline{z})}$, what are $\underline{J_s}(y)$ and $\underline{\sigma_s}$ on the surface of the perfectly conducting medium 2? This question relates to the figure above

Problem 4.4

A TE wave is incident upon medium 2 at angle θ_i. If $\varepsilon_2 = 9\varepsilon_o = 9\varepsilon_1$, and $\mu_1 = \mu_2 = \mu_o$, then,



a) What is the angle of transmission θ_t?

b) If the same wave were incident at θ_i from medium 2, what would the critical angle θ_c be? A quantitative answer is sought.

c) If the incident wave for part (b) arrived at a 45-degree angle, what would be the decay rate α along the x axis, where the electric field in medium 1 decays as $e^{-\alpha x}$? What is α in units of λ^{-1}?

Problem 4.5

Marconi wants to bounce radio waves off the ionosphere, assuming it is modeled as a thick ionized slab with 10^{12} electrons/m^3 (in daylight on a good day).

a) What is the maximum frequency f(Hz) for which waves at normal incidence will be perfectly reflected?

b) What is the maximum frequency f_5(Hz) at which waves are perfectly reflected if they are incident upon this planar ionosphere at five degrees grazing angle? This corresponds to long ionospheric radiowave bounces across the ocean.