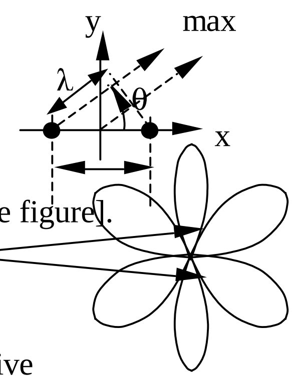
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

Problem Set 4 Supplementary Solutions — Available in Tutorials: March 11, 2002

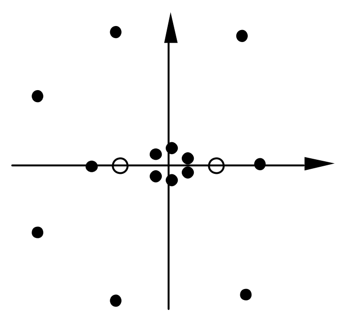
Problem 4.1



a) We are asked only to sketch the patterns, not to compute them, so we need only to find the angles of the maxima and minima. Clearly the two in-phase signals will add along the y axis and at $\theta = \pm\cos^{-1}(1/1.5) = \pm 48.2^o$ and $\pm 131.8^o$ [see figure]. We have nulls at $\theta = 0, \pi$, and $\pm\cos^{-1}(0.5/1.5) = \pm 70.5^o$ and $\pm 109.5^o$. All maxima are equal, and all nulls are zero.

b) The same method above for excitations 1 and 2j yields relative gains along the x and y axes of $|1 + 2j|^2$ and $|1 - 2j|^2 = 5$. Maximum gain is $(1+2)^2 = 9$, and the minima have gain $(2-1)^2 = 1$. Maxima occur at angles where the two radiated beams add in phase; this occurs when the ray from the left dipole lags the ray from the first spatially by 0.75λ (the phase of the right dipole = 2j ⇒ π/2 phase lead), or leads by 0.25 or 1.25 λ. Thus full maxima occur (for $\theta < \pi/2$) at angles $\theta = \pm\cos^{-1}(0.75/1.5)$; these angles are $\pm 60^O$. For $\theta > \pi/2$, maxima occur when $\theta = \pm(\pi/2 + \sin^{-1} b)$, where b = (0.25/1.5) or (1.25/1.5); these angles are $\pm 99.6^o$ and $\pm 146.4^o$. A local (weaker) maximum occurs at θ = 0, along the x axis. Minima occur for left-dipole lags of 0.25λ and 1.25λ, and leads of 0.75λ. The corresponding angles are $\pm 33.6, \pm 80.4$, and $\pm 120^o$. A local minimum occurs on the negative x axis. The radiation formula (2.4.8) in the text gives the an analytic expression for the pattern for two dipoles, but the locations and magnitudes of each maximum and minimum, and their magnitudes, should be sufficient to sketch the pattern, suggested below.

λ



c) The main idea here is explained in Figure 9.18 in the text (page 437); the first null occurs when $\theta \cong \lambda/L$ and the second one when $\theta \cong 2\lambda/L$. The idea is that we can group the radiators in pairs that cancel at some angle, and collectively they cancel. These pairs are separated by L/2, and the phase offset is λ/2, so the angle of the first null is $\sin^{-1}[(\lambda/2)/(L/2)] \cong \lambda/L$ radians. At the second null each canceling pair is separated by L/4, so its angle $\cong 2\lambda/L$ radians.