MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

	Issued:	March 5, 2002
Problem Set 5	Due in Recitation:	March 13, 2002

Suggested Reading: Text: 4.3, 9.4 p.430-6, 9.5, (9.7)[1], 9.8≠pp474-5

Problem 5.1

Assume a certain brand of plate glass has permittivity $\varepsilon = 2\varepsilon_o$, and we wish to build a solar house with it. At the surface of the earth direct sunlight conveys roughly one kilowatt per square meter.

a) When the sunlight enters roughly normal to the glass, approximately what fraction of the solar energy enters the house? Assume the glass is totally transparent at all important energy-bearing wavelengths. Remember a sheet of glass has two sides.

b) When the light enters at 60 degrees from normal, what fraction of the TE wave energy is reflected by the top surface of the glass?

c) What is Brewster's angle for this glass?

Problem 5.2

Sally (class of '85) has a new 50-foot sailboat with a spinning radar antenna high on the mast that operates near 10 GHz; its aperture measures 15×90 cm. Above this radar is a radar retro-reflector that makes Sally's boat more visible to other radars.

a) Would you recommend spinning this antenna about a vertical axis with the short dimension (15 cm) oriented horizontal or vertical? Briefly explain the reason for your choice.

b) At what angle (radians) from the beam axis is the first null of the antenna transmitter pattern?

c) If the retro-reflector can be modeled as a flat square sheet of metal, 40 cm on a side, that is always oriented to retro-reflect perfectly all incident radar signals, what is its scattering cross-section σ_s at 10 GHz? Note that σ_s is much larger than the physical size of the device.

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[1] Reading suggestions that are enclosed in parentheses are quite useful, but are unlikely to be on any quiz or final exam.

d) Sally's 10-watt radar is sensitive to echoes stronger than 10^{-16} watt. What is the maximum range r at which this radar can detect a small object having $\sigma_s = 1\ m^2$?

Problem 5.3

Roughly 100 years ago Thomas Edison built a large multi-turn loop antenna to receive solar radiation at low frequencies before it was known the ionsphere existed and would block them. Although he circled an entire iron mine with loops of wire, let's consider a modern low-budget version of his concept. Assume a coil 1 meter in diameter has 100 turns of wire. Let's assume the incoming solar radiation to which our receiver is sensitive is near 30 MHz and has intensity 10^{-15} [Wm^{-2}].

a) Ignoring ionospheric effects and reflections from the ground, how should this loop antenna be oriented with respect to the receiver-sun axis in order to maximize reception?

b) What is the rms strength of the incoming wave's [$10^{-15}\ Wm^{-2}$] magnetic field $\overline{H}(t)$?

c) What then is the maximum possible rms open-circuit voltage across the loop antenna terminals due to this solar radiation?

d) If at these frequencies we model the terrestrial surface as a perfectly conducting mirror that is oriented perpendicular to the incoming radiation, name at least two places where can we position this loop antenna to maximize its open-circuit voltage. Briefly explain your reasoning.

e) Repeat part (d) for the case where the incoming TE polarized radiation is arriving at an angle of 60 degrees with respect to the surface normal. How should the loop antenna be oriented then?

f) Repeat part (e) for TM polarization.

g) Calculations and experiments both yield the fact that the gain of small loop antennas is $1.5 \sin^2\theta$, the same as for a Hertzian dipole, where θ is the angle from the loop axis. This relation and the results from (c) enable us to compute the radiation resistance R_r of small loop antennas as a function of their diameter D, number of turns N, and radio frequency f. What is this R_r [ohms]?