MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

Problem Set 6 Solutions — Available in Tutorials: April 1, 2002

Problem 6.1

a) Charge relaxation time (p. 88): $\boxed{\tau = \frac{\varepsilon}{\sigma} = \frac{4.8\times10^{-19}}{10^{-14}} = 4.8\times10^{-5}\,\mathrm{s}}$

b) Resistance of the parallel plate structure: $\boxed{R = \frac{d}{A\sigma} = \frac{1}{10\times10^{-14}} = 1\times10^{13}\,\Omega}$

c) Capacitance of the structure: $\boxed{C = \frac{\varepsilon A}{d} = \frac{4.8\times10^{-19}(10)}{1} = 4.8\times10^{-18}\,\mathrm{F}}$

d) For N capacitors in parallel, we have $C_{eq} = NC$ and $R_{eq} = \frac{R}{N}$. From circuit theory we know that $\tau = R_{eq}C_{eq}$ and we notice that the N's cancel and that $\tau = RC$ and from lecture 10 we know that $\boxed{\tau = RC = \frac{\varepsilon}{\sigma} = 4.8\times10^{-5}\,\mathrm{s}}$

e) For M capacitors in series, we have $C_{eq} = \frac{C}{M}$ and $R_{eq} = RM$. From circuit theory we know that $\tau = R_{eq}C_{eq}$ and we notice that the M's cancel and that $\tau = RC$ and as before $\boxed{\tau = RC = \frac{\varepsilon}{\sigma} = 4.8\times10^{-5}\,\mathrm{s}}$

Note: that for the same capacitors in series or parallel the charge relaxation time is the same.

Problem 6.2

For a solenoid filled with permeability μ,

$$L = \frac{mN^2A}{2pR} = \frac{mN^2A}{4p\sqrt{A}} = \boxed{\frac{mN^2\sqrt{A}}{4p}}$$

For a toroidal solenoid with gap d,

$L = \frac{m_0 N^2 A}{d}$ if $d \geq 2pR(m_0/m)^2$ as stated on Lecture Slide 11-9.

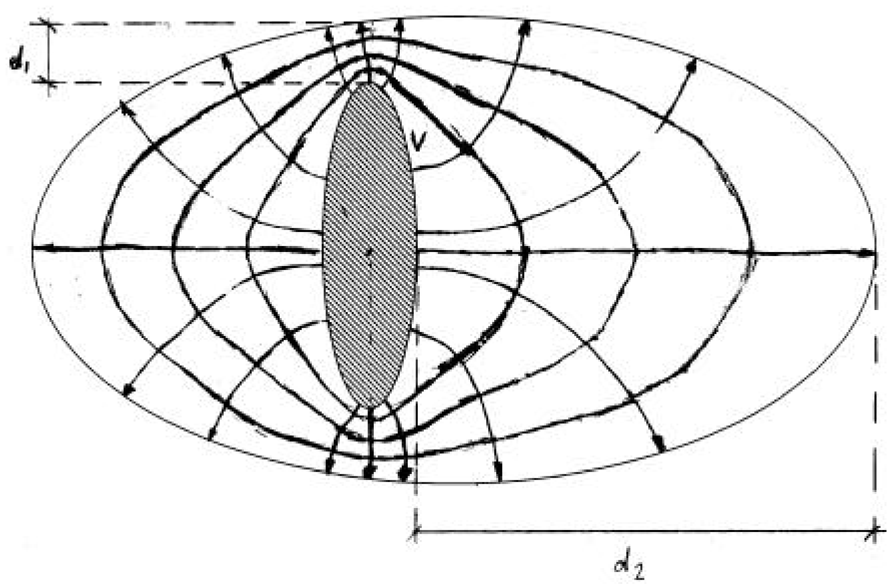
Since $d = \frac{2pRm_0}{m} > 2pR\left(\frac{m_0}{m}\right)^2$ for $m = 10^5 m_0$,

$$L = \frac{m_0 N^2 A}{d} = \frac{mN^2 A}{2pR} = \frac{mN^2 A}{4p\sqrt{A}} = \boxed{\frac{mN^2\sqrt{A}}{4p}}.$$

Therefore, the inductances for the two configurations are equal. The ratio between the two inductances is 1.

Note: Even though the magnetic energy stored in the gap comprises about half the total stored magnetic energy ($d = 2pR(m_0 / m)$, the magnetic energy density in the gap still dominates the magnetic energy density in the torus.

Problem 6.3



Approach: Draw radial lines outward emanating from the center of the inner ellipsiod. Then divide the resulting radial lines into four equal distance sections. Connecting the i^{th} equal distance points of all the radial lines the i^{th} equipotential line can be drawn. The E-field lines are then drawn such that it is directed from the inner conductor towards the outer conductor. The E-field lines are orthogonal to both conductor surfaces and the equipotentail lines.

$E_{max} = \frac{V}{d_1}$ and $E_{min} = \frac{V}{d_2}$ hence we have $\boxed{\frac{E_{max}}{E_{min}} = \frac{d_2}{d_1}}$.

Problem 6.4

a) $\Phi(x,y) = (A + Bx)(C + Dy)$

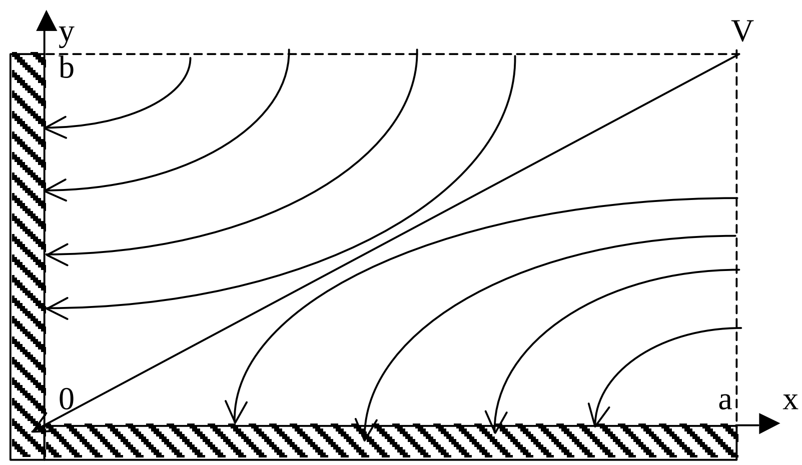
Using $\Phi(x,b) = \frac{x}{a}V$

$\Phi(a,y) = \frac{y}{b}V$

$\Phi(0,0) = 0$ and

$\Phi(a,b) = V$, we can conclude that $\boxed{\Phi(x,y) = \frac{xy}{ab}V}$.

b)



$$\overline{E} = -\nabla\Phi = -\frac{V}{ab}(y\hat{x} + x\hat{y})$$