MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

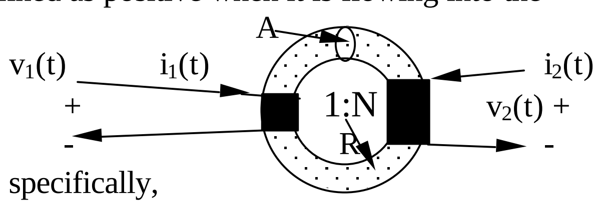
6.014 Electrodynamics

	Issued:	March 20, 2002
Problem Set 7	Due in Recitation:	April 3, 2002

Suggested Reading: Text: Sections 5.1, 5.2, 5.5, and 6.5<267, plus notes 1.2R and 3.2R.

Problem 7.1

Consider an ideal step-up transformer wound on a high-permeability iron toroid with a turns ratio 1:N. The radius of the toroid is R and its cross-sectional area is A, as illustrated. Its permeability is $\mu >> \mu_o$. If the power dissipated in the toroid itself is negligible compared to the power flow into the load at the transformer output, then the power into the transformer approximates the power out to the load, i.e. $v_1 i_1 \cong -v_2 i_2$, where the sign is reversed on i_2 because it is defined as positive when it is flowing into the transformer, as illustrated:



a) How does the average energy stored in the toroid vary with its permeability μ? Assume $\mu >> \mu_o$. More specifically, as $\mu \to \infty$ does this stored energy decline toward zero or increase toward ∞, for a given v(t)? Assume $i_2(t) = 0$ for simplicity. Briefly explain your reasoning or calculations.

b) Assume the polarity of the windings is such that a positive step voltage $v_1(t)$ produces a positive step in $v_2(t)$ when $i_2(t)$ is zero (open-circuit). If the input is now open-circuited and $i_2(t)$ is a square wave oscillating at 1 kHz between 1 and 2 milliamps, approximately what is $v_1(t)$? Indicate your answer by a simple dimensioned sketch of $v_1(t)$.

c) Assume the output i_2 is terminated with a resistor R that has a value such that the energy storage and dissipation within the toroid can be neglected in comparison to that dissipated in R. What now is the Thevenin equivalent of the passive toroid-plus-R circuit as seen from terminals 1?

Problem 7.2

Integrated circuits can be manufactured in a variety of ways. Let's assume that the wires connecting the transistors: 1) are formed with thickness 0.1 micron (a micron is 10^{-6} meters) and have width 0.4 micron, 2) are deposited on an insulating dielectric 0.1 microns thick with $\varepsilon/\varepsilon_o = 4$, and 3) have conductivity $\sigma = 4\times10^{7}$ Sm^{-1}, as does the associated thick ground plane. The breakdown field strength for this thin dielectric is a million volts/cm.

a) What is the maximum operating voltage this circuit can use? Margins of safety can reduce this specification.

b) What is the characteristic impedance Z_o of this TEM line?

c) How long can these wires be before their series resistance is comparable to Z_o?

d) If the highest frequencies (harmonics due to sharp edges of square waves) handled by this circuit are 10 GHz, how long is a quarter-wavelength in this TEM line for this maximum frequency?

e) What is the inductance L (Henries/m) for this line?

f) Assuming the assumptions above are realistic, as an IC designer, which of your answers above might concern you, if any, and in what way?

<u>Problem 7.3</u>

A certain computer drives a flip-flop at the end of a 5-cm long 50-ohm TEM transmission line having $\mu = \mu_o,$ and $\varepsilon = 4\varepsilon_o$. The Thevenin source impedances for the line driver and the flip-flop are both 200 Ω. The equivalent Thevenin voltage source for the line driver is a 10-volt step function, and the flip-flop switches state when its input reaches 5 volts.

a) Sketch and dimension the voltage distribution V(z) on the line at $t = 10^{-10}$ seconds.

b) Repeat (a) for $t = 5\times10^{-10}$ seconds. Also sketch the current distribution I(z) at 5×10^{-10} seconds.

c) Sketch and dimension the load voltage $V_L(t)$ as a function of time from t = 0 until the flip-flop switches state.

d) What is the asymptotic value of $V_L(t)$ as $t \to \infty$?

e) Note that triggering is excessively delayed by this transmission line. What is the minimum TEM line impedance that would solve this excess delay problem for the given source and load?