MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

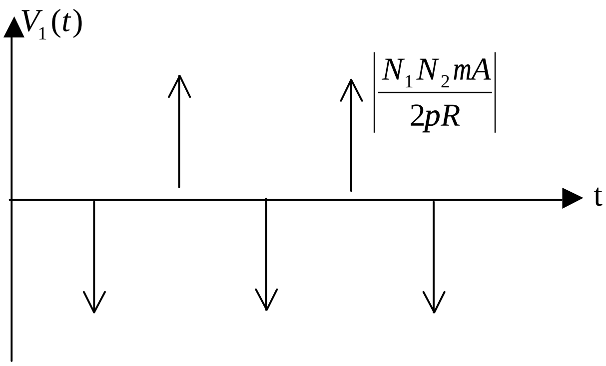
Problem Set 7 Solutions Available in Tutorials: April 7, 2002

Problem 7.1

a)
$$w_m = \int_v \frac{1}{2} m \, |\overline{H}|^2 \; dv$$
$$= \frac{1}{2} m \, |\overline{H}|^2 \; (2pRA)$$
$$= pRAm \, |\overline{H}|^2$$
$$\oint_c \overline{H} \cdot d\,\bar{l} = N_1 i_1 - N_2 i_2$$
$$|\overline{H}| \, 2pR = N_1 i_1 - N_2 i$$
$$|\overline{H}| = \frac{N_1 i_1 - N_2 i_2}{2pR}$$
$$\therefore w_m = pRAm \left(\frac{N_1 i_1 - N_2 i_2}{2pR} \right)^2 = \frac{Am\left(N_1 i_1 - N_2 i_2\right)^2}{4pR}$$
Therefore, $w_m \to \infty$ as $m \to \infty$.

b)
$$V_1 = N_1 \frac{d\Phi}{dt} \text{ and } V_2 = N_2 \frac{d\Phi}{dt}$$
$$\oint_c \overline{E} \cdot d\bar{l} = -\int_S \frac{d\overline{B}}{dt} \cdot d\bar{s} = N_2 mA \frac{d\,|\overline{H}|}{dt}$$
$$\therefore \frac{d\Phi}{dt} = mA \frac{d\,|\overline{H}|}{dt}$$
$$|\overline{H}| = \frac{N_1 i_1 - N_2 i_2}{2pR}$$
$$\therefore \frac{d\Phi}{dt} = \frac{mA}{2pR} \frac{d\left(N_1 i_1 - N_2 i_2\right)}{dt}$$
$$V_1 = \frac{N_1 mA}{2pR} \frac{d\left(N_1 i_1 - N_2 i_2\right)}{dt}$$
$$V_2 = \frac{N_2 mA}{2pR} \frac{d\left(N_1 i_1 - N_2 i_2\right)}{dt}$$
If V_2 is positive when $i_2 = 0$, then $\frac{di_1}{dt} > 0$.

Therefore, $V_1(t) = \frac{N_1 mA}{2pR} \frac{d\left(N_1 i_1 - N_2 i_2\right)}{dt} = -\frac{N_1 N_2 mA}{2pR} \frac{di_2}{dt}$



f=1kHz

c) $R_{1eff} = \frac{V_1}{i_1} = \frac{(N_1 / N_2)V_2}{(N_2 / N_1)i_2} = \left(\frac{N_1}{N_2}\right)^2 R_L$

Problem 7.2

In all what follows: the subscript d correspond to the dielectric slab; the subscript w correspond to the conductor wire.

a) The magnitude of the E-field for the parallel plate transmission line is:

$$\boxed{E = \frac{V}{d_d} \Rightarrow V_{max} = \left(\frac{1\times10^6}{1\times10^{-2}}\right)(0.1\times10^{-6}) = 10V}$$

b) The dielectric is not lossy, hence $\eta = \sqrt{\frac{\mu}{\varepsilon}}$ is real. We know that $Z_0 = \frac{\eta d_d}{W}$ where in this problem we have that $\mu = \mu_0$ and $\varepsilon = 4\varepsilon_0$. Substituting this into the above relationships we find that $\boxed{Z_0 = \frac{(377)(0.1\times10^{-6})}{2(0.4\times10^{-6})} = 47.125\Omega}$

c) The resistance of the wire strip is $R = \frac{\ell}{\sigma A}$. The resistance of the ground path of the circuit is taken to be zero; the reason being that when $A_{ground} \to \infty \Rightarrow R_{ground} = 0$. We can calculate the length from setting $|R| = |Z_0| \Rightarrow \ell = |Z_0| \sigma d_w W = 75.4\mu m$

d) For $f = 10GHz$ the wavelength of the EM wave is $\lambda = \frac{c}{f} = \frac{1}{\sqrt{\mu\varepsilon} f} = 1.5\times10^{-2} m$. Thus a quarter wavelength is $\lambda / 4 = 3.75mm$.

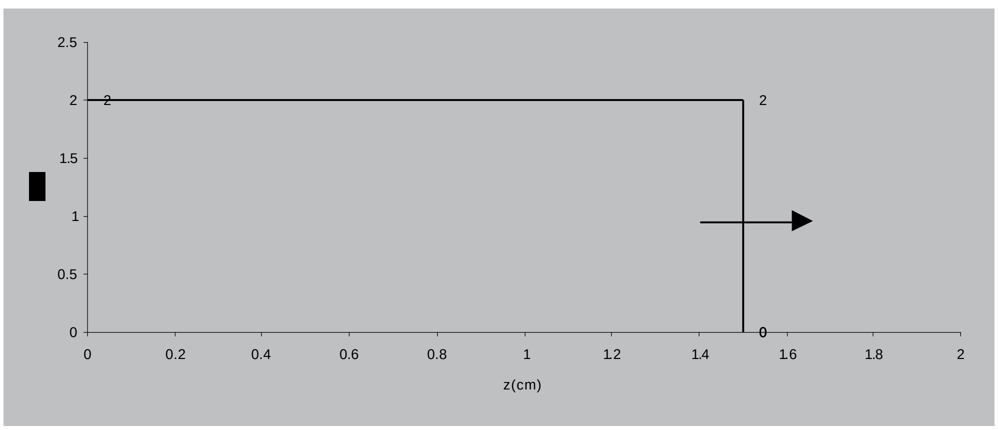
e) $L = \frac{m_0 d}{W} = 0.31416mH$

f) Answers that might concern me …

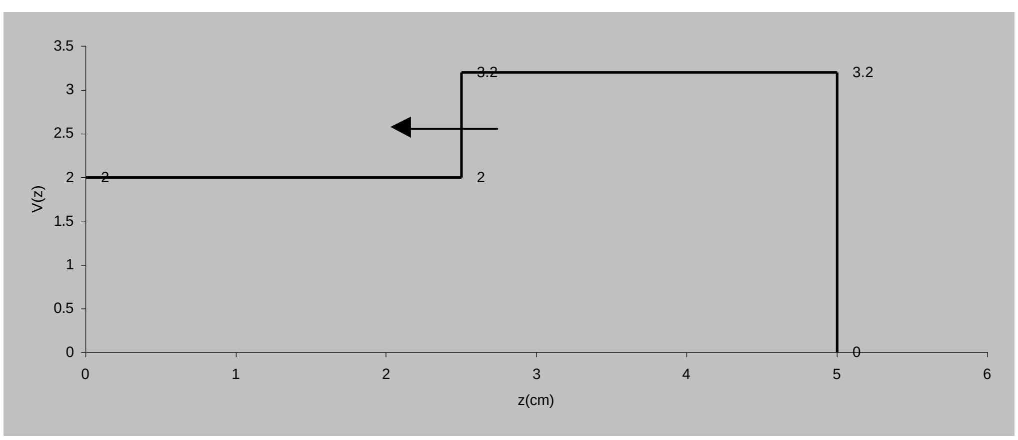
- We ignored fringing effects. The latter might decrease V_{max} . It is conceivable that we can have circuits operating near $\pm 10V$ (for instance the RS232 12V serial comminucation bus in a computer). We surely don't want dielectric breakdown of the material.
- The length of the wire for which its resistance becomes comparable to the characteristic impedance of the line is short. It is not hard to imagine that printed circuit board will have wires in excess of this length. When that is the case the I^2R losses might be significant.
- The calculated quarter wavelength is comparable to the physical dimension of a printed circuit board. Thus it is not unreasonable to doubt the validity of lumped parameter approximation of circuit parameters.

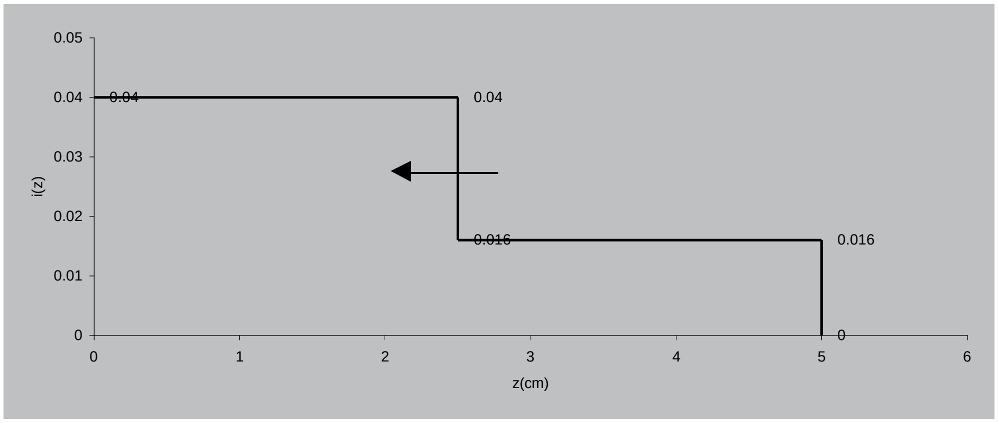
<u>Problem 7.3</u>

a)

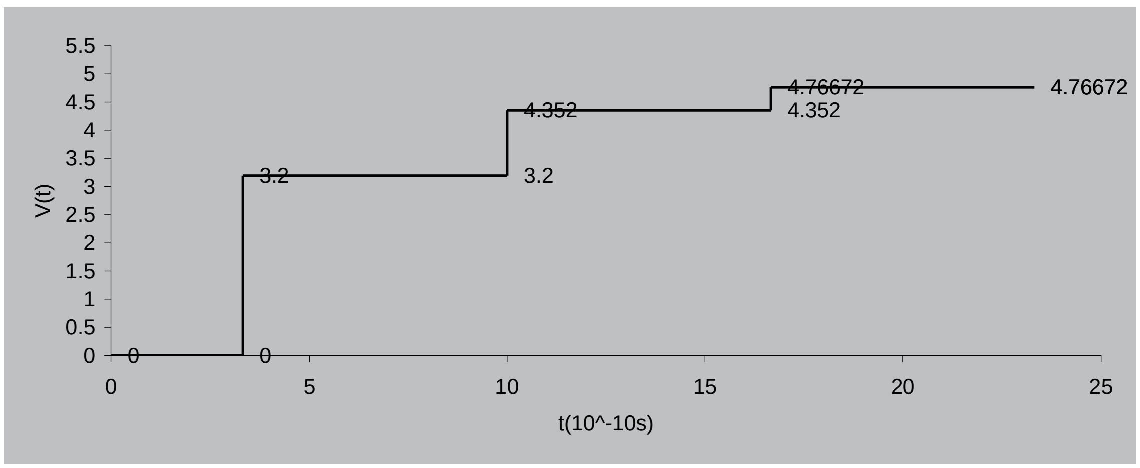


b)





c)



$V(t) \rightarrow 5\,\text{V}$ as $t \rightarrow \infty$. Therefore, the flip flop never switches.

d) $$V_L(t \rightarrow \infty) = \frac{Z_L}{Z_L + Z_s} V_s = 5\text{V}$$

e) If the TEM line impedance $Z_0 = Z_L$ then $\Gamma_L = 0$ (no reflections). Since $Z_s = Z_L$, $V_+ = 5\text{V}$. Therefore, with $Z_0 = Z_L$ we have a forward traveling wave of magnitude 5V. Therefore, the flip flop switches when this wave reaches z = 5cm at t = $3.33 \times 10^{-10}\, s$.