MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

Problem Set 9 Solutions   Available in Tutorials: April 22, 2002

Problem 9.1

Using $V_{Th} = \left(\dfrac{-jZ_0 \csc kl}{Z_L - jZ_0 \cot kl} \right) V_0$ and $Z_{Th} = Z_0 \left(\dfrac{Z_L + jZ_0 \tan kl}{Z_L + jZ_0 \tan kl} \right)$

a) $Z_{Th} = 100j$, $V_{Th} = 0$

b) $Z_{Th} = 80 + 60j$, $V_{Th} = 0$

c) $Z_{Th} = 100$, $V_{Th} = -j\underline{V_0}$

Problem 9.2

a) The reflection coefficient at the load is $\Gamma_L = \dfrac{R_L - Z_0}{R_L + Z_0} = -\dfrac{1}{3}$. The power absorbed in the load can be calculated as $P_d = \dfrac{1}{2Z_0} |V_+|^2 (1 - |\Gamma_L|^2)$ and the incident power at the load is $\dfrac{1}{2Z_0} |V_+|^2$, thus the power reflected at the load is

$\boxed{\dfrac{1}{2Z_0} |V_+|^2 |\Gamma_L|^2 \Rightarrow \text{ratio power reflected} = |\Gamma_L|^2 = \dfrac{1}{9}}$.

b) Now we have $\Gamma_L = \dfrac{R_L - Z_0}{R_L + Z_0} = -3$. The $\boxed{\text{ratio power reflected} = |\Gamma_L|^2 = 9}$, but this does not make sense because you are reflecting more than the power incident at the load. Notice that the load is no longer a passive element but rather an active element. Thus the load is adding power to the circuit.

Problem 9.3

a) $\boxed{D = \dfrac{l}{4}}$

$Z_A = \sqrt{Z_0 R} = \sqrt{100 \cdot 200} = \boxed{100\sqrt{2}}$

b) In order to match impedences, we start at $Z_{Ln} = (100 + 100j)/100 = 1$+j on the Smith Chart. We then rotate towards the generator until we reach the R=1 circle in the lower half of the Smith Chart (we end up at 1-j). The required rotation corresponds to

$$\boxed{D = \frac{l}{6}}$$

Furthermore, $-jwL = -jZ_0$

$$\boxed{\therefore L = \frac{100}{w}}$$

c) Since the inductor is connected in parallel, it is easier to look at the admittance (admittances add in parallel). In order to match admittances, we start at $Y_{Ln} = 100/(100+100j) = (1-j)/2$. We then rotate towards the generator until we reach the G=1 circle on the upper half of the Smith Chart. The required rotation corresponds to

$$\boxed{D = \frac{l}{4}}$$

$$\boxed{L = \frac{100}{w}}$$

Problem 9.4

a) Series RLC resonator: The resonant (natural) frequency of the oscillator is $\omega_0 = \frac{1}{\sqrt{LC}} = 20\pi \times 10^6 \ldots(1)$. For the series circuit we know that the decay rate of the resonator is $\alpha_0 = \frac{R}{2L}$. Furthermore we have $Q = \frac{\omega_0}{2\alpha_0} = \frac{1}{R}\sqrt{\frac{L}{C}} = 100\ldots(2)$. Thus we have two equations and we solve for L and C. From (1) we find $L = \left((20\pi \times 10^6)^2 C\right)^{-1}$ and using this relationship in (2) we find $C = \left((100)(20\pi \times 10^6)R\right)^{-1}$. Substituting $R = 1\Omega$ into the latter we find $\boxed{C = \frac{1}{2\pi}\text{nF}}$ and $\boxed{L = \frac{10}{2\pi}\mu\text{H}}$.

b) Parallel R(G)LC resonator: The resonant frequency is unchanged. For the parallel circuit we know that the decay rate of the resonator is $\alpha_0 = \frac{G}{2C}$ (discharging of the capacitor). Furthermore we have $Q = \frac{\omega_0}{2\alpha_0} = \frac{1}{G}\sqrt{\frac{C}{L}} = 100\ldots(2)$. Thus we have two equations and we solve for L and C. From (1) we find $L = \left((20\pi \times 10^6)^2 C\right)^{-1}$ and using this relationship in (2) we find $C = \left((100)/(20\pi \times 10^6)\right)G$. Substituting

$G = 1\text{mho}$ into the latter we find $\boxed{C = \frac{10}{2\pi}\mu F}$ and $\boxed{L = \frac{1}{2\pi}nH}$. (Note that the conversion from series to parallel resonator is obtained by interchanging the variables as follows $G \leftrightarrow R$ and $L \leftrightarrow C$ (see page 362)).

<u>Problem 9.5</u>

a) Connect the capacitor in series with the inductor in order to achieve an open-circuit. For $l = l/4$ and R=100, $Z_{Th} = 100$ (the resistance of the lumped circuit, therefore, is $200\,\Omega$).

Since $Q = \frac{w_0 L}{R + R_{Th}}$,

$$L = \frac{Q \cdot R}{w_0} = \frac{100 \cdot 200}{2p \cdot 10^7} = \boxed{318mH}.$$

Furthermore, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$,

$$\therefore C = \frac{L}{(QR)^2} = \boxed{0.79\,pF}.$$

b)

$$\boxed{Q_I = \frac{w_0 L}{R_I} = 200}$$

$$\boxed{Q_E = \frac{w_0 L}{R_E} = 200}$$

c)

$$a = a_0 = \frac{w_0}{2Q_L} = \frac{2p \cdot 10^7}{200} = \boxed{p \cdot 10^5}$$