MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.014 Electrodynamics

	Issued:	April 25, 2002
Problem Set 11	Due in Recitation:	May 1, 2002

Suggested Reading: Text: 7.1, 7.2, skim 7.4, Notes for Lecture 18, Prof. Bers's notes pp 1-5, 23-56.

Problem 11.1

Sketch the $\overline{E}$ and $\overline{H}$ field lines for the TE_1 mode beyond cutoff for $\omega t = 0, \pi/2$ ($\omega < \omega_{co}$, analogous to Figure 7.2(c) on page 289 in the text).

Problem 11.2

a) Sketch the wall currents at t = 0 associated with the TE_{10} rectangular waveguide mode for a monochromatic wave propagating in the +z direction.

b) Sketch all the places thin slots could be cut in this waveguide without significantly interrupting (cutting across) any wall currents at any time t. Thin probes can be inserted through such slots to observe the electromagnetic fields inside without substantially disturbing the passing waves; such probes can also be used as antennas to transmit and receive signals in the waveguide.

Problem 11.3

A certain dielectric-slab waveguide of thickness d lies in the y-z plane centered at x = 0 with electric and magnetic fields:

$$\overline{H} = \hat{y} A \sin k_x x \, e^{-jk_z z} \quad \text{and} \quad \overline{E} = (\hat{x} B \sin k_x x - \hat{z} jC \cos k_x x) e^{-jk_z z} \quad \text{for } |x| \le d/2$$

$$\overline{H} = \hat{y} D e^{-\alpha x} e^{-jk_z z} \quad \text{and} \quad \overline{E} = (\hat{x} F e^{-\alpha x} - \hat{z} jG e^{-\alpha x}) e^{-jk_z z} \quad \text{for } |x| > d/2$$

a) If $\lambda/2 < d < \lambda$, where λ is the wavelength inside the dielectric, what mode is propagating? (e.g. TE_m or TM_m; m = ?)

b) Sketch the $\overline{E}$ fields at t = 0. Assume A, B, etc. are real quantities.

c) The fields inside the slab can be thought of as trapped waves bouncing back and forth inside at an angle $\theta > \theta_c$, where θ_c is the critical angle. What is θ in terms of the parameters given here?

Problem 11.4

A certain optical fiber has dispersion characterized by:

$$k \cong \beta_o\omega_o + \beta_1(\omega - \omega_o) + \beta_2(\omega - \omega_o)^2/2$$

a) In what way does the distance that optical pulses can propagate on this line before distorting depend on the constants β_o and β_1? Explain briefly.

b) Approximately how far can 1-GHz pulse trains with ~10-GHz bandwidth propagate on this fiber before pulse distortion becomes significant? Assume these pulses represent amplitude modulation of a laser operating at $f = c/\lambda = 3\times10^{14}$ Hz. Briefly explain your method. We are concerned here about the distortion of a sinusoid that is contained largely within the band $3\times10^{14} \pm 10^{10}$ Hz. Distortion occurs when the highest velocity signals have moved ahead of the slowest signals by a distance that is perhaps one-half the width of a single pulse.