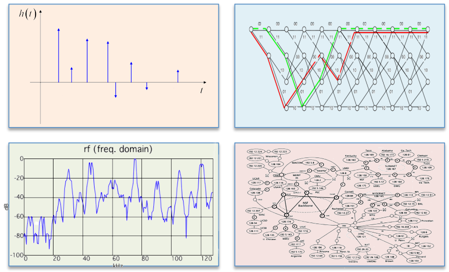


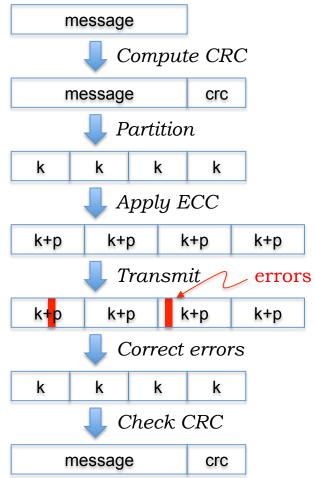
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2009 Lecture #8

- using SEC/CRC in digital transmissions
- impulse noise, burst errors, interleaving
- Reed-Solomon coding

Digital Transmission using ECC



- Start with original message
- Add CRC to enable verification of error-free transmission
- Apply ECC, adding parity bits to each k-bit block of the message. Our ECCs were designed for *single-bit error correction*. Number of parity bits (p) depends on code:
  - Replication: p grows as O(k)
  - Rectangular: p grows as O(√k)
  - Hamming: p grows as O(log k)
- After xmit, correct errors
- Verify CRC, fails if undetected/ uncorrectable error
- Deliver or discard message

Is Single-bit Error Correction Enough?

p(2 or more errors) = 1 – p(no errors) – p(exactly one error)

$$= 1 - (1 - \text{BER})^k - k*\text{BER}*(1-\text{BER})^{k-1}$$

p(≥2 errors), k \ BER	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
8	2.8e-05	2.8e-07	2.8e-09	2.8e-11	2.8e-13
32	4.9e-04	5.0e-06	5.0e-08	5.0e-10	5.0e-12
256	2.8e-02	3.2e-04	3.3e-06	3.3e-08	3.3e-10
1024	2.7e-01	4.9e-03	5.2e-05	5.2e-07	5.2e-09
8192	1.0e+00	2.0e-01	3.2e-03	3.3e-05	3.4e-07

Conclusion: Yes, SEC is okay if BER isn't too big and we keep k small. Some errors still get through but are caught by CRC check; deal with discarded messages at higher level of protocol.

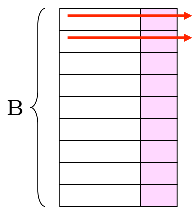
Noise models

- Gaussian noise
  - Equal chance of noise at each sample
  - Gaussian PDF: low probability of large amplitude
  - Good for modeling total effect of many small, random noise sources
- Impulse noise
  - Infrequent bursts of high-amplitude noise, e.g., on a wireless channel
  - Some number of consecutive bits lost, bounded by some burst length B
  - Single-bit error correction seems like it's useless for dealing with impulse noise…
    or is it???

Correcting single-bit errors is nice, but in many situations errors come in bursts many bits long (e.g., damage to storage media, burst of interference on wireless channel, ...). How does single-bit error correction help with that?

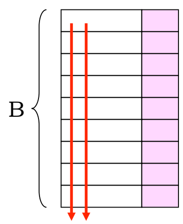
Dealing with Burst Errors

Well, can we think of a way to turn a B-bit error burst into B single-bit errors?



Row-by-row transmission order

(parity bits are shown shaded)

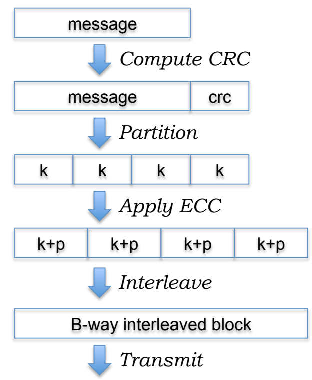


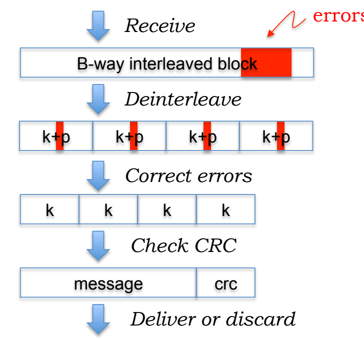
Col-by-col transmission order

Problem: Bits from a particular codeword are transmitted sequentially, so a B-bit burst produces multi-bit errors.

Solution: interleave bits from B different codewords. Now a B-bit burst produces 1-bit errors in B different codewords.

Interleaving





Framing

- The receiver needs to know
  - the beginning of the B-way interleaved block in order to do deinterleaving
  - the beginning of each ECC block in order to do error correction.
  - Since the interleaved block is made up of B ECC blocks, knowing where the interleaved block begins automatically supplies the necessary start info for the ECC blocks
- Framing is accomplished by having the transmitter insert sync sequences to mark beginnings…
  - Data and parity bits must not have patterns that can be confused with the sync pattern
  - Syncs are themselves subject to error
  - Some channels have natural boundary indicators, e.g., the beginning of transmission.

Sync techniques

- Recode bit stream to ensure sync uniqueness
- 8b/10b recoding provides for several unique patterns useful for sync and other out-of-band information
  - Used on wired channels where BER is small, which means sync is seldom corrupted during transmission
- Choose sync pattern that has, say, 5 1's in a row
  - To prevent sync from appearing in message, "bit-stuff" 0's after any sequence of four 1's in the message.
  - This step is easily reversed at receiver (just remove 0 after any sequence of four consecutive 1's in the message).
  - Creates variable-length blocks, a slight pain
  - Less overhead than 8b/10b if you don't need 8b/10b's other benefits

Remaining agenda items

- With M ECC blocks per message, we can correct somewhere between 1 and M errors depending on where in the message they occur.
  - Can we make an ECC that corrects up to E errors without any constraints where errors occur?
  - Yes! Reed-Solomon codes, discussed next
- Framing is necessary, but the sync itself can't be protected by an ECC scheme that requires framing.
  - This makes life hard for channels with higher BERs
  - Is there an error correction scheme that works on un-framed bit streams?
  - Yes! Convolutional codes: encoding and the clever decoding scheme will be discussed in the next lectures.

In search of a better code

- Problem: information about a particular message unit (bit, byte, ..) is captured in just a few locations, i.e., the message unit and some number of parity units. So a small but unfortunate set of errors might wipe out all the locations where that info resides, causing us to lose the original message unit.

- Potential Solution: figure out a way to spread the info in each message unit throughout *all* the code words in a block. Require only some fraction good code words to recover the original message.

Thought experiment…

- Suppose you had two 8-bit values to communicate: A, B
- We'd like an encoding scheme where each transmitted value included information about both A and B
  - How about sending y = Ax + B for various values of x?
  - Standardize on a particular sequence for x, known to both the transmitter and receiver. That way, we don't have to actually send the x's – the receiver will know what they are. For example, x = 1, 2, 3, 4, …
  - How many values do you need to solve for A and B?
  - We'll send extra to provide for recovery from errors…

Example

- Suppose you received four values from the transmitter y = 73, 249, 321, 393, corresponding to x = 1, 2, 3 and 4
  - 4 Eqns: A·1+B=73, A·2+B=249, A·3+B=321, A·4+B=393
- We need two of these equations to solve for A and B; there are six possible choices for which two to use
- Take each pair and solve for A and B

$A \cdot 1 + B = 73$ $A \cdot 2 + B = 249$ *A=175, B=-102*	$A \cdot 1 + B = 73$ $A \cdot 3 + B = 321$ *A=124, B=-51*	$A \cdot 1 + B = 73$ $A \cdot 4 + B = 393$ *A=106.6, B=-33.6*
$A \cdot 2 + B = 249$ $A \cdot 3 + B = 321$ *A=72, B=105*	$A \cdot 2 + B = 249$ $A \cdot 4 + B = 393$ *A=72, B=105*	$A \cdot 3 + B = 321$ $A \cdot 4 + B = 393$ *A=72, B=105*

- Majority rules: A=72, B=105
  - The received value 73 had an error
  - If no errors: all six solutions for A and B would have matched

Spreading the wealth…

- Generalize this idea: oversampled polynomials. Let

  $P(x) = m_0 + m_1x + m_2x^2 + \ldots + m_{k-1}x^{k-1}$

  where $m_0, m_1, \ldots, m_{k-1}$ are the *k* message units to be encoded. Transmit value of polynomial at *n* different predetermined points $v_0, v_1, \ldots, v_{n-1}$:

  $P(v_0), P(v_1), P(v_2), \ldots, P(v_{n-1})$

  Use any *k* of the received values to construct a linear system of *k* equations which can then be solved for *k* unknowns $m_0, m_1, \ldots, m_{k-1}$. Each transmitted value contains info about all m_i.
- Note that using integer arithmetic, the P(v) values are numerically greater than the m_i and so require more bits to represent than the m_i. In general the encoded message would require a lot more bits to send than the original message!

Solving for the m_i

- Solving k *linearly independent* equations for the k unknowns (i.e., the m_i):

$$\begin{pmatrix} 1 & v_0 & v_0^2 & \cdots & v_0^{k-1} \\ 1 & v_1 & v_1^2 & \cdots & v_1^{k-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & v_{k-1} & v_{k-1}^2 & \cdots & v_{k-1}^{k-1} \end{pmatrix} \begin{pmatrix} m_0 \\ m_1 \\ \vdots \\ m_{k-1} \end{pmatrix} = \begin{pmatrix} P(v_0) \\ P(v_1) \\ \vdots \\ P(v_{k-1}) \end{pmatrix}$$

- Solving a set of linear equations using Gaussian Elimination (multiplying rows, switching rows, adding multiples of rows to other rows) requires add, subtract, multiply and divide operations.

- These operations (in particular division) are only well defined over *fields,* e.g., rational numbers, real numbers, complex numbers -- not at all convenient to implement in hardware.

Finite Fields to the Rescue

- Reed's & Solomon's idea: do all the arithmetic using a finite field (also called a Galois field). If the m_i have B bits, then use a finite field with order 2^B so that there will be a field element corresponding to each possible value for m_i.

- For example with B = 2, here are the tables for the various arithmetic operations for a finite field with 4 elements. Note that every operation yields an element in the field, i.e., the result is the same size as the operands.

+	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

*	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	3	1
3	0	3	1	2

A	-A	A^{-1}
0	0	0
1	1	1
2	2	3
3	3	2

$A + (-A) = 0$ $A * (A^{-1}) = 1$

How many values to send?

- Note that in a Galois field of order 2^B there are at most 2^B unique values v we can use to generate the P(v)
  - if we send more than 2^B values, some of the equations we might use when solving for the m_i may not be linearly independent and we won't have enough information to find a unique solution for the m_i.
  - Sending P(0) isn't very interesting (only involves m_0)

- Reed-Solomon codes use $n = 2^B-1$ (n is the number of P(v) values we generate and send).
  - For many applications B = 8, so n = 255
  - A popular R-S code is (255,223), i.e., a code block consisting of 223 8-bit data bytes + 32 check bytes

Use for error correction

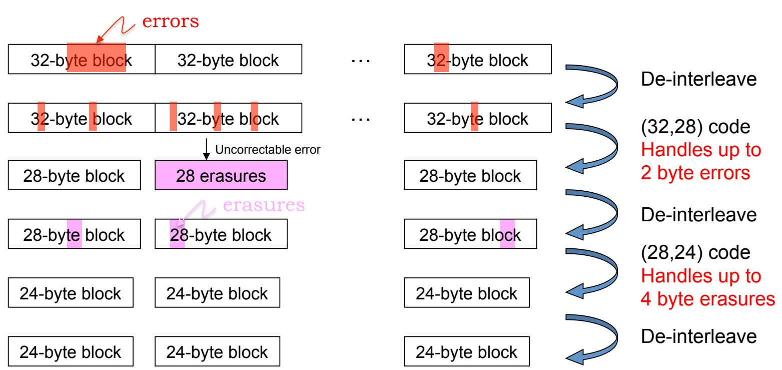
- If one of the $P(v_i)$ is received incorrectly, if it's used to solve for the m_i, we'll get the wrong result.
- So try all possible (n choose k) subsets of values and use each subset to solve for m_i. Choose solution set that gets the majority of votes.
  - No winner? Uncorrectable error… throw away block.
- (n,k) code can correct up to (n-k)/2 errors since we need enough good values to ensure that the correct solution set gets a majority of the votes.
  - R-S (255,223) code can correct up to 16 symbol errors; good for error bursts: 16 consecutive symbols = 128 bits!

Erasures are special

- If a particular received value is known to be erroneous (an "erasure"), don't use it all!
  - How to tell when received value is erroneous? Sometimes there's channel information, e.g., carrier disappears.
  - See next slide for clever idea based on concatenated R-S codes
- (n,k) R-S code can correct n-k erasures since we only need k equations to solve for the k unknowns.
- Any combination of E errors and S erasures can be corrected so long as 2E + S ≤ n-k.

Example: CD error correction

- On a CD: two concatenated R-S codes



Result: correct up to 3500-bit error bursts (2.4mm on CD surface)