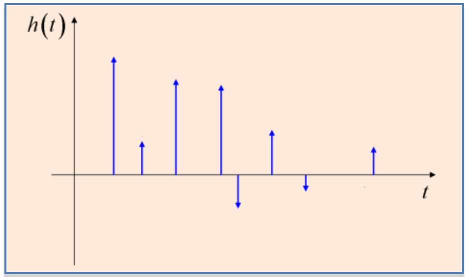
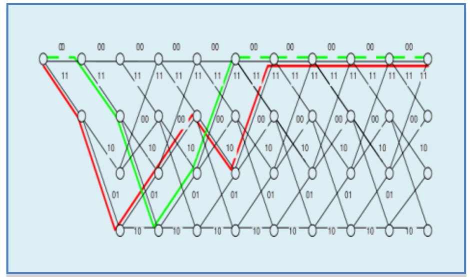
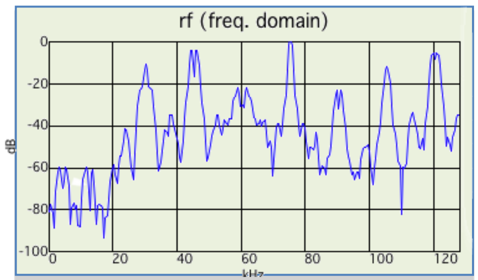
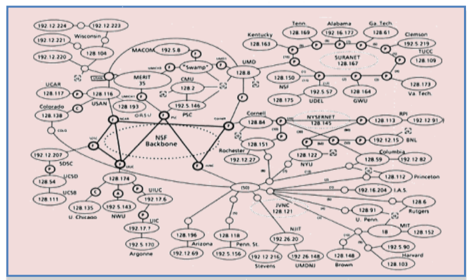








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

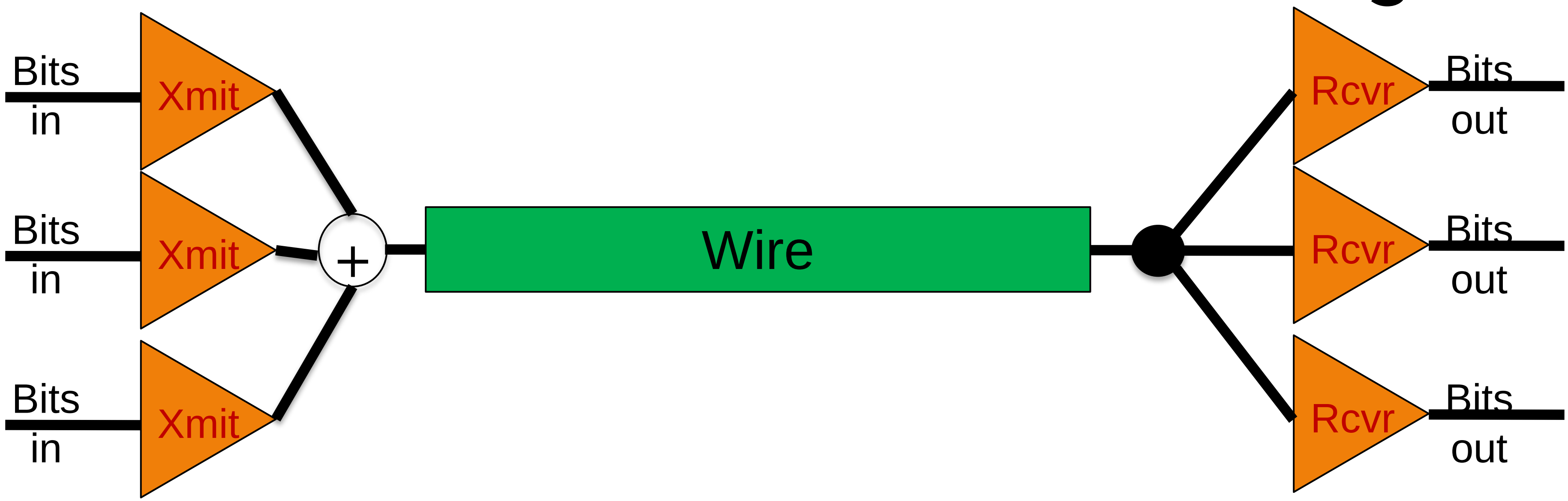
6.02 Fall 2010
Lecture #12

- Summary of progress
- The problem of sharing in frequency domain
- Sines → LTI → Sines by experiment
- Discrete-Time Complex Exponentials
- Frequency Response

First Problem - Low BER on Wires

- Problems and Analysis Techniques:
  - Intersymbol Interference (ISI)
    - LTI Systems and Unit Sample Responses
    - Eye Diagrams
  - Noise
    - Probability Density and Cumulative Distribution Functions
    - Normal (Gaussian) random variables
- Solution Approaches
  - Techniques based on LTI models
    - Deconvolution
  - Error Detection and Correction Codes
    - Parity bits, Convolutional Code, Viterbi Algorithm

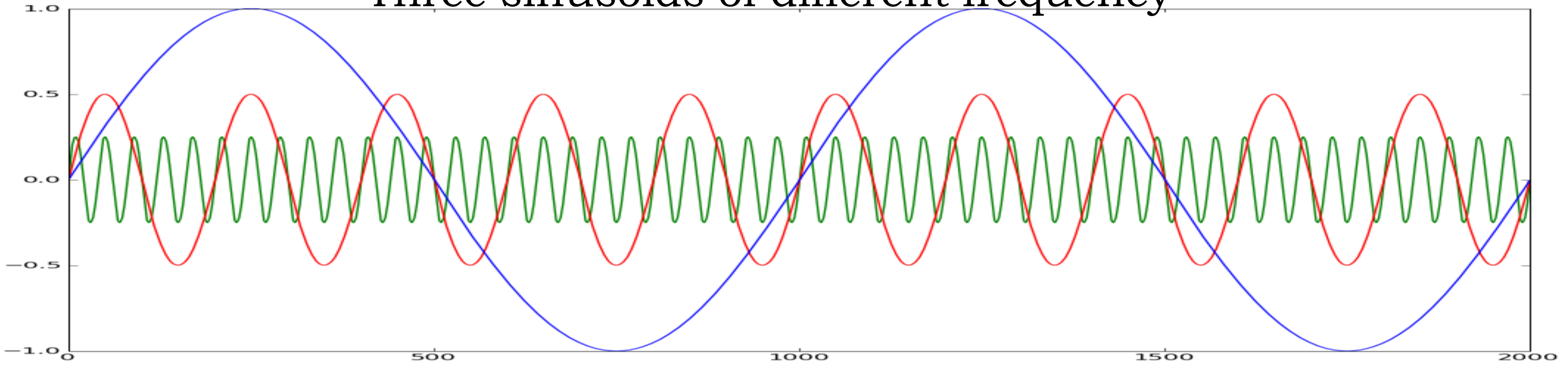
Second Problem – Resource Sharing



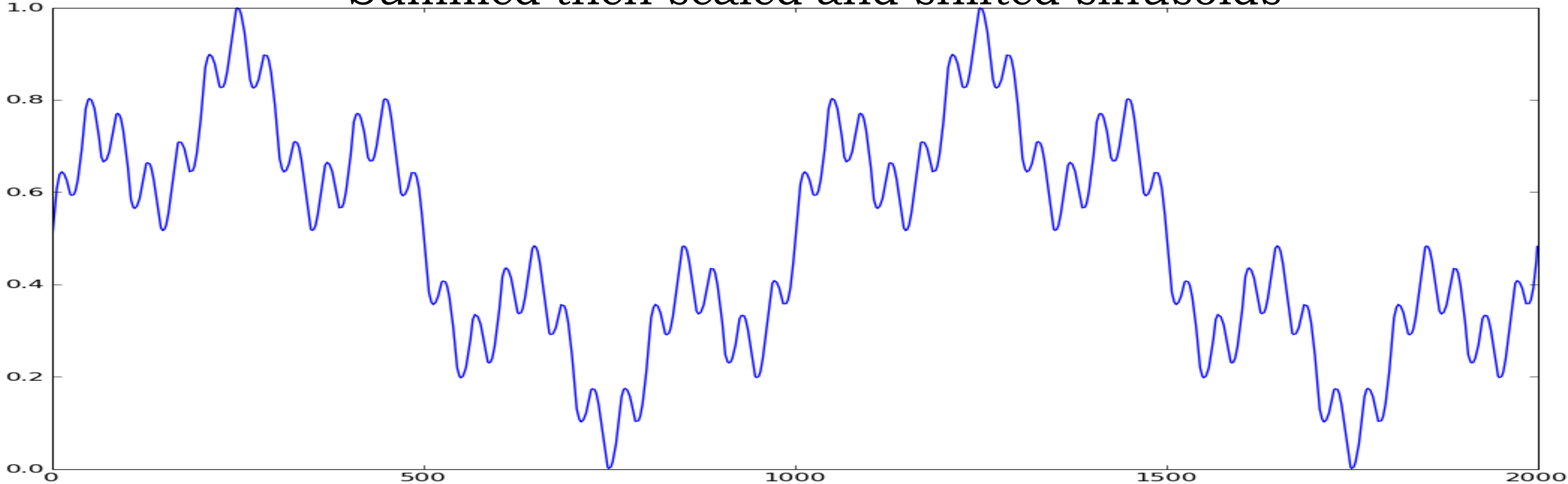
- Two Approaches
  - Time Sharing (e.g. TDMA)
    - Xmit-Rcvr pair gets a dedicated time slot or contend for slots
    - Used in both wired and wireless networks
  - Frequency Sharing (aka Frequency Division Multiplexing)
    - Each Xmit-Rcvr gets part of the spectrum (explained later)
    - Used in Broadcast TV, cell phones and WiFi.
  - Most WiFi systems use TDMA and FDM
- Next Two Weeks on issues in FDM

Sinusoids and LTI Systems

Three sinusoids of different frequency

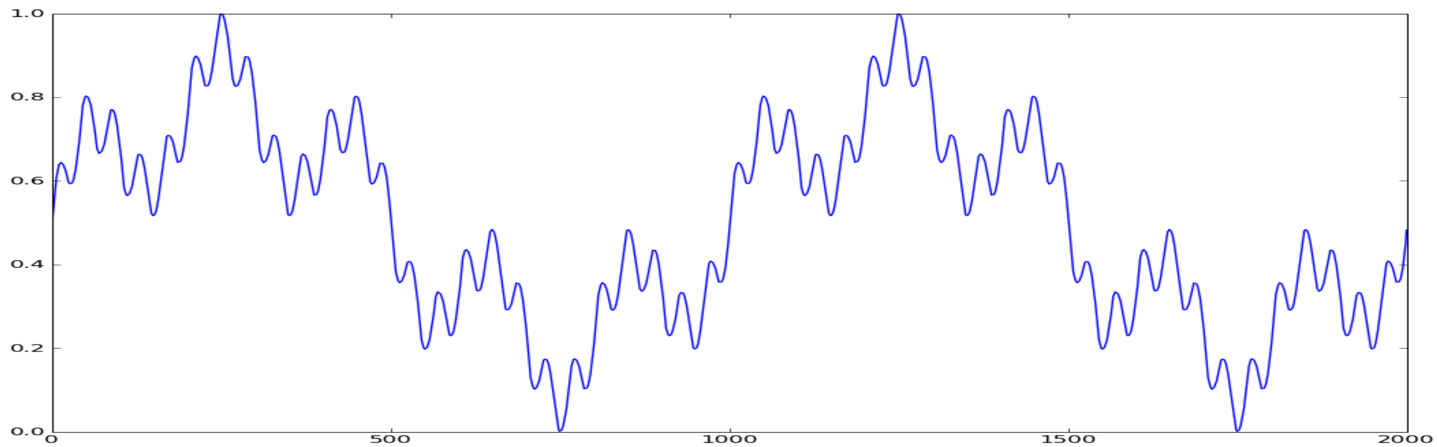


Summed then scaled and shifted sinusoids

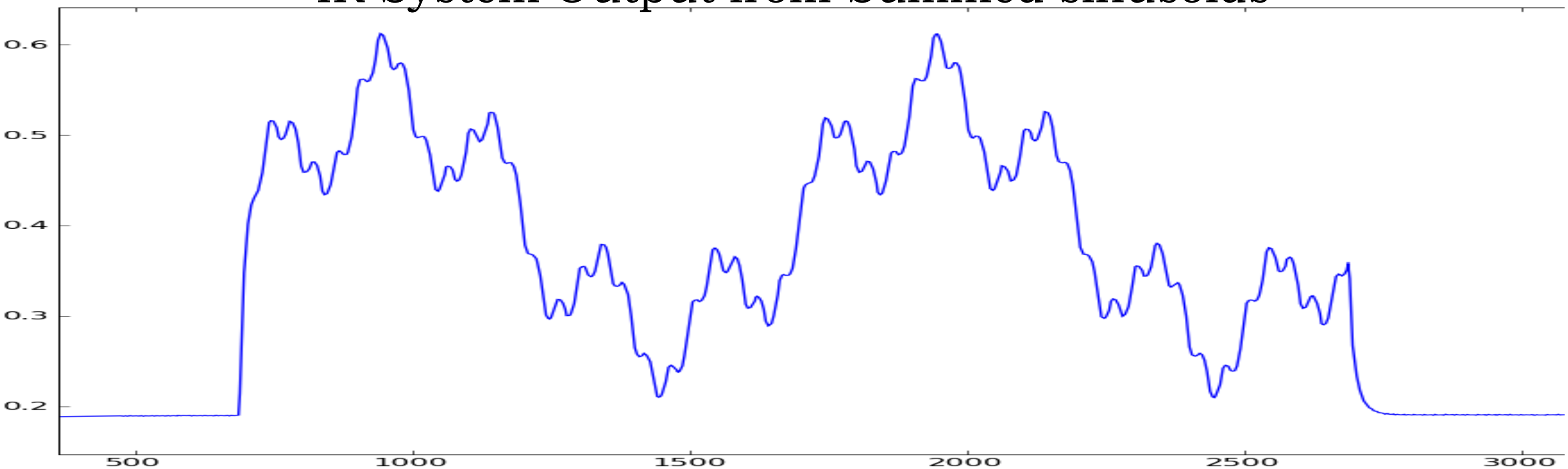


Sinusoids and LTI Systems

Summed then scaled and shifted sinusoids

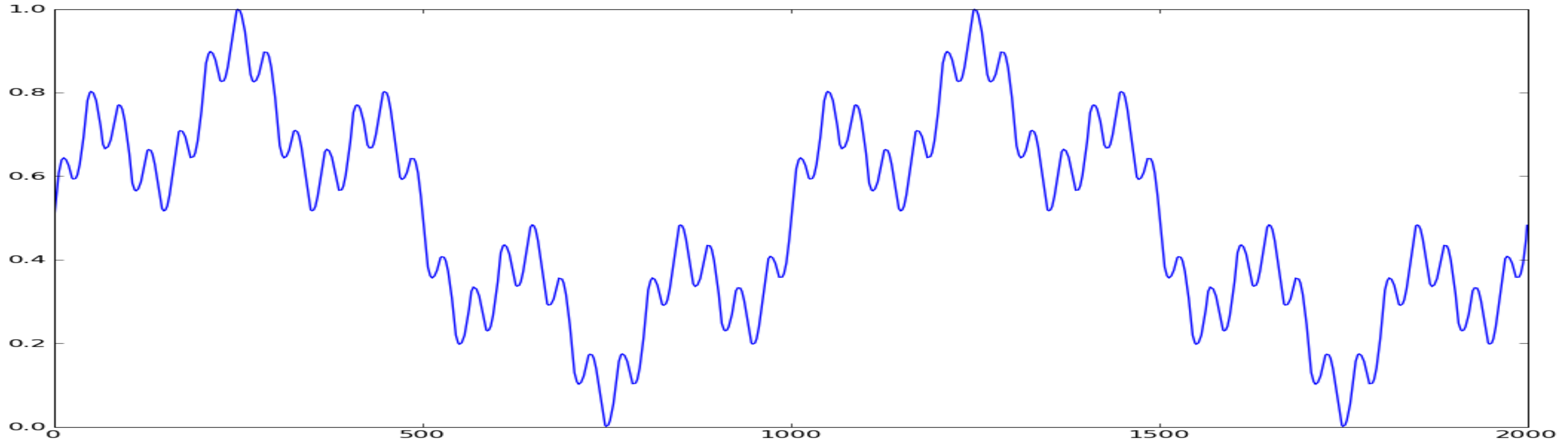


IR System Output from Summed sinusoids

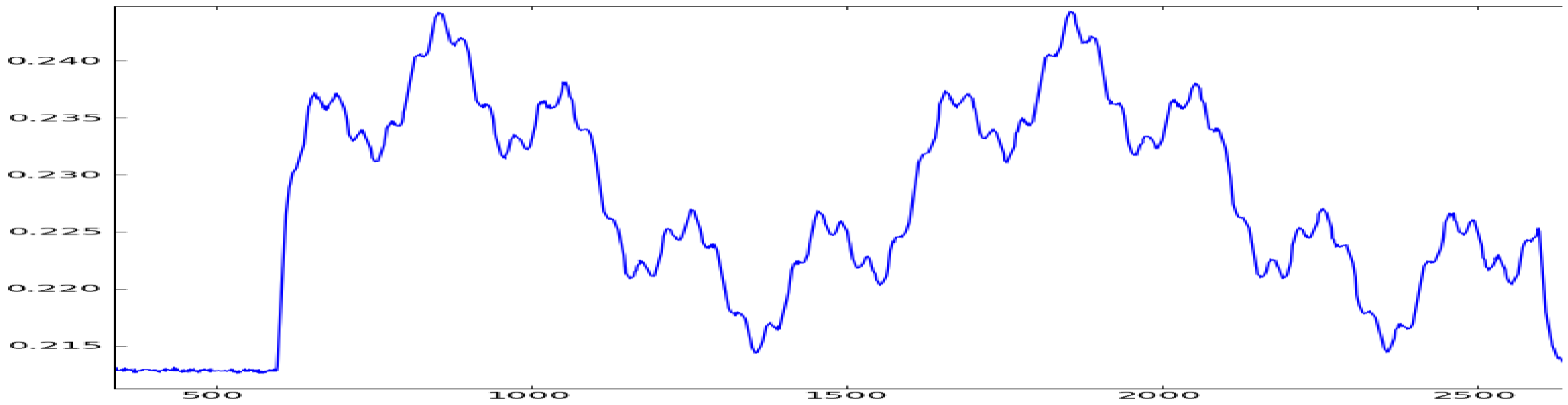


Sinusoids and LTI Systems

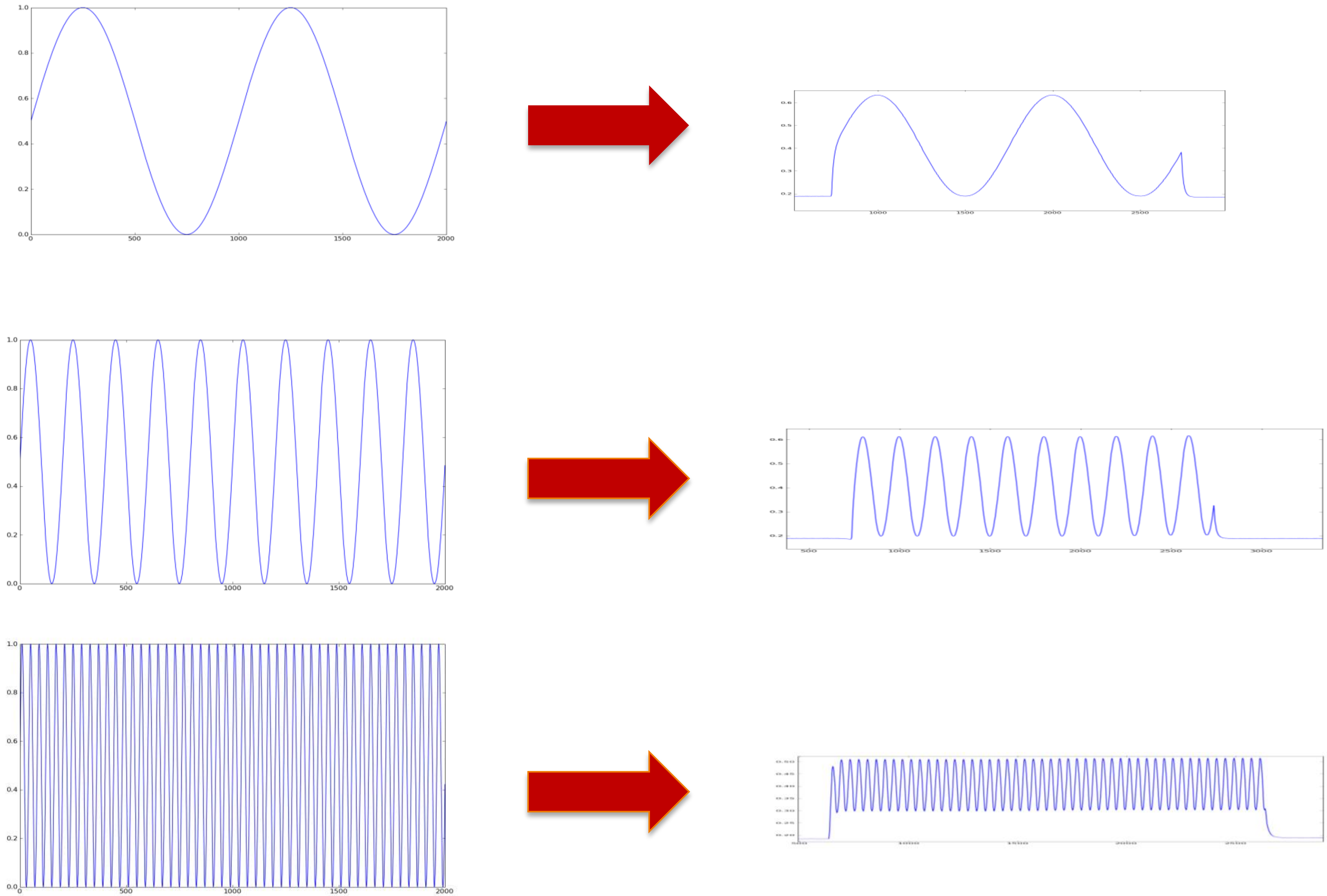
Summed then scaled and shifted sinusoids



IR System (off ceiling) Output from Summed sinusoids



IR System Response to Different Sines

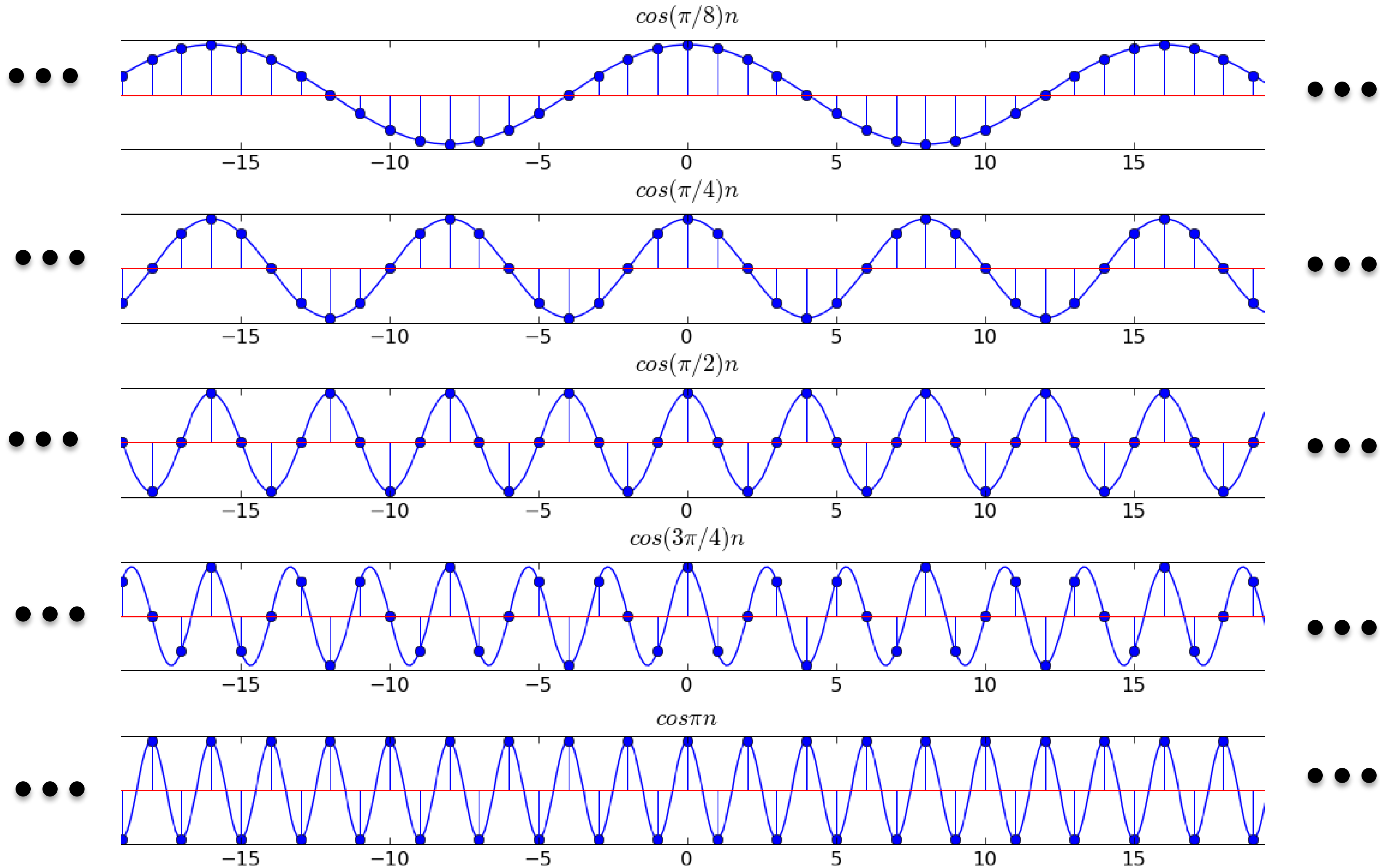


Sinusoids in LTI Systems

- Sinusoids → LTI Systems → Sinusoids
  - Different Frequencies do not seem to mix
    - Xmit: encode data on different frequencies (Modulation)
    - Rcvr: decode data from different frequences (Filtering)
  - Different Frequencies have different "Gain"
    - E.G. the IR Channel:
      - higher frequencies → lower "gain".
- Analyzing Sinusoids in LTI systems
  - Model as Eternal signals (-infinity < n < infinity)
  - Difficulty Computing response to $x[n] = \cos \Omega n$
  - Simplify response computation using complex exponentials

$$\cos \Omega n = \frac{e^{j\Omega n} + e^{-j\Omega n}}{2} \qquad \sin \Omega n = \frac{e^{j\Omega n} - e^{-j\Omega n}}{2j}$$

Discrete Time Cosines $-\infty < n < \infty$



Why is pi highest frequency?

LTI Systems for Eternal Sines and Cosines

- Convolution Changes a little

x[n] is eternal

$$y[n] = \sum_{m=0}^{\infty} h[m]x[n-m] = \sum_{m=0}^{\infty} h[m]\cos\Omega[n-m]$$

h[m] is still causal

- If unit sample response finite length (L)

$$y[n] = \sum_{m=0}^{L} h[m]\cos\Omega[n-m]$$

How do we simplify?

Frequency Response with Complex Exponentials

- From convolution

$$y[n] = \sum_{m=0}^{L} h[m] e^{j\Omega(n-m)}$$

Reorganizing

$$y[n] = \underbrace{\left(\sum_{m=0}^{L} h[m] e^{-j\Omega m}\right)}_{\text{A } \textit{complex} \text{ number if the sum converges}} e^{j\Omega n}$$

$$y[n] = H\left(e^{j\Omega}\right) e^{j\Omega n}$$

$\boxed{H(e^{j\Omega}),\ -\pi < \Omega < \pi, \text{ is the frequency response}}$

Going back to Cosines

- Suppose

$$x[n] = \cos \Omega n = \frac{1}{2} e^{j\Omega n} + \frac{1}{2} e^{-j\Omega n}$$

$$\Downarrow$$

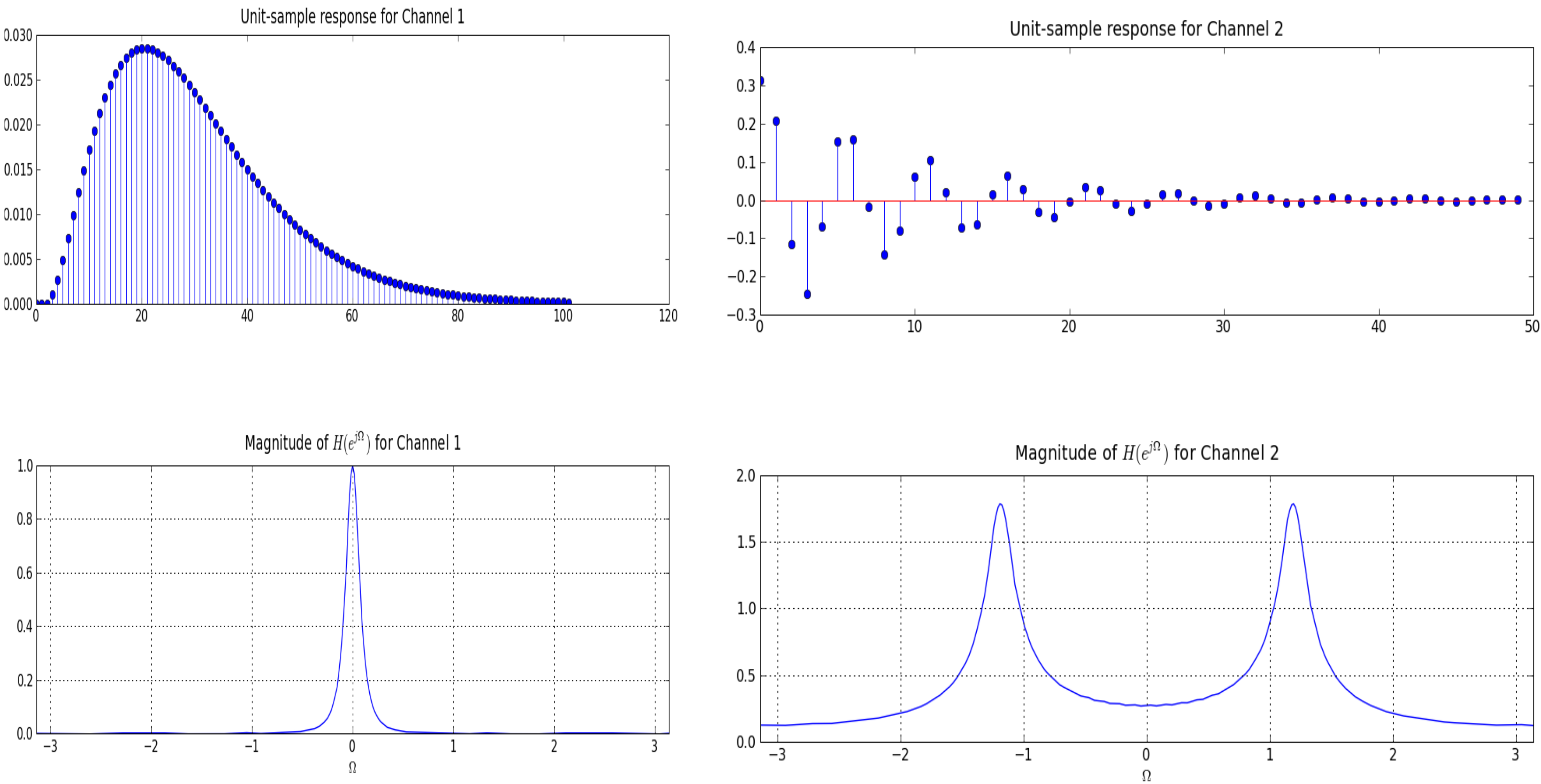
$$y[n] = \left(\sum_{m=0}^{L} h[m] e^{-j\Omega m} \right) \frac{e^{j\Omega n}}{2} + \left(\sum_{m=0}^{L} h[m] e^{j\Omega m} \right) \frac{e^{-j\Omega n}}{2}$$

$$\Downarrow$$

$$y[n] = \frac{1}{2} H\left(e^{j\Omega}\right) e^{j\Omega n} + \frac{1}{2} H\left(e^{-j\Omega}\right) e^{-j\Omega n}$$

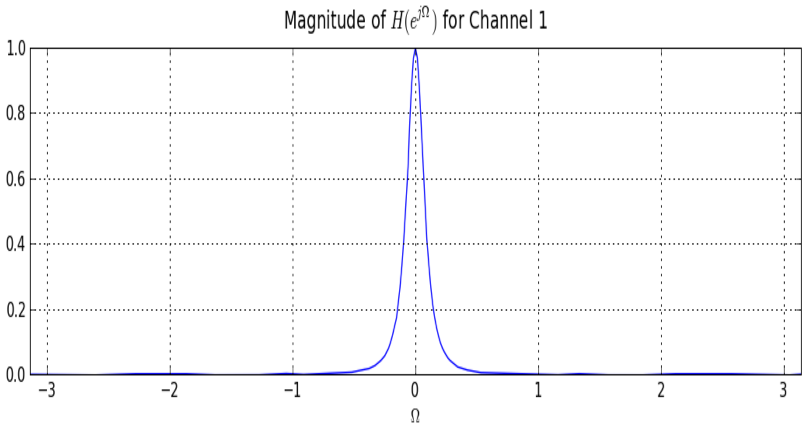
- Just using LTI superposition property!

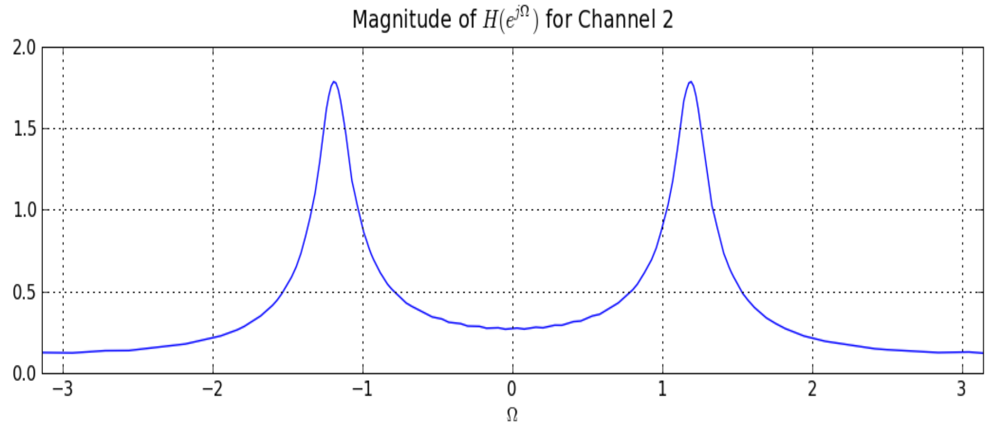
Channel 1 and 2 Frequency Responses



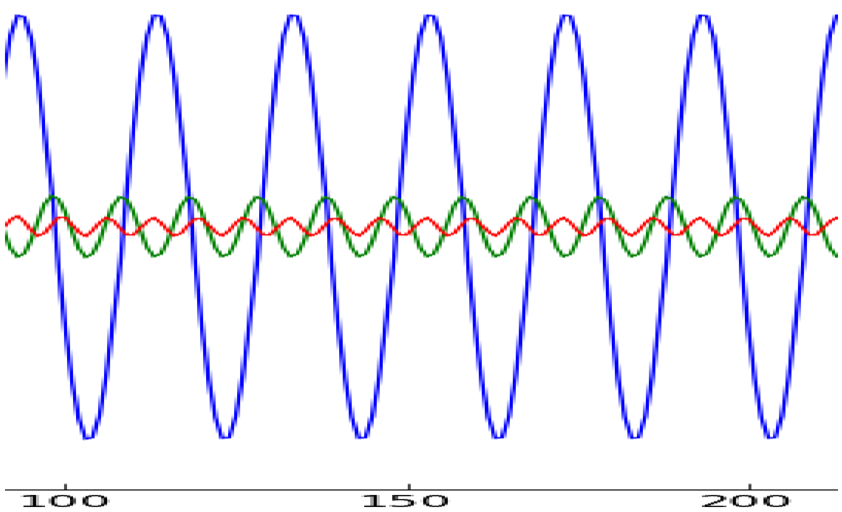
Channel 1 and 2 Frequency Responses

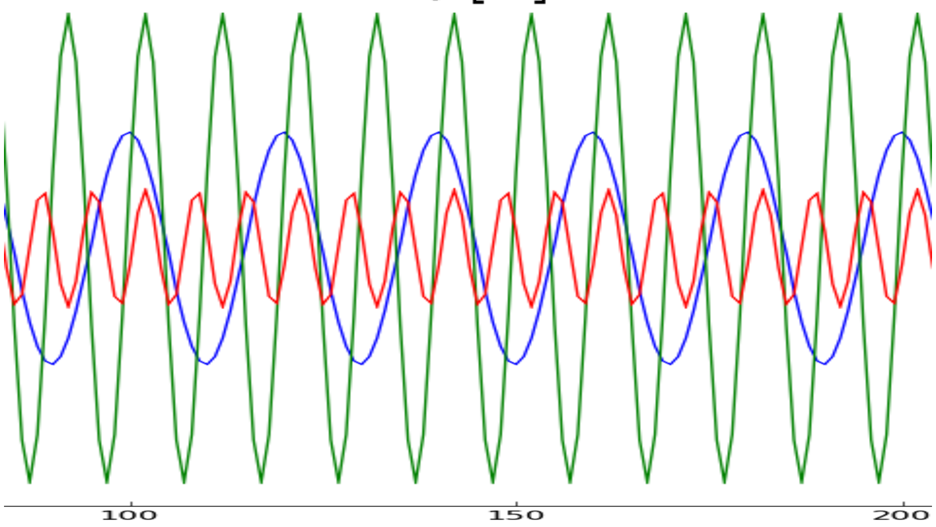
$$x[n] = \cos(\Omega n) \qquad \Omega = \left\{ \frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10} \right\}$$





$$y[n] \qquad y[n]$$

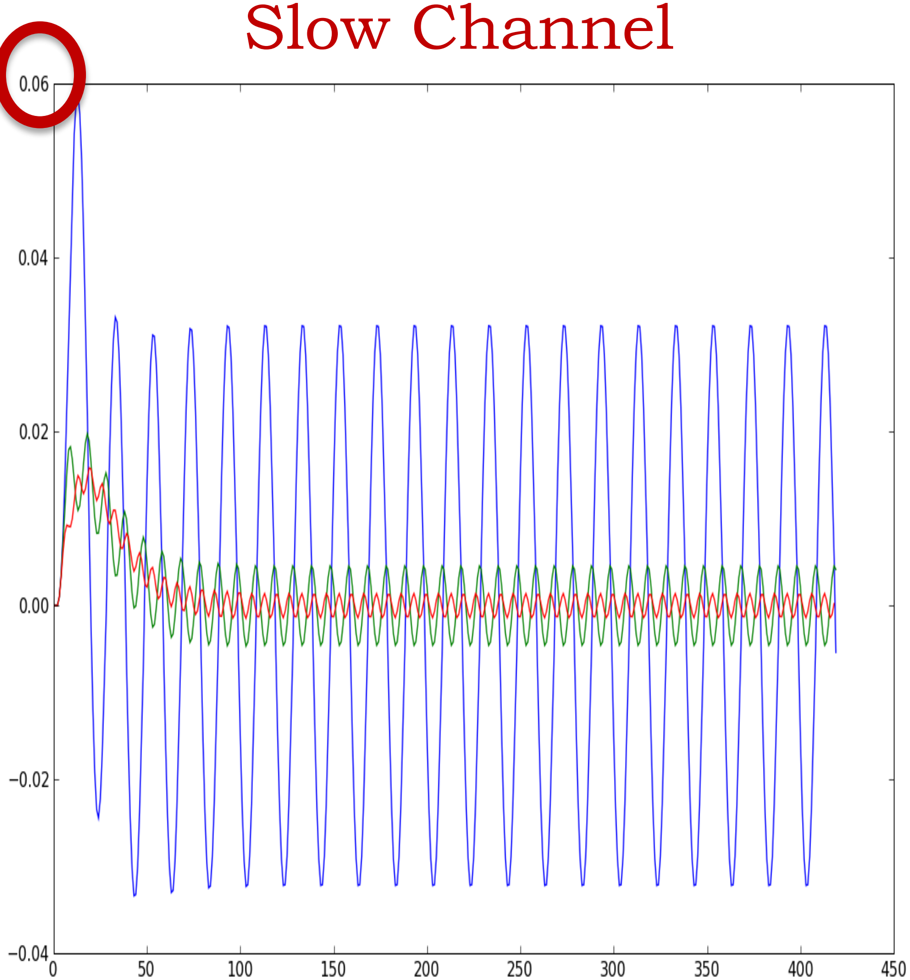




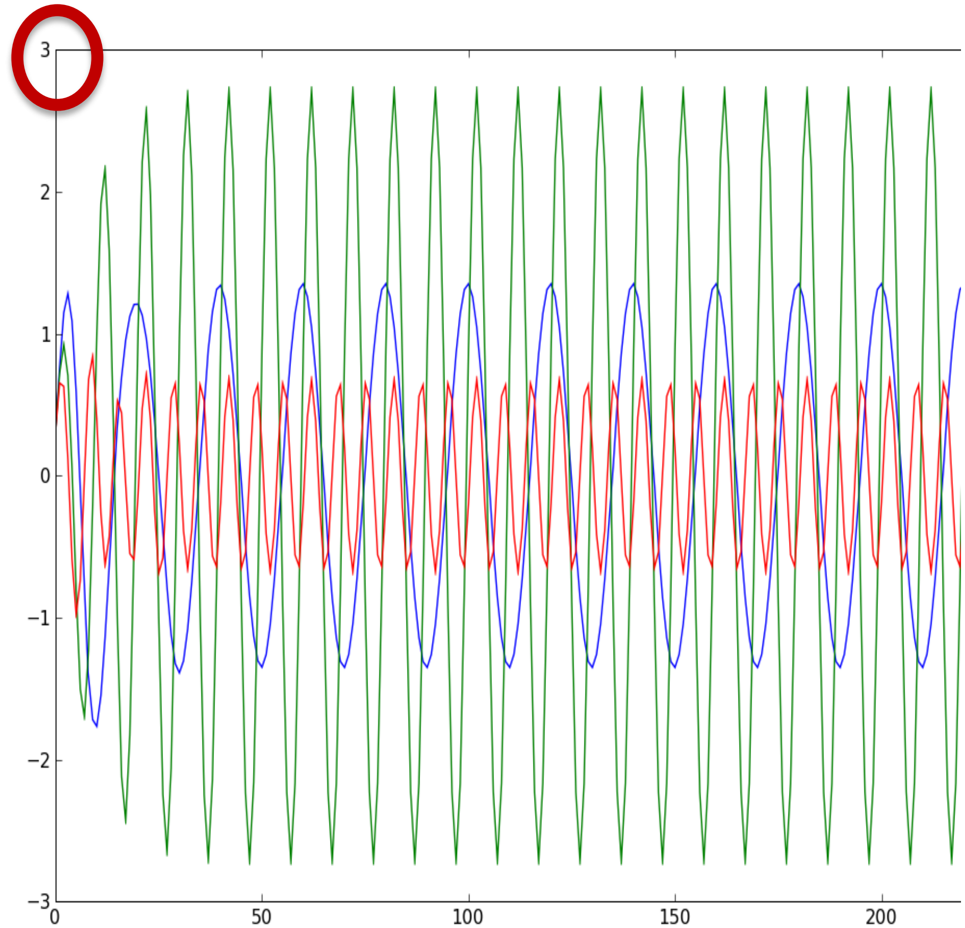
Non-eternal Example: Cosine starts at zero

$$x[n] = \cos(\Omega n)u[n] \;\; \Omega = \{\frac{\pi}{10}, \frac{2\pi}{10}, \frac{3\pi}{10}\}$$

Slow Channel



Fast Channel



Summary and Next Time

- Frequency Division Multiplexing
  - Eternal frequencies do not mix when input to LTI systems
  - Frequency division multiplexing
    $$x[n] = A_1 e^{j\Omega_1 n} + \ldots + A_K e^{j\Omega_K n}$$
  - Frequency Response
    $$H\left(e^{j\Omega}\right) \quad \Omega \in [-\pi, \pi]$$
- Next Two Weeks
  - Need Filters at receiver to separate messages.
  - How do we analyze more complicated messages
    $$x[n] = x_1[n] e^{j\Omega_1 n} + \ldots + x_K[n] e^{j\Omega_K n}$$
    - Do the modulated messages still stay separated?
  - Fourier Analysis and Spectrum
  - Modulation Schemes