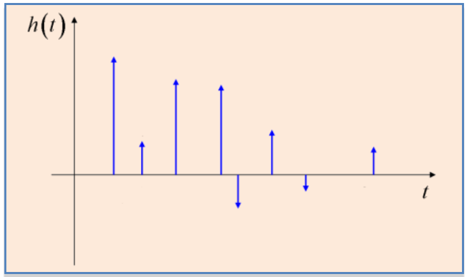
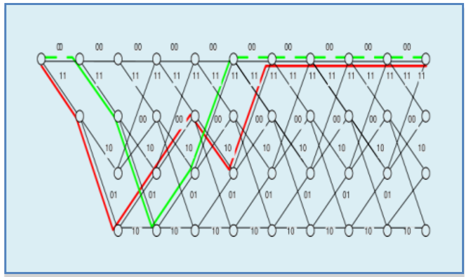
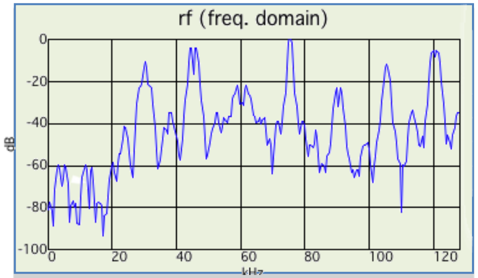
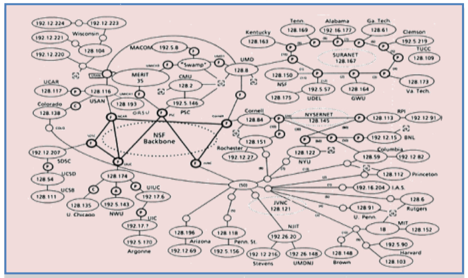








INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2010
Lecture #13

- Frequency Response and Filters.
- Equations for Filter Design.
- Effects of low-pass, high-pass and band-pass filters
- Series and Parallel Composition.

Basic Tools: DT Sinusoids and Complex Exponentials

- Continuous Time to Discrete Time Sinusoids

$$x(t) = \cos 2\pi f t$$

Sample Period

$$x(nT_s) = \cos 2\pi f T_s n$$

Sample Rate

$$x(n\frac{1}{f_s}) = \cos 2\pi \frac{f}{f_s} n$$

$$x[n] = \cos 2\pi \frac{f}{f_s} n = \cos \Omega n$$

- Complex Exponentials

$$cos\Omega n = \frac{1}{2}e^{j\Omega n} + \frac{1}{2}e^{-j\Omega n} \quad sin\Omega n = \frac{1}{2j}e^{j\Omega n} - \frac{1}{2j}e^{-j\Omega n}$$

Basic Tools: Frequency Response

- Frequency Response

$$x[n] = e^{j\Omega n} \quad y[n] = \sum_{m=0}^{L} h[m]x[n-m]$$

$$y[n] = H\left(e^{j\Omega}\right) e^{j\Omega n} \quad H\left(e^{j\Omega}\right) = \sum_{m=0}^{L} h[m]e^{-j\Omega m}$$

Complex Number

$$\boxed{H(e^{j\Omega}),\ -\pi < \Omega < \pi, \text{ is the frequency response}}$$

$$cos\Omega n = \frac{1}{2}e^{j\Omega n} + \frac{1}{2}e^{-j\Omega n} \quad sin\Omega n = \frac{1}{2j}e^{j\Omega n} - \frac{1}{2j}e^{-j\Omega n}$$

Designing Filter's Unit Sample Response

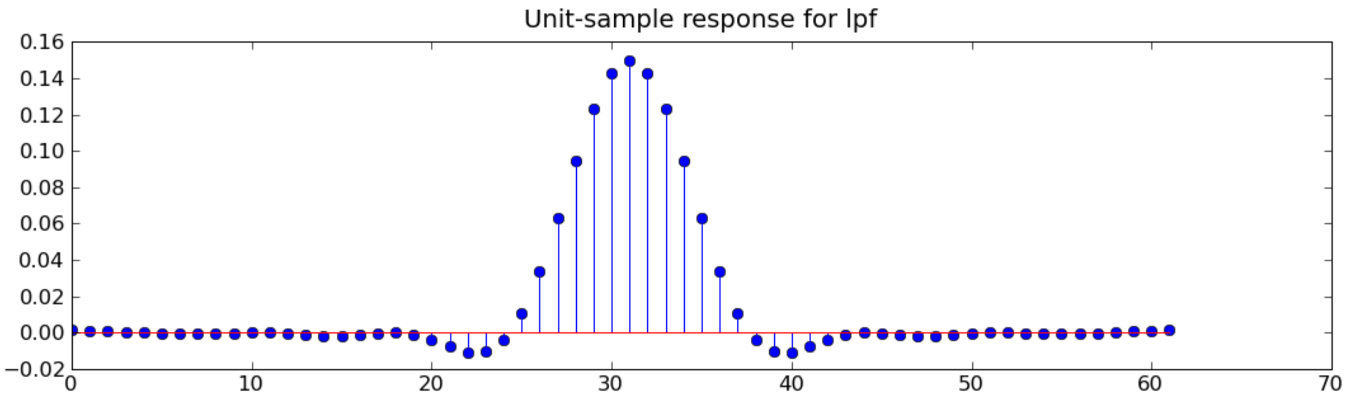
- Design a Unit Sample response with L+1 nonzero values with a given set of frequency responses.

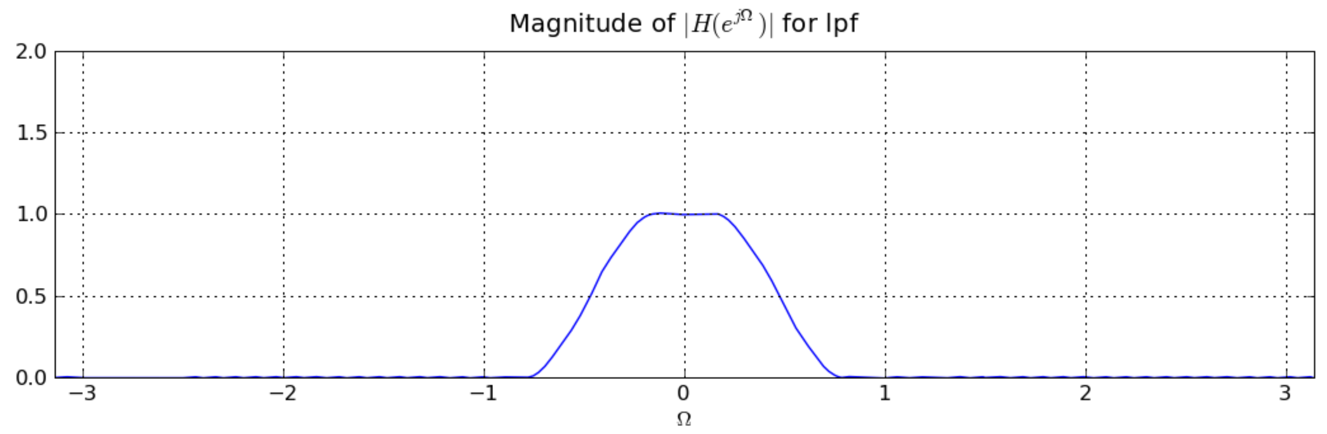
$$\begin{bmatrix} 1 & e^{j\Omega_1} & e^{j\Omega_1 2} & \dots & e^{j\Omega_1(L-1)} & e^{j\Omega_1 L} \\ 1 & e^{-j\Omega_1} & e^{-j\Omega_1 2} & \dots & e^{-j\Omega_1(L-1)} & e^{-j\Omega_1 L} \\ 1 & e^{j\Omega_2} & e^{j\Omega_2 2} & \dots & e^{j\Omega_2(L-1)} & e^{j\Omega_2 L} \\ 1 & e^{-j\Omega_2} & e^{-j\Omega_2 2} & \dots & e^{-j\Omega_2(L-1)} & e^{-j\Omega_2 L} \\ \vdots & \vdots & & \vdots & \vdots & \\ 1 & e^{j\Omega_K} & e^{j\Omega_K 2} & \dots & e^{j\Omega_K(L-1)} & e^{j\Omega_K L} \\ 1 & e^{-j\Omega_K} & e^{-j\Omega_K 2} & \dots & e^{-j\Omega_K(L-1)} & e^{-j\Omega_K L} \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \\ \vdots \\ h[L-1] \\ h[L] \end{bmatrix} = \begin{bmatrix} H\left(e^{-j\Omega_1}\right)) \\ H\left(e^{j\Omega_1}\right)) \\ H\left(e^{-j\Omega_2}\right)) \\ H\left(e^{j\Omega_2}\right)) \\ \vdots \\ H\left(e^{-j\Omega_K}\right)) \\ H\left(e^{j\Omega_K}\right)) \end{bmatrix}$$

- 2K constraints (desired frequency responses)
- L+1 Unknowns (values of h to be designed)

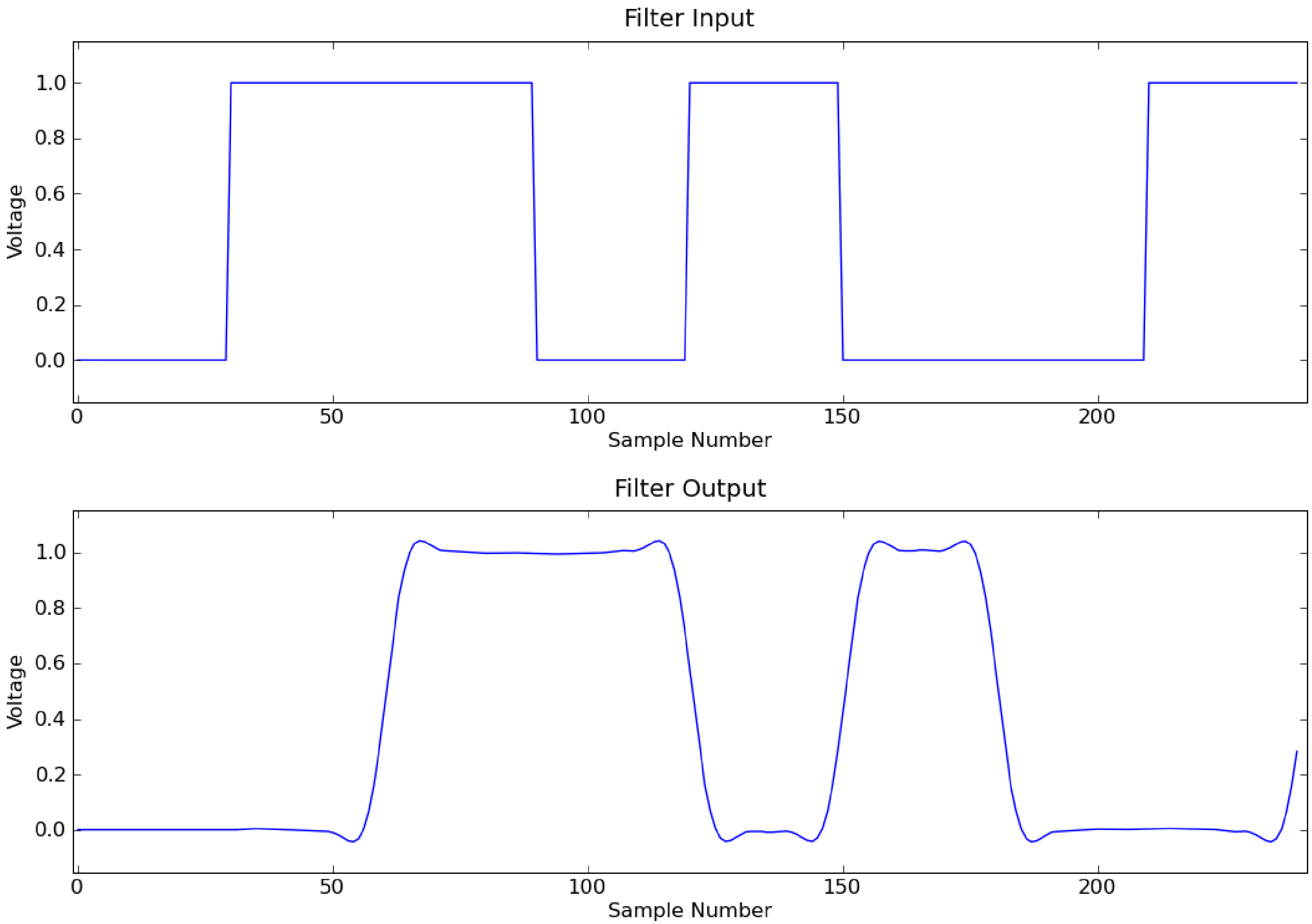
#Eqns = # Unknowns →2K = L+1

Low Pass Filter

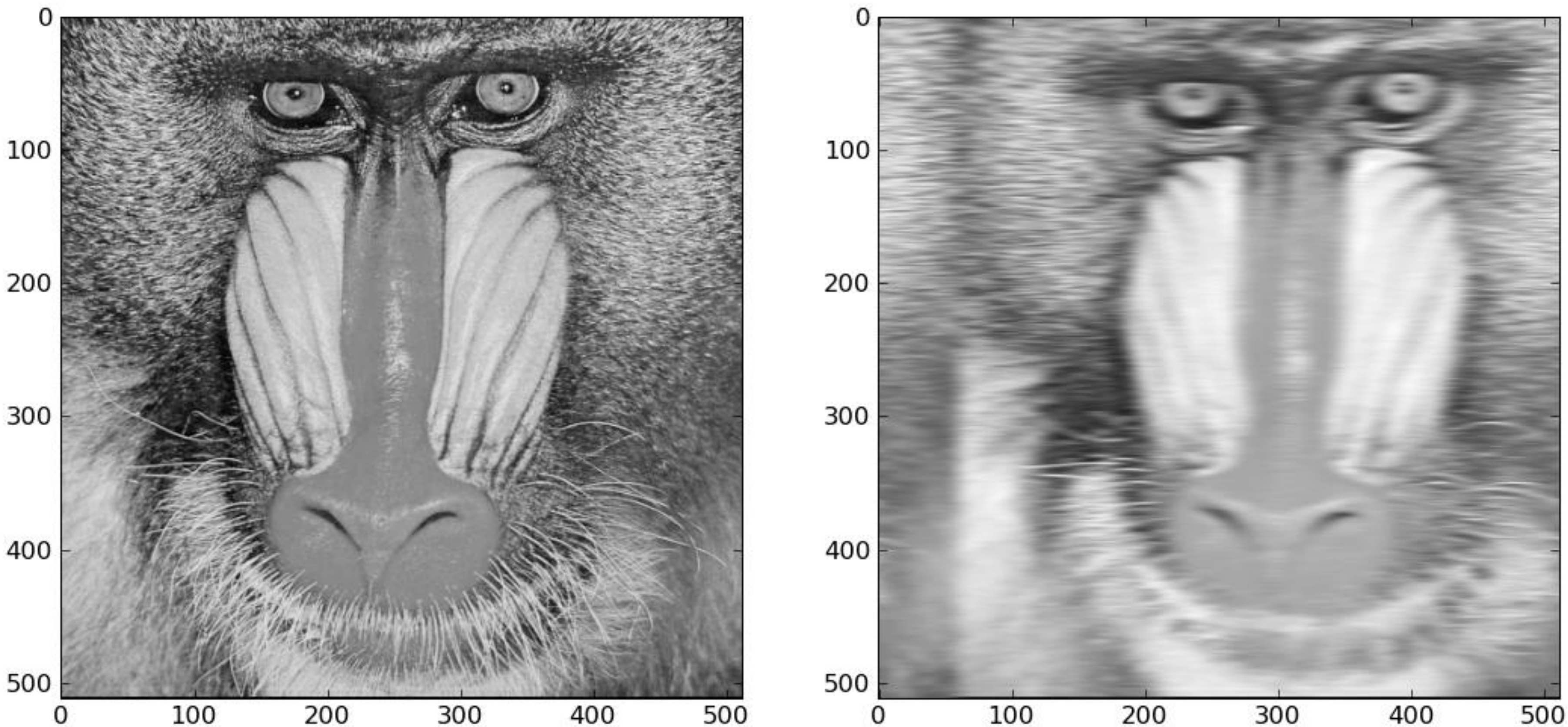




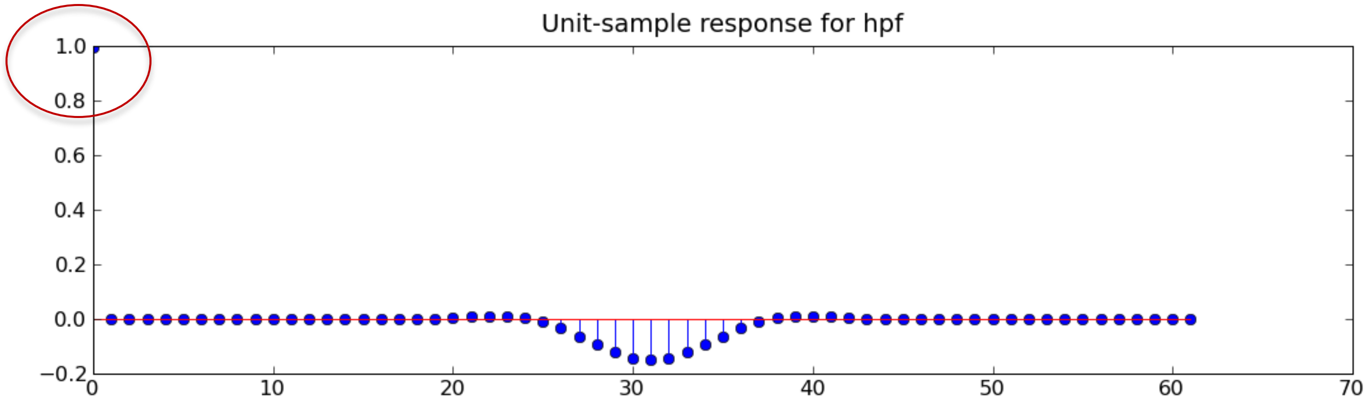
Low Pass Filter Applied to Bits

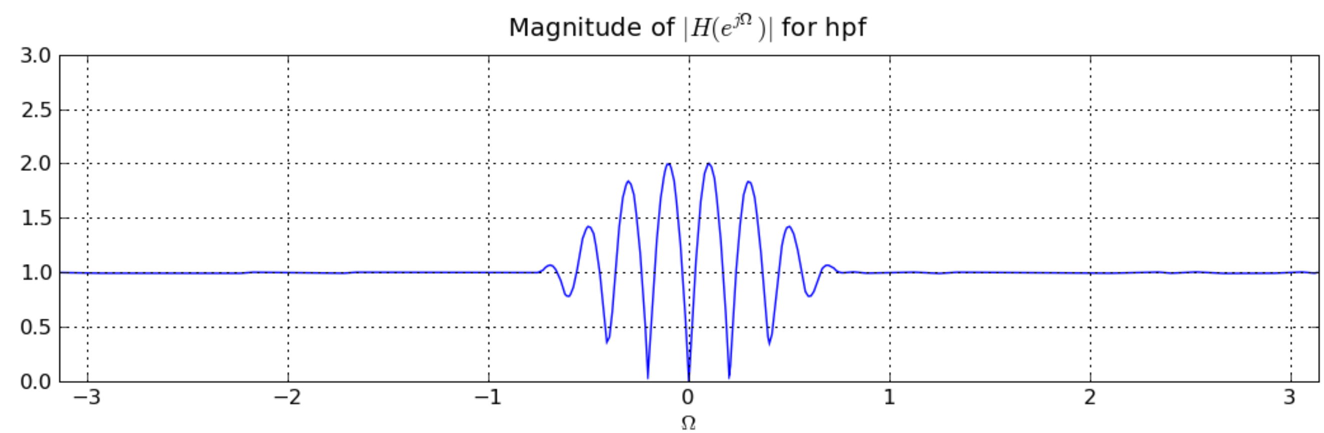


Low Pass Filter Applied to Image

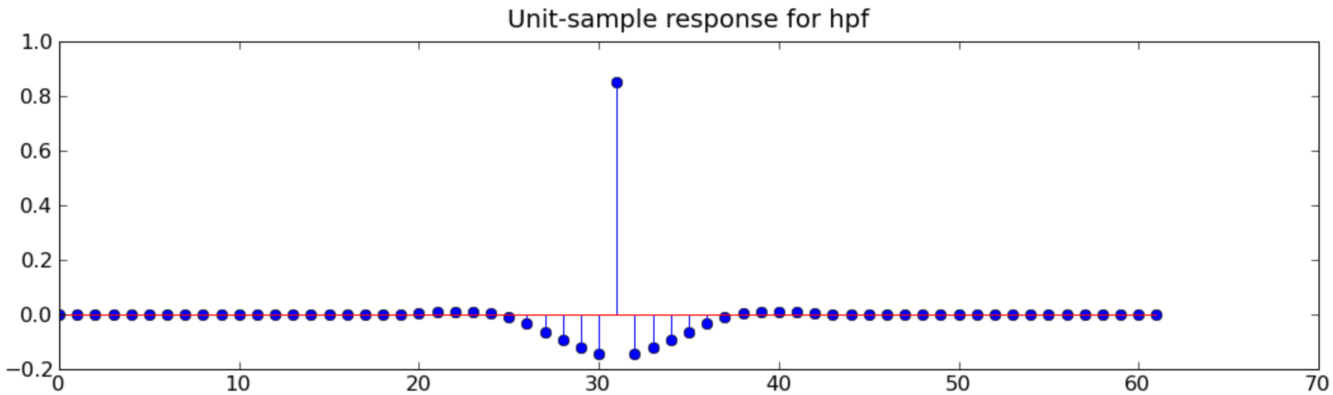


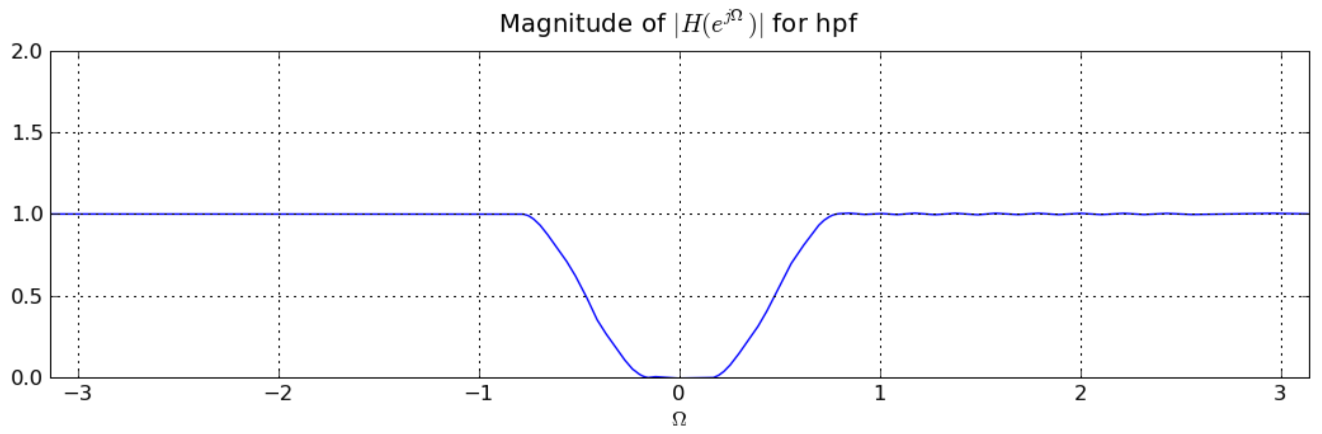
“Wrong” High Pass Filter



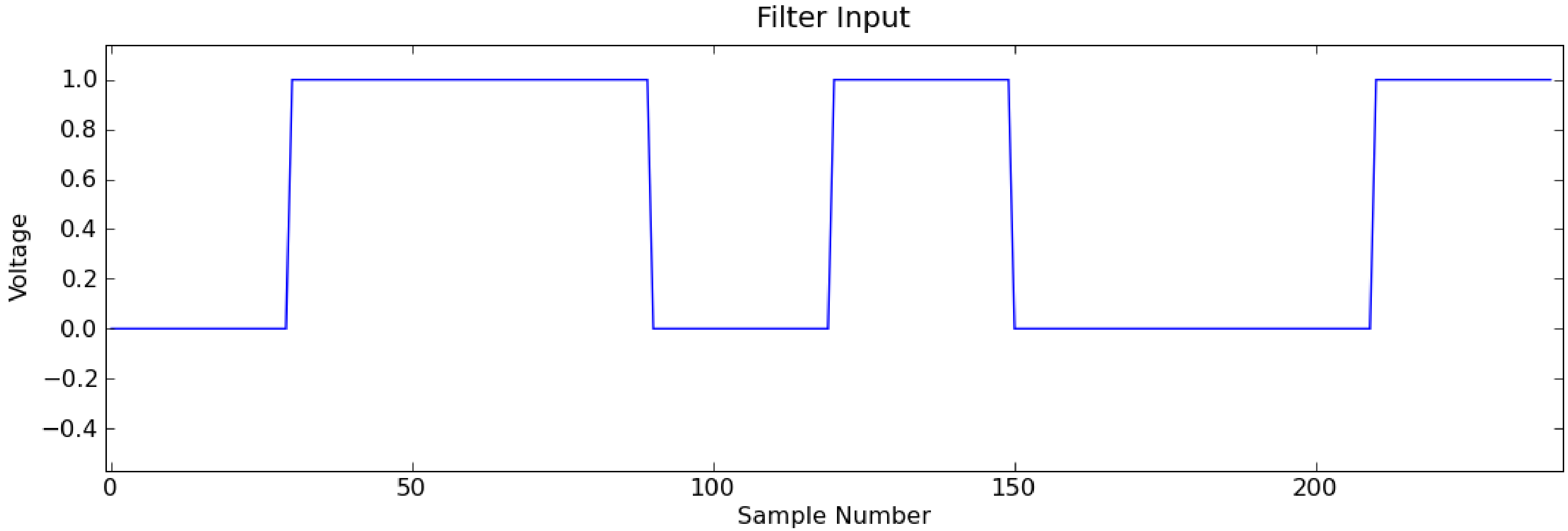


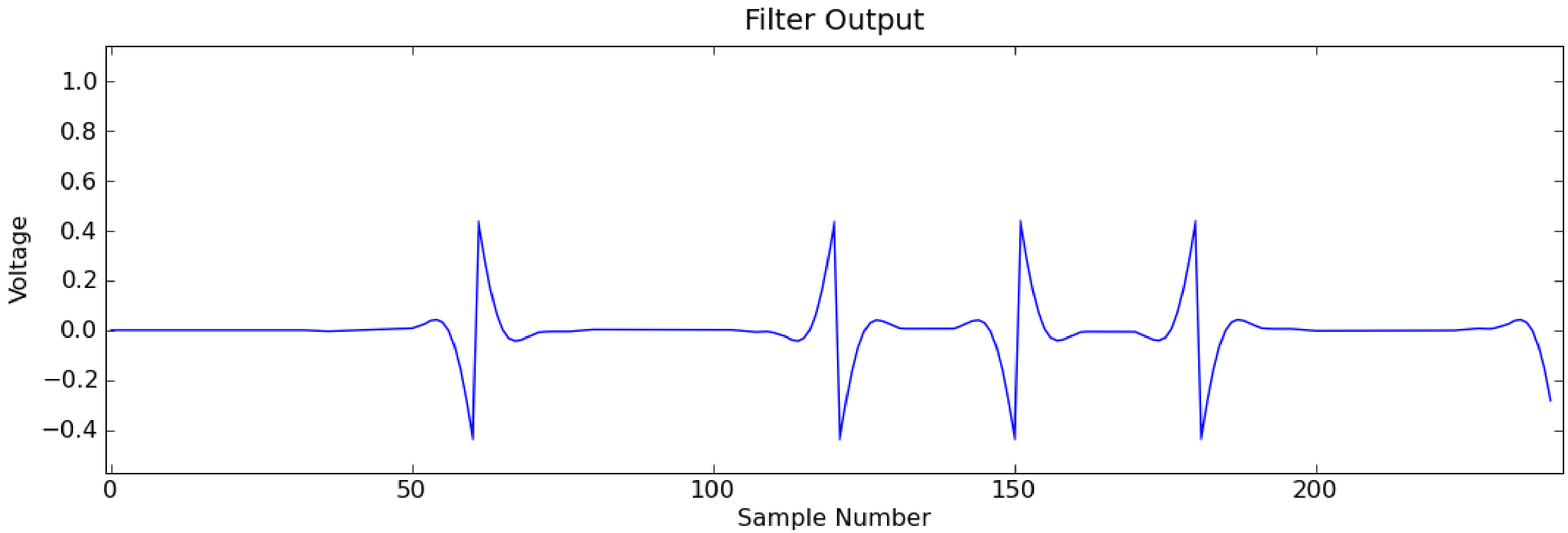
High Pass Filter



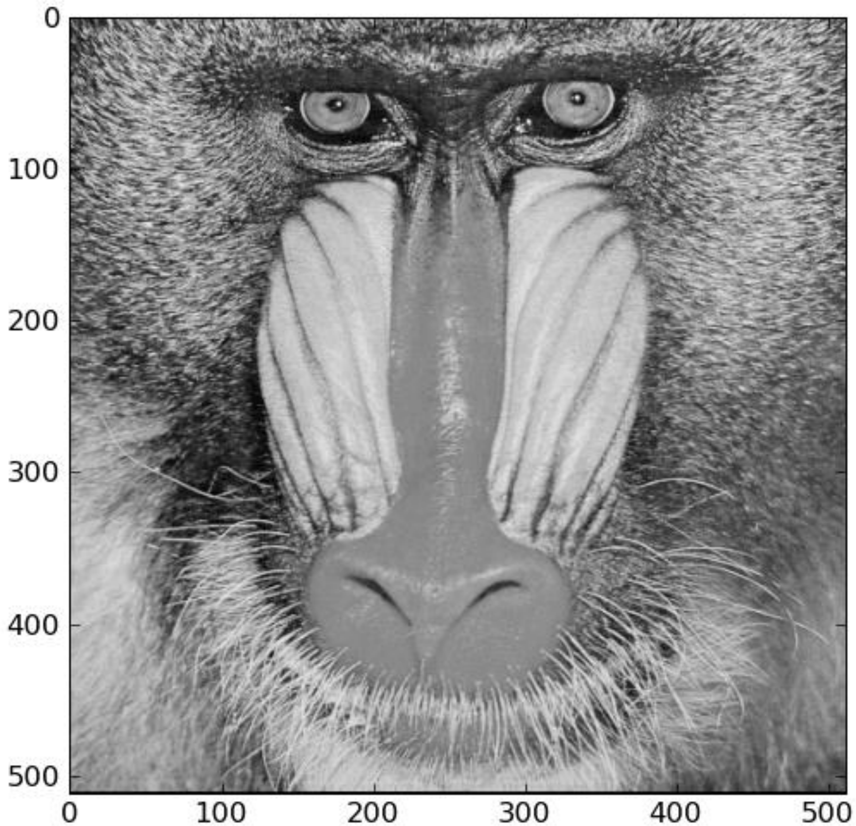


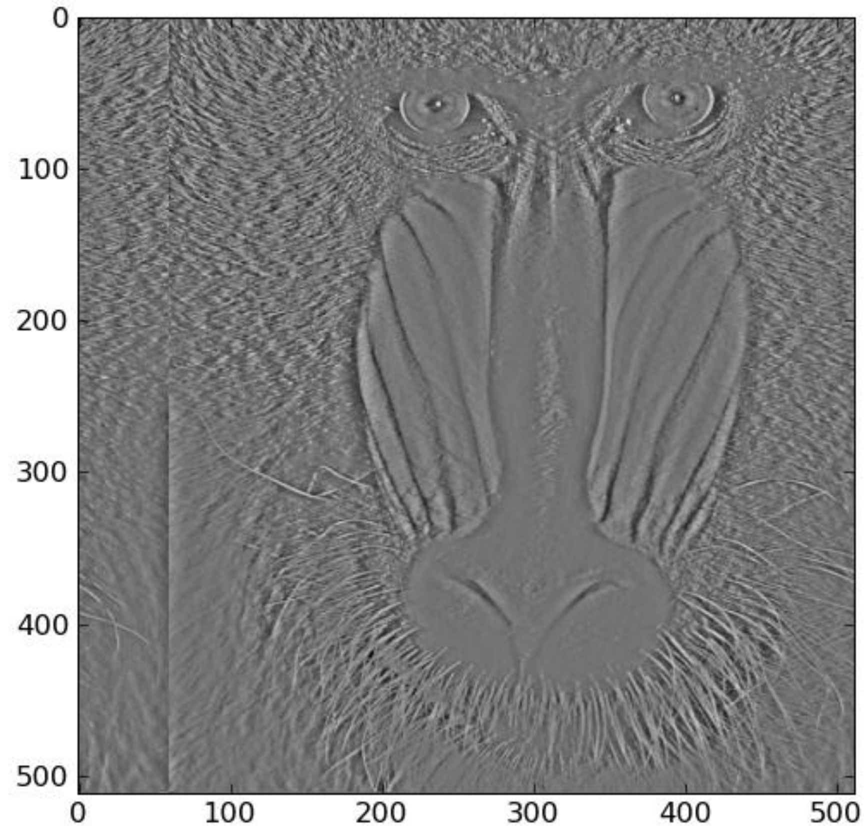
High Pass Filter Applied to Bits





High Pass Filter Applied to Image





Band Pass Filter