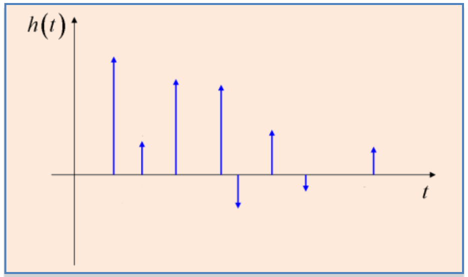
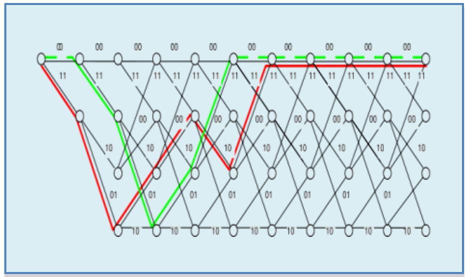
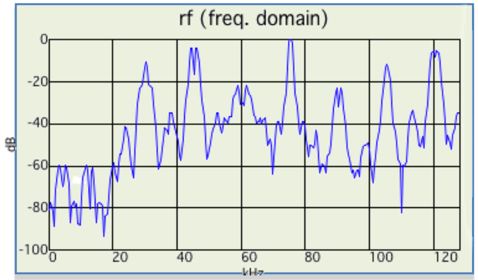
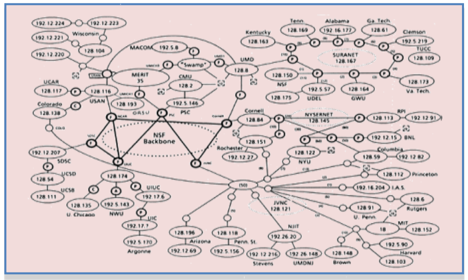








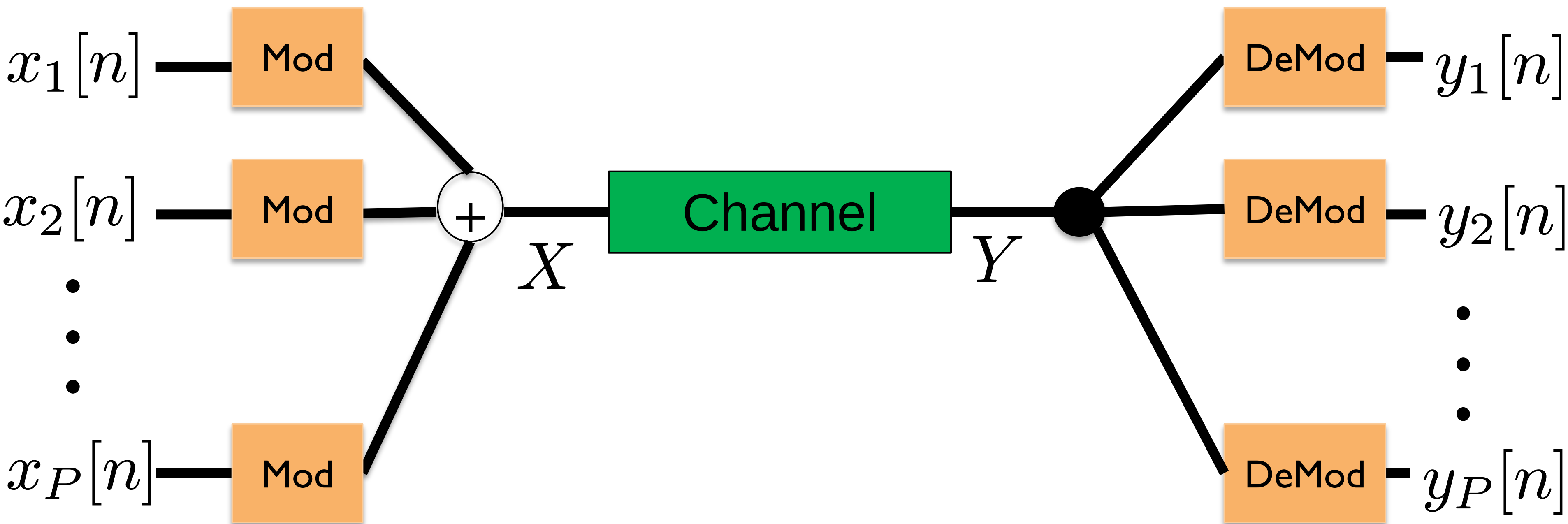
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2010
Lecture #15

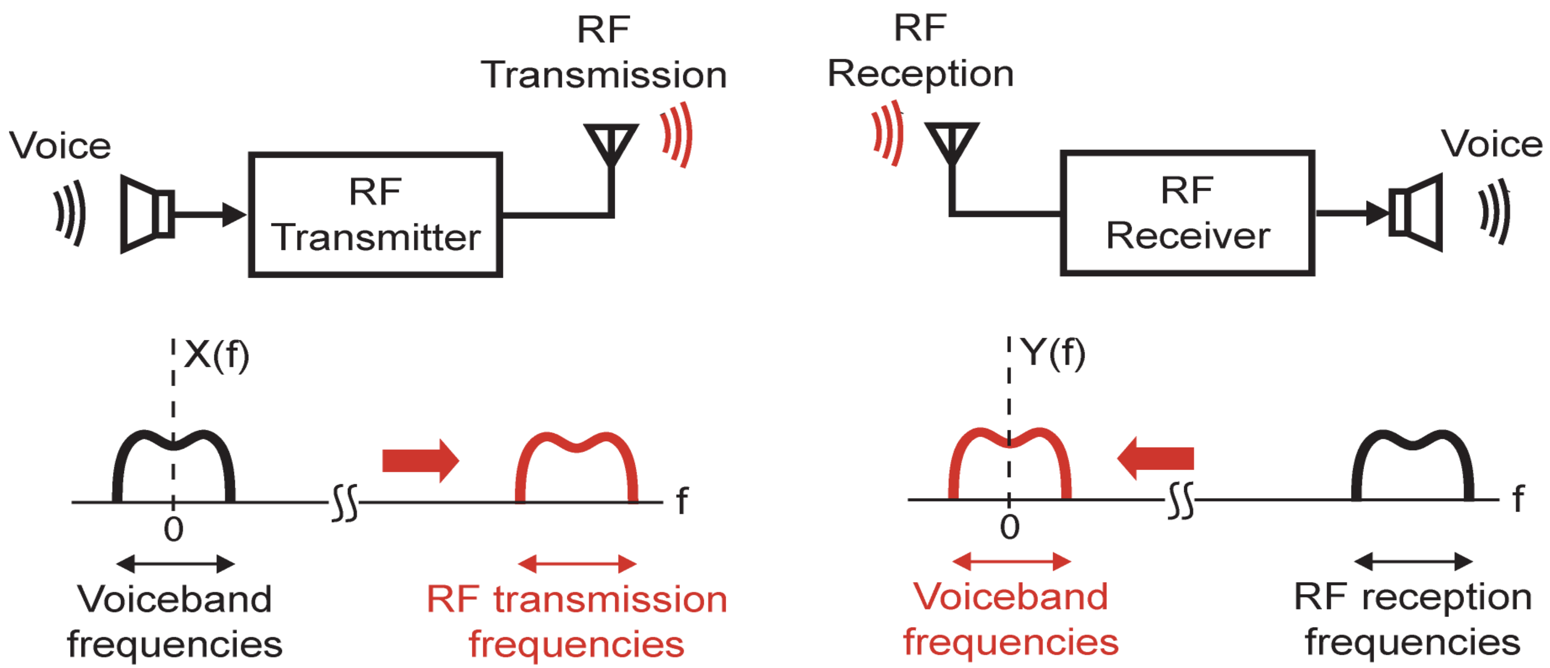
- Reasons for Modulations
- Reminder about Fourier Series
- Alternate Frequency Axes
- Key Modulation Formula
- Modulation Examples

Frequency Domain Sharing - Big Picture



- Questions
  - What is Modulation (Mod in figure)?
  - What do typical X's look like?
  - What should the Demodulator be?
  - What is the relation between $x_i[n]$ and $y_i[n]$

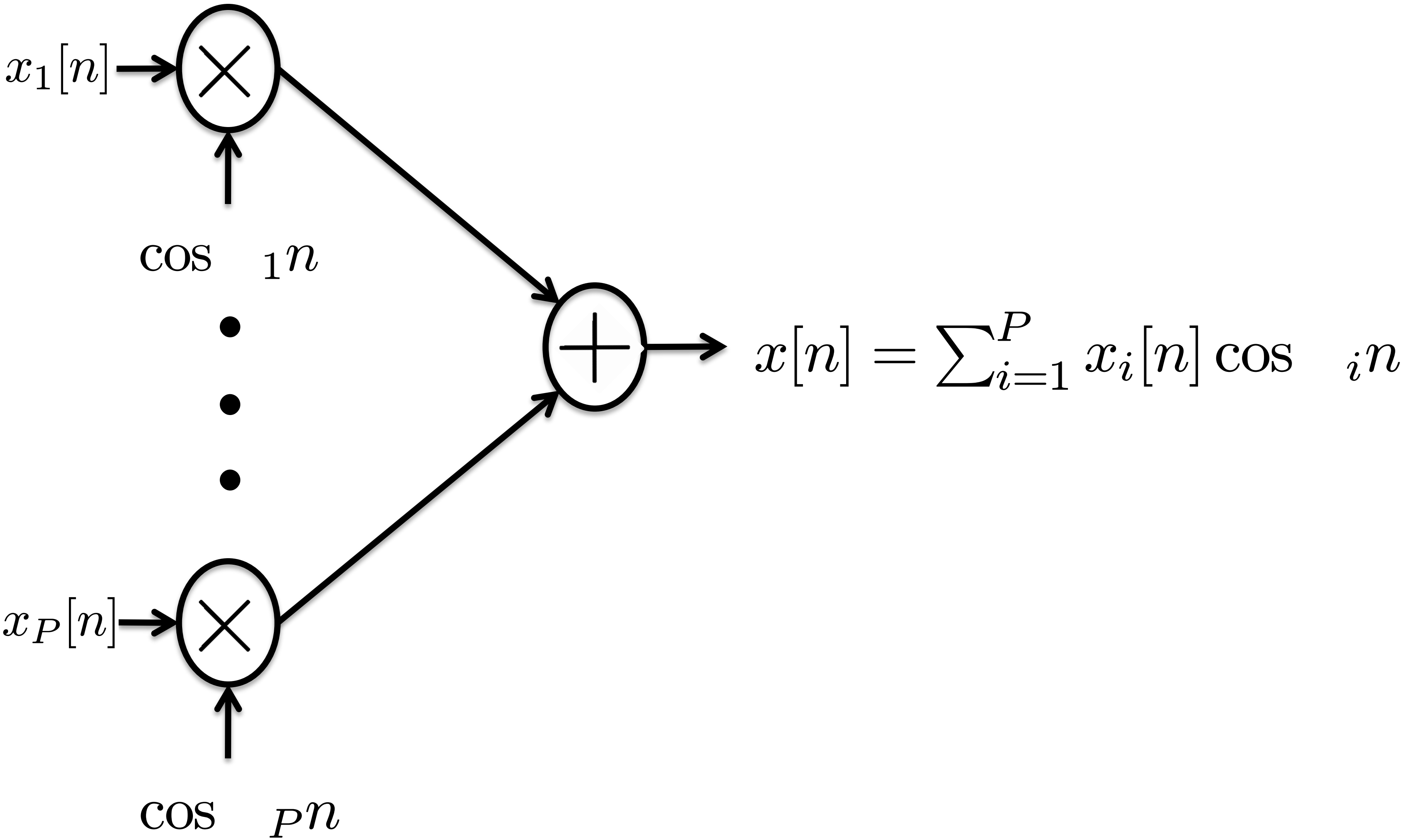
Motivation for High Frequency Modulation



- Modulation is used to change the frequency band of a signal
  - Enables RF communication in different frequency bands
    - Used in cell phones, AM/FM radio, WLAN, cable TV, ….
  - Note: higher frequencies lead to smaller antennas
    - 3GHz, 10cm wavelength, 2.5cm antenna (1/4 wavelength)

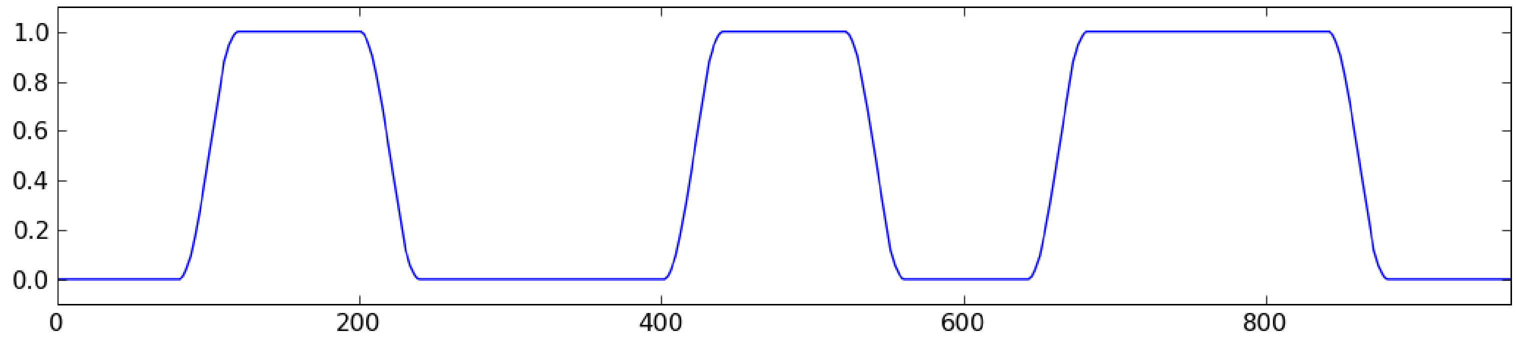
Slide Thanks to M. Perrott and C. Sodini

Modulation by Cosine Multiplication

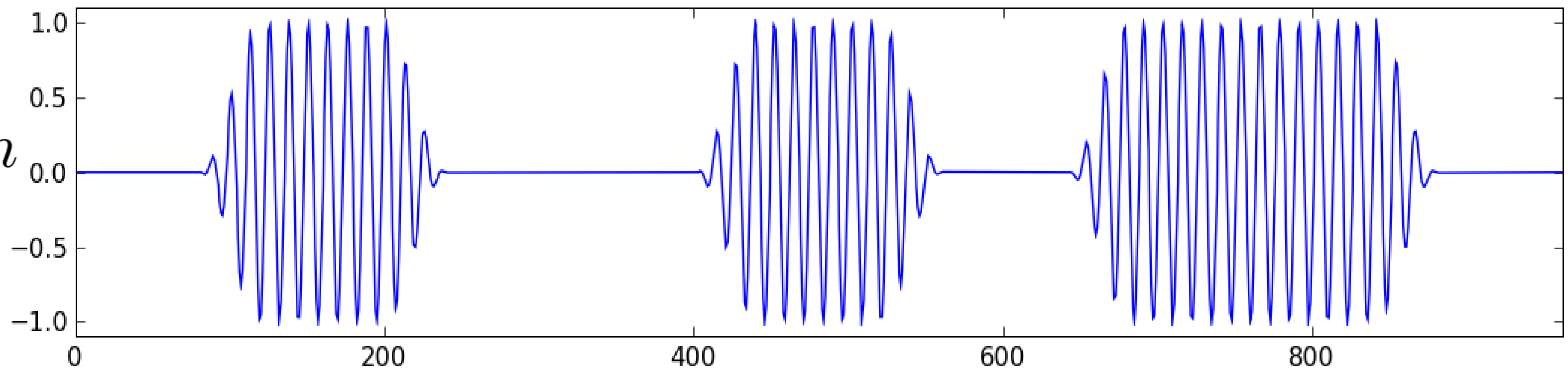


What does a modulated X look like?

$x_i[n]$



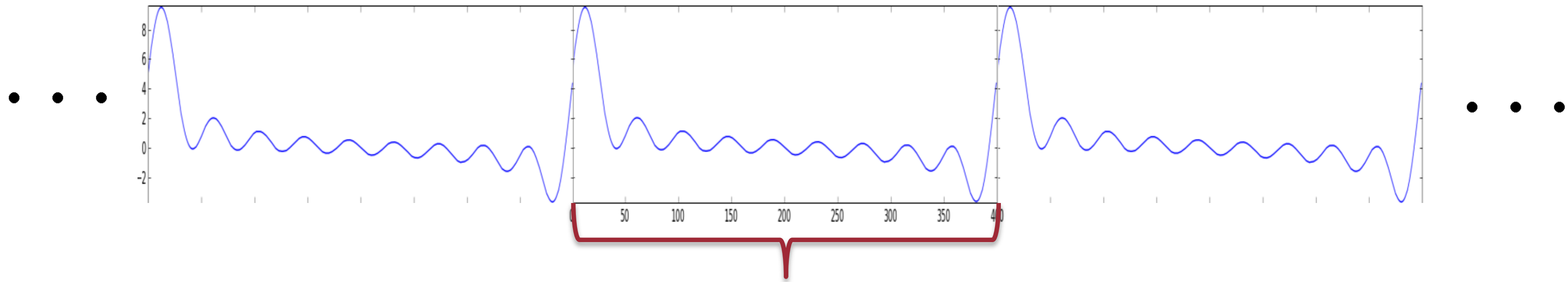
$x_i[n] \cos 0.5n$



Can Fourier Series Help us understand Modulation?

Periodic Assumption (with period *N*)

$$x[n] = x[n+N]$$



One Period

$N = 400$

- Periodic with Period N (N even) means
  - Only certain frequency complex exponentials

$$\Omega \in \left\{0, \pm\frac{2\pi}{N}, \pm 2\frac{2\pi}{N}, \ldots, \pm\left(\frac{N}{2}-1\right)\frac{2\pi}{N}, \pm\pi\right\}$$

  - x[n] can be represented with a Fourier series

Discrete-Time Fourier Series

$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j\frac{2\pi}{N}kn} \qquad K = \frac{N}{2}$$

OR

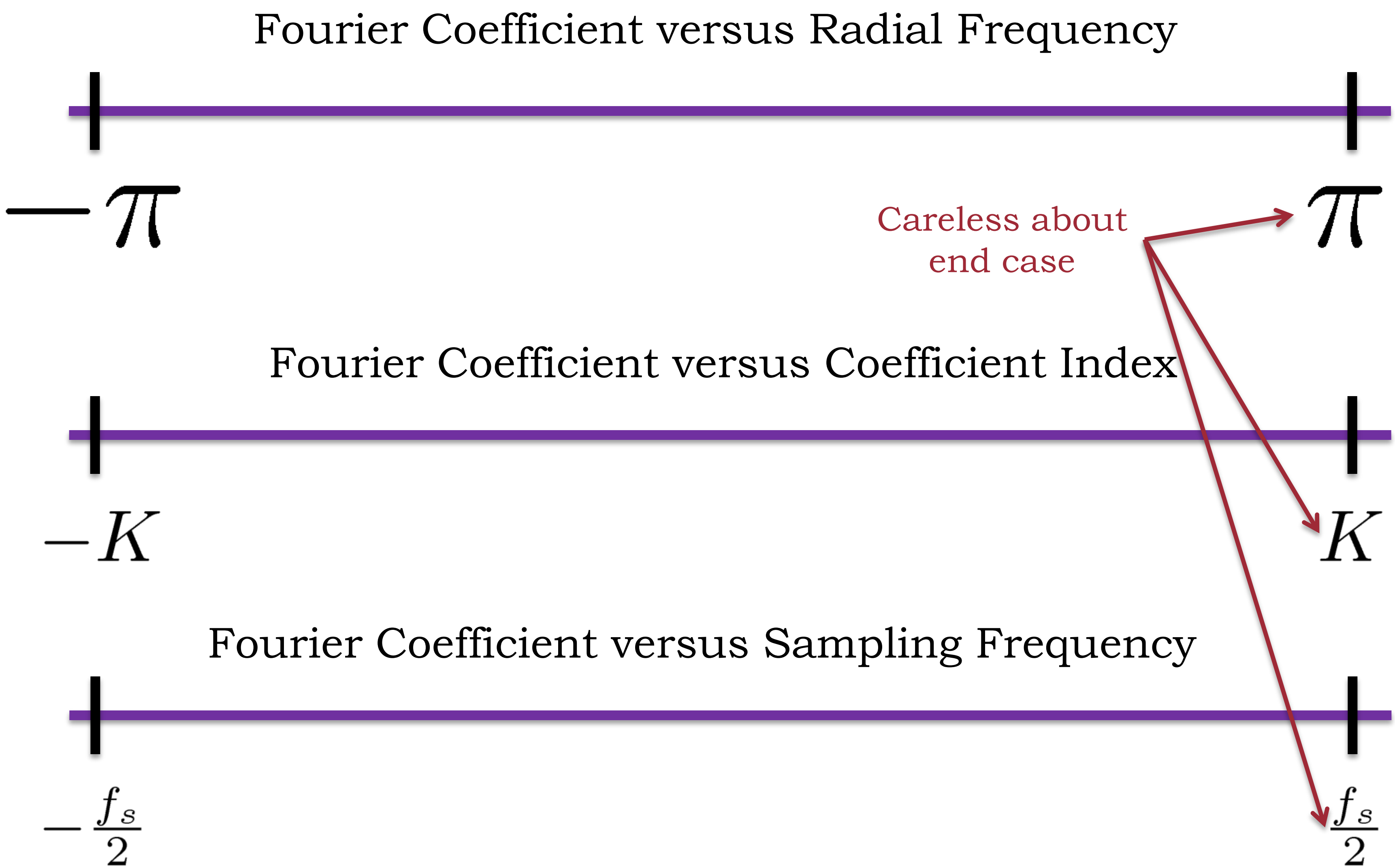
$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j\Omega_k n} \qquad -\pi \leq \Omega_k \leq \pi$$

Key DTFS Modulation Identity

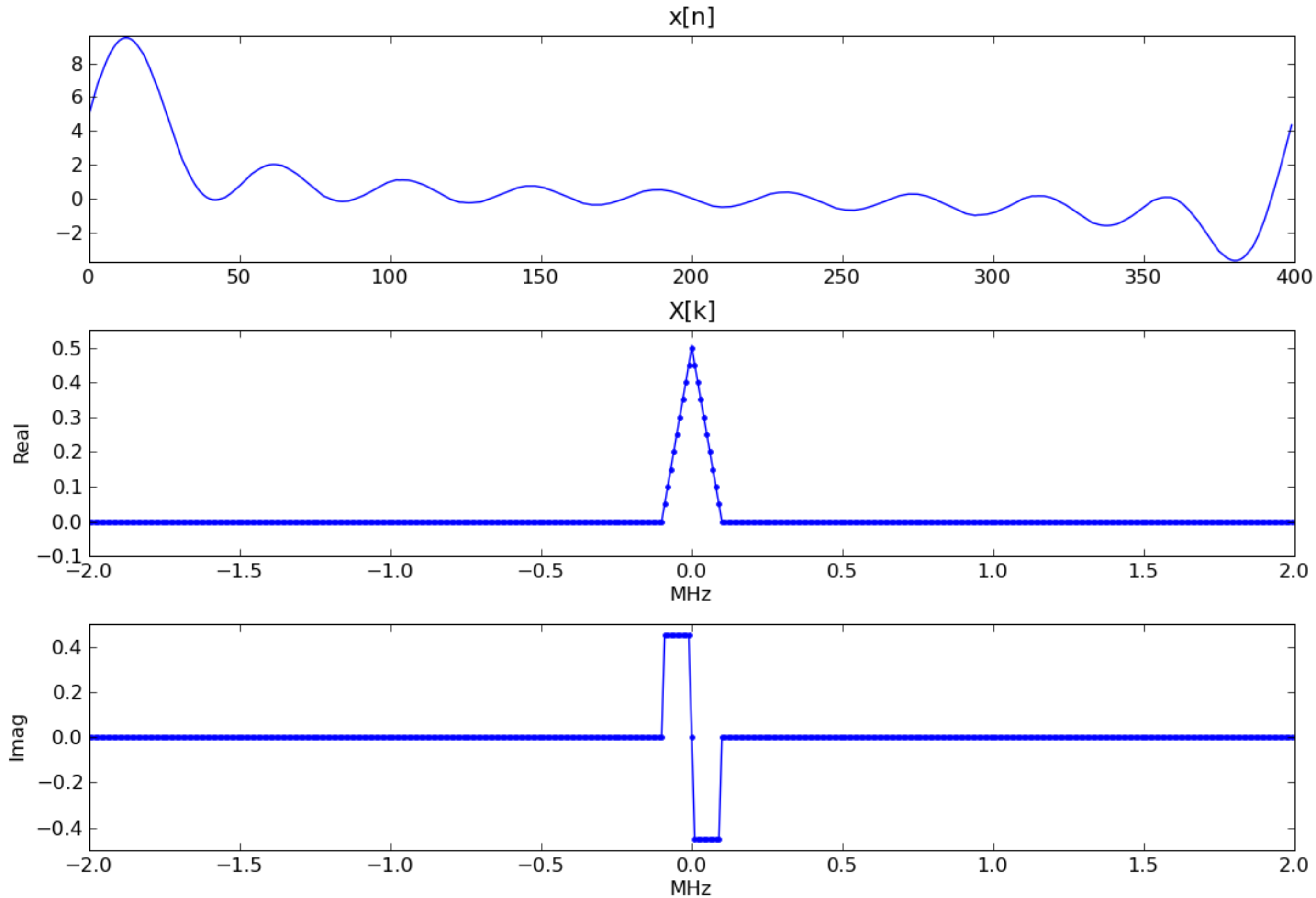
$$x[n] \cos \Omega_m [n] = \left(\sum_{k=-K}^{K-1} X[k] e^{j\Omega_k n} \right) \left(\frac{1}{2} e^{j\Omega_m n} + \frac{1}{2} e^{-j\Omega_m n} \right)$$

$$= \frac{1}{2} \sum_{k=-K}^{K-1} X[k] e^{j(\Omega_k + \Omega_m)n} + \frac{1}{2} \sum_{k=-K}^{K-1} X[k] e^{j(\Omega_k - \Omega_m)n}$$

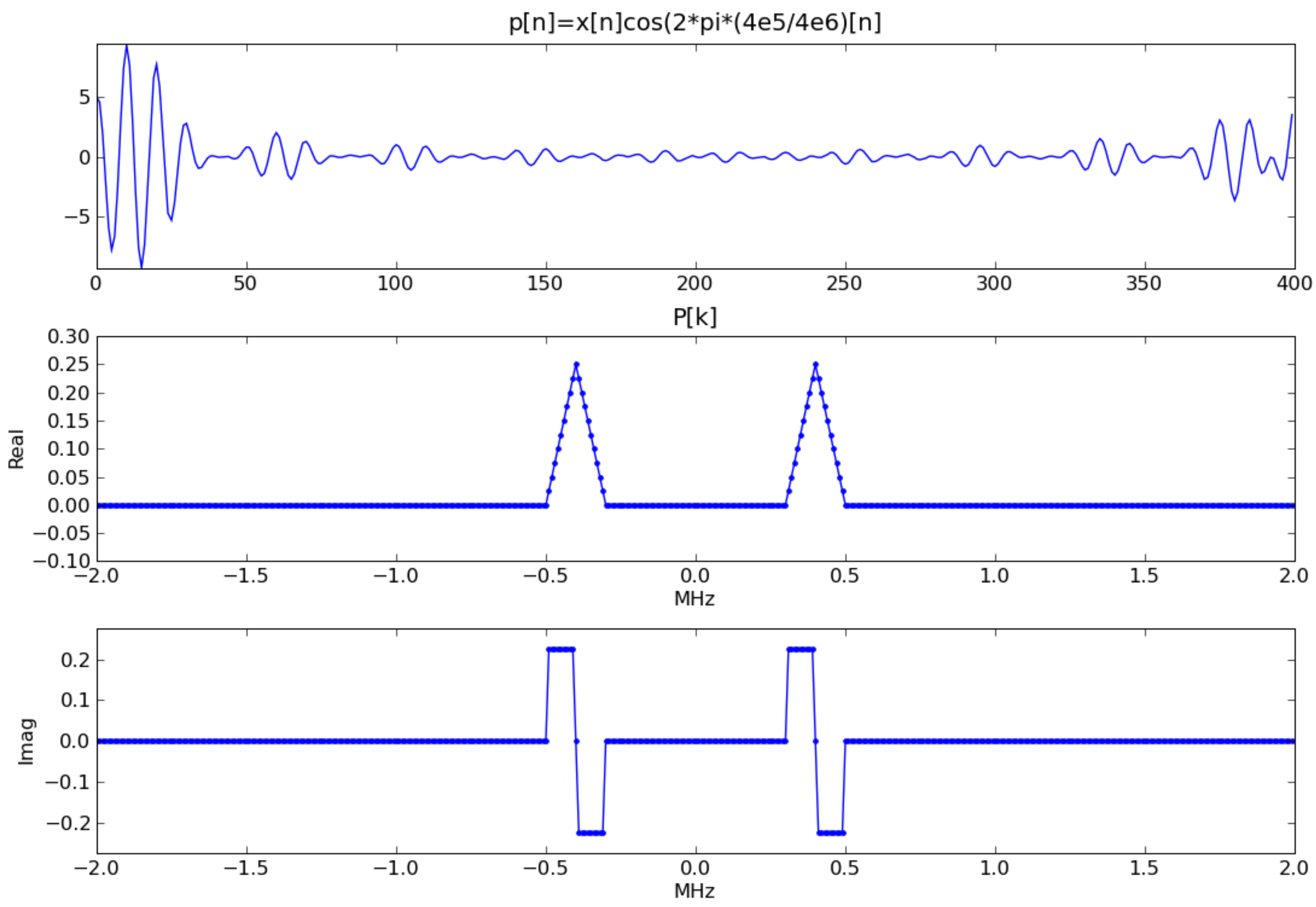
Frequency Axes Alternatives



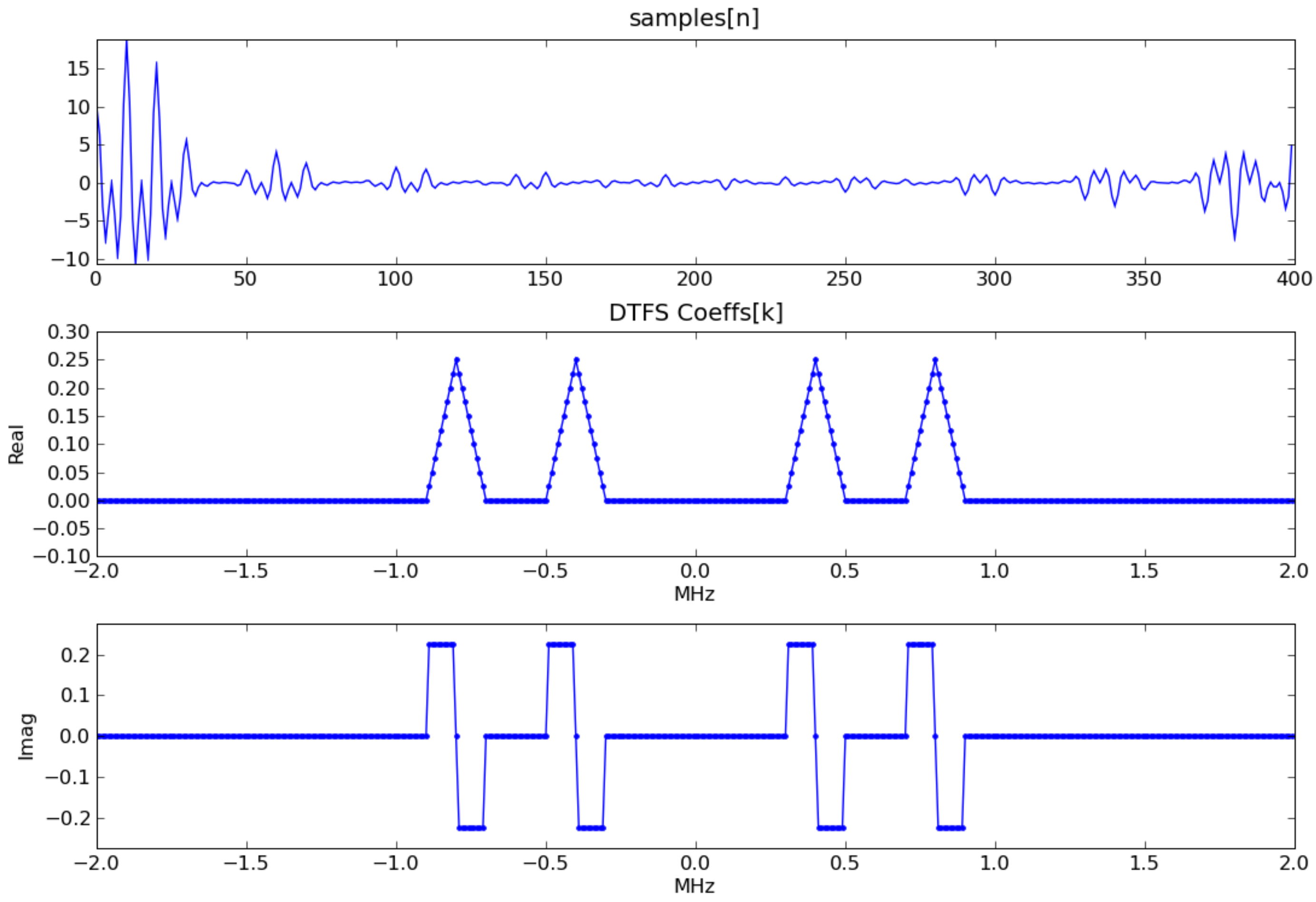
DTFS of x (versus 4Mhz sampling frequency)



DTFS of x after 400khz cosine multiplication

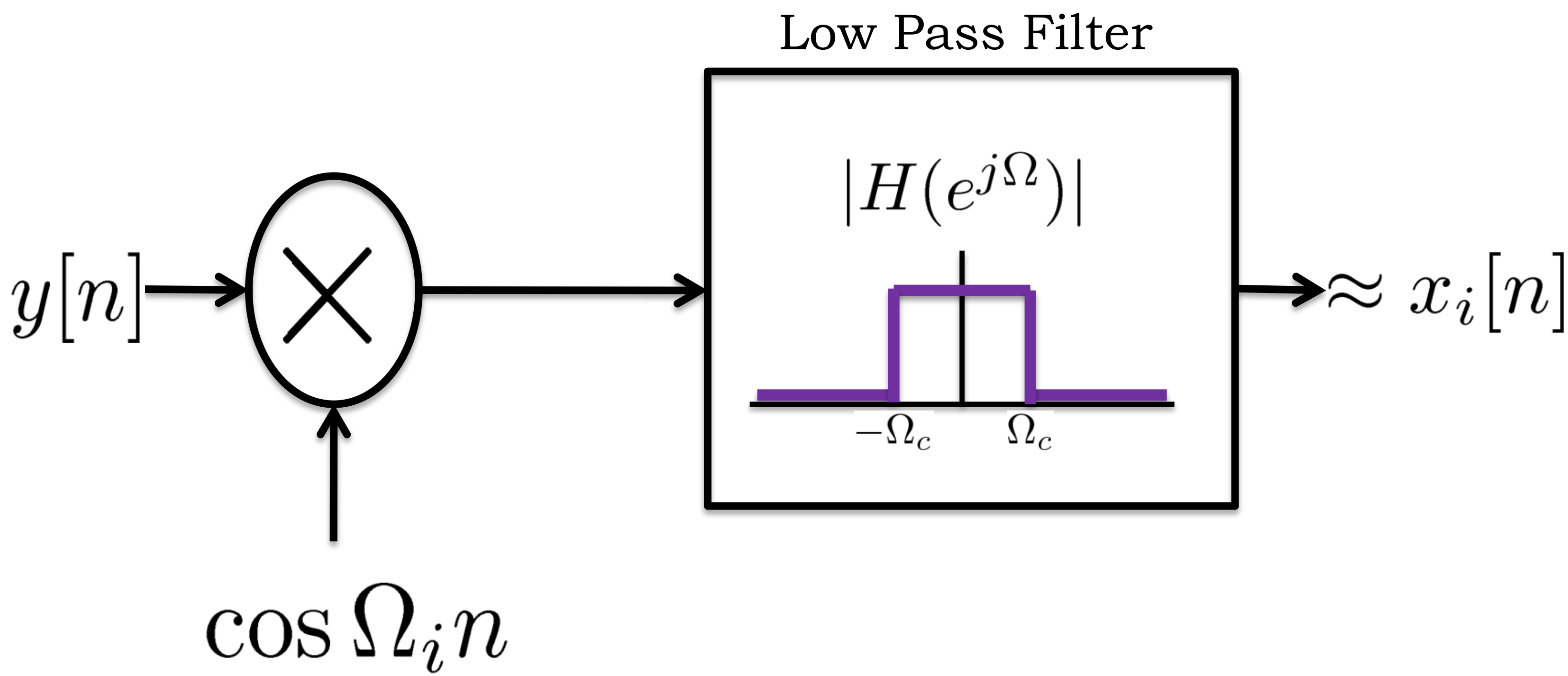


DTFS of x after 400khz and 800khz cosine multiplication

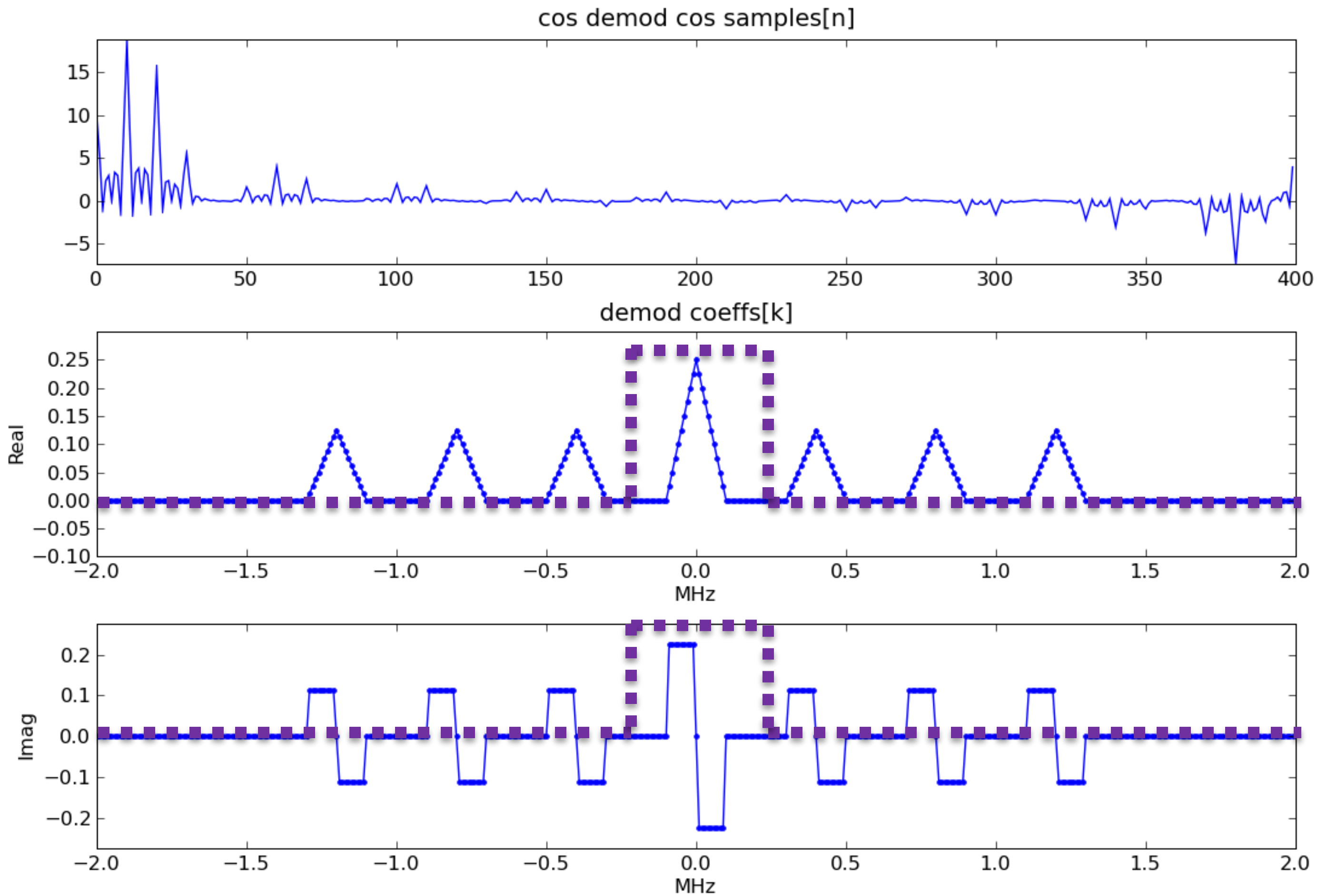


Demodulation by cosine multiplication and LPF (Ideal Channel)

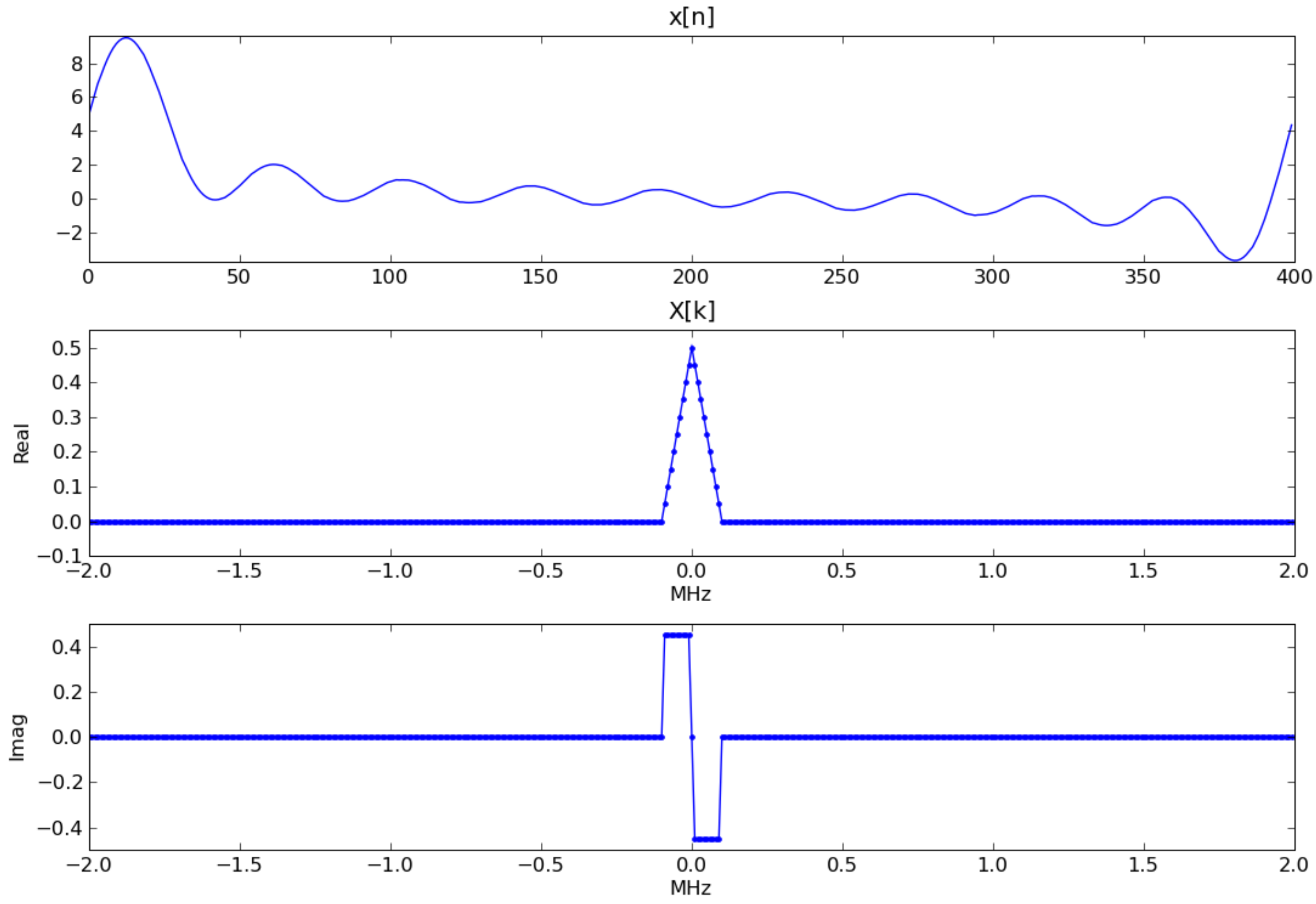
$$y[n] = x[n] = \sum_{i=1}^{P} x_i[n] \cos \Omega_i n$$



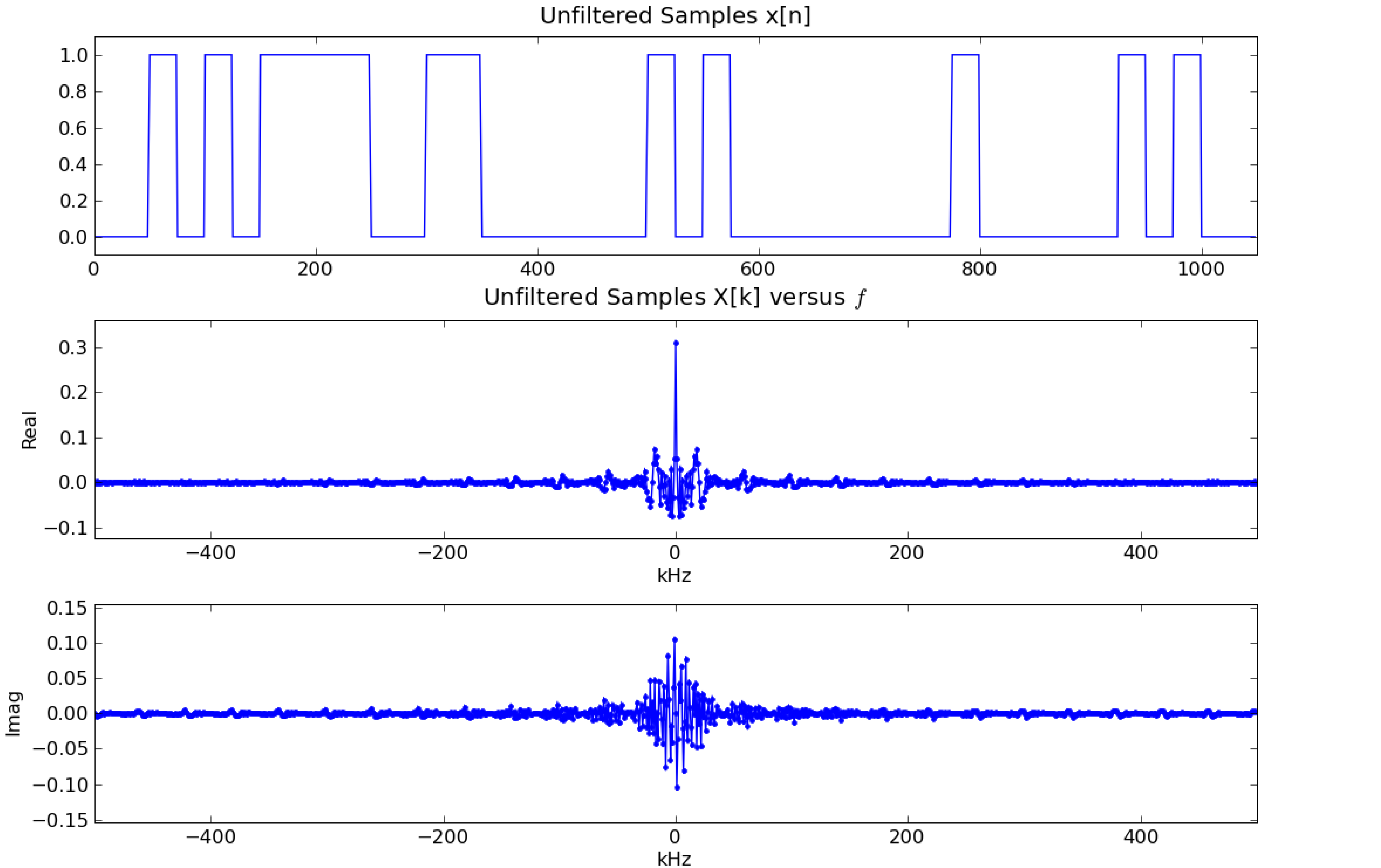
Mod by 400khz and 800khz cos multiplication, demod by 400khz cos multiplication



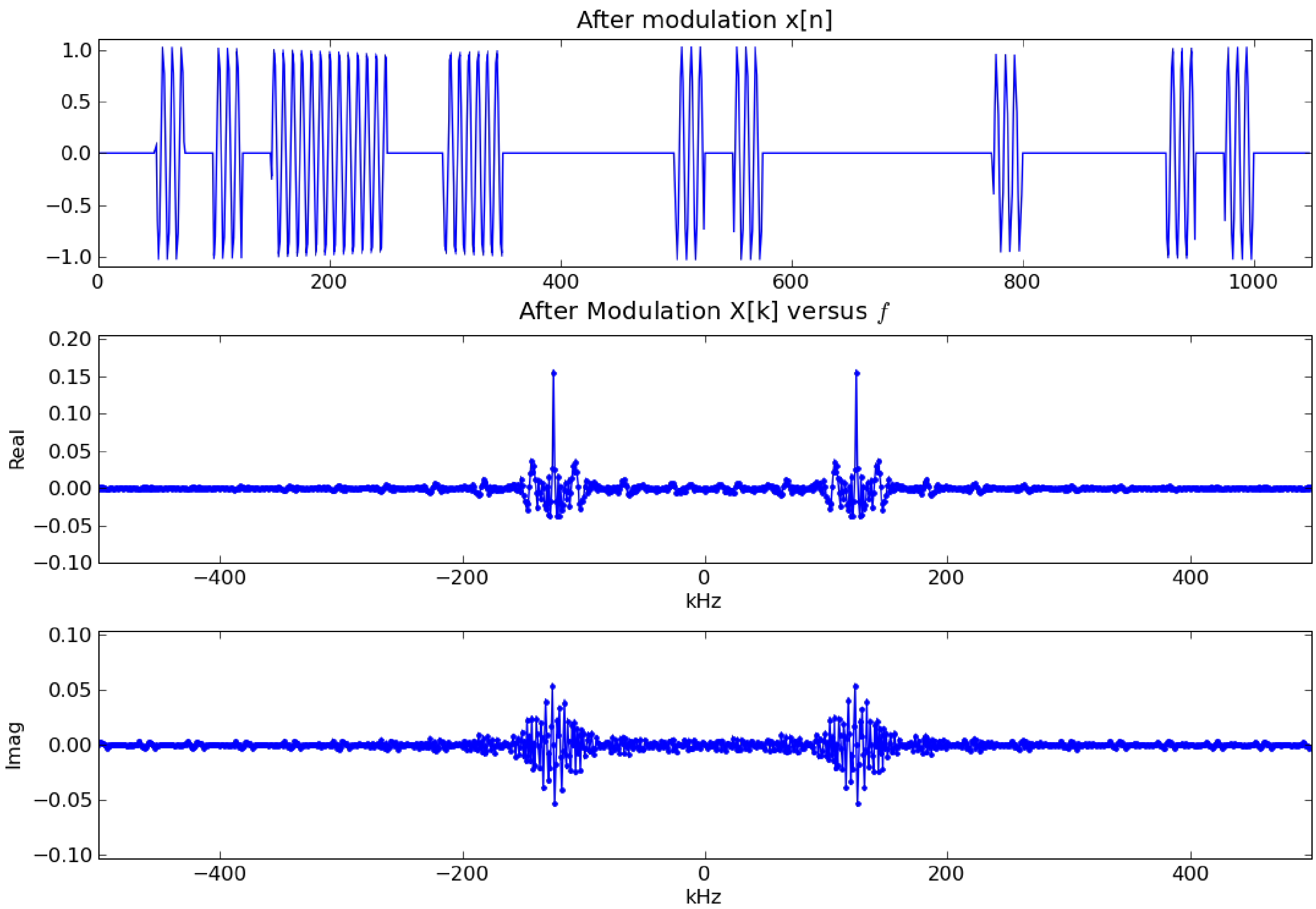
DTFS of x (versus 4Mhz sampling frequency)



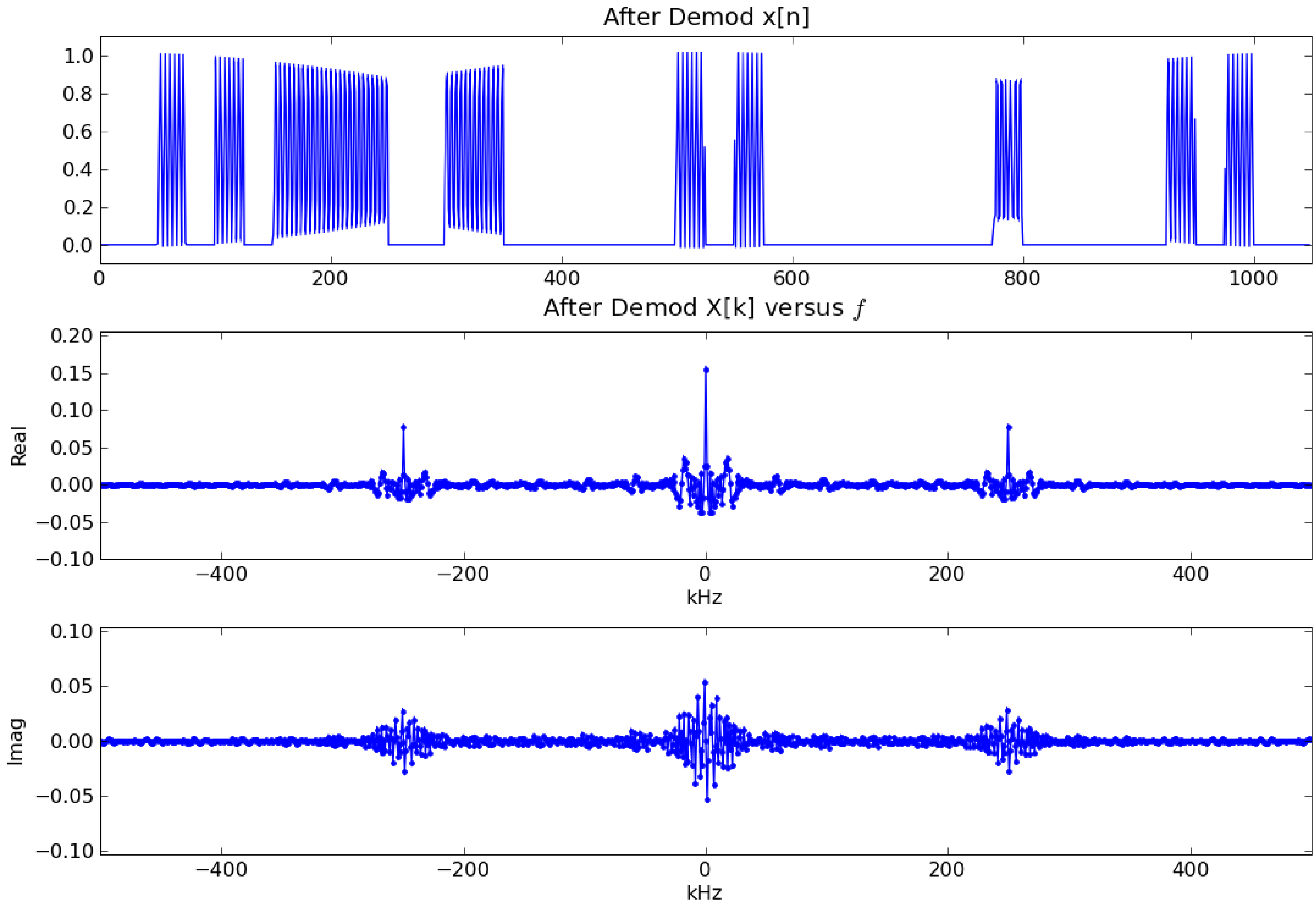
40000 Bits/Sec, 1M samples/sec



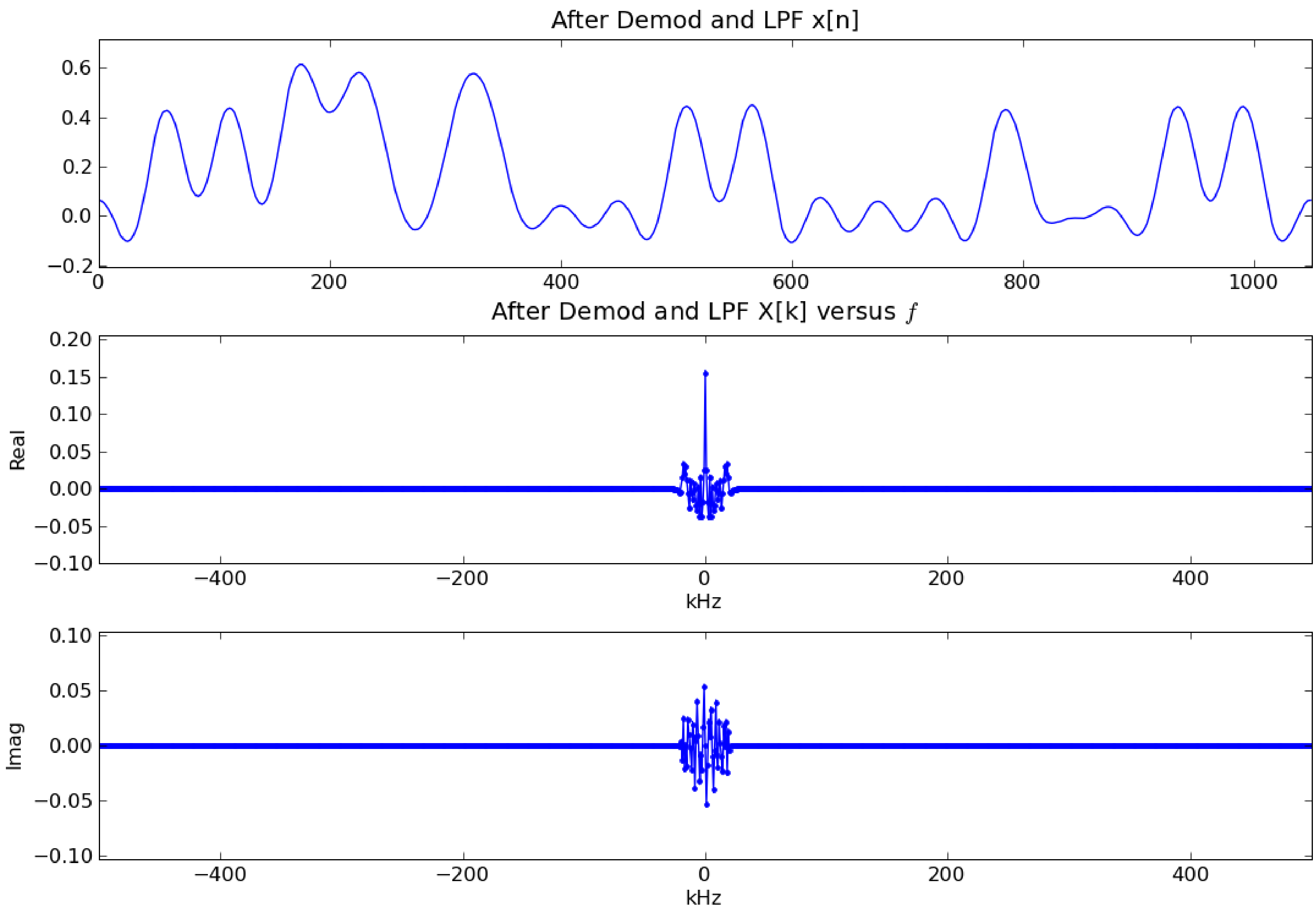
Mod by 125Khz cos mult. (1M Sample Rate)



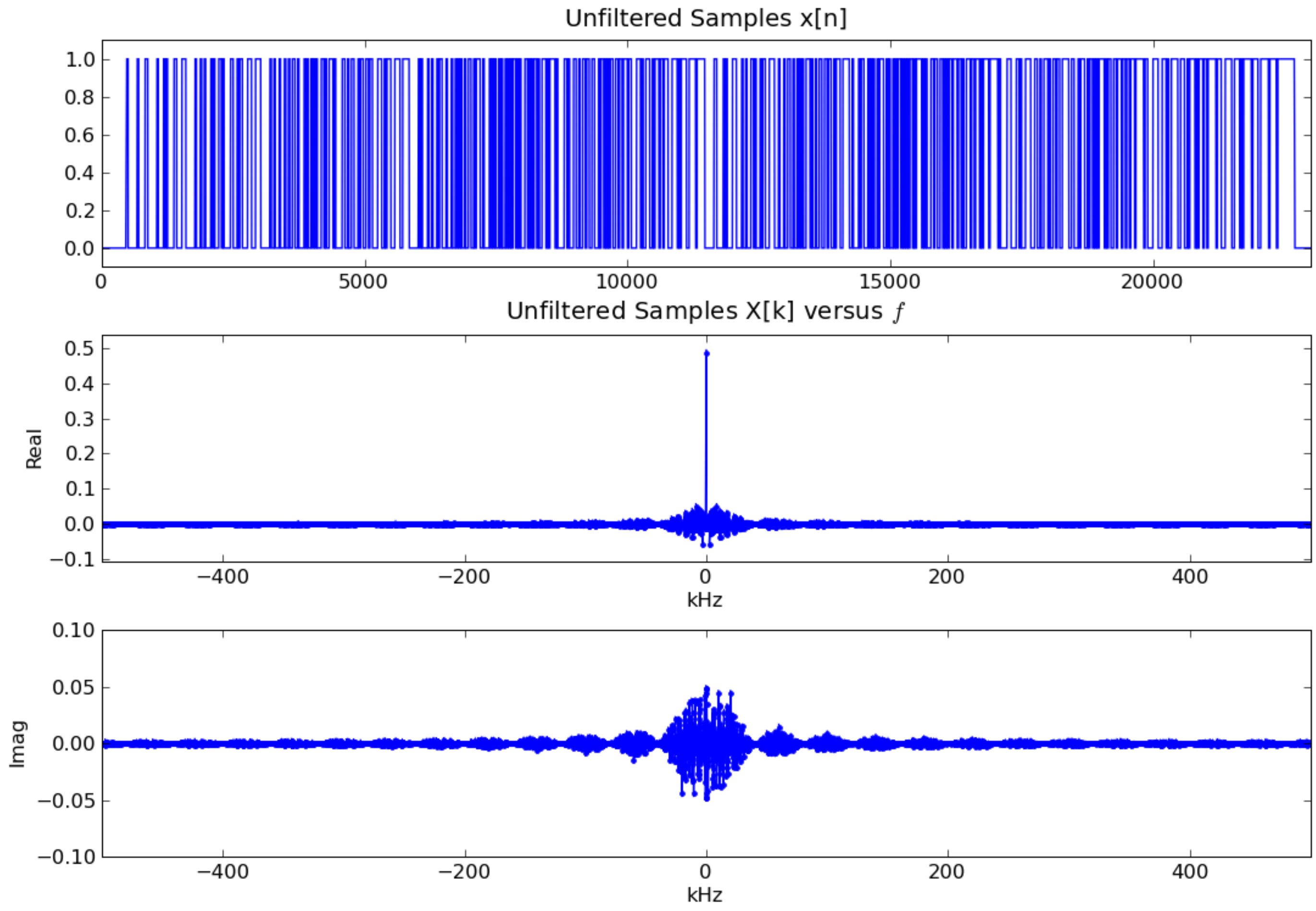
Demodulation by cosine multiplication



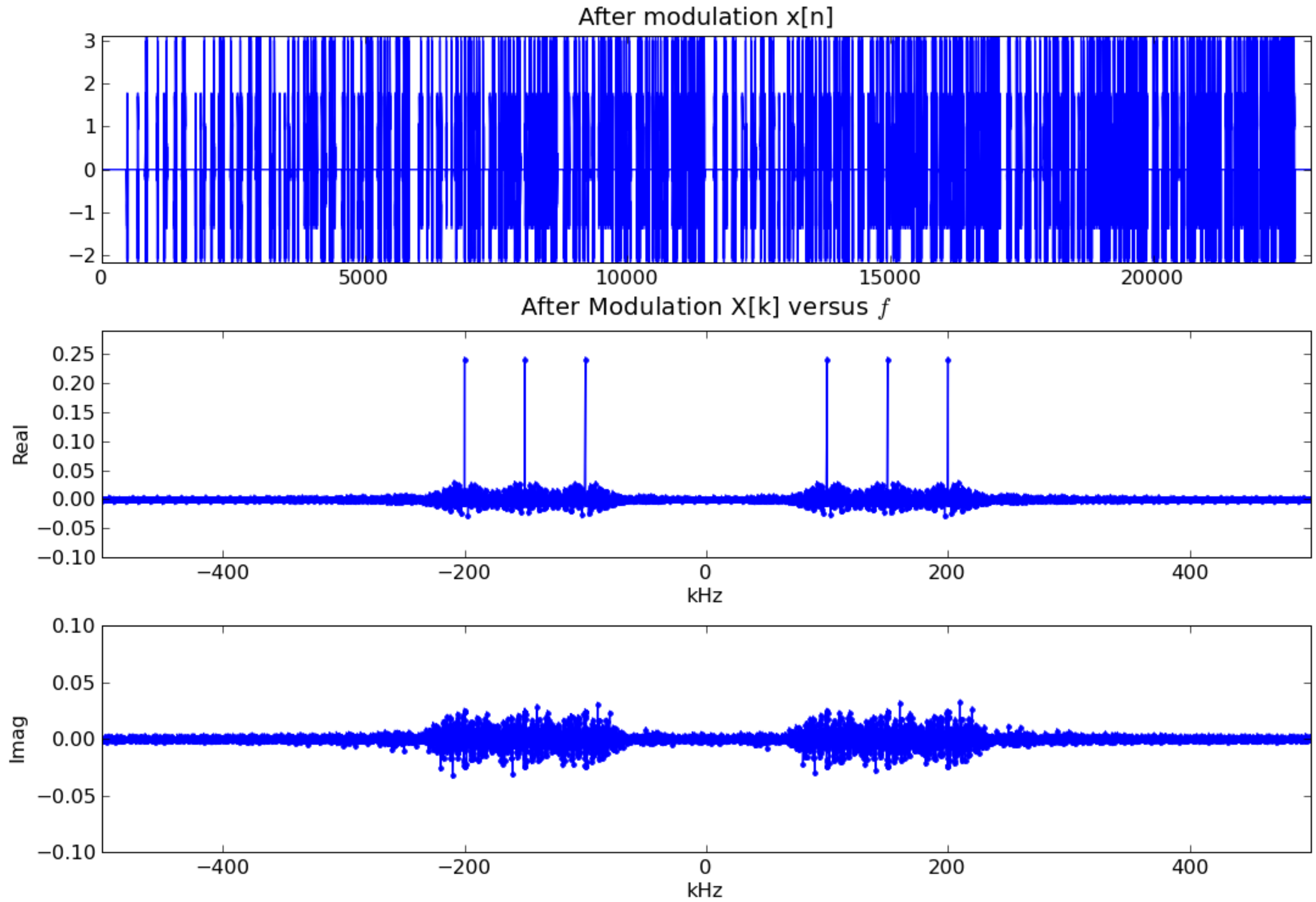
Demod and LPF'd Signal



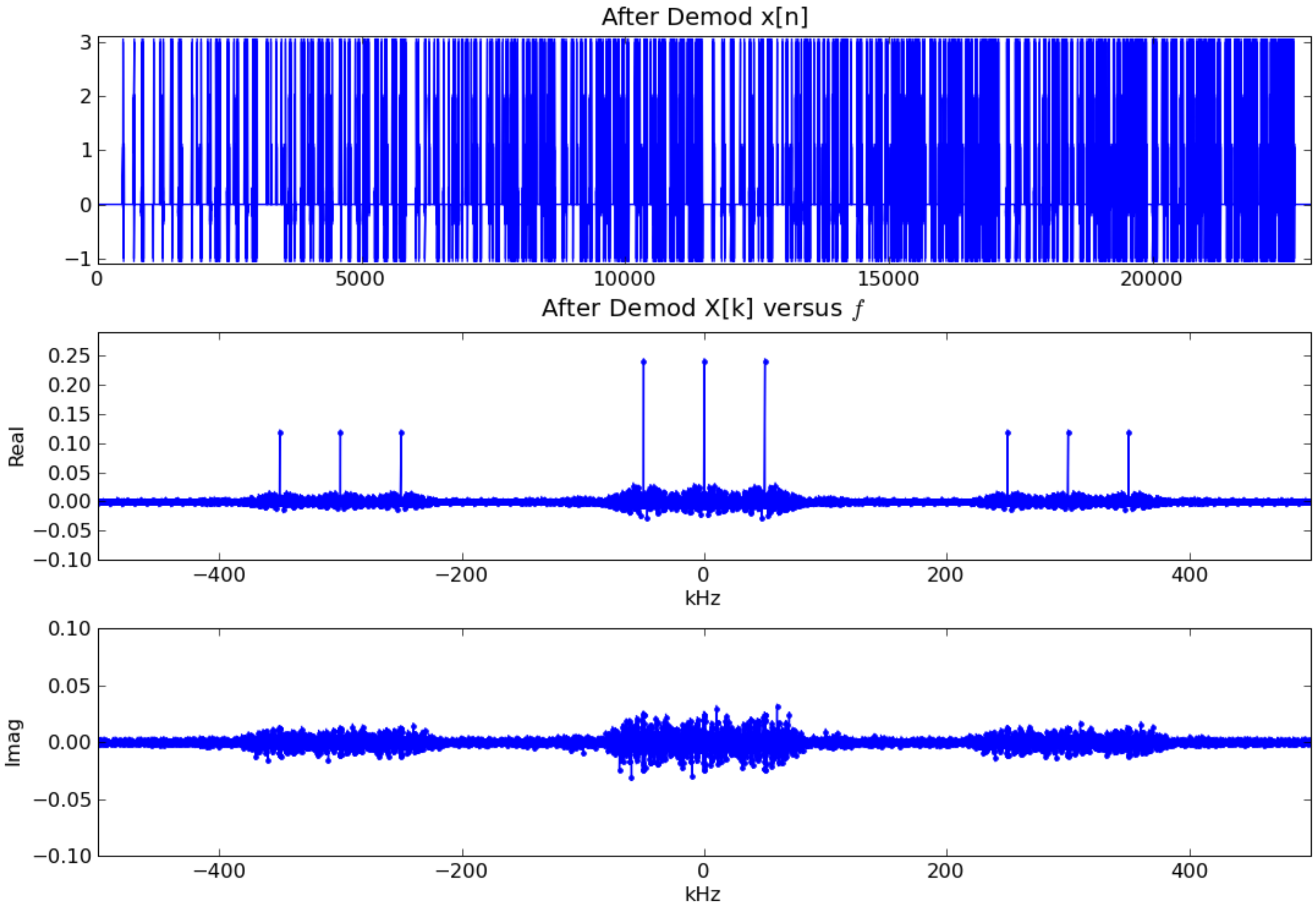
Long (920 bit) Transmission (25 samples/bit)



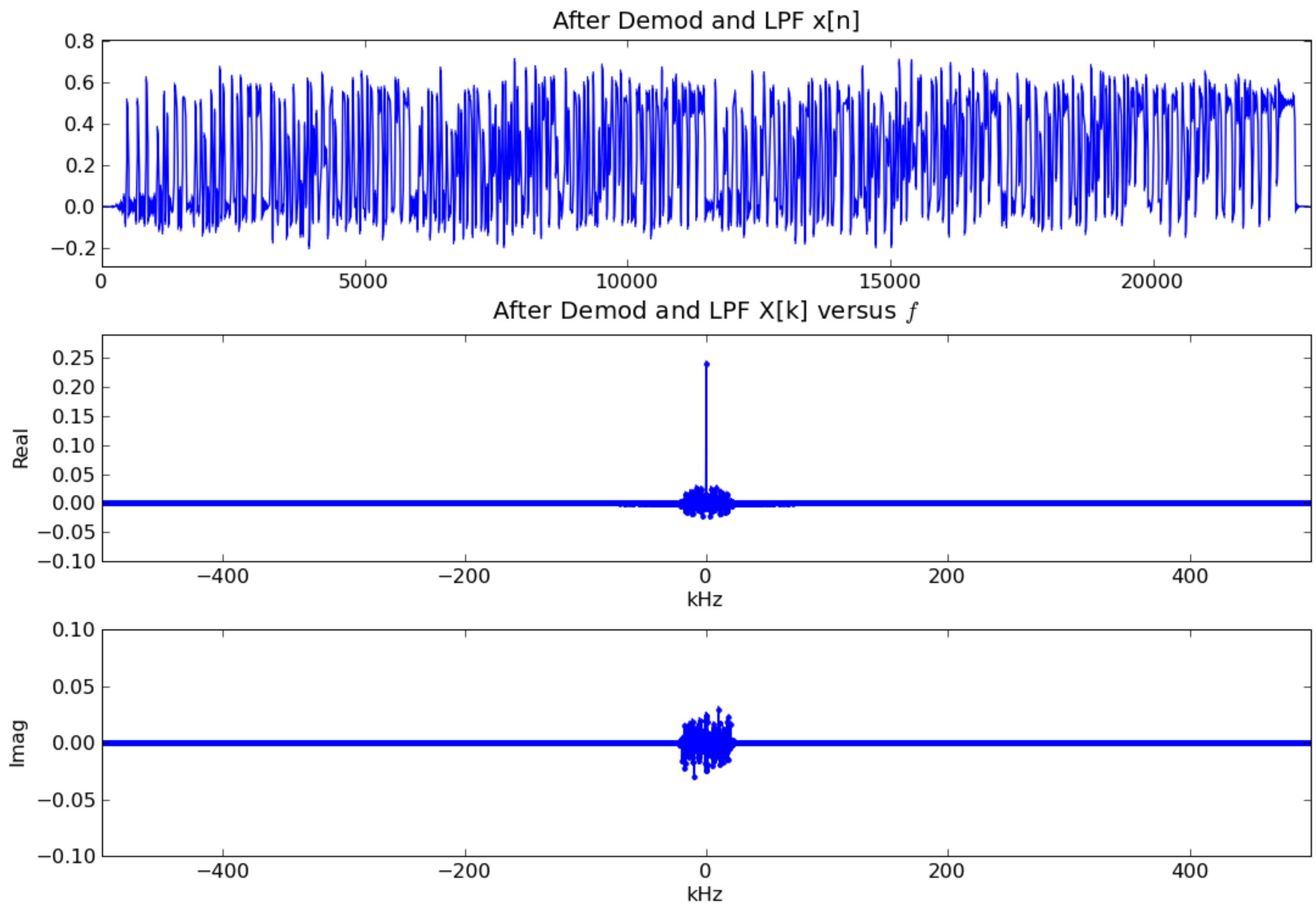
3 Simultaneous Transmitters: 100,150,200Khz



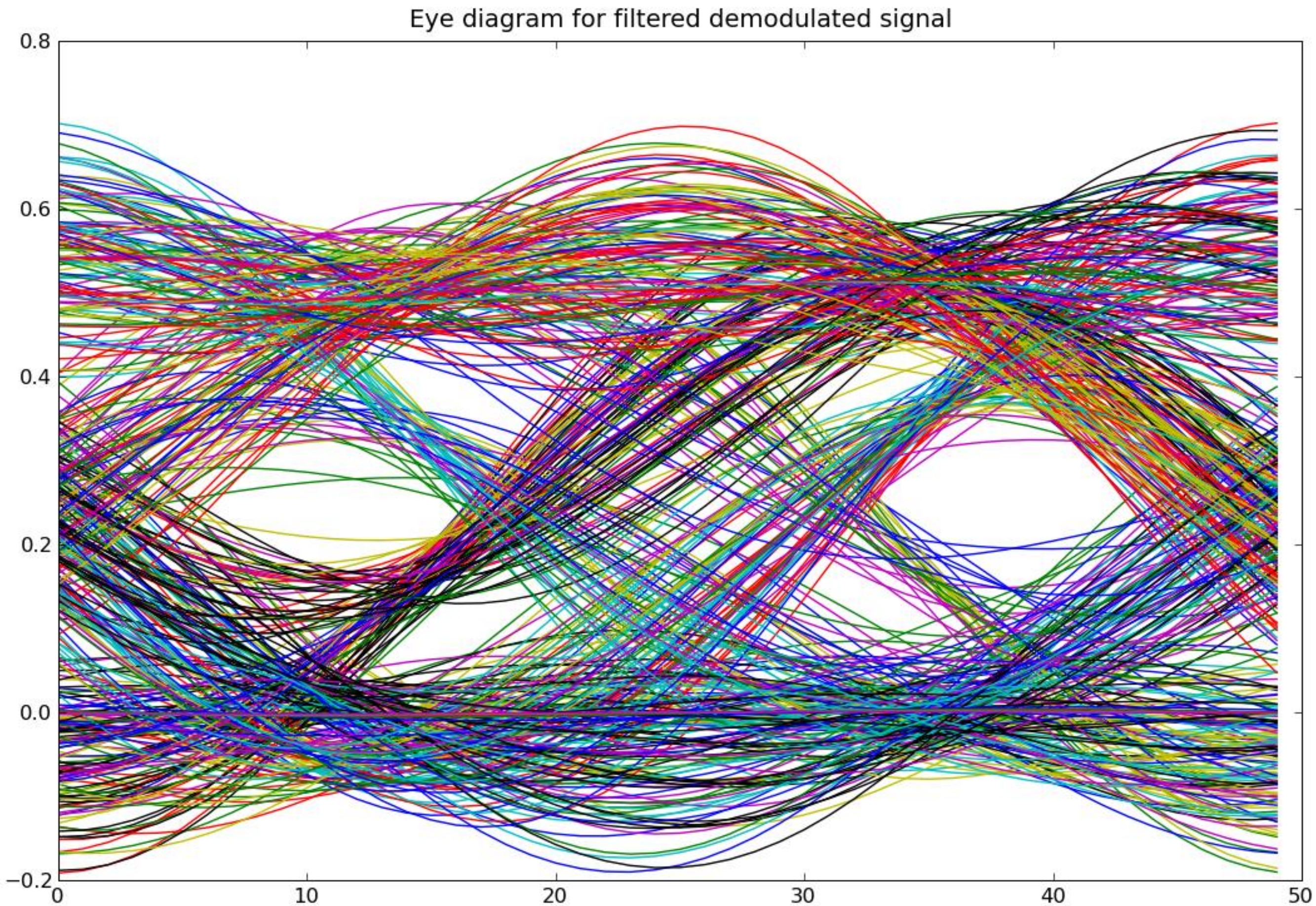
3 Transmitters after Demod by 150khz cosine multiplication



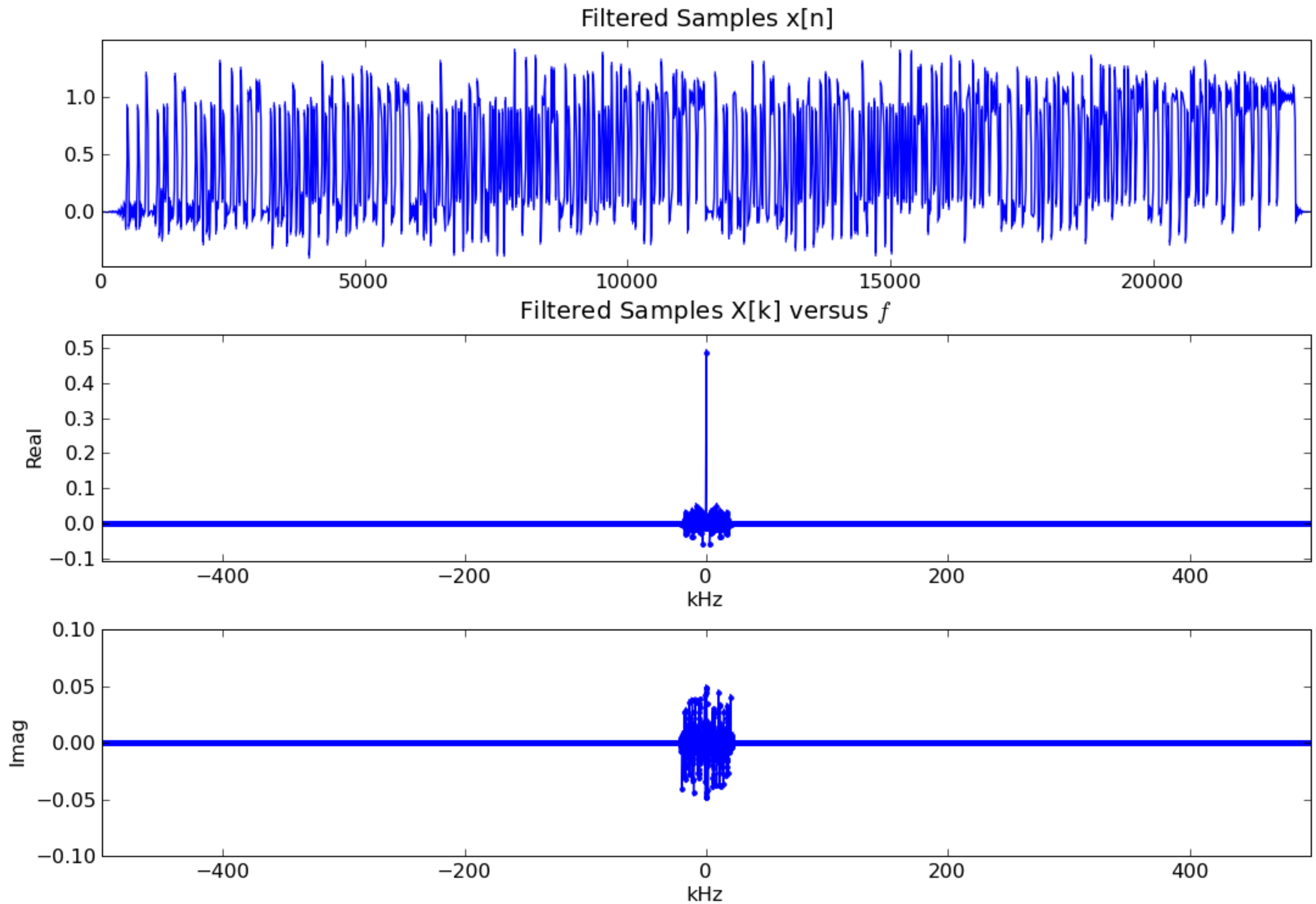
After Low-pass Filter (cutoff freq = 21Khz)



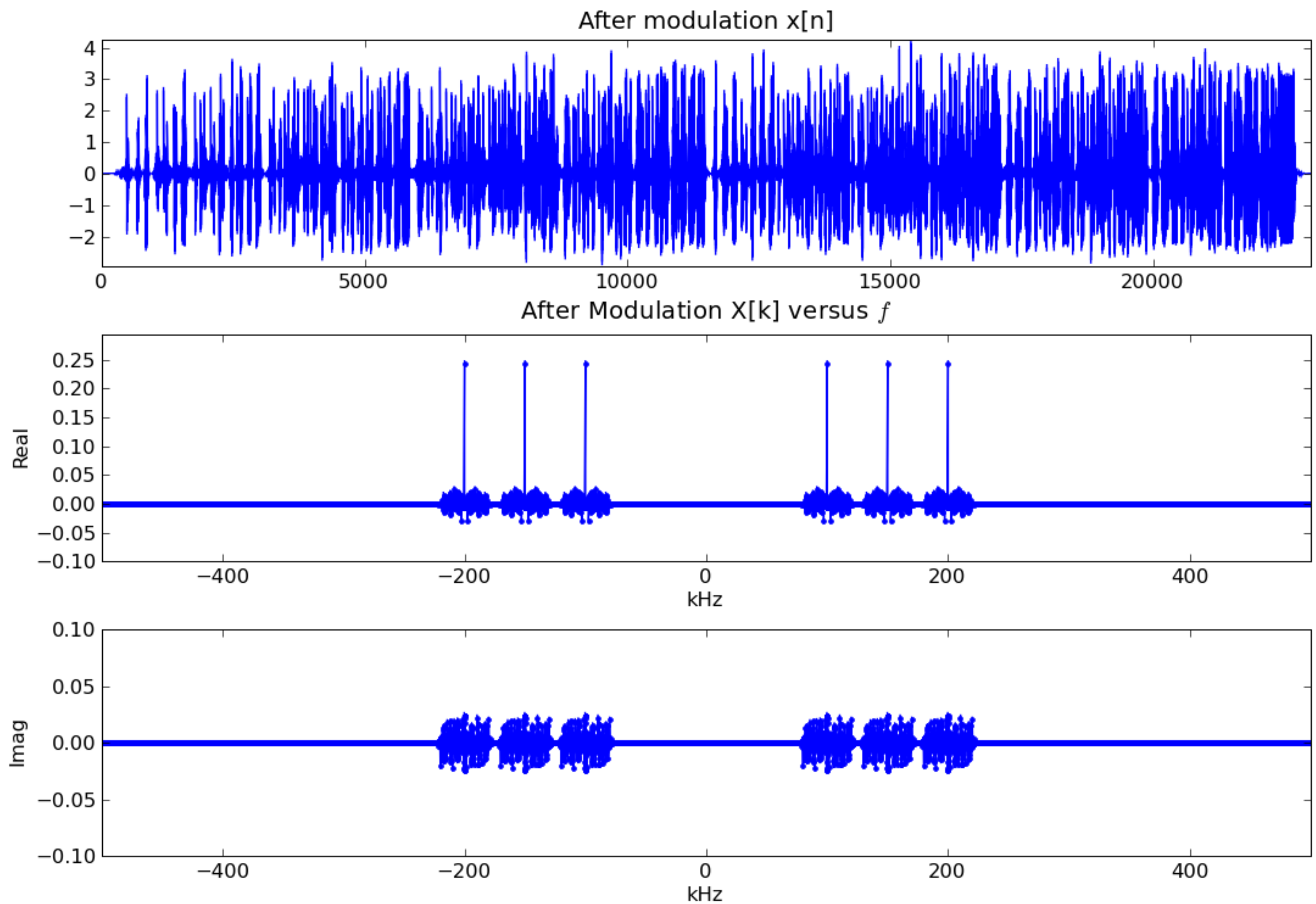
Eye Diagram for Received Bits



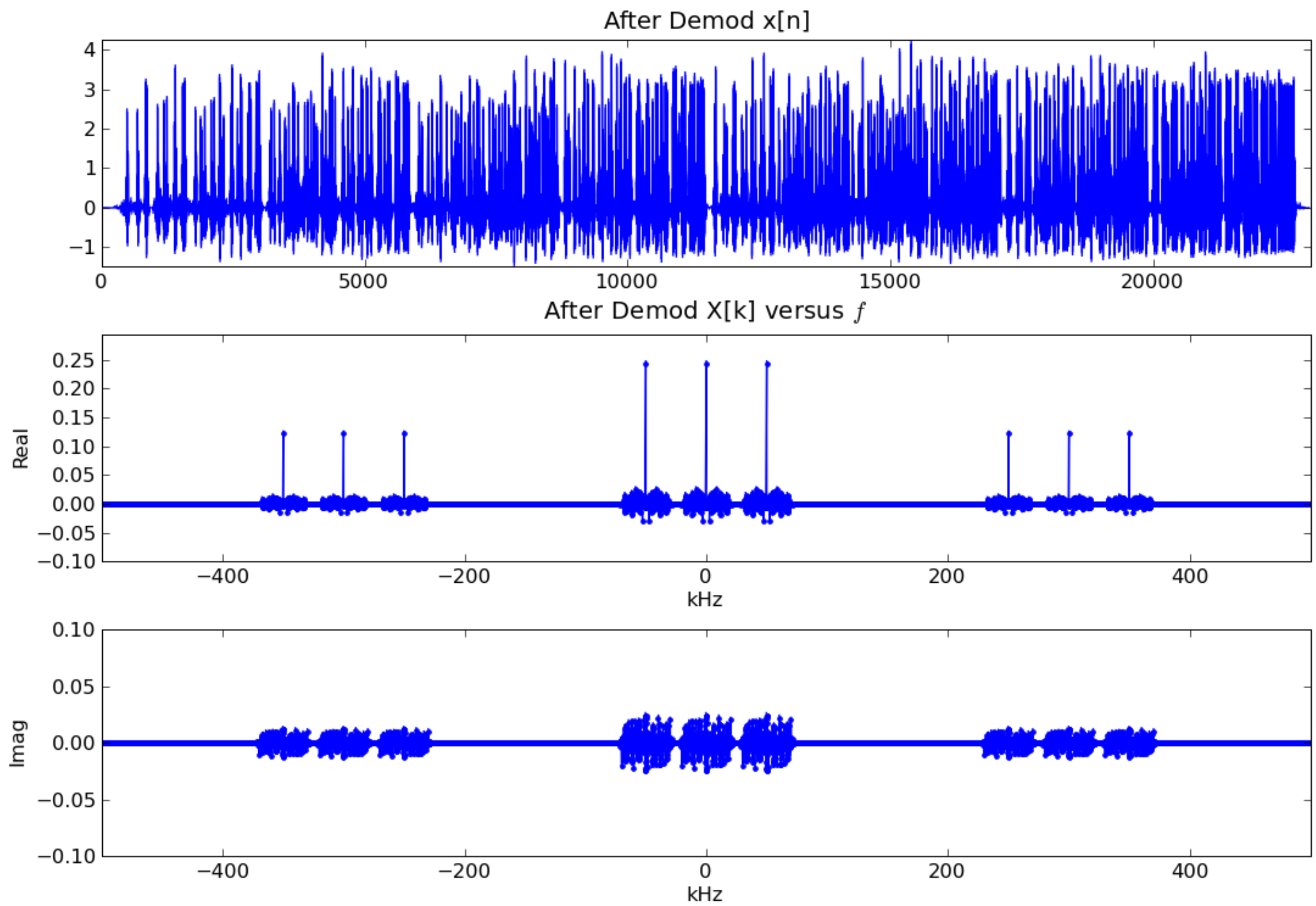
Long (920 bit) Transmission (25 samples/bit) with 21khz bandwidth transmit filter



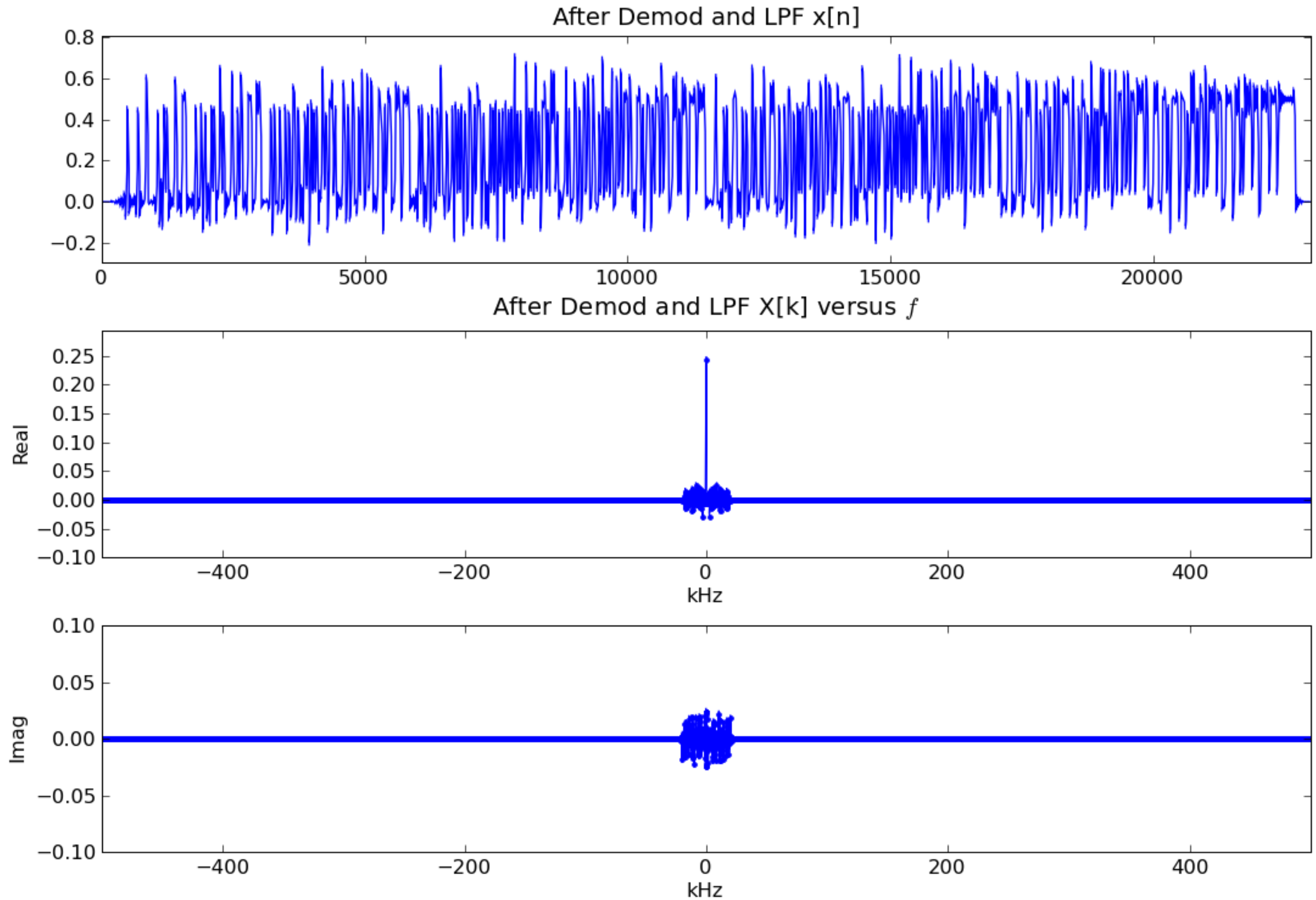
3 Simultaneous Transmitters: 100,150,200Khz with transmit filtered signals



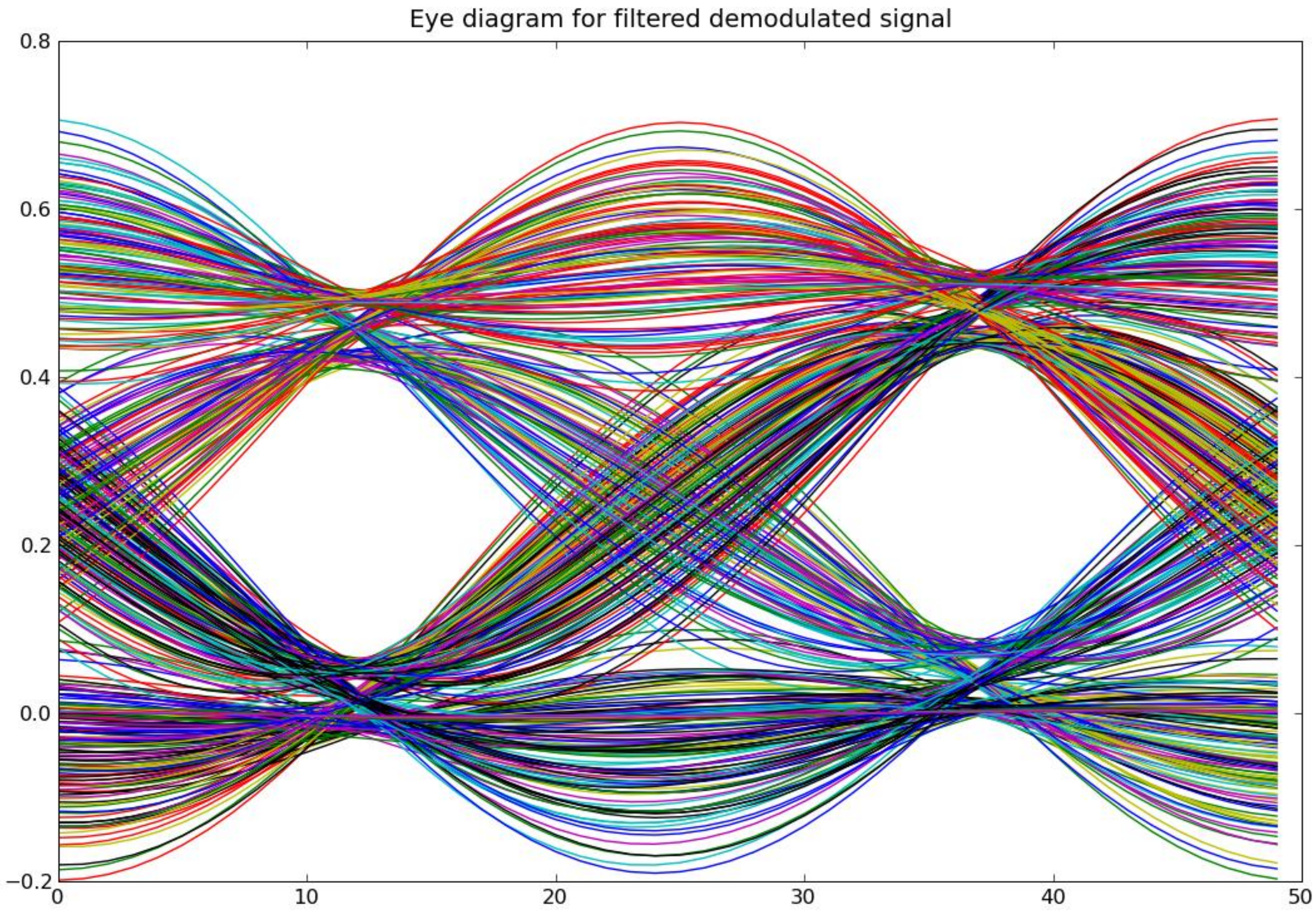
3 Transmitters after 150khz Demod with transmit filter



After Low-pass Filter (cutoff freq = 21Khz)



Eye Diagram received bits, transmit filters



Compare Eyes to See Channel Inteference