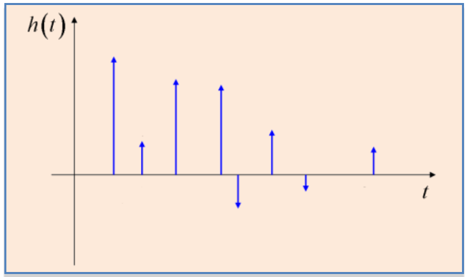
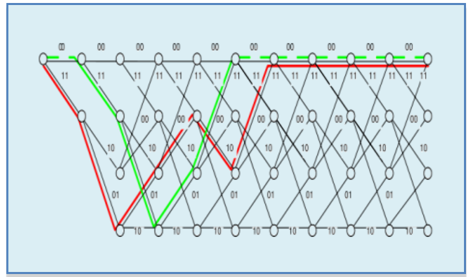
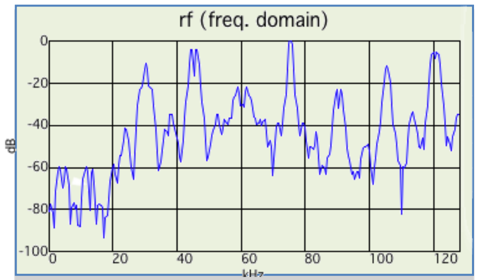
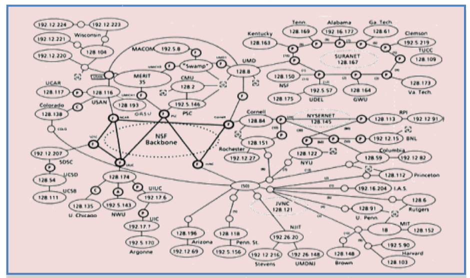








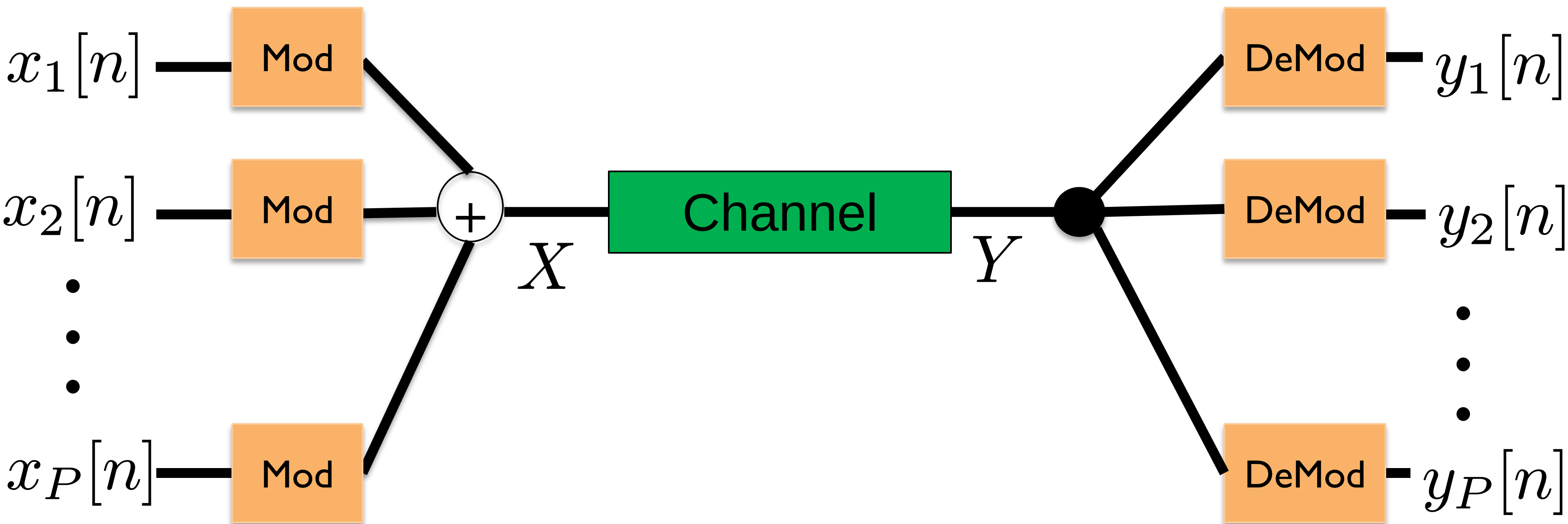
INTRODUCTION TO EECS II

DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2010
Lecture #16

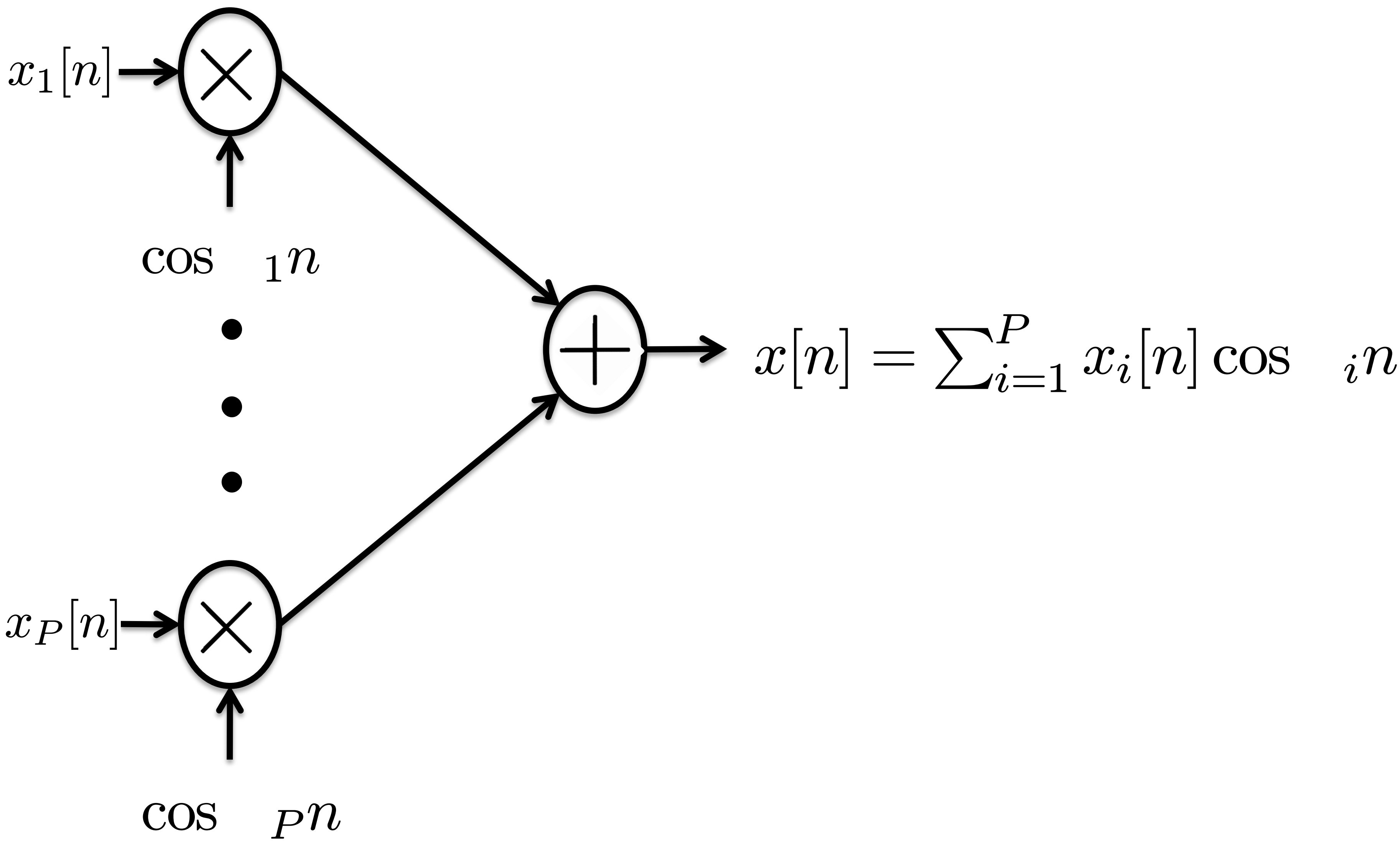
- Reminders
  - FD sharing
  - Fourier Series and Pictures
  - Mod and Demod
- New issue, sine mod -> cosine demod
  - Channel Delay

Frequency Domain Sharing - Big Picture



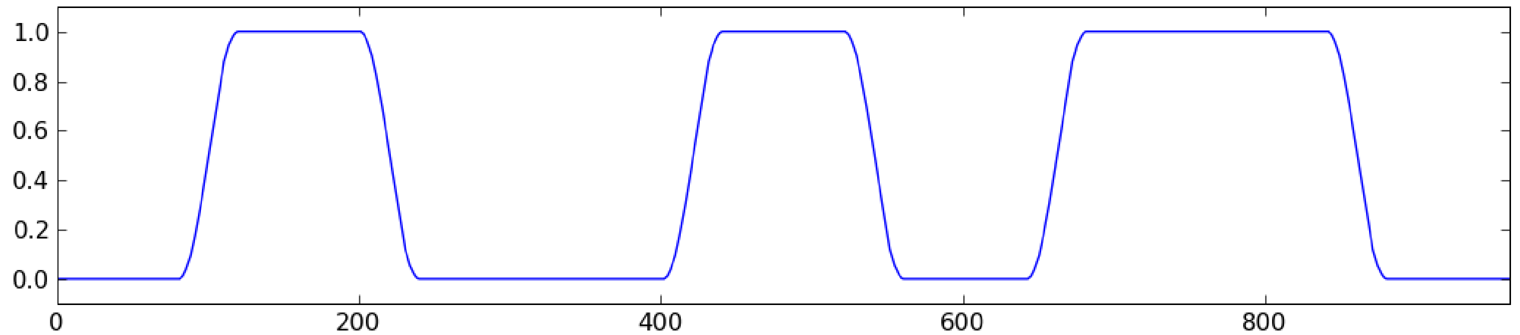
- Questions
  - What is Modulation (Mod in figure)?
  - What do typical X's look like?
  - What should the Demodulator be?
  - What is the relation between $x_i[n]$ and $y_i[n]$
  - What happens if there is channel delay (NEW)

Modulation by Cosine Multiplication

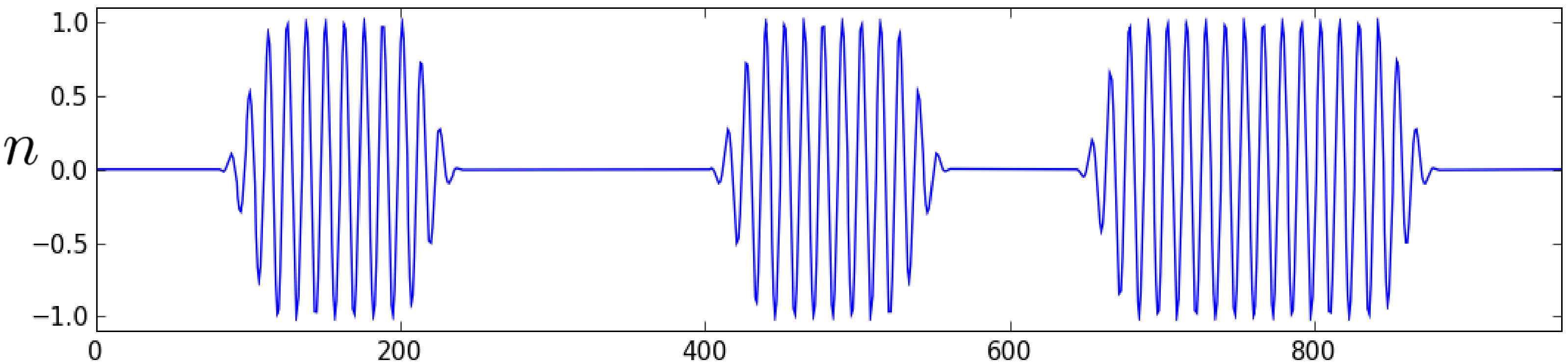


What does a modulated X look like?

$$x_i[n]$$



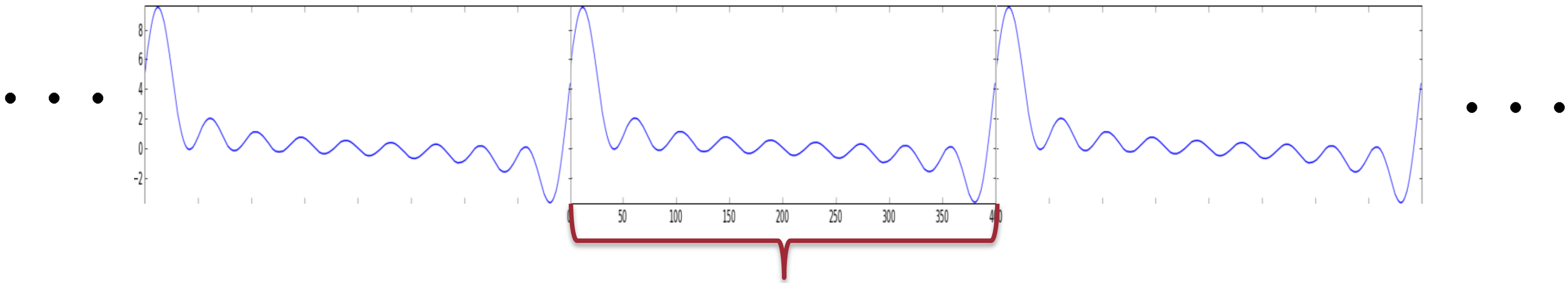
$$x_i[n] \cos 0.5n$$



Can Fourier Series Help us understand Modulation?

Periodic Assumption (with period *N*)

$$x[n] = x[n+N]$$



One Period

$N = 400$

- Periodic with Period N (N even) means
  - Only certain frequency complex exponentials

$$\Omega \in \left\{0, \pm\frac{2\pi}{N}, \pm 2\frac{2\pi}{N}, \ldots, \pm\left(\frac{N}{2}-1\right)\frac{2\pi}{N}, \pm\pi\right\}$$

  - x[n] can be represented with a Fourier series

Discrete-Time Fourier Series

$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j\frac{2\pi}{N}kn} \qquad K = \frac{N}{2}$$

OR

$$x[n] = \sum_{k=-K}^{K-1} X[k] e^{j\Omega_k n} \qquad -\pi \leq \Omega_k \leq \pi$$

Key DTFS Modulation Identity

$$x[n]\cos\Omega_m[n] = \left(\sum_{k=-K}^{K-1} X[k]e^{j\Omega_k n}\right)\left(\frac{1}{2}e^{j\Omega_m n} + \frac{1}{2}e^{-j\Omega_m n}\right)$$

$$= \frac{1}{2}\sum_{k=-K}^{K-1} X[k]e^{j(\Omega_k+\Omega_m)n} + \frac{1}{2}\sum_{k=-K}^{K-1} X[k]e^{j(\Omega_k-\Omega_m)n}$$

Key DTFS Modulation Identity

Modulation Frequency

$$x[n]\cos\Omega_m[n] = \left(\sum_{k=-K}^{K} X[k]e^{j\Omega_k n}\right)\left(\frac{1}{2}e^{j\Omega_m n} + \frac{1}{2}e^{-j\Omega_m n}\right)$$

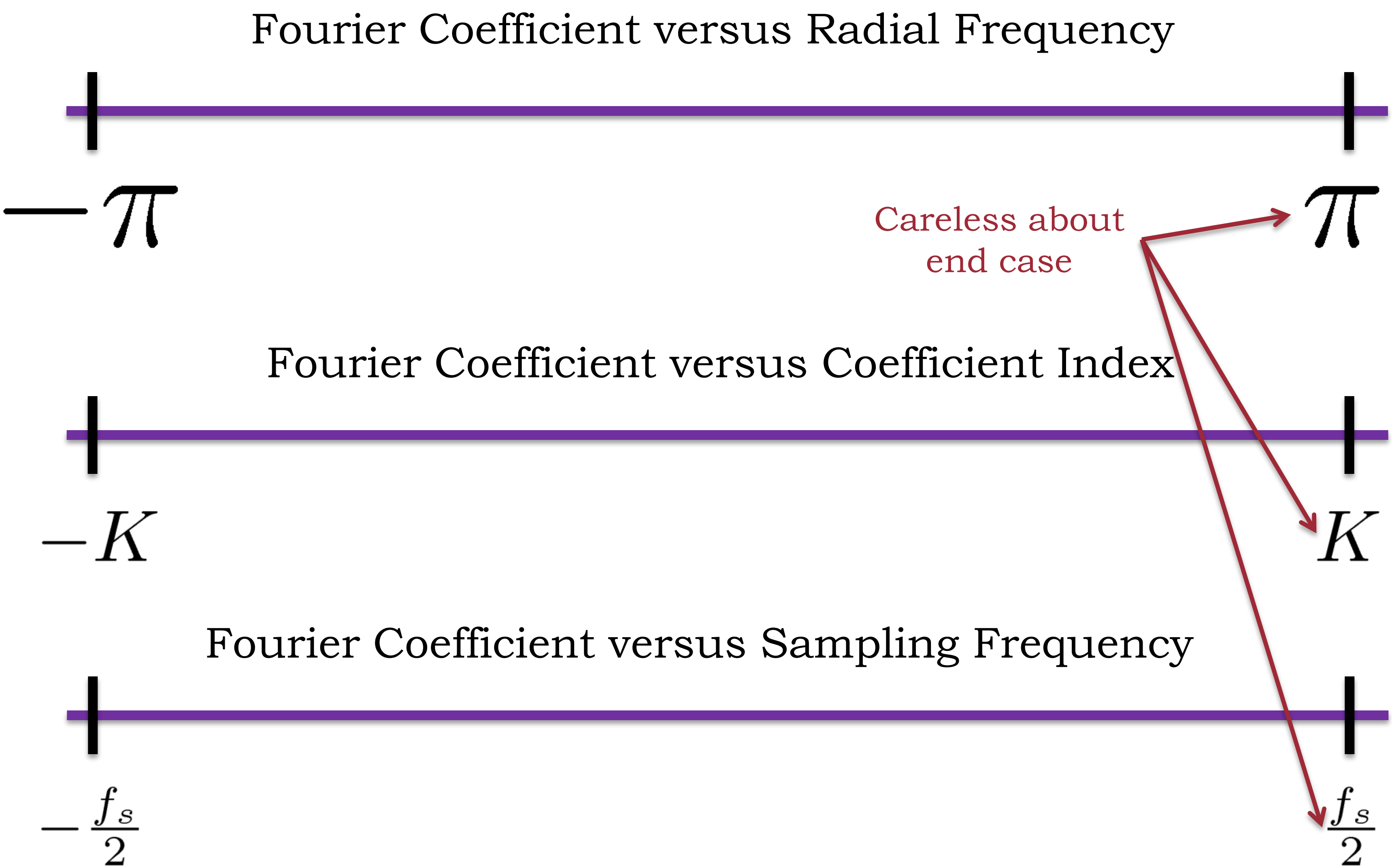
<u>Note:</u> $\Omega_K = \frac{2\pi}{N}K \quad \Omega_M = \frac{2\pi}{N}M$ } <u>Discrete Frequenc</u>

$$= \frac{1}{2}\sum_{k=-K}^{K} X[k]e^{j(\Omega_k+\Omega_m)n} + \frac{1}{2}\sum_{k=-K}^{K} X[k]e^{j(\Omega_k-\Omega_m)n}$$

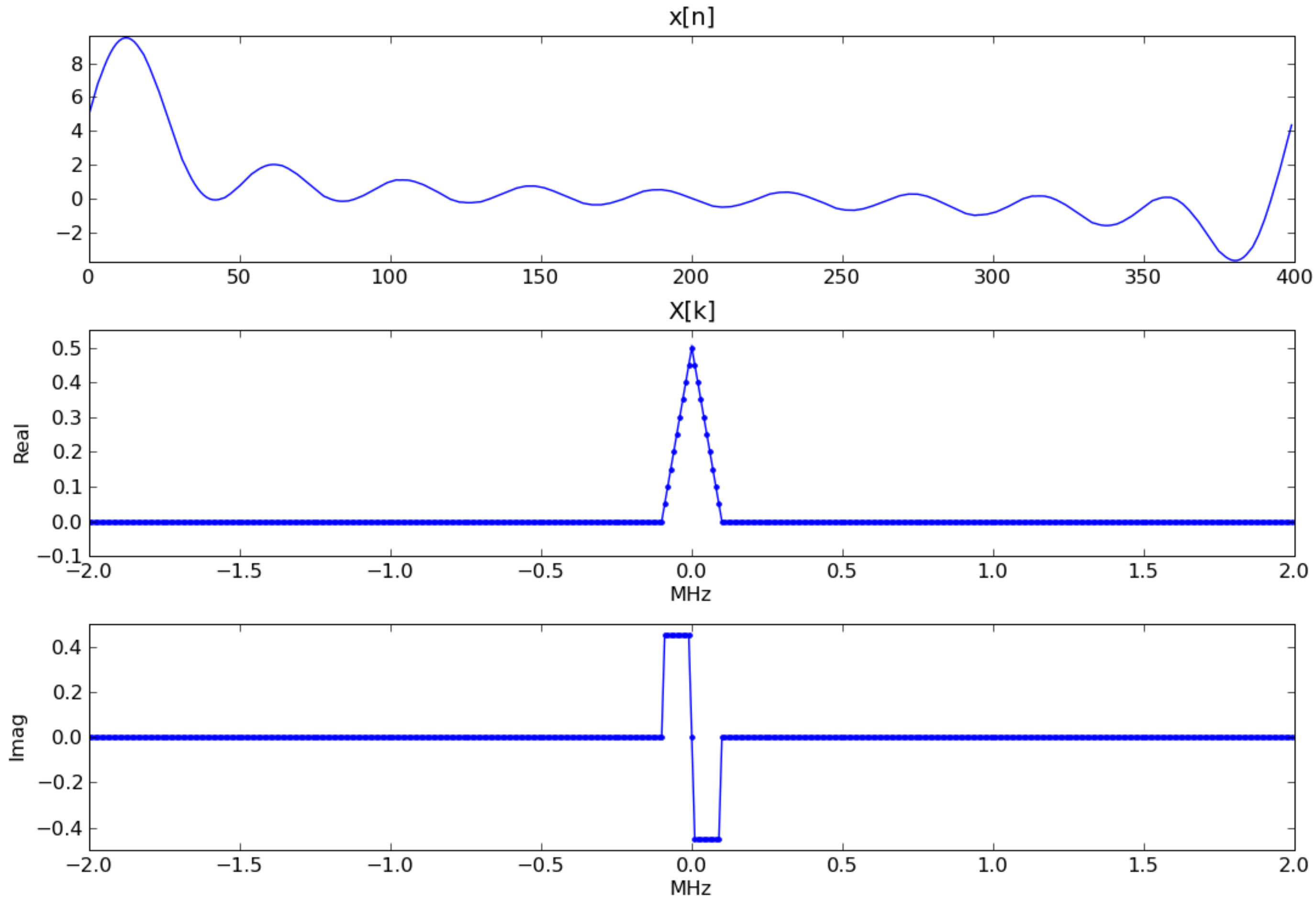
$$x[n]\sin\Omega_M[n] = \left(\sum_{k=-K}^{K} X[k]e^{j\Omega_k n}\right)\left(\frac{-j}{2}e^{j\Omega_m n} + \frac{j}{2}e^{-j\Omega_m n}\right)$$

$$= \frac{-j}{2}\sum_{k=-K}^{K} X[k]e^{j(\Omega_K+\Omega_M)n} + \frac{j}{2}\sum_{k=-K}^{K} X[k]e^{j(\Omega_K-\Omega_M)}$$

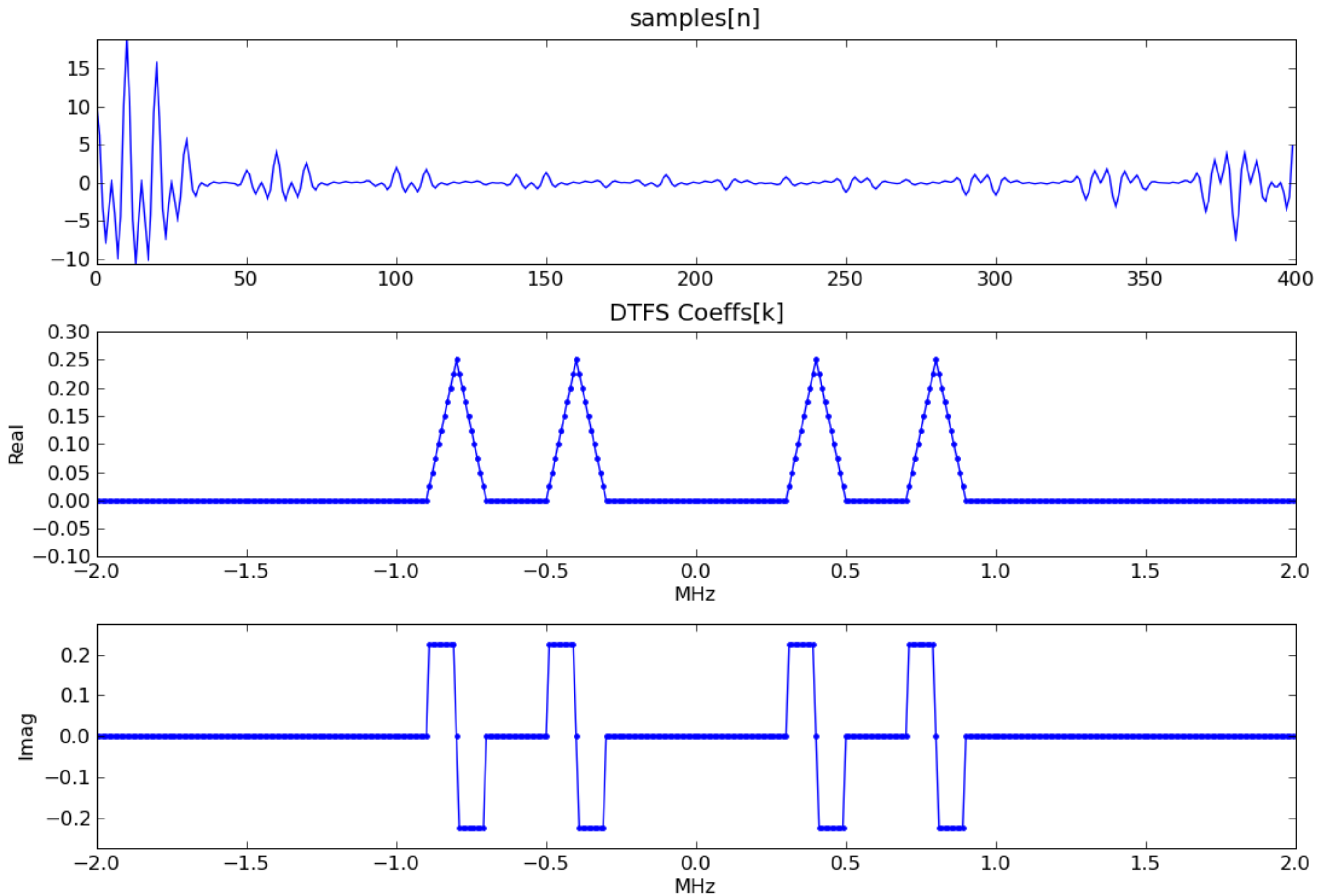
Frequency Axes Alternatives



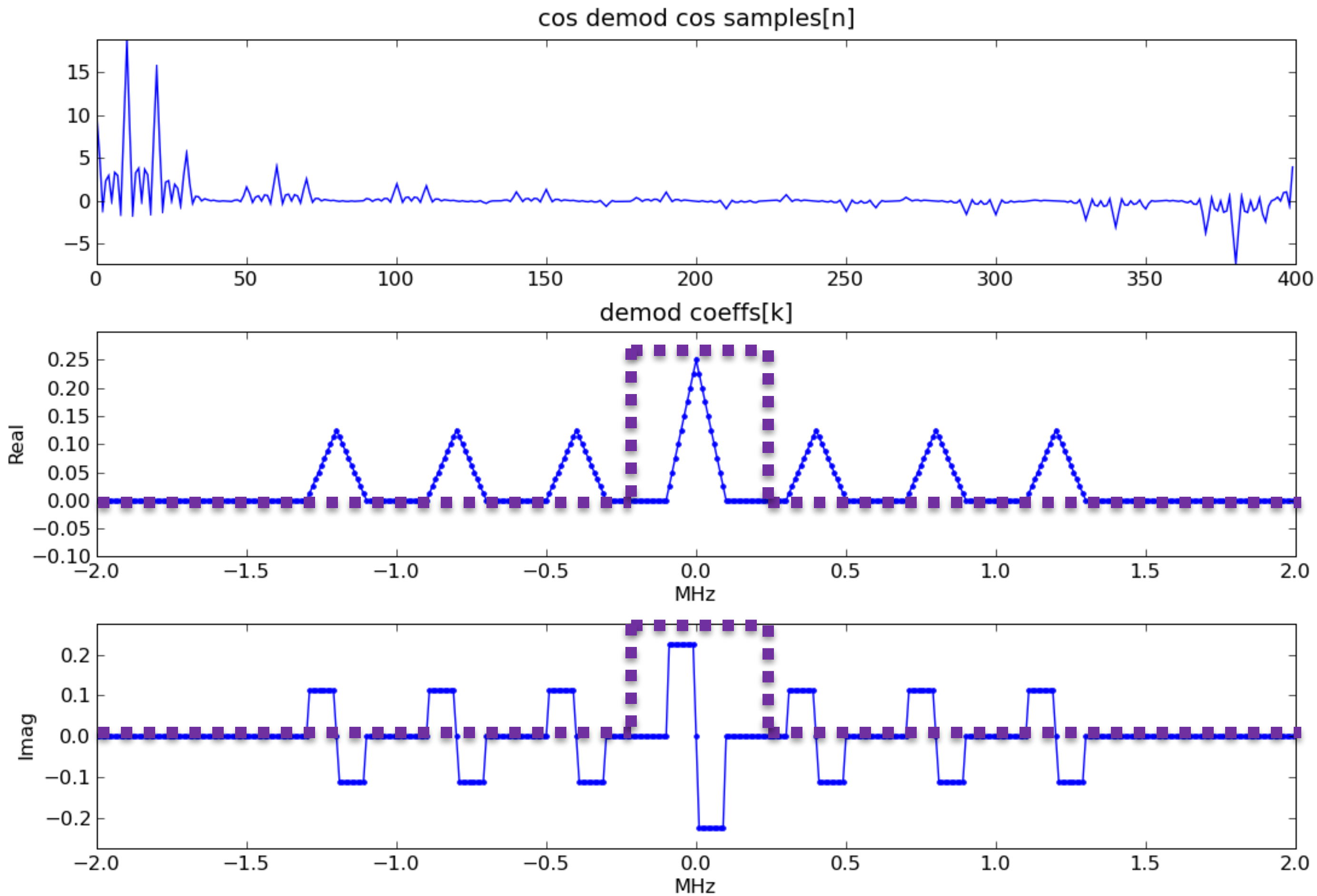
DTFS of x (versus 4Mhz sampling frequency)



DTFS of modulation using 400khz and 800khz cosine multiplication

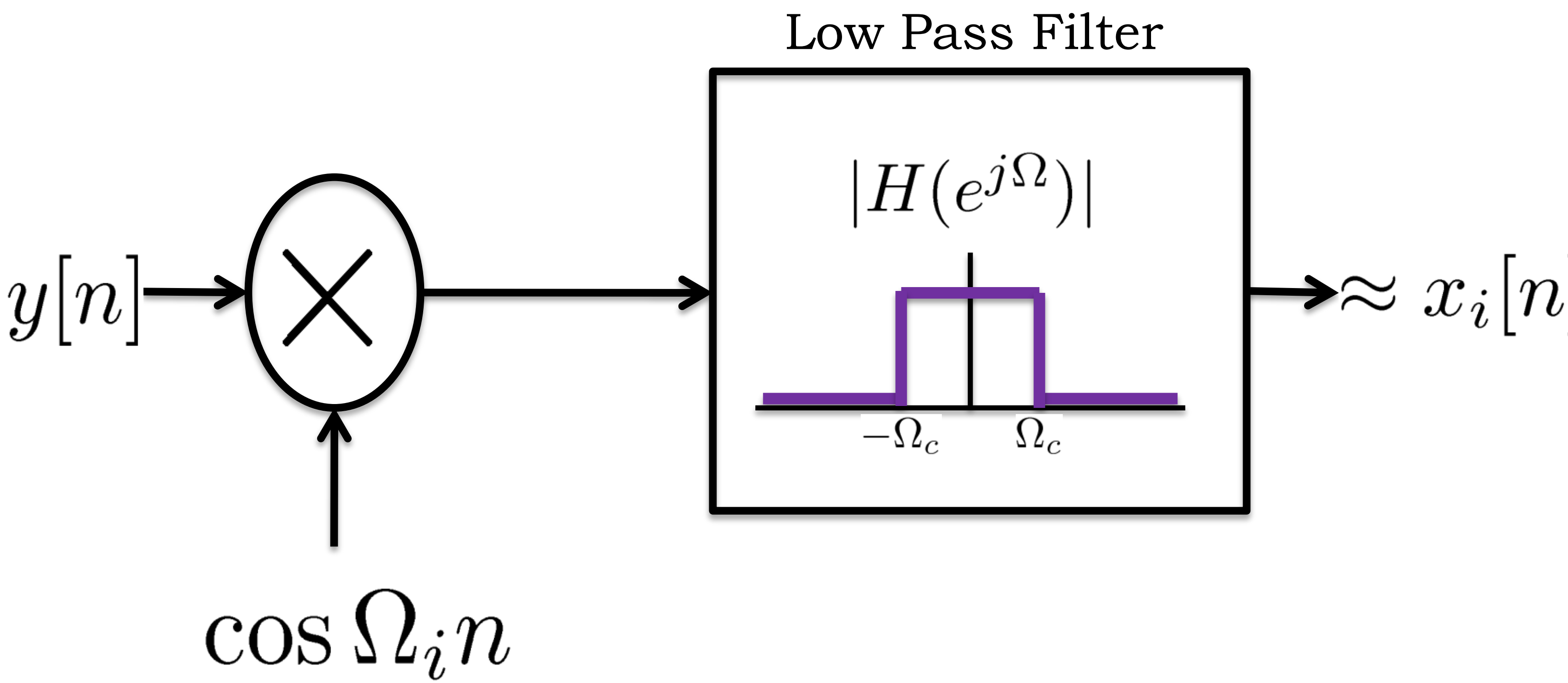


Mod with 400khz and 800khz cosines
Demod with a 400khz cosine

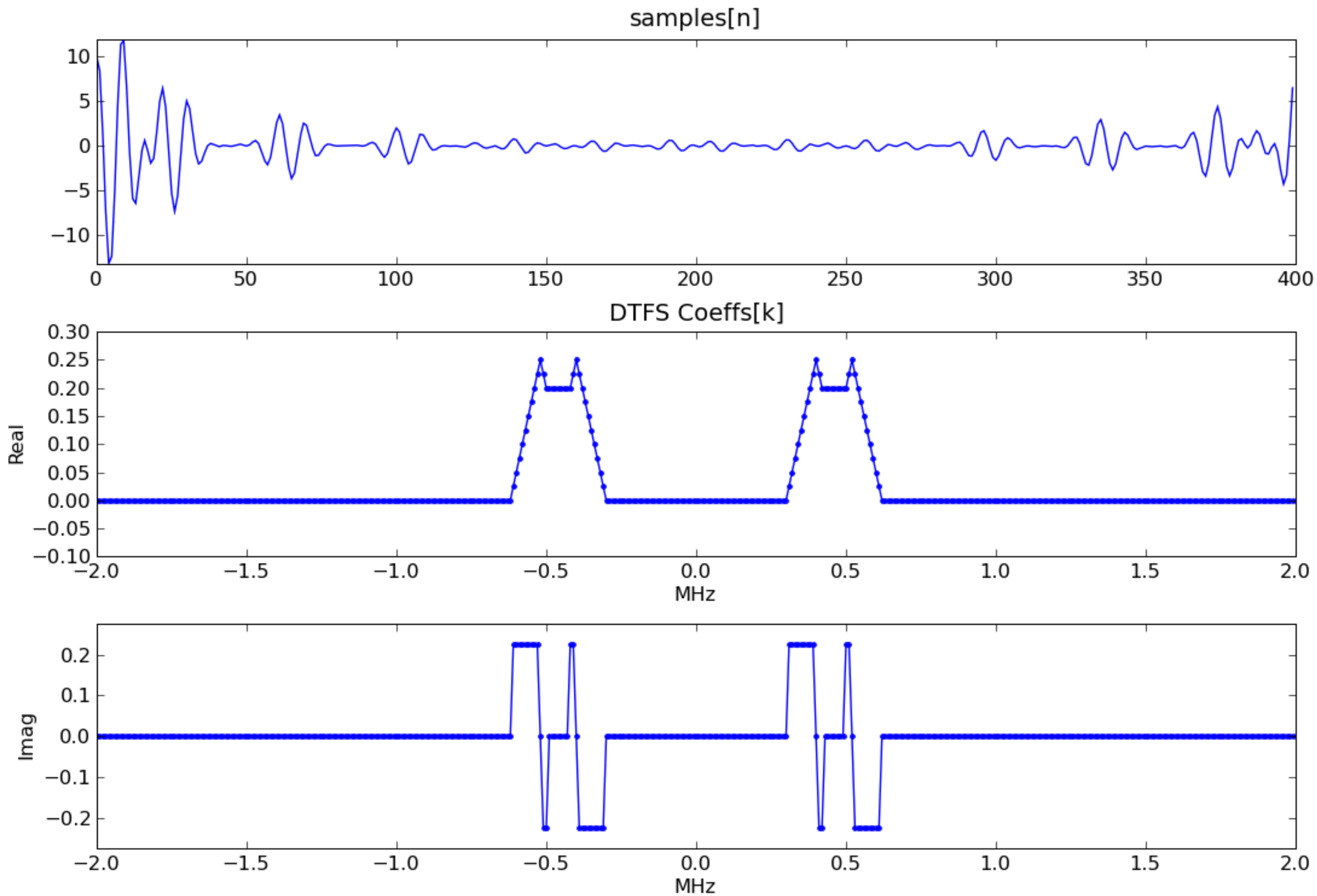


Demodulation using cosine multiplication and a Low-pass Filter (Ideal Channel)

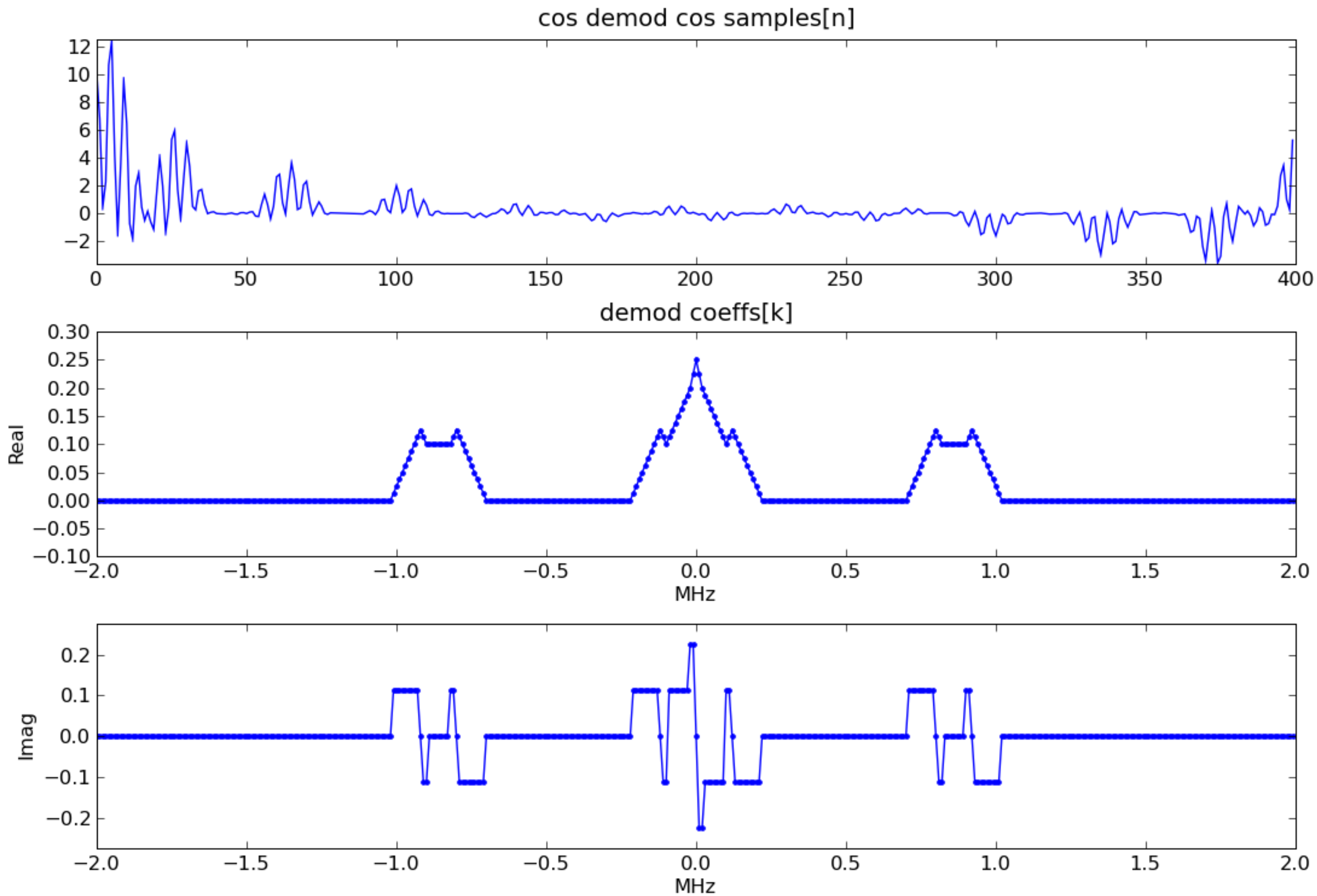
$$y[n] = x[n] = \sum_{i=1}^{P} x_i[n] \cos \Omega_i n$$



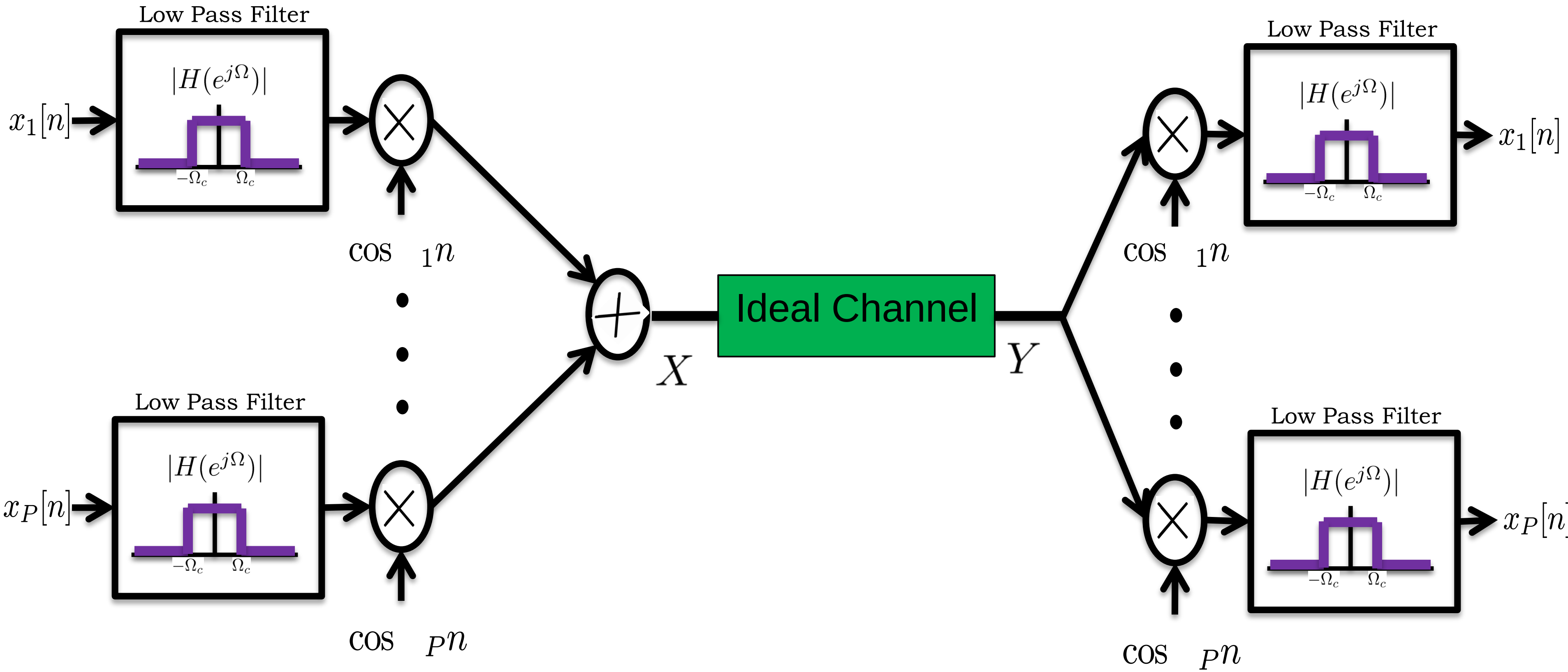
Mod by multiplication with nearby cosines (400khz, 520khz)



Demod of nearby cosine case (at 400khz)

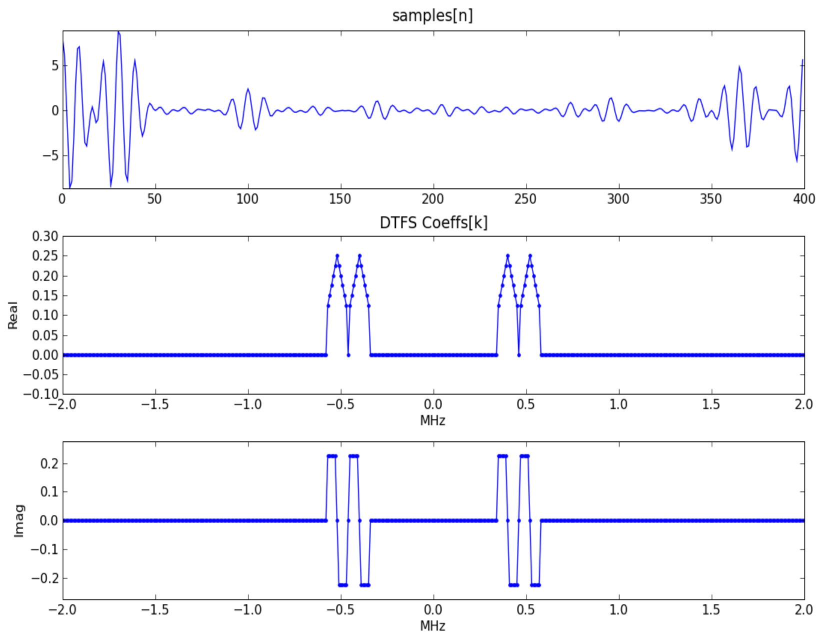


Modulation/Demodulation using cosine multiplication and I/O LPF's

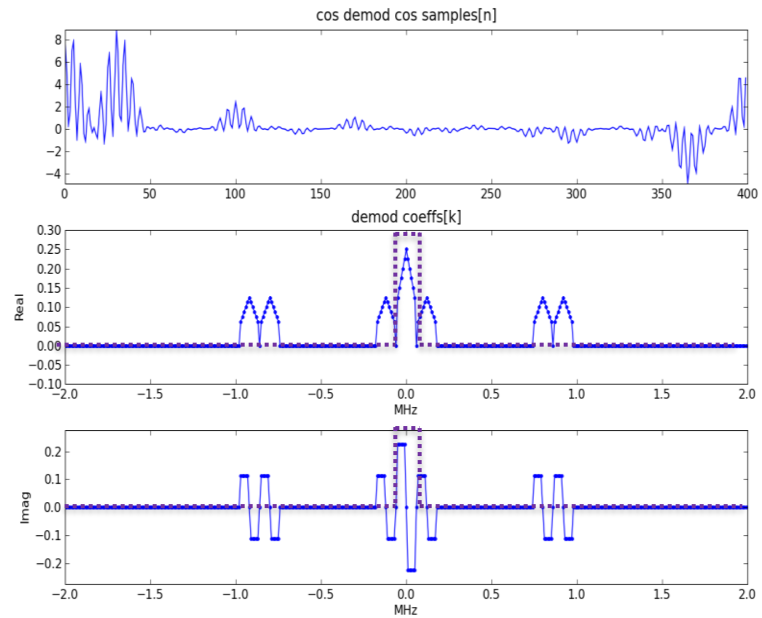


LPF'd before cosine multiplication case

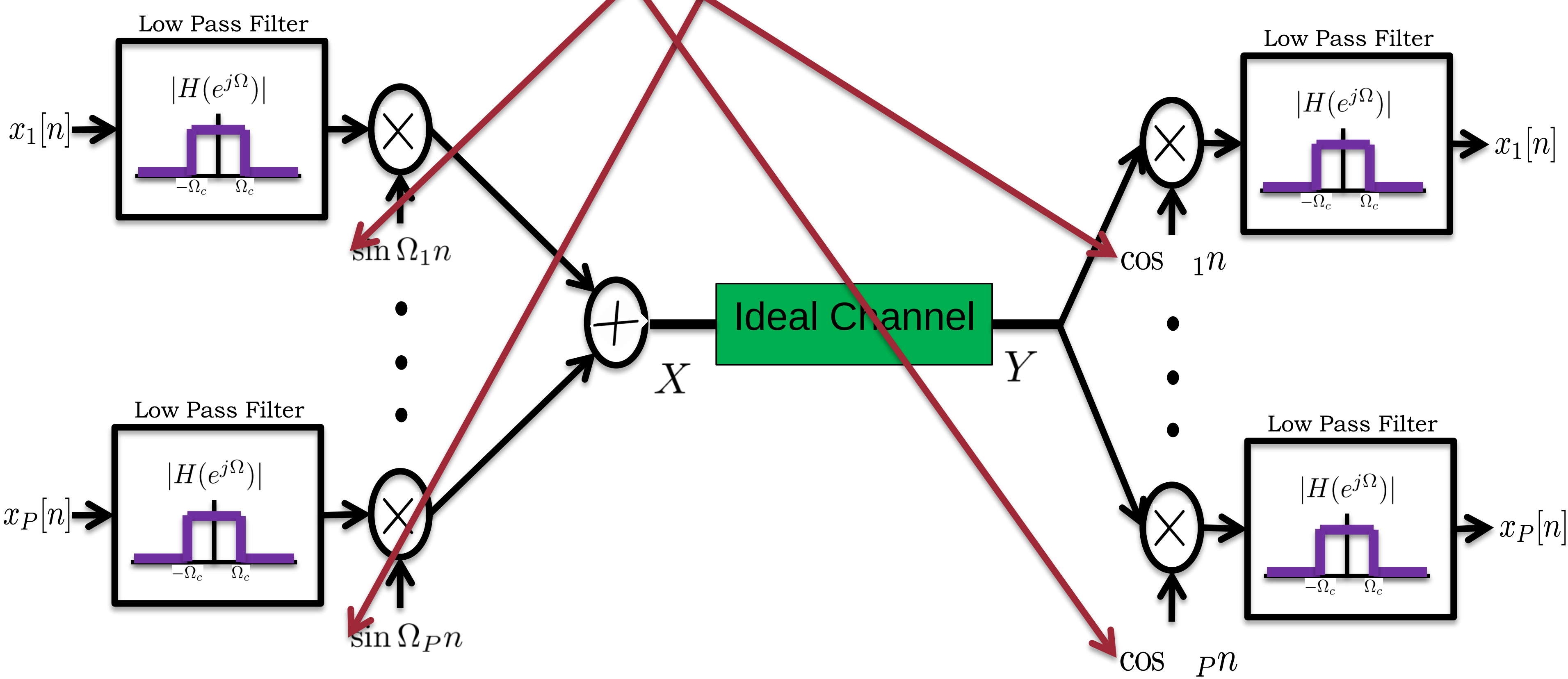
LPF then Modulation by Cosine Multiplication



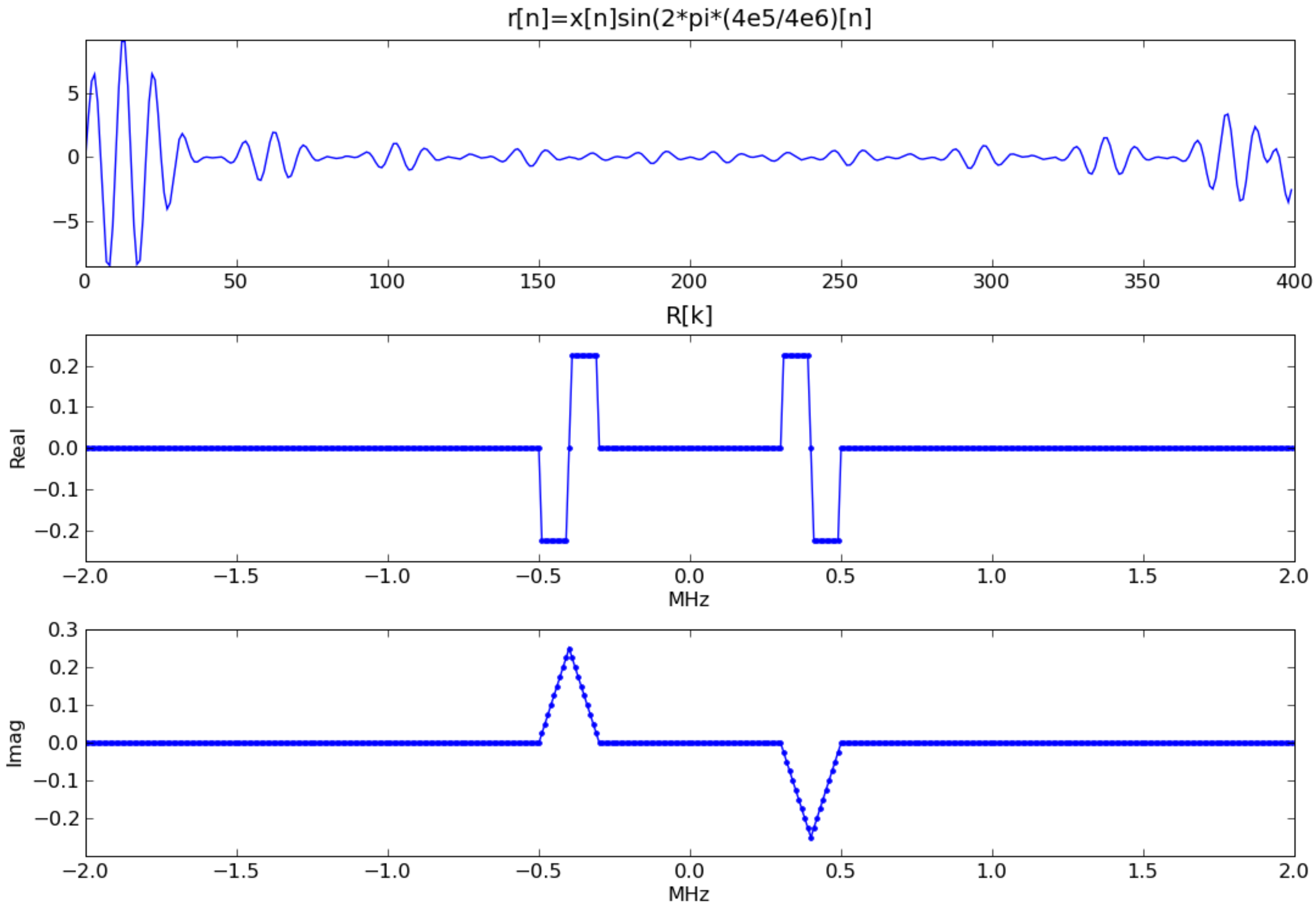
After Demodulation by Cosine Multiplication



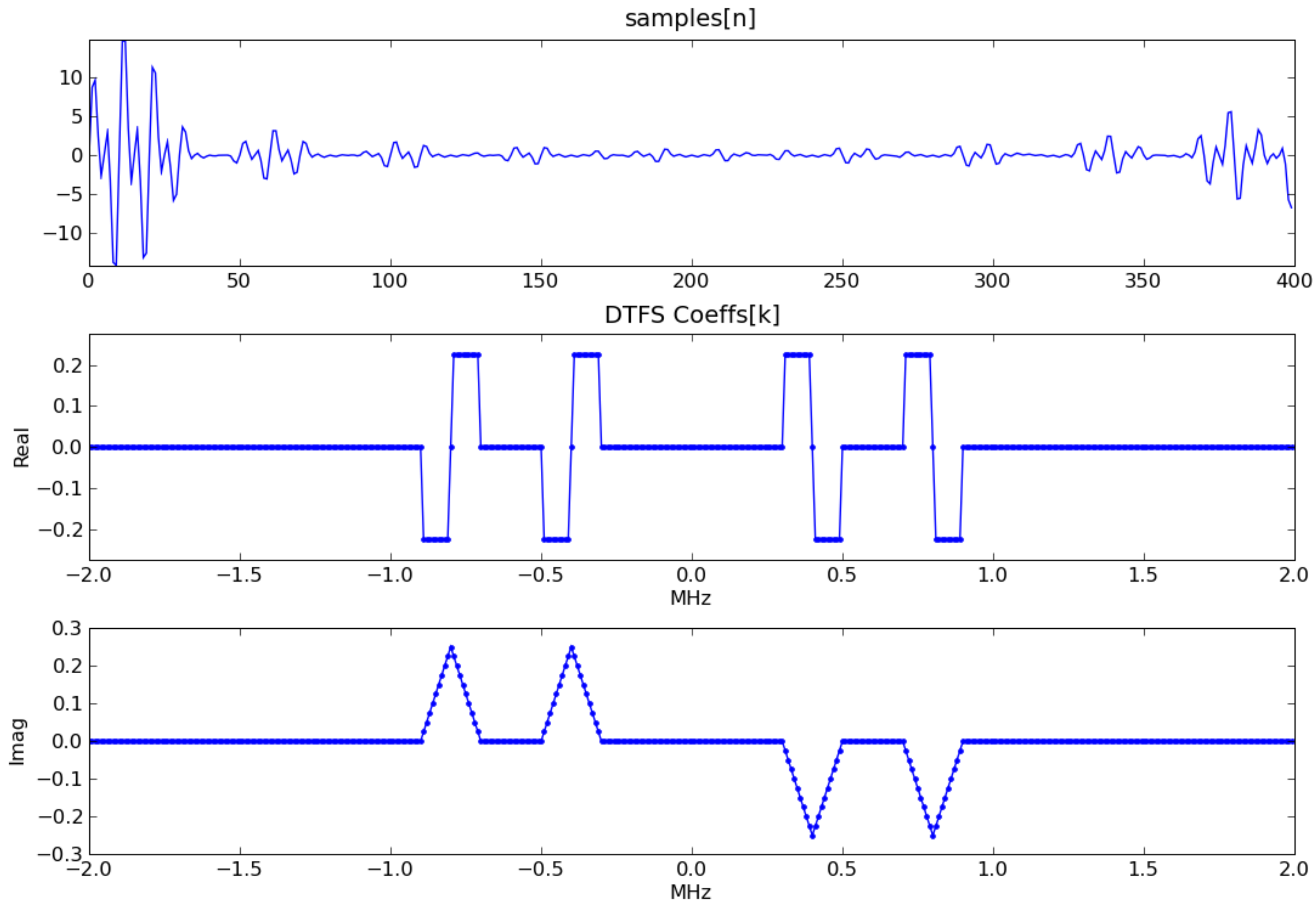
Mod by LPF then Sine Multiplication, Demod by Cosine Multiplication then LPF



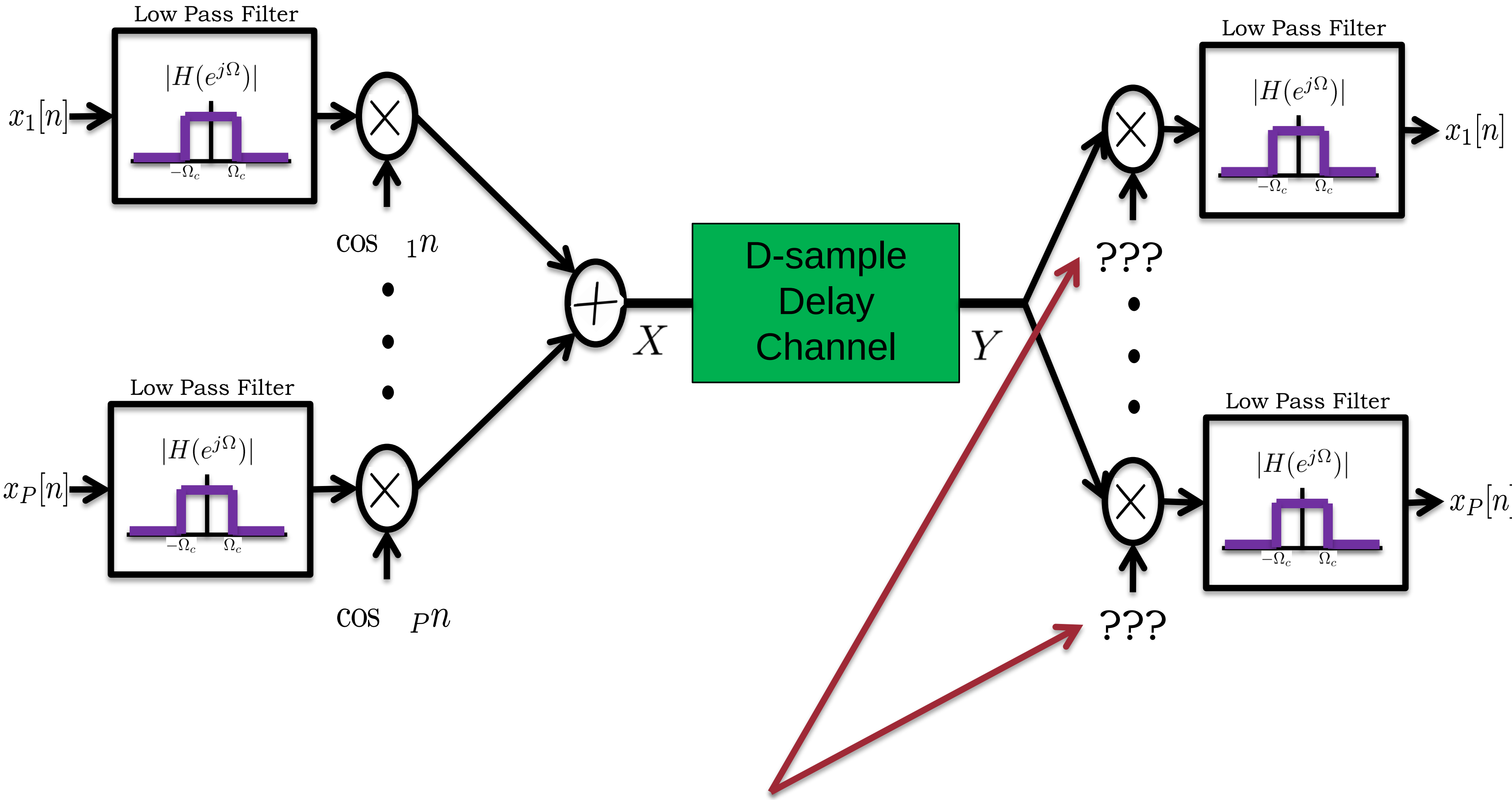
DTFS of Mod by sine multiplication



DTFS of Demod by cos multiplication, From Mod by sin multiplication



Mod by Cos Multiplication with Channel Delay



With what shall I demodulation it dear Liza, dear Liza?

Analyzing Wire Delay

Modulated signal

$$x[n]\cos\Omega n = x[n]\left(\tfrac{1}{2}e^{j\Omega n} + \tfrac{1}{2}e^{-j\Omega n}\right)$$

Modulated and Delayed signal

D = # samples of Delay

Received Signal

$$x[n-D]\cos(\Omega(n-D)) =$$

$$x[n-D]\left(\tfrac{1}{2}e^{-j\Omega D}e^{j\Omega n} + \tfrac{1}{2}e^{j\Omega D}e^{-j\Omega n}\right)$$

Demodulate with Cosine

$$x[n-D]\left(\frac{e^{-j\Omega D}e^{j\Omega n}}{2} + \frac{e^{j\Omega D}e^{-j\Omega n}}{2}\right)\left(\frac{e^{j\Omega n}}{2} + \frac{e^{-j\Omega n}}{2}\right)$$

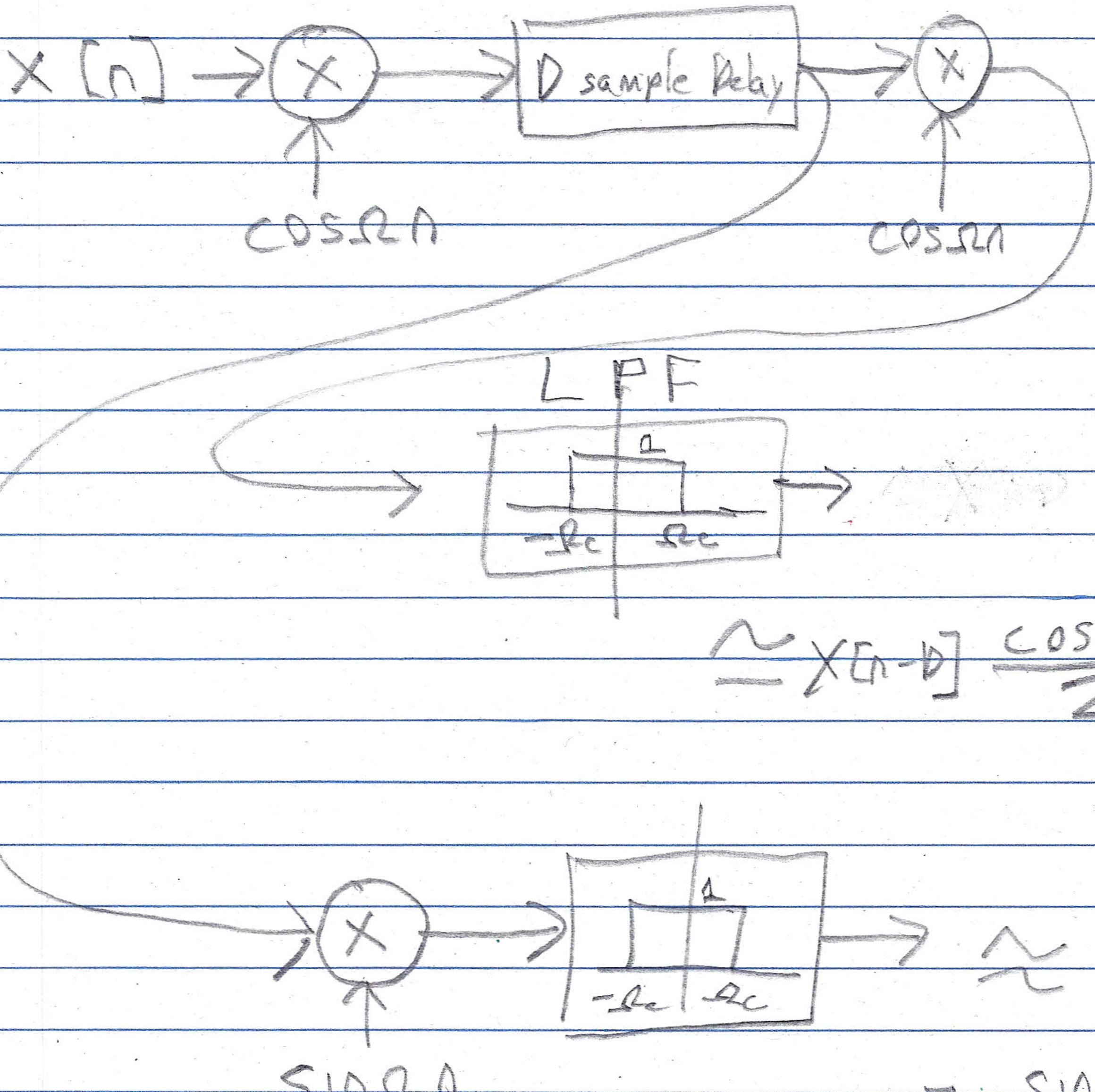
$$x[n-D]\left(\frac{e^{j2\Omega n}}{4}e^{-j\Omega D} + \frac{e^{-j2\Omega n}}{4}e^{j\Omega D} + \frac{1}{4}e^{j\Omega D} + \frac{1}{4}e^{-j\Omega D}\right)$$

Demod with sine

$$x[n-D]\left(\quad \cdot \quad \right)\left(\frac{-j}{2}e^{j\Omega n} + \frac{j}{2}e^{j\Omega n}\right)$$

Assuming $X[n]$ is band limited

$$\Omega_c < \Omega$$



$$\simeq X[n-D] \frac{\cos \Omega D}{2}$$

$$\simeq X[n-D] \frac{\sin \Omega D}{2}$$

If $D = 0$: $X[n-D] \frac{\cos \Omega D}{2} = \frac{1}{2} X[n-D]$

$X[n-D] \frac{\sin \Omega D}{2} = 0$

If $D\Omega = \pi/2$: $X[n-D] \frac{\cos \Omega D}{2} = 0$

$X[n-D] \frac{\sin \Omega D}{2} = \frac{1}{2} X[n-D]$

Example: Consider modulation and demodulation using sine and cosine multiplication. The signal, x[n], is multiplied by a cosine, then transmitted through a channel with a D-sample delay, then multiplie d by a sine and a cosine.

For this example, we will assume periodicity, with a N=1000 sample period. Then, the only frequencies that are periodic with period 1000 are multiples of

$$\Omega_1 = \frac{2 * \pi}{1000}.$$

As a specific case, let x[n] be given by

$$x[n] = 0.3 + 0.5\cos\Omega_1 n + 0.4\cos 2\Omega_1 n + 0.3\cos 3\Omega_1 n + 0.3\cos 4\Omega_1 n + 0.3\cos 5\Omega_1 n,$$

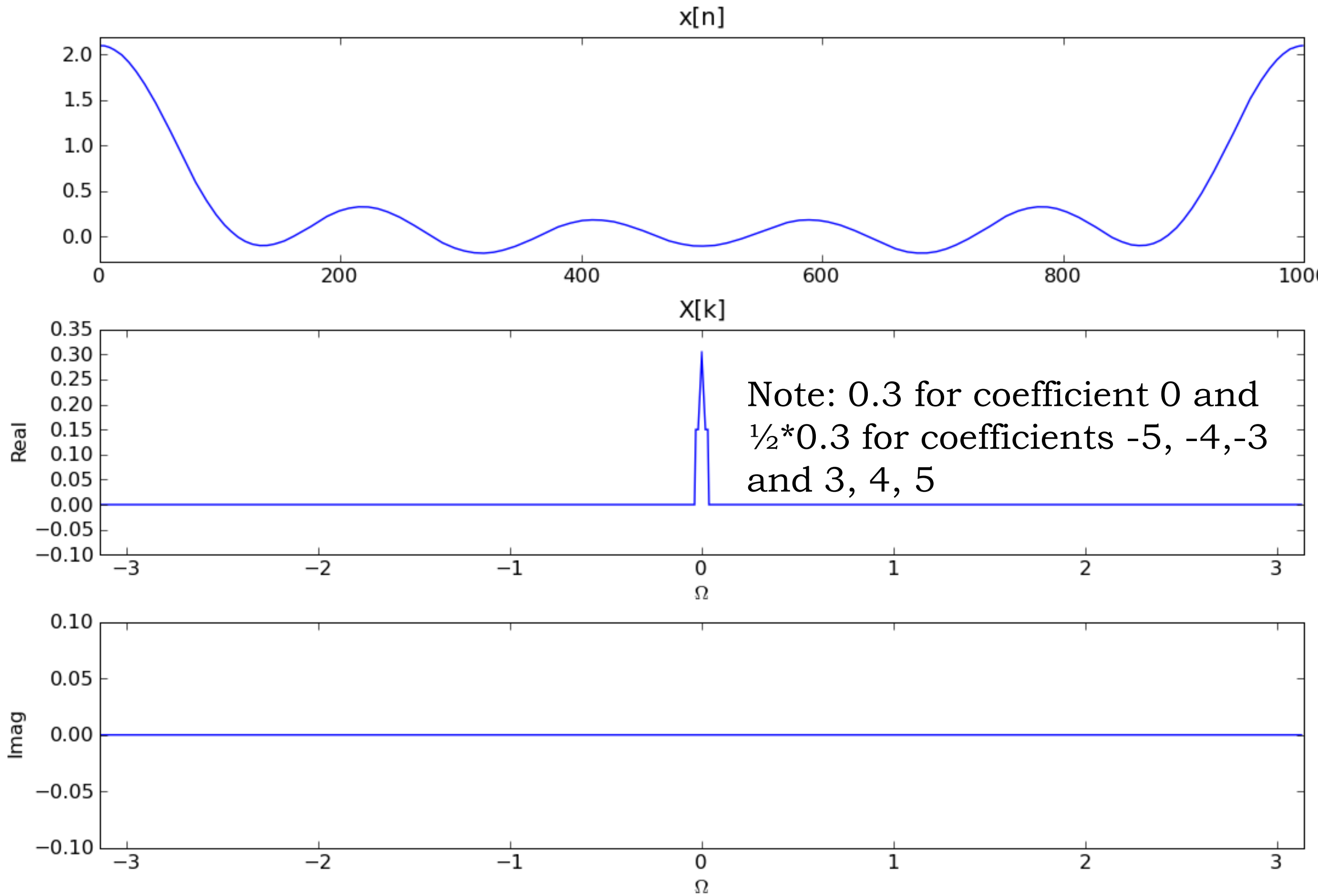
and be modulated by $\cos\Omega_{mod} n$

Note that if the channel has a five-sample delay, then

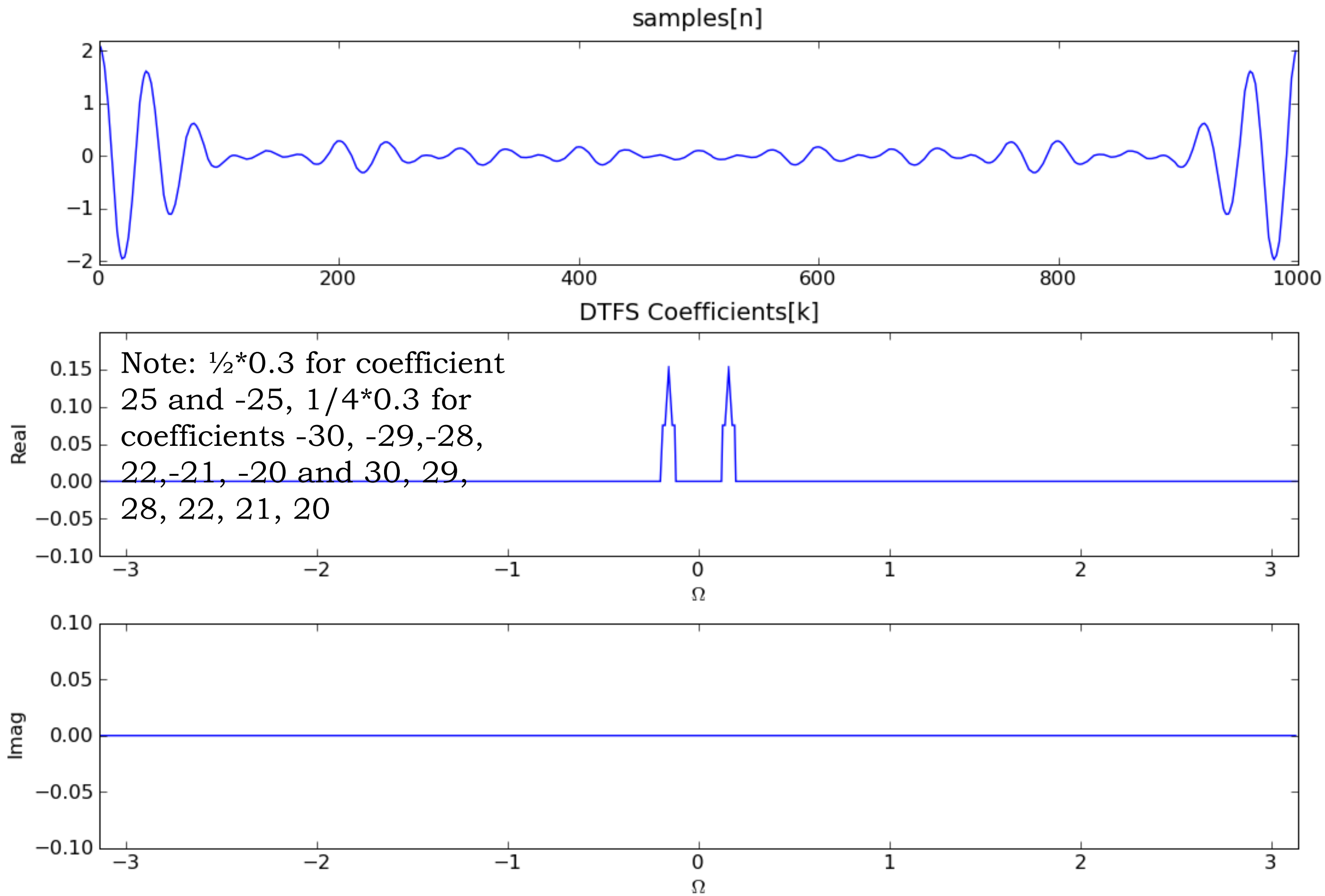
$$25\Omega_1 D = \frac{\pi}{2}$$

The plots on the following slides show the impact of a five-sample channel delay on modulation and demodulation for four different modulation frequencies (but always a five-sample delay). The different modulation frequencies will change the phase shift introduced by the delay, and generate different outputs from sine and cosine demodulation. In particular, the modulation frequencies considered are 25, 50, 75 and 100 times Ω_1.

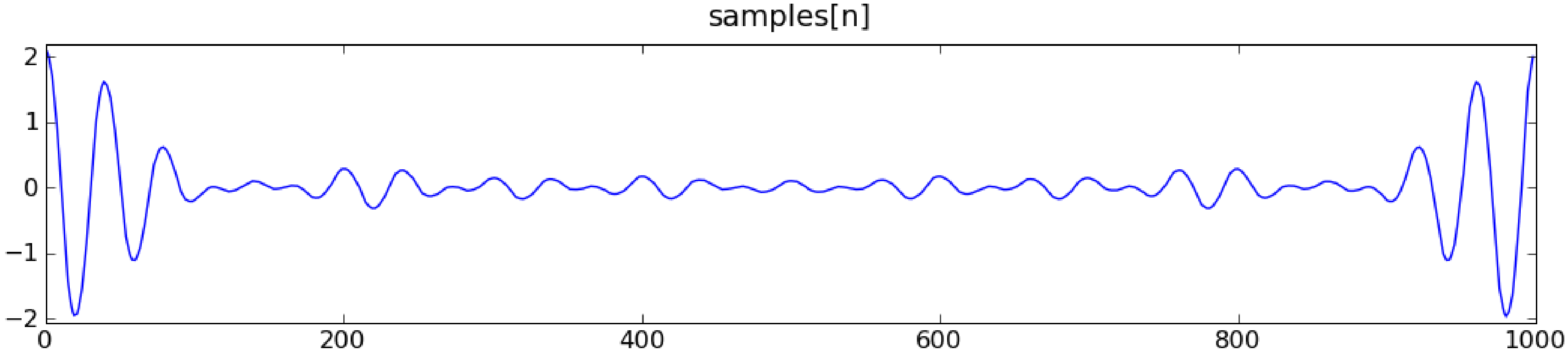
Plots of x[n] versus n, Real and Imaginary parts of X[k] versus Ω

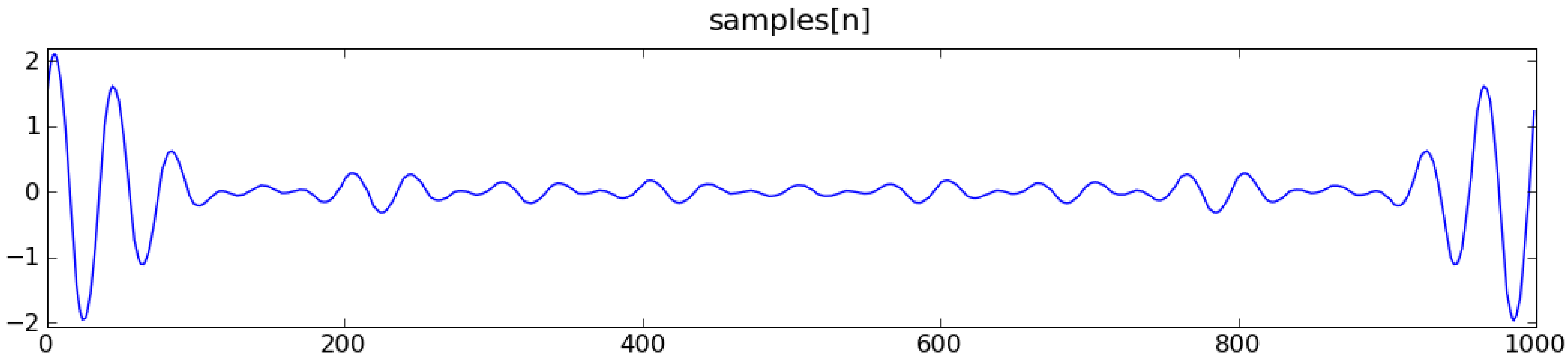


Modulation using multiplication by Cos25 Omega_1 n

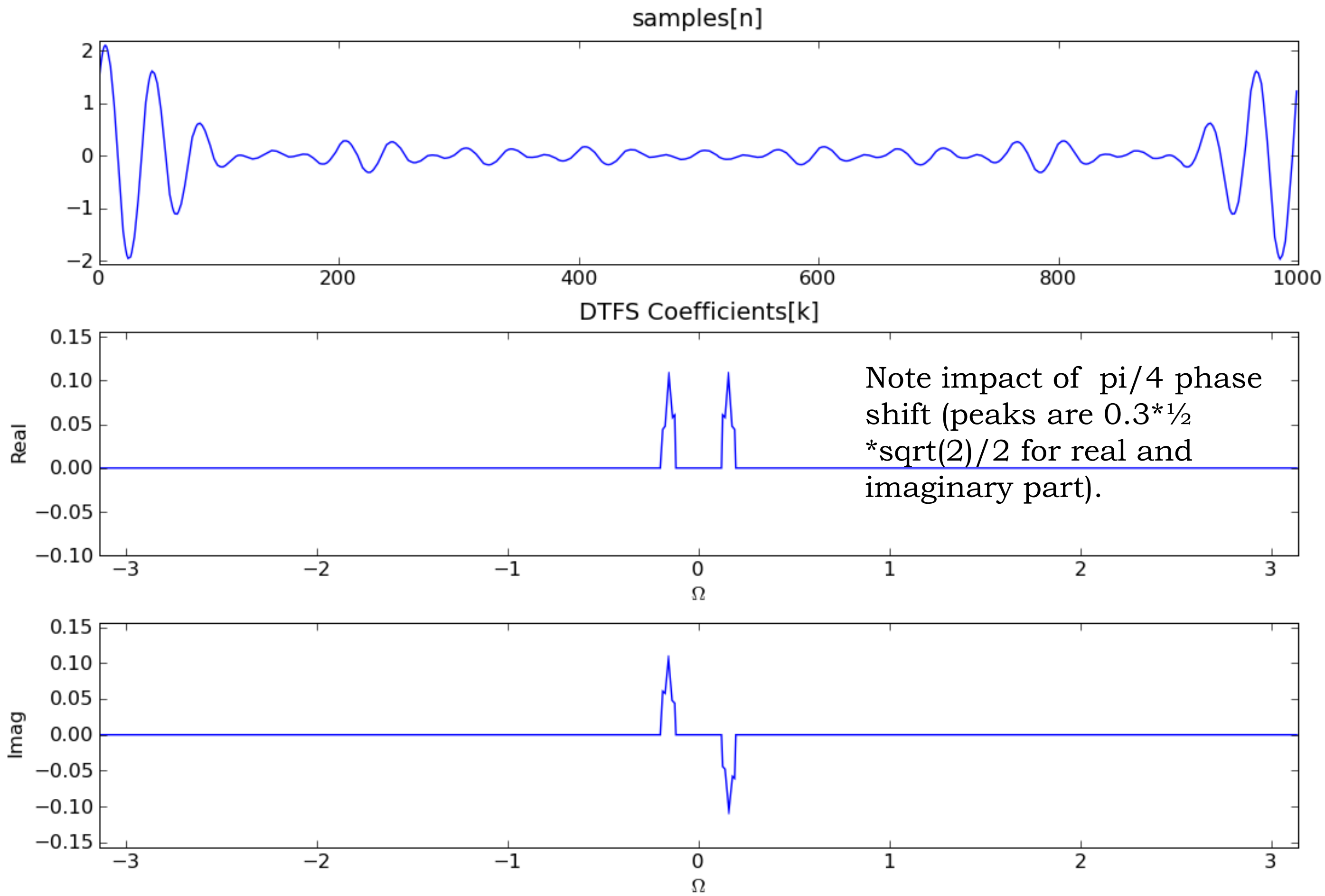


Plots of x after modulation without and with 5 sample delay (note periodicity!)

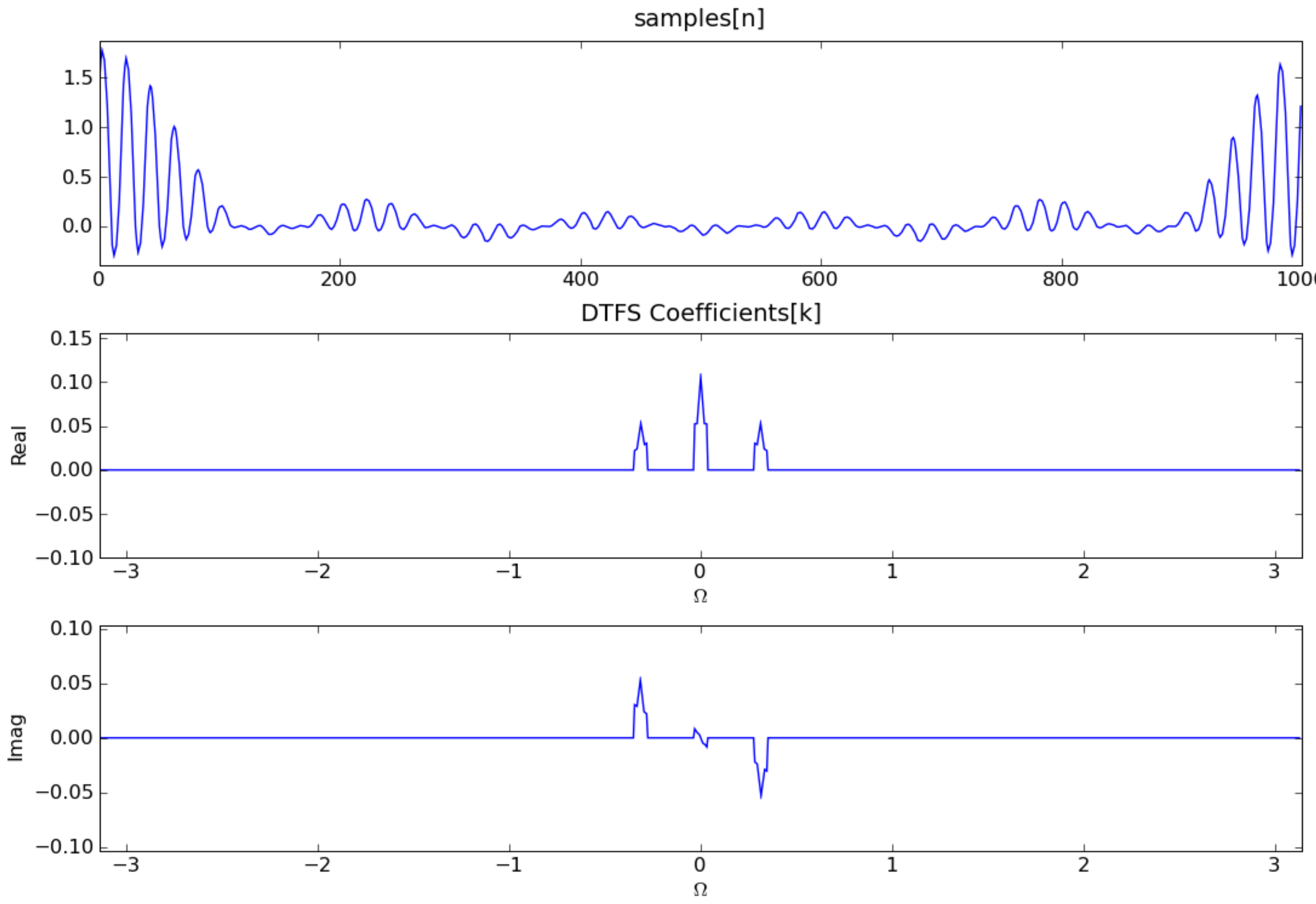




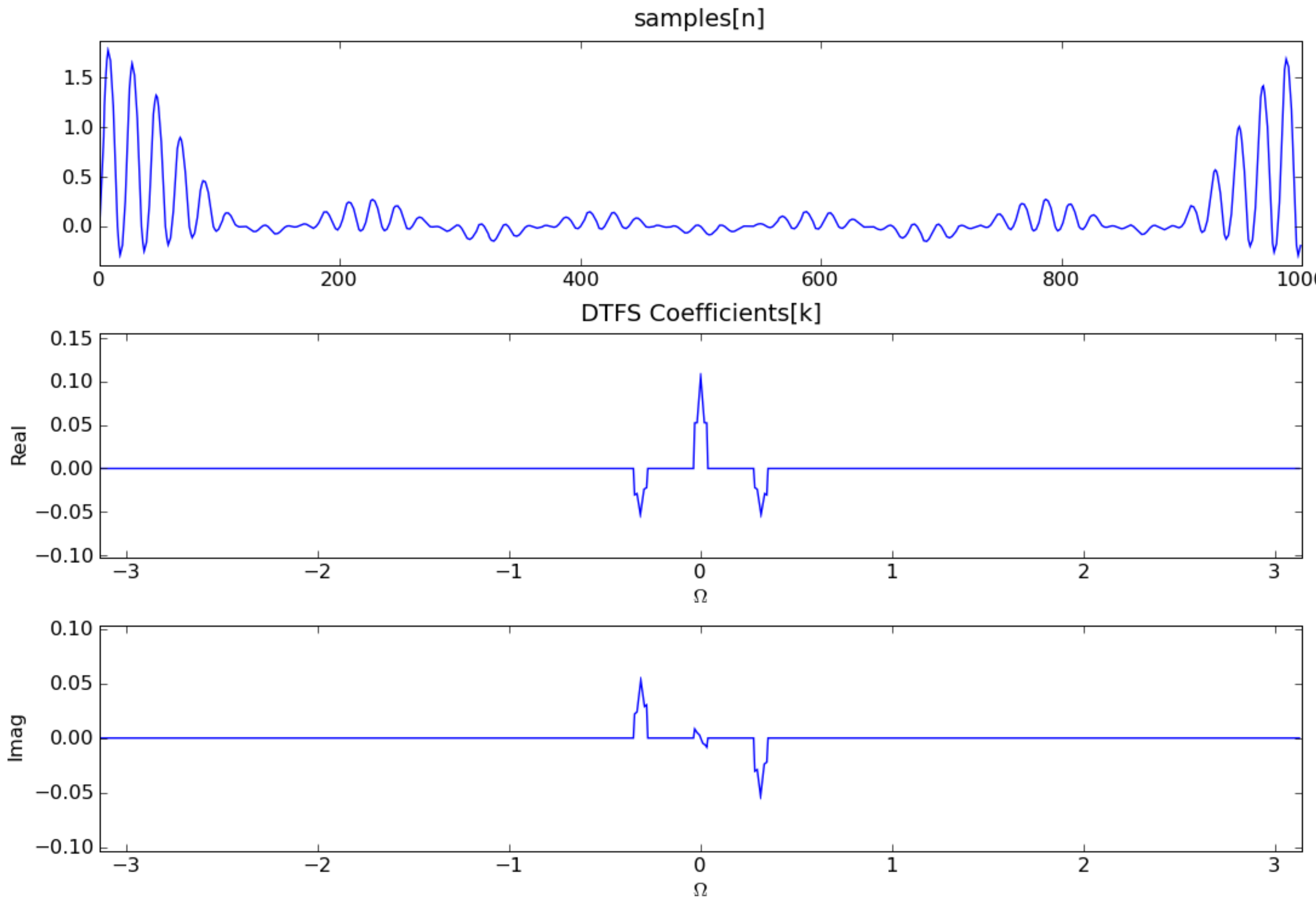
Modulation with Cos25 Omega_1 n multiplication followed by 5 sample delay



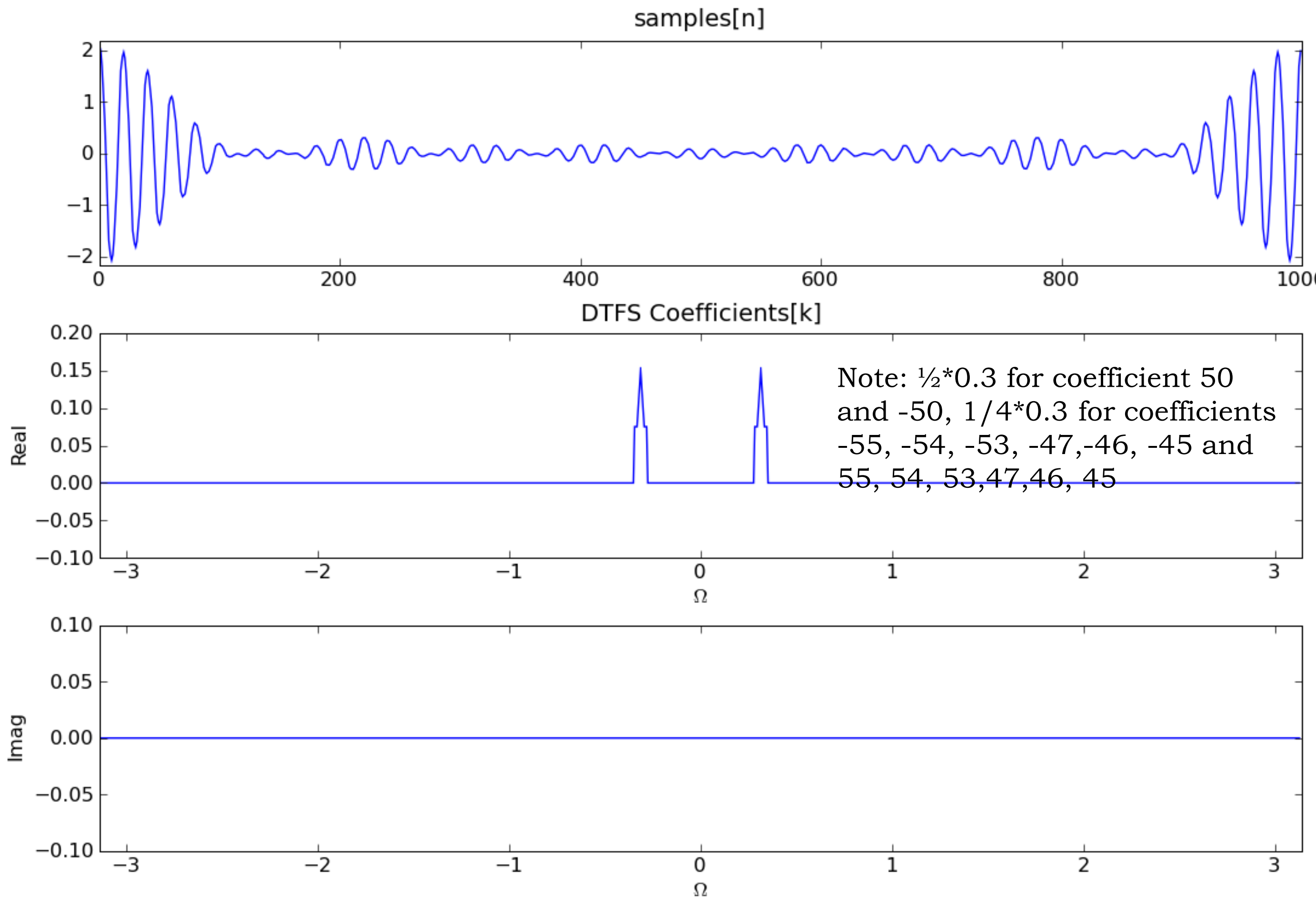
Demod by Cos25 Omega_1 n multiplication after delay



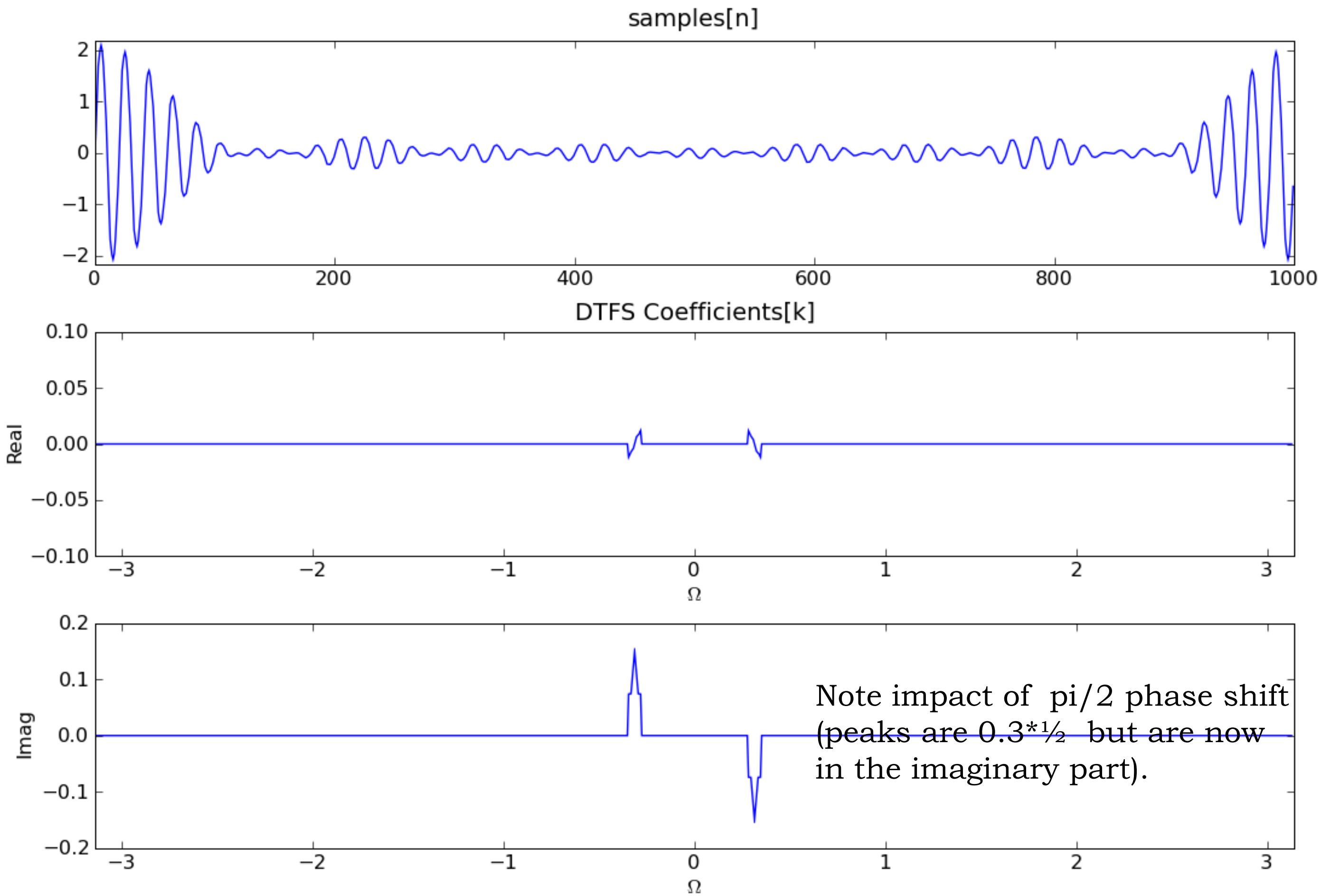
Demod by Sin 25 Omega_1 n multiplication after delay



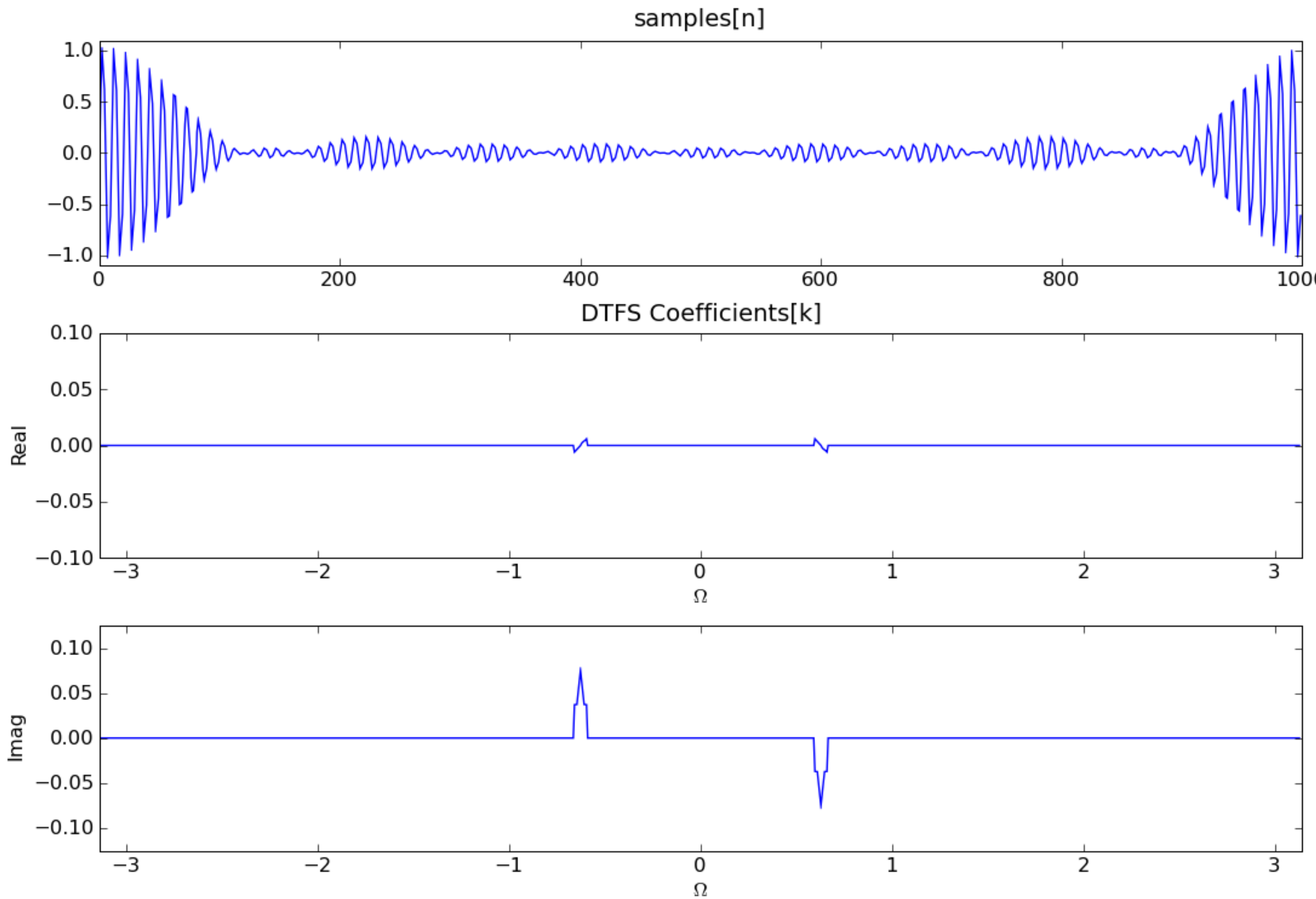
Modulation by Cos50 Omega_1 n multiplication



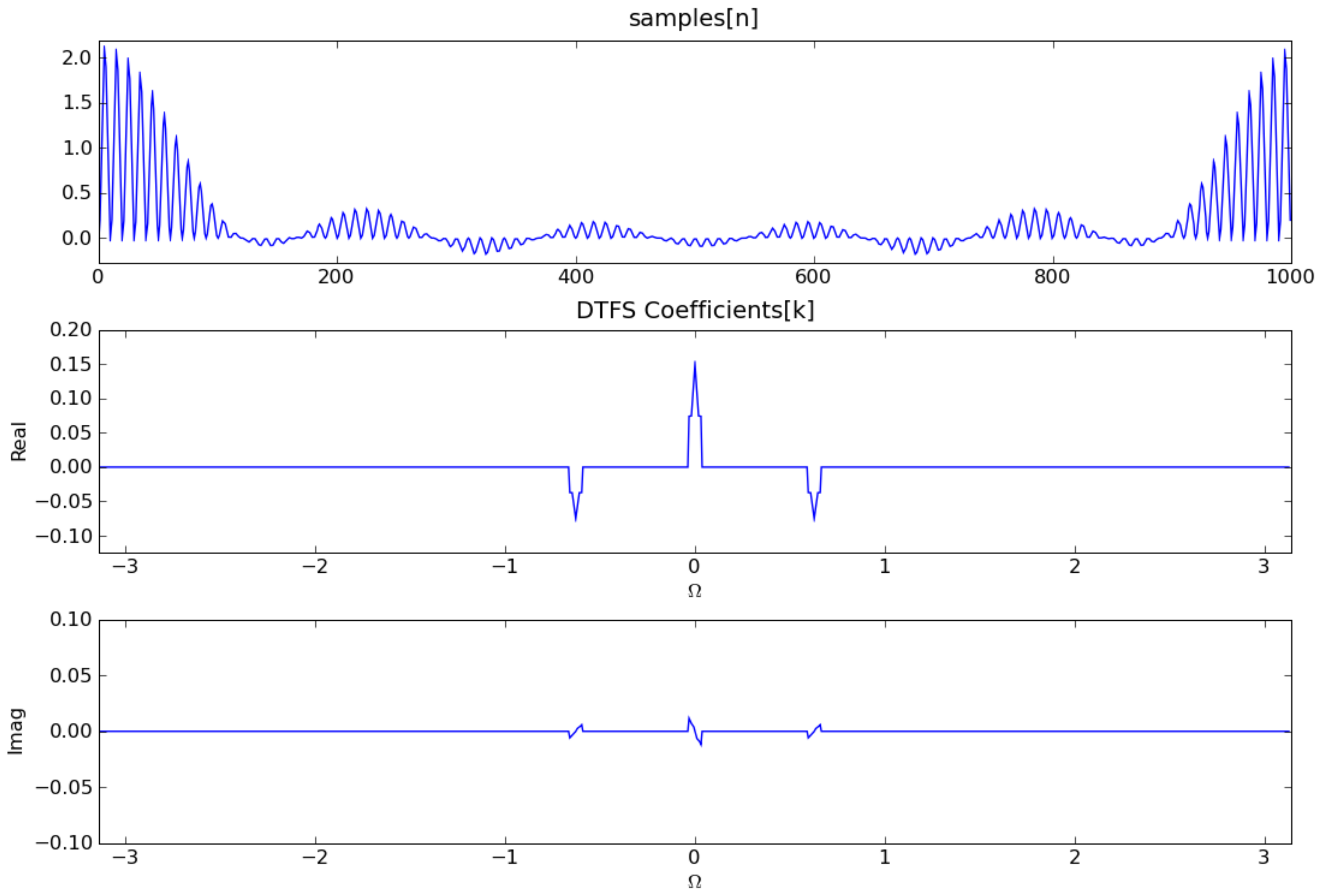
Modulation by Cos50 Omega_1 n multiplication followed by 5 sample delay



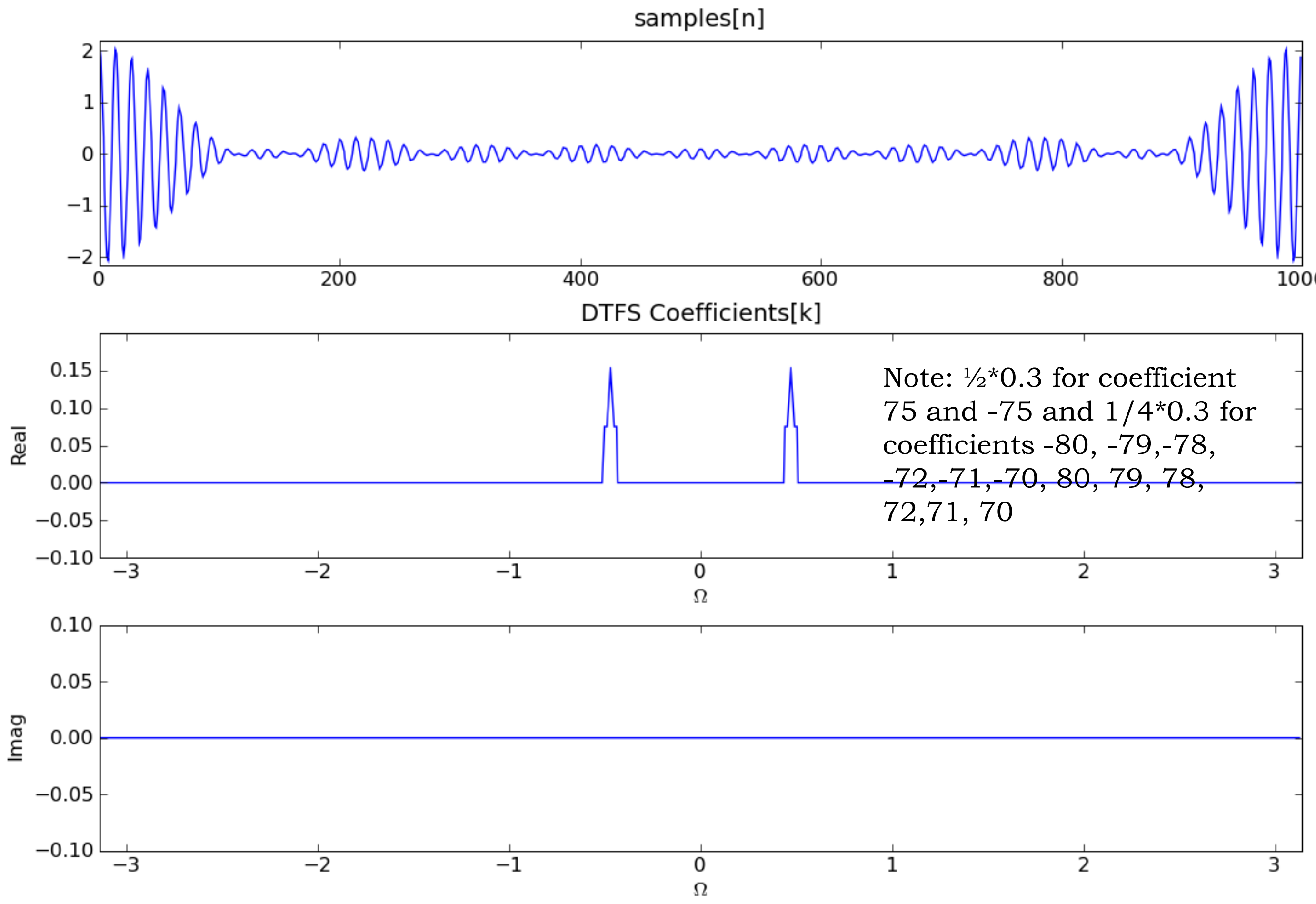
Demod by Cos50 Omega_1 n multiplication after delay



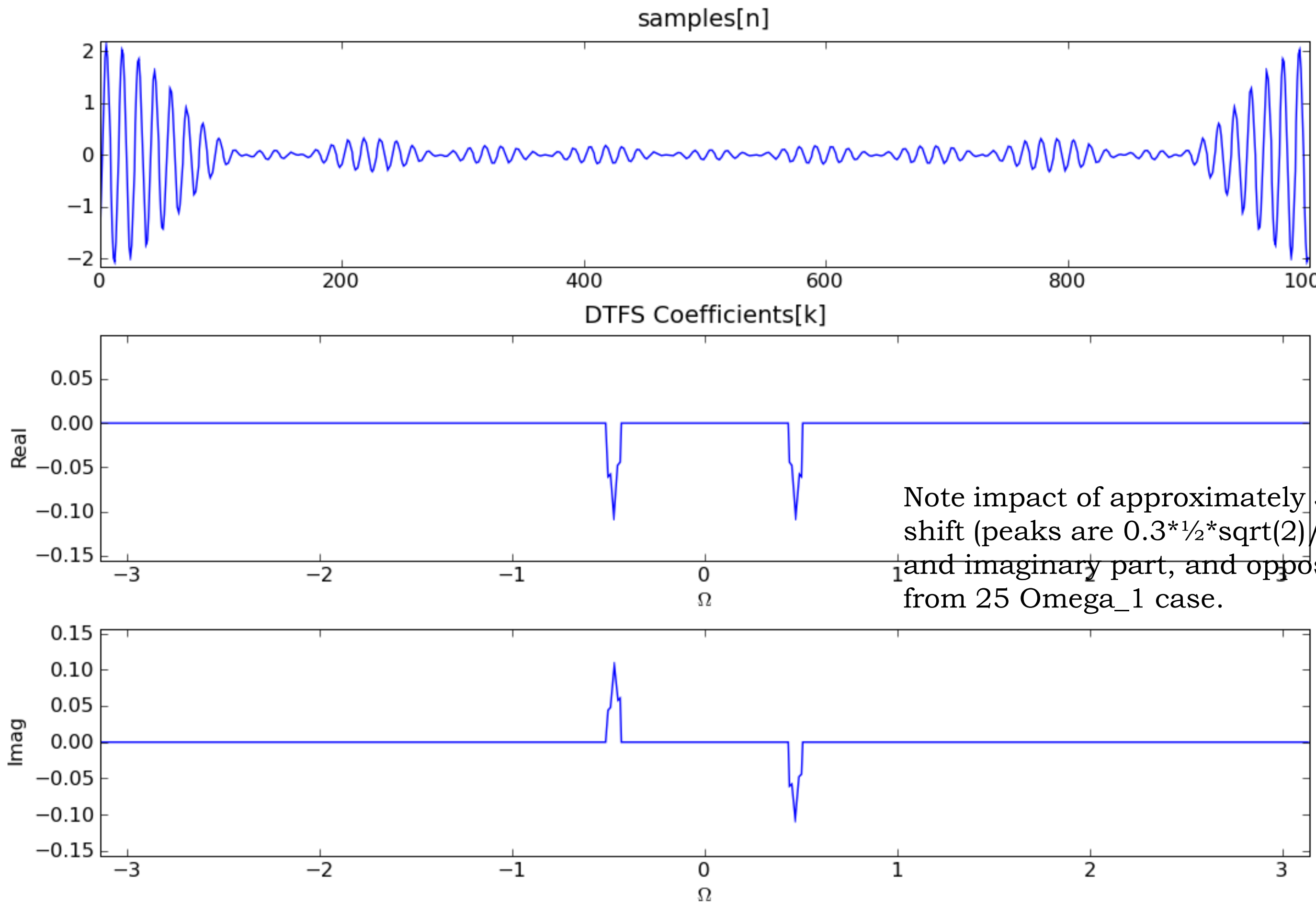
Demod by Sin 50 Omega_1 n multiplication after delay



Modulation by Cos75 Omega_1 n multiplication

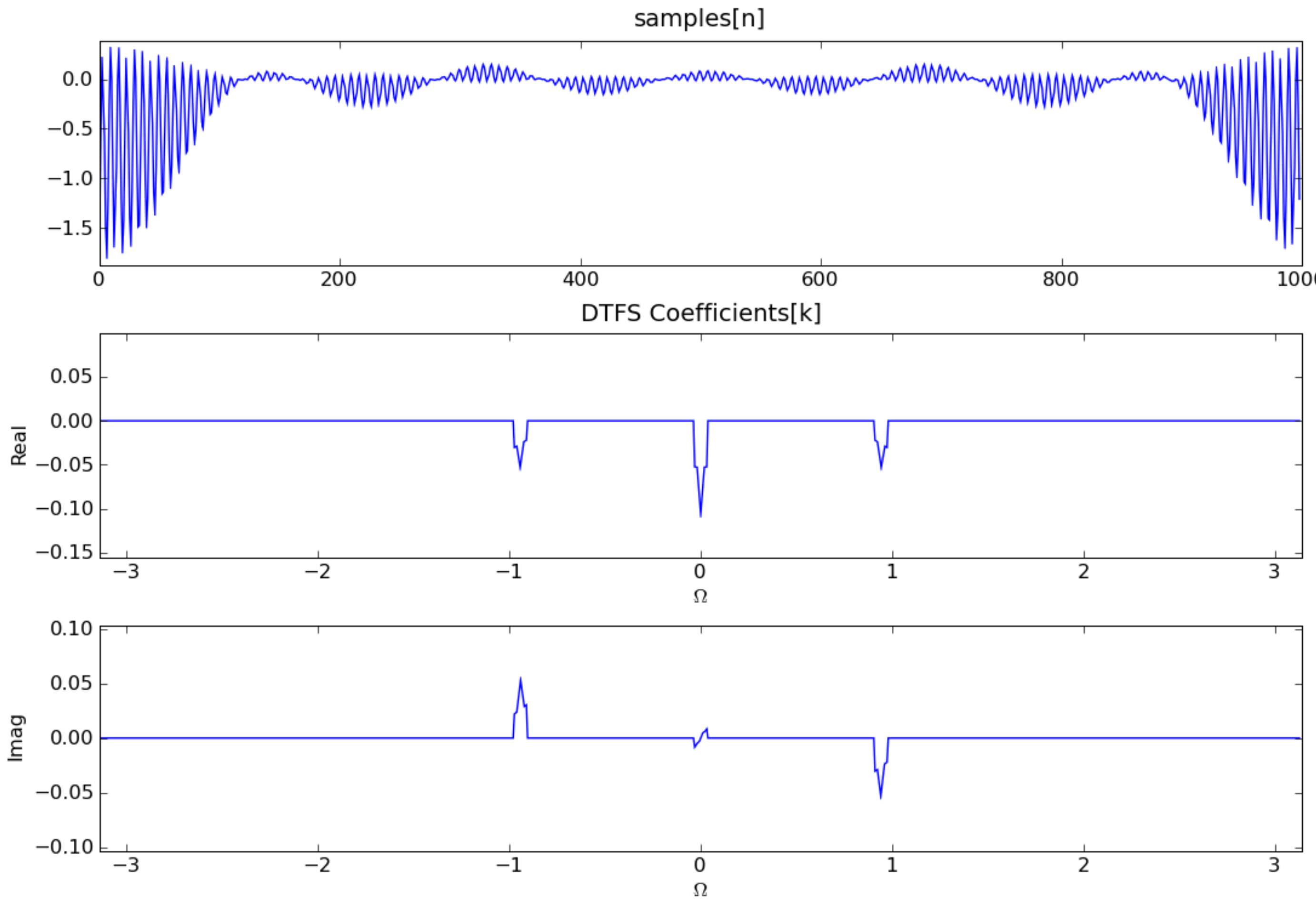


Modulation by Cos 75 Omega_1 n multiplication followed by 5 sample delay

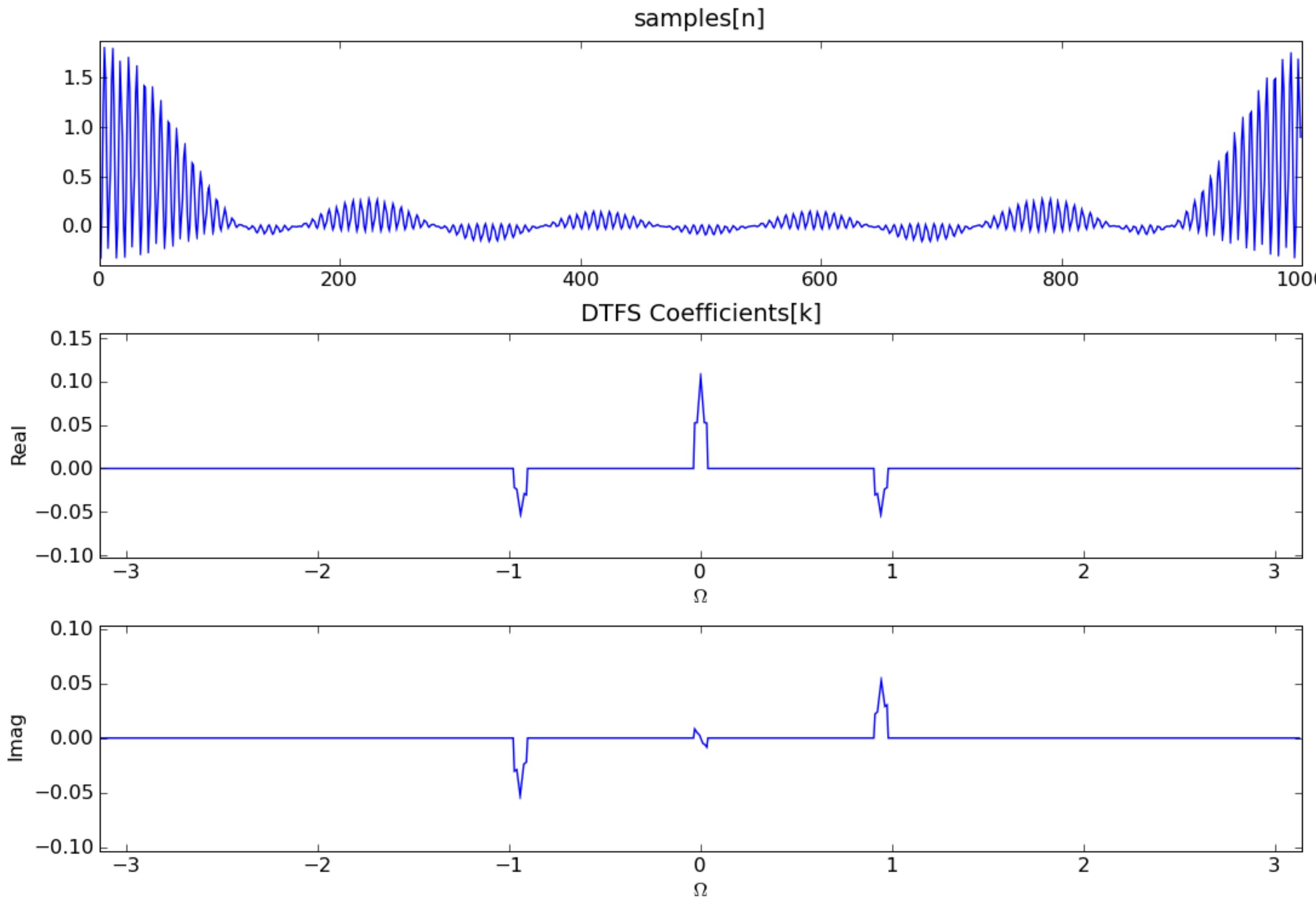


Note impact of approximately 3pi/4 phase shift (peaks are 0.3*½*sqrt(2)/2 for real and imaginary part, and opposite in sign from 25 Omega_1 case.

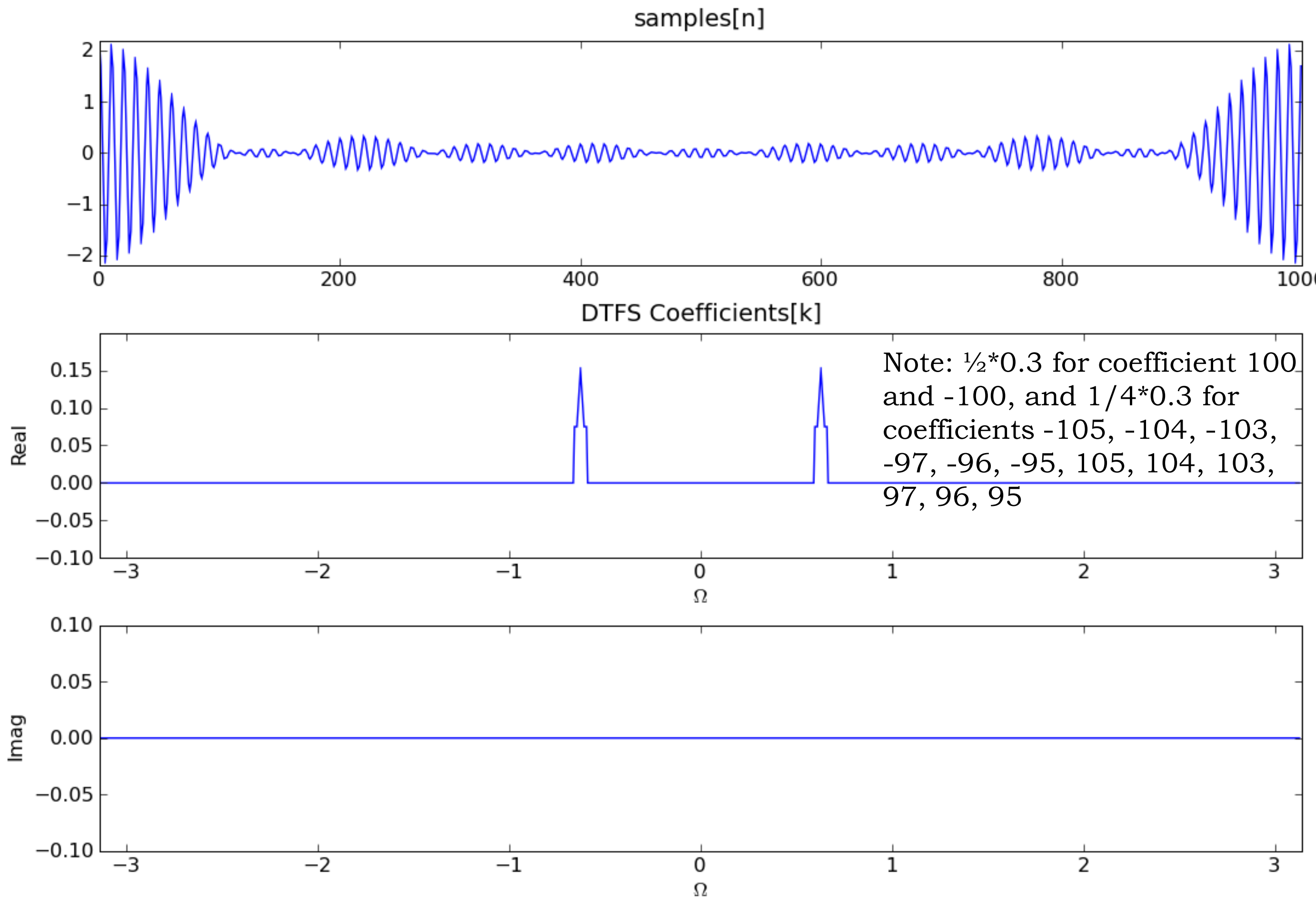
Demod by Cos75 Omega_1 n multiplication after delay



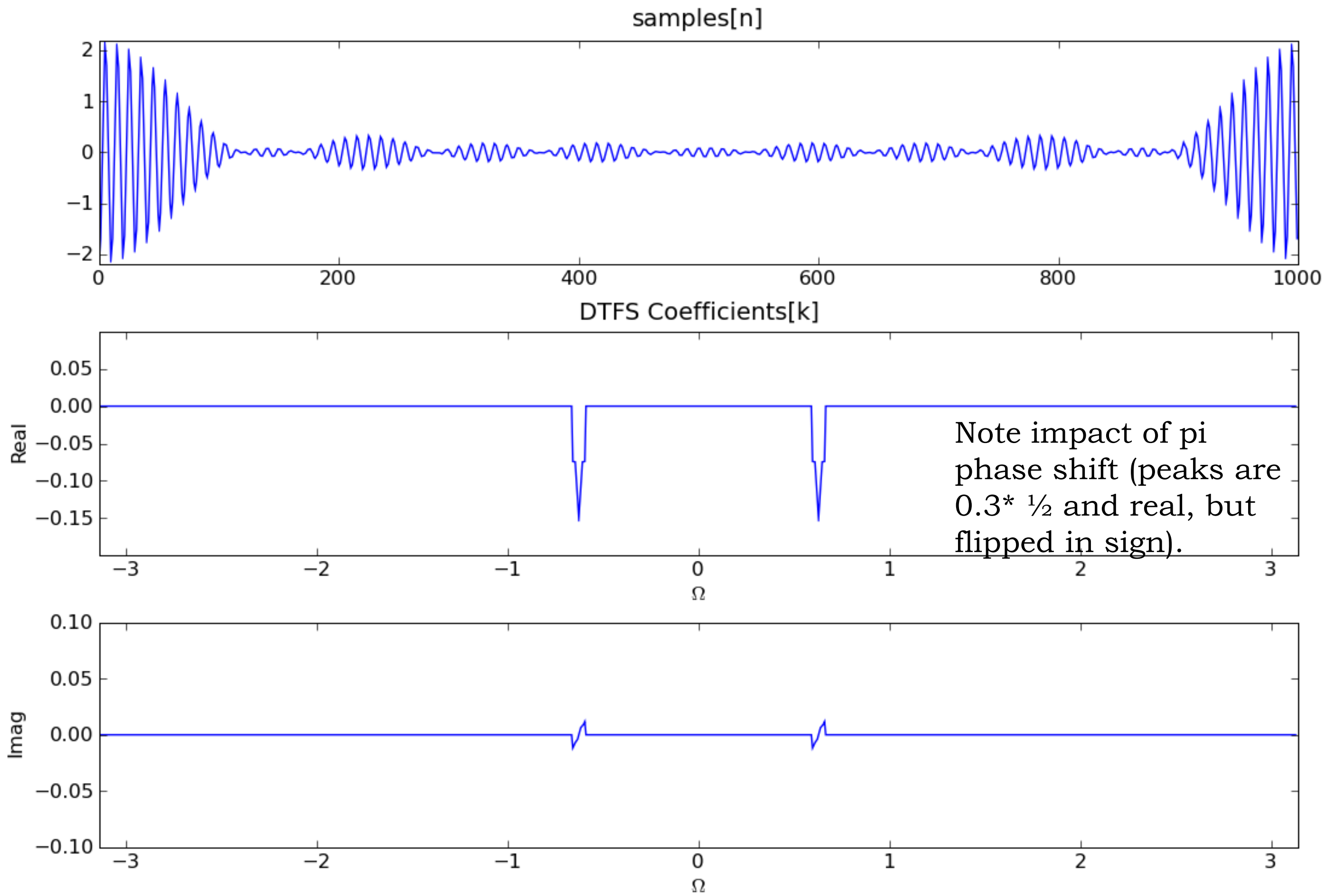
Demod by Sin 75 Omega_1 multiplication after delay



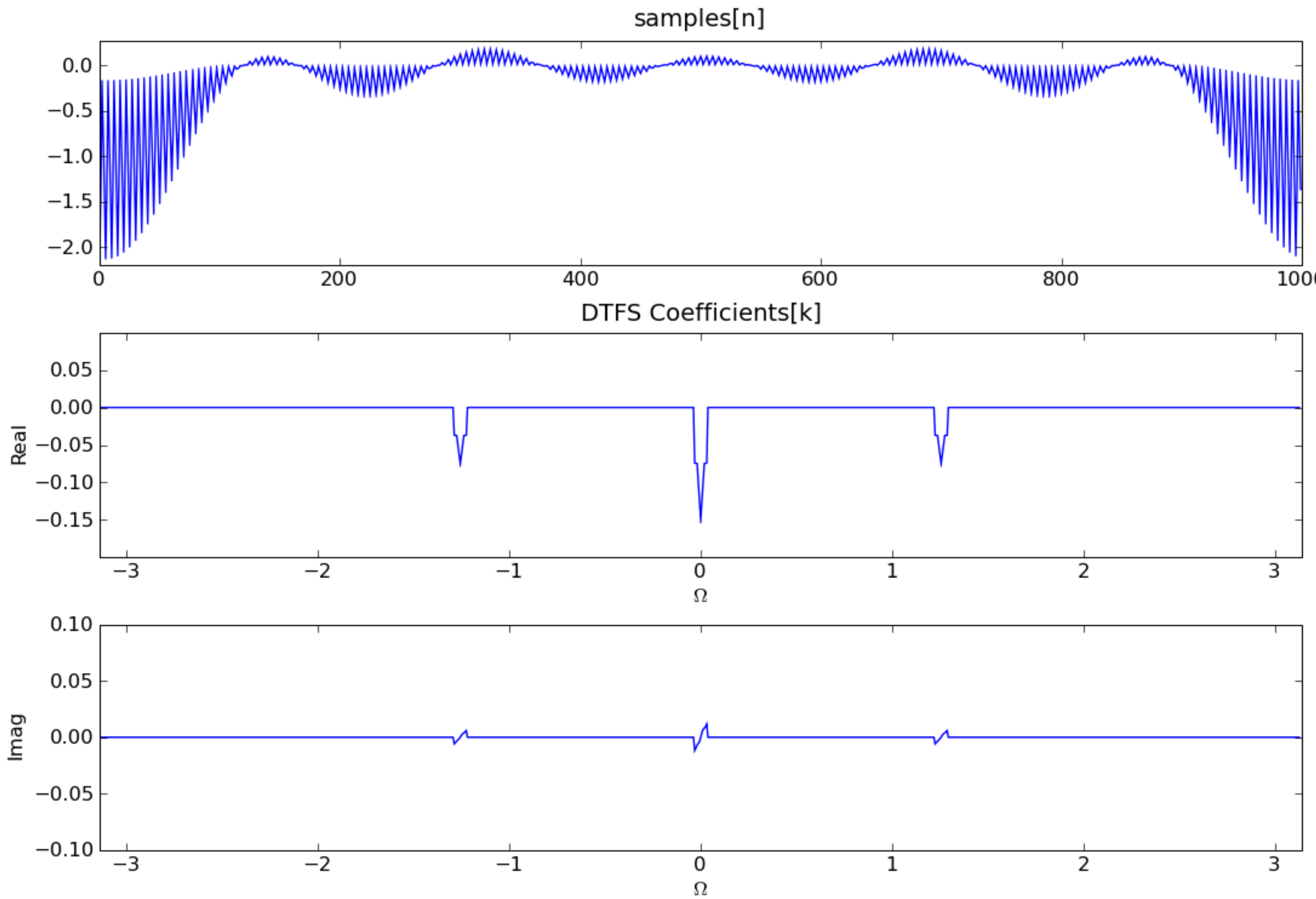
Modulation by Cos100 Omega_1 n multiplication



Mod by Cos 100 Omega_1 n multiplication followed by 5 sample delay



Demod by Cos100 Omega_1 n multiplication after delay



Demod by Sin 100 Omega_1 n multiplication after delay