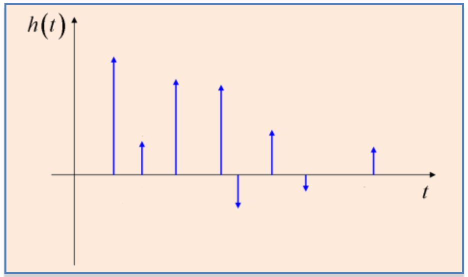
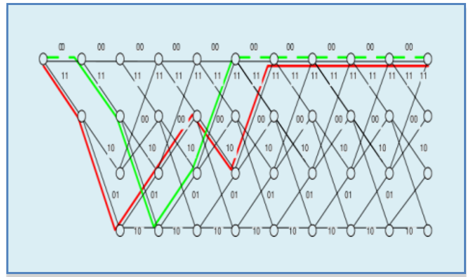
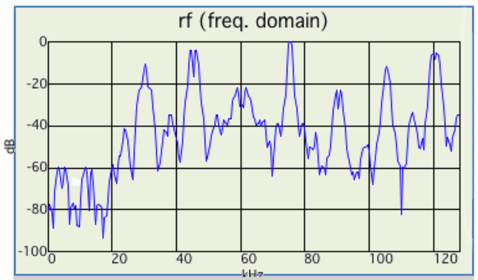
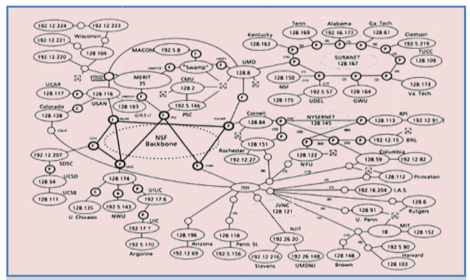








INTRODUCTION TO EECS II

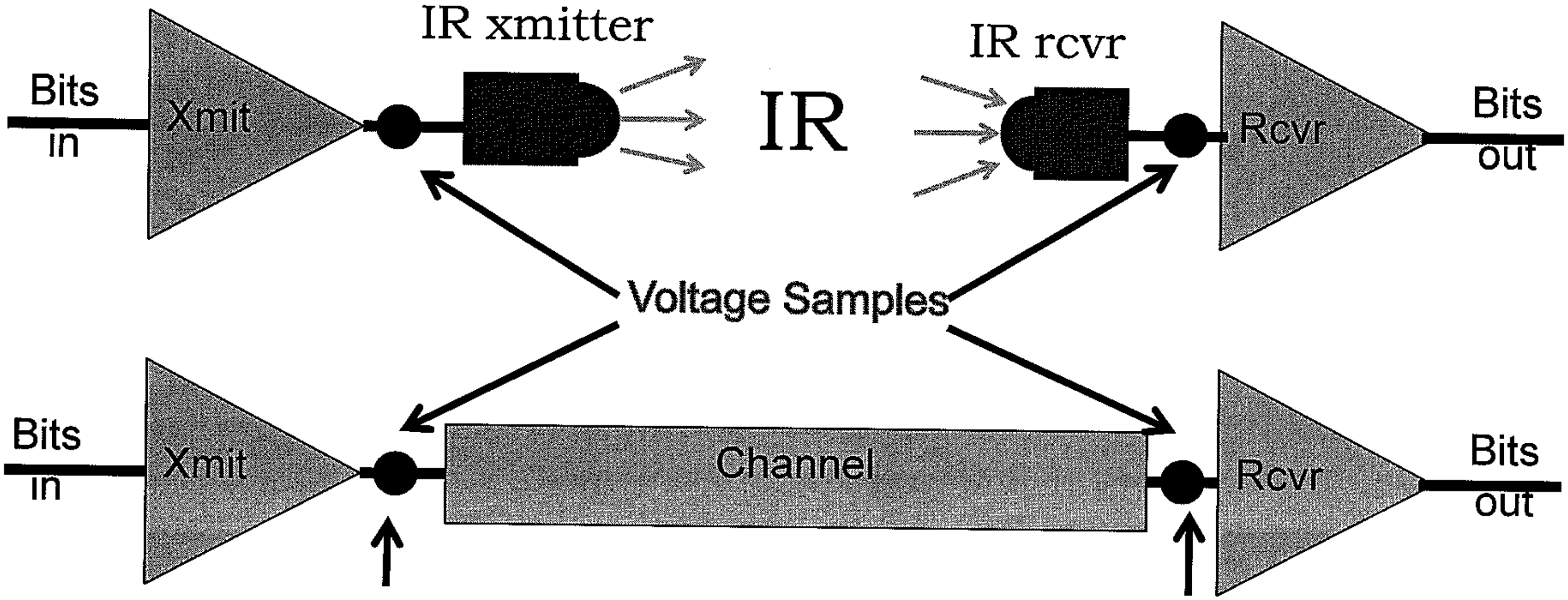
DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2010
Lecture #3

- Good Channel → Eye's wide open
- LTI Channel ⇔ Convolution with Unit Sample Response
- Use USR to Deconvolve: Bad Channel → Good Channel
- Deconvolution and Noise

2

Bits → Samples {X} → Channel → Samples {Y} → Bits

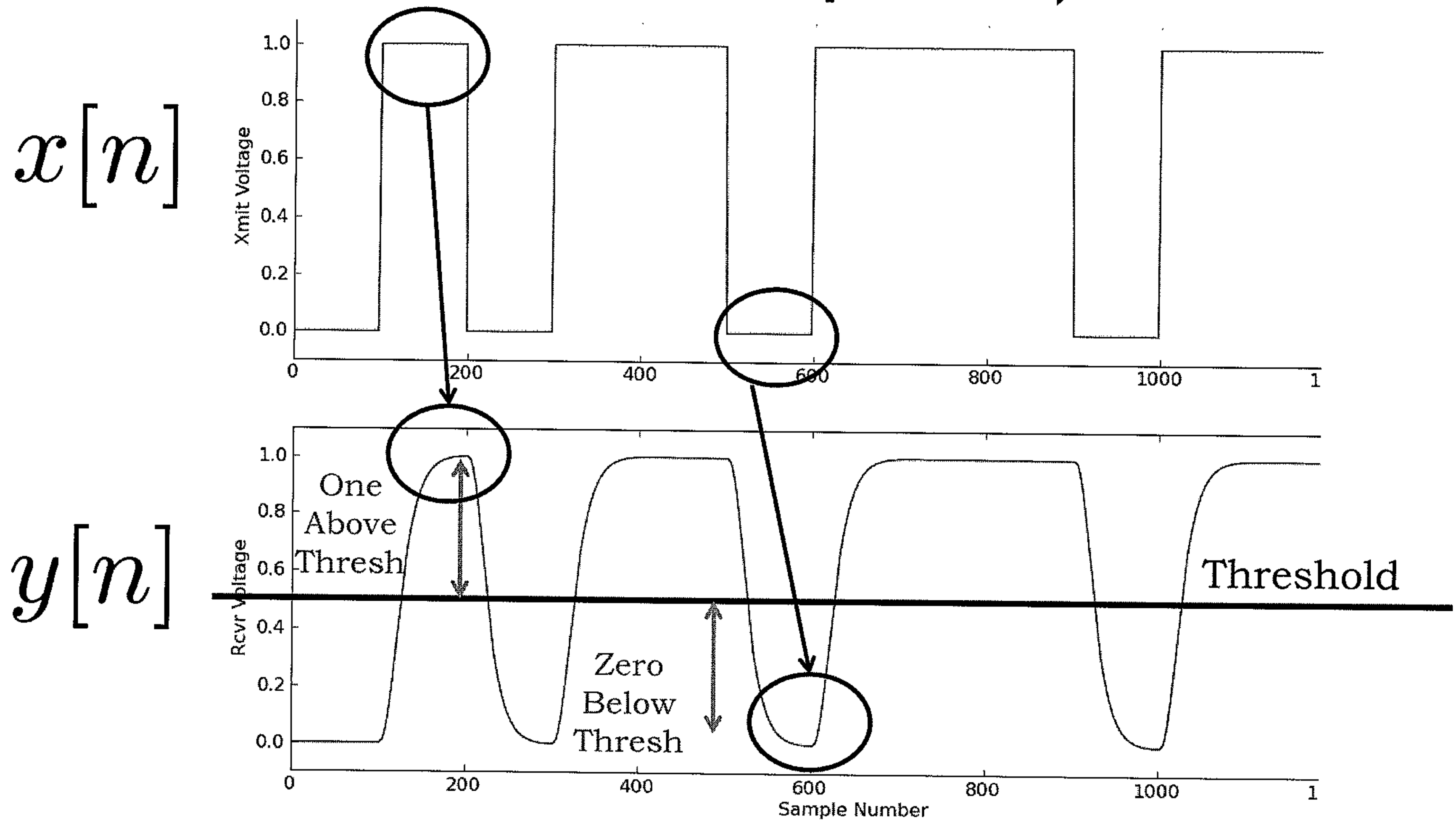


$X \equiv$ entire sequence
$x[n] \equiv n^{th}$ sample value

$Y \equiv$ entire sequence
$y[n] \equiv n^{th}$ sample value

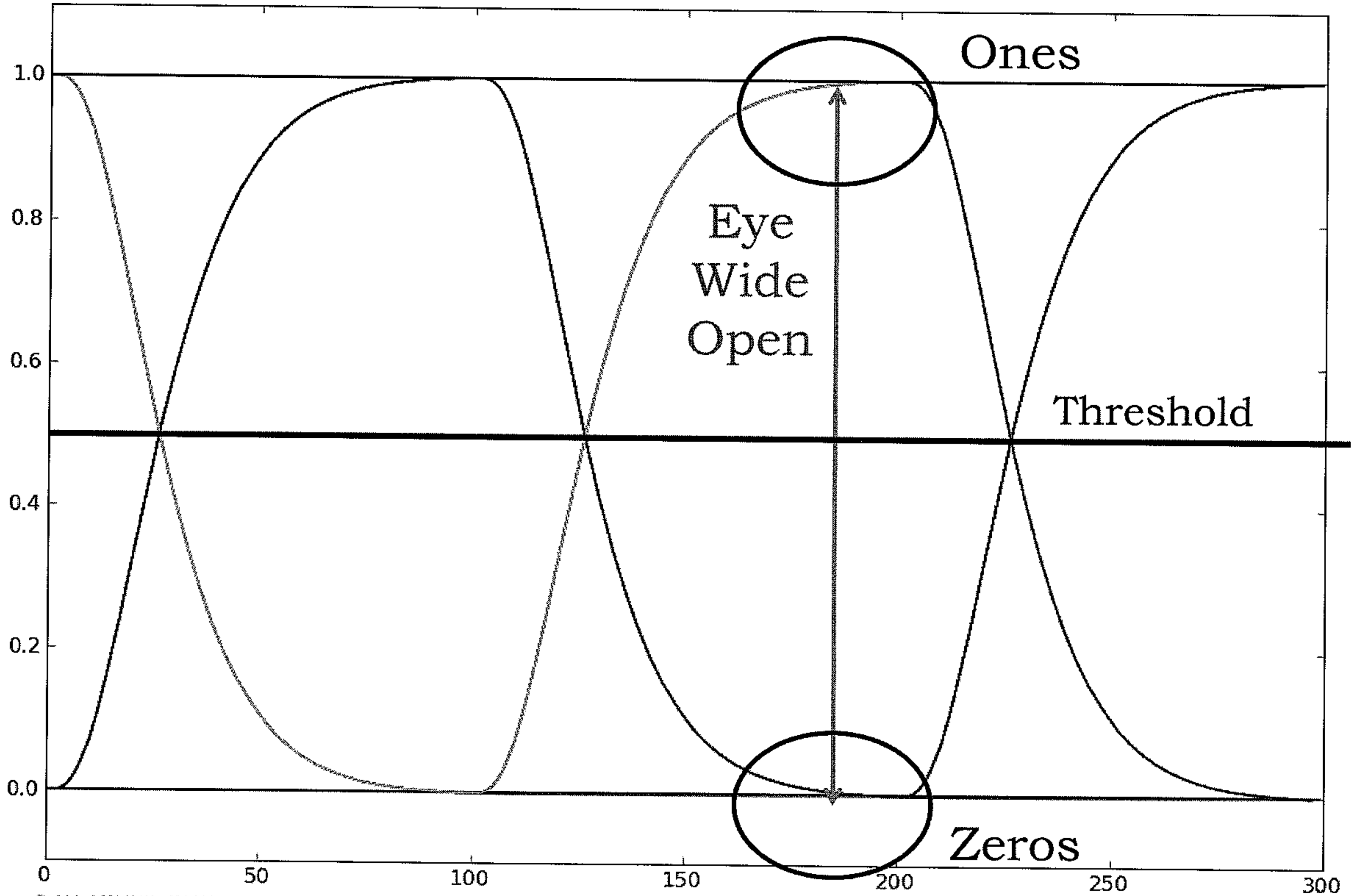
Sample Rates:
- 4 million Samples/Second (IR Transceiver)
- Up to gigaSamples/Second (Fastest)

Excellent Channel Transmission (using 100 Samples/bit)



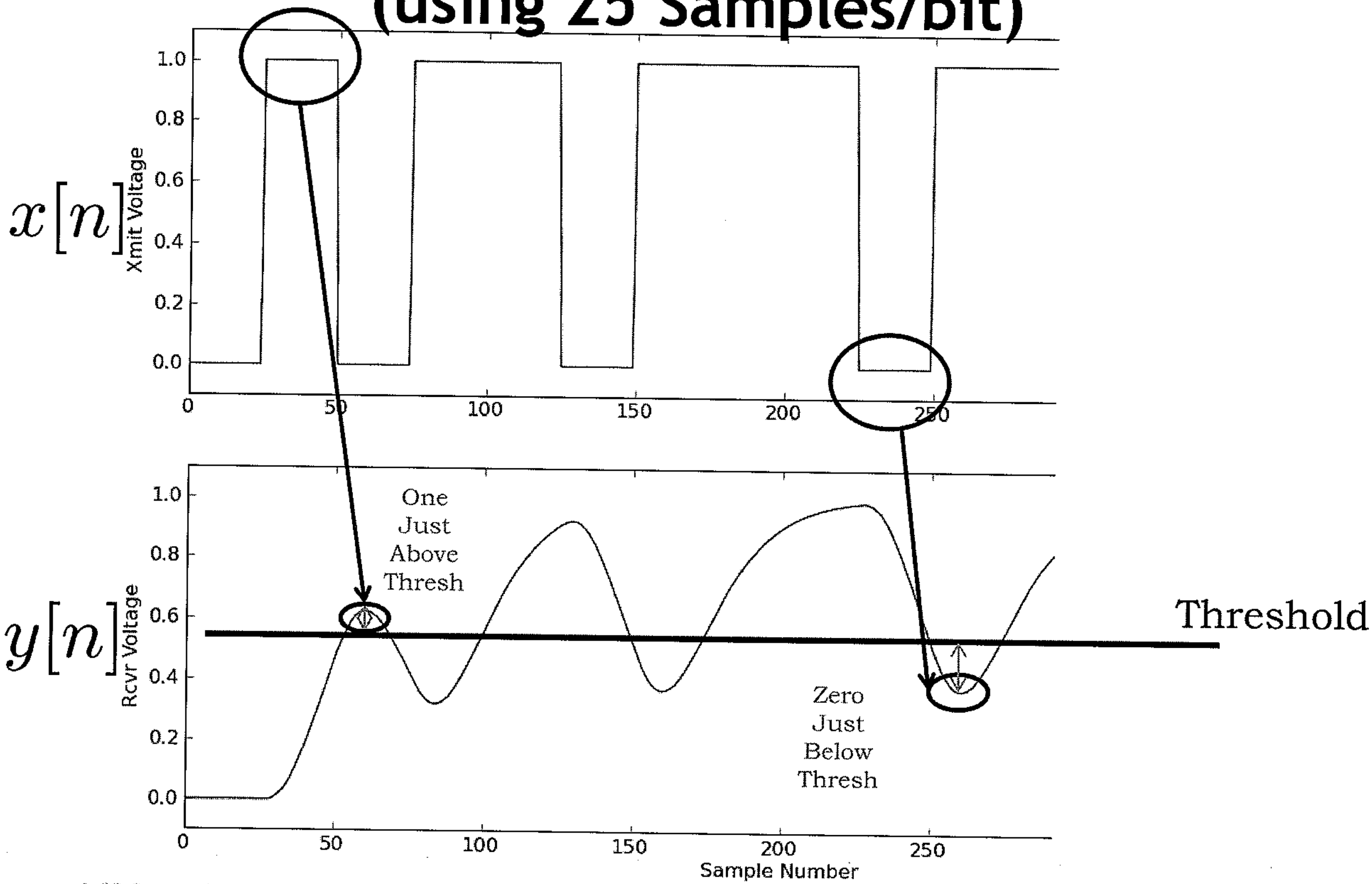
4

Excellent Channel Transmission Eye Diagram



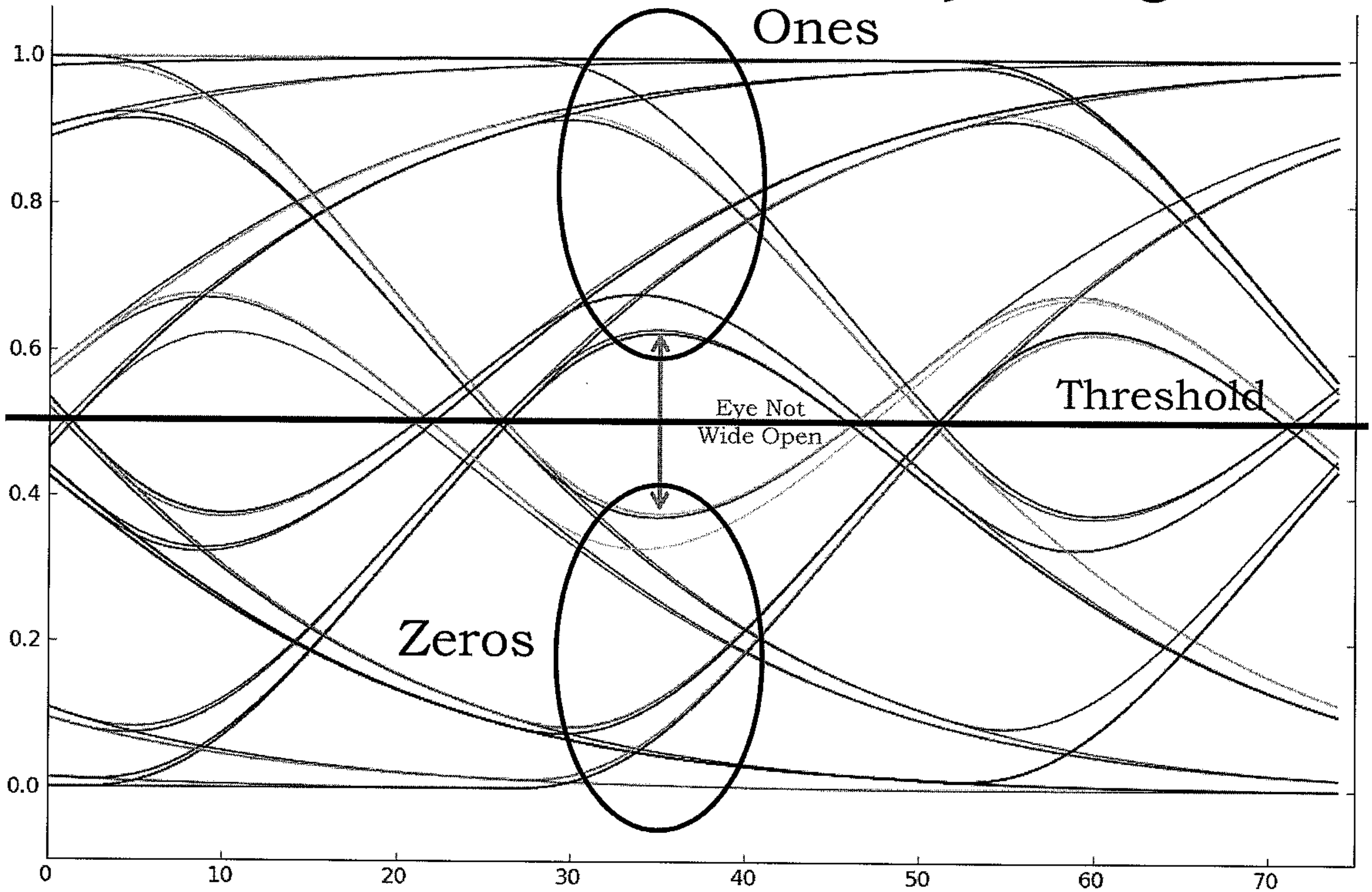
Okay Channel Transmission (using 25 Samples/bit)

5



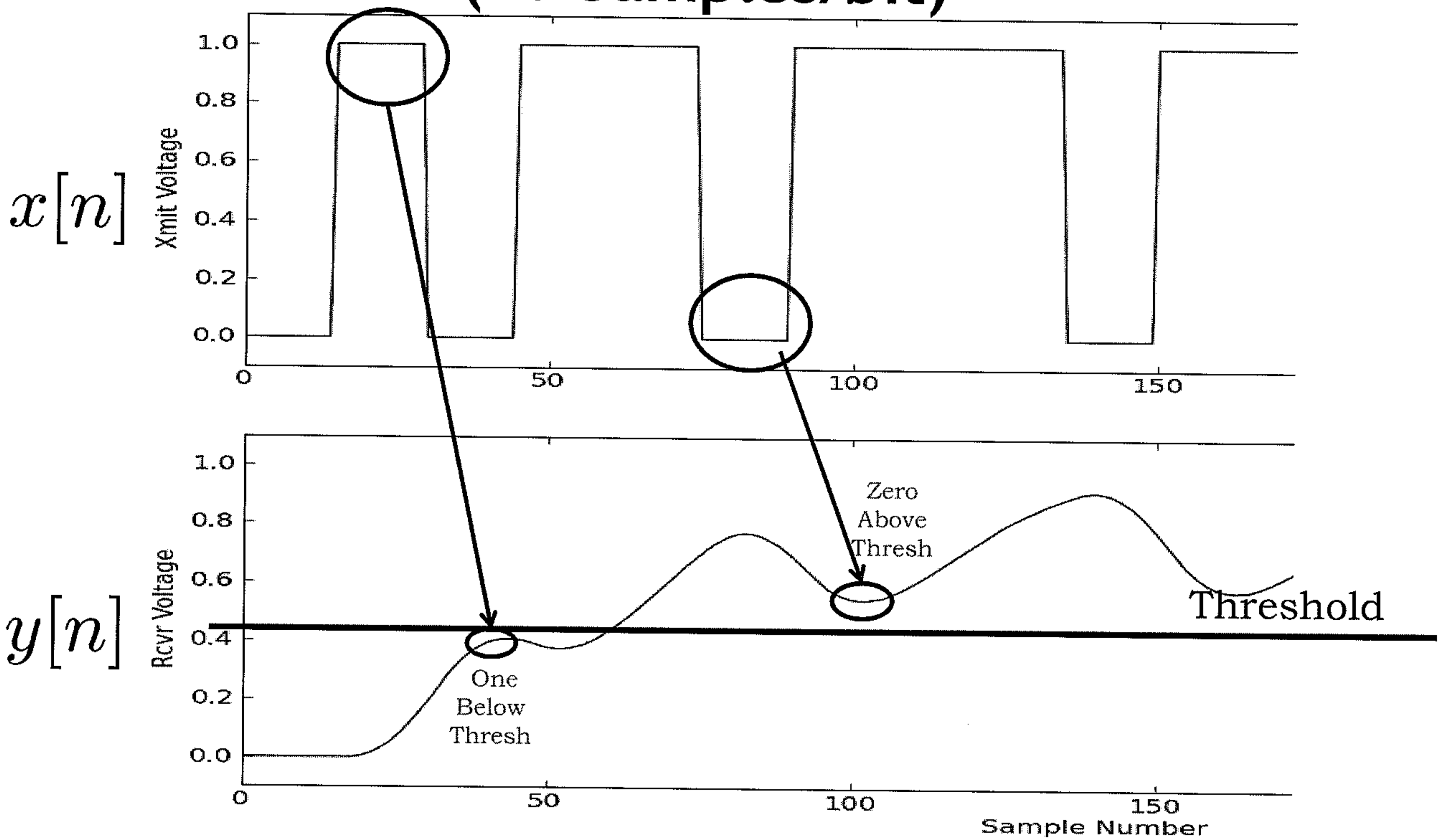
6

Okay Channel Transmission Eye Diagram



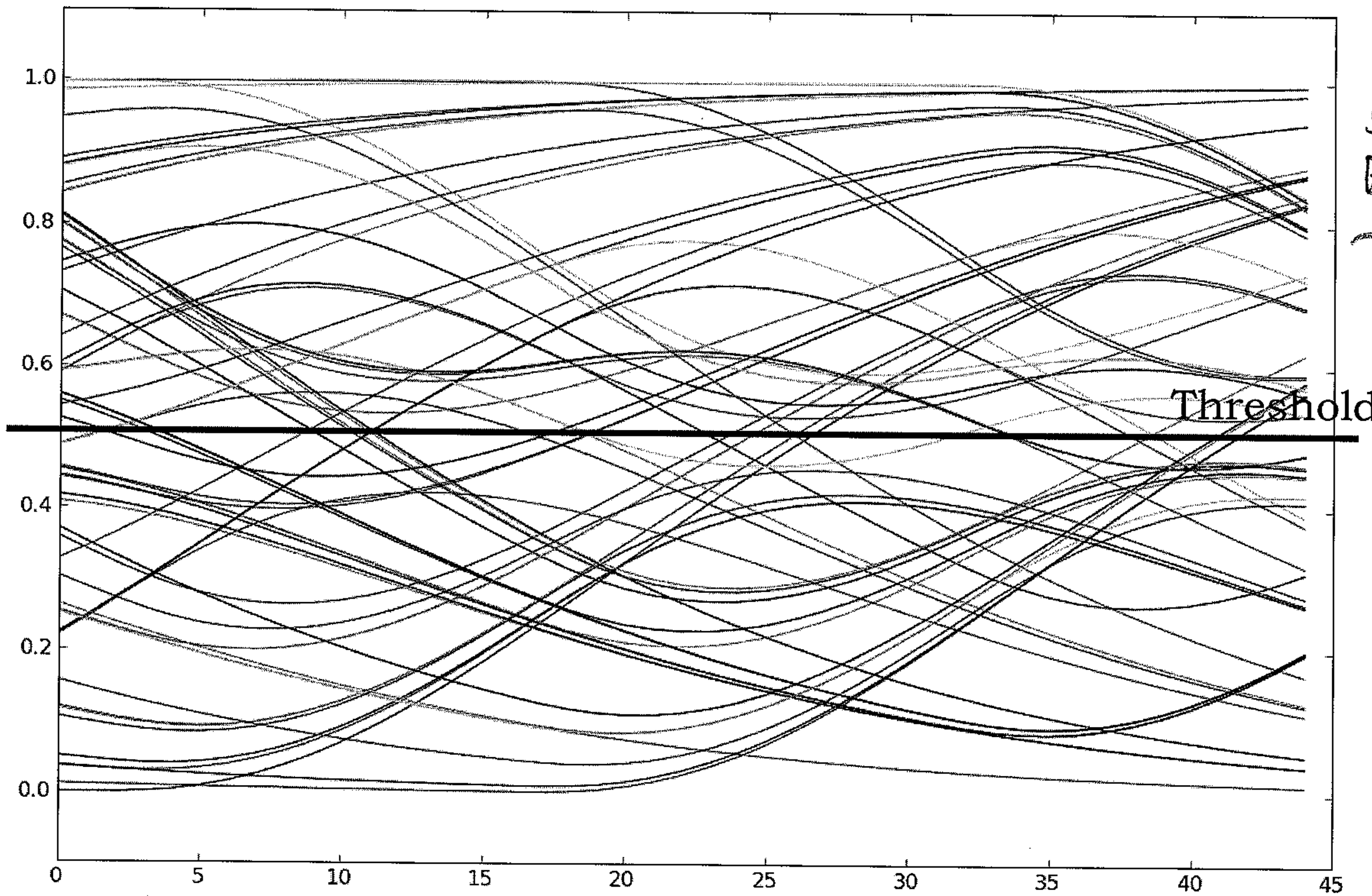
Unusable Channel Transmission (15 Samples/bit)

7



Unusable Channel Transmission Eye Diagram

8



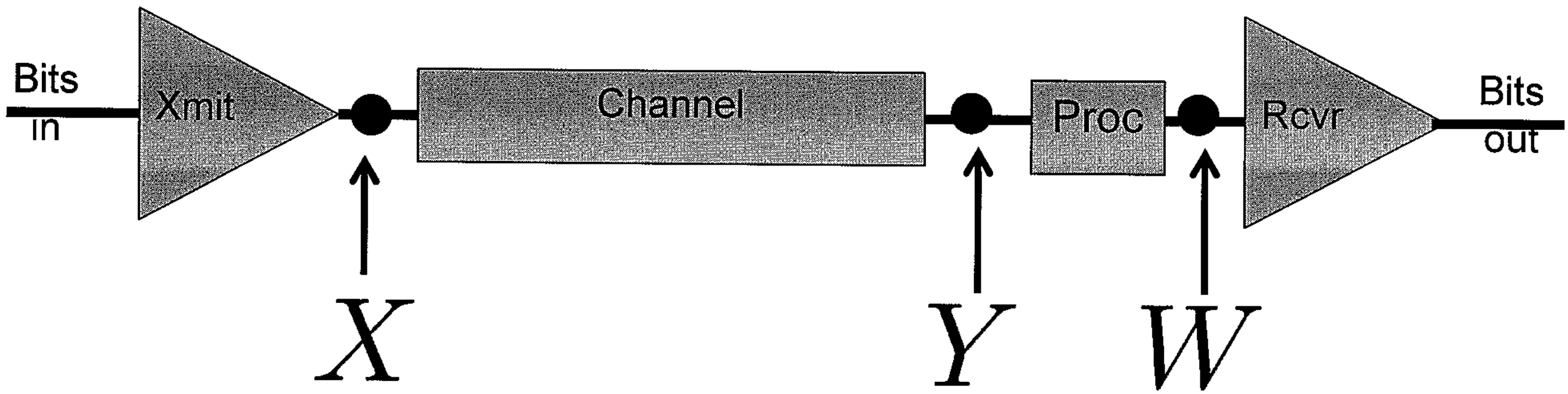
No Eye!

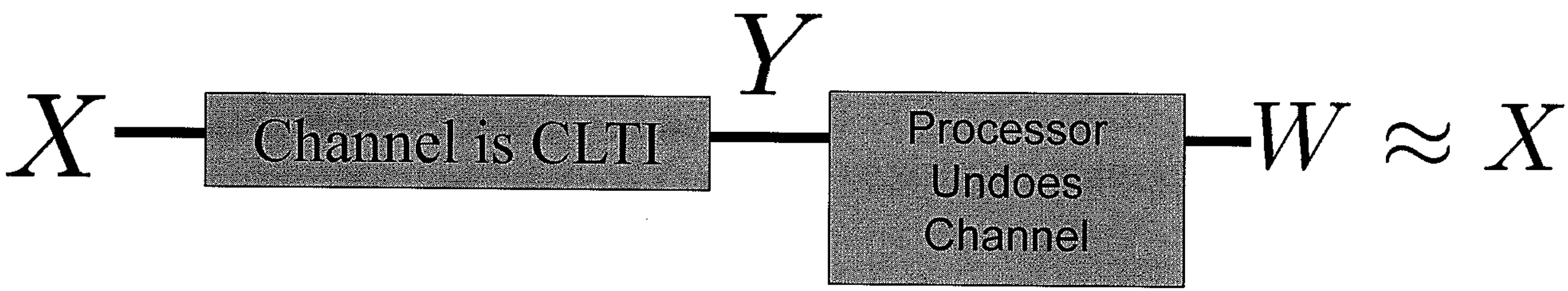
What does an Eye Diagram Tell You?

9

- How many samples to use per bit
  - More samples/bit:
    - More open eye → Easier detection ⇧
    - Fewer bits/second → Slower bit rate ⇩
- Where to put the 1-0 threshold
  - The threshold should evenly divide the upper and lower parts of the eye.
- How to estimate errors
  - Next Week.

Faster Bit Rate with Post-Channel Processing?





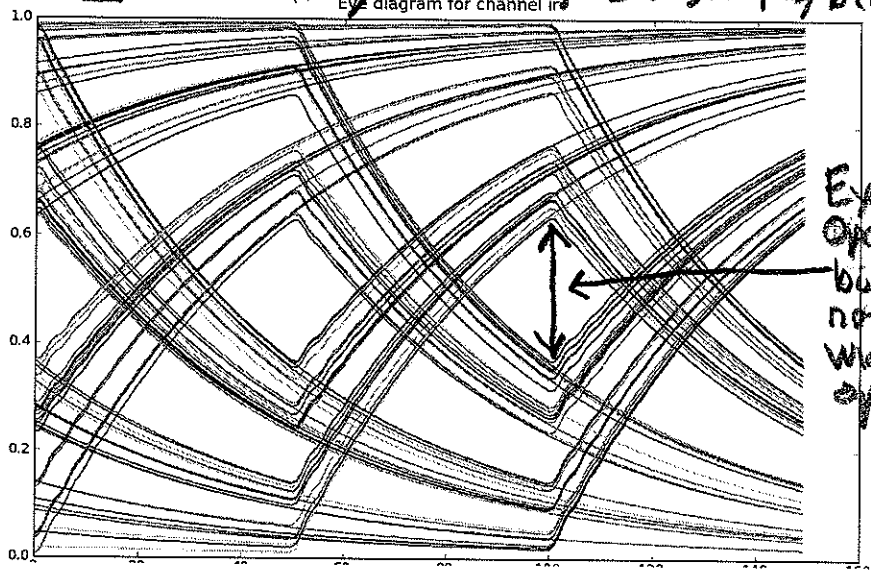
- Model Wire as Causal, Linear Time-Invariant (CLTI)
- Use Unit Sample Response to Undo Channel Effects

If W = X then only
need 1 sample/bit
(Perfect deconvolution)

10A

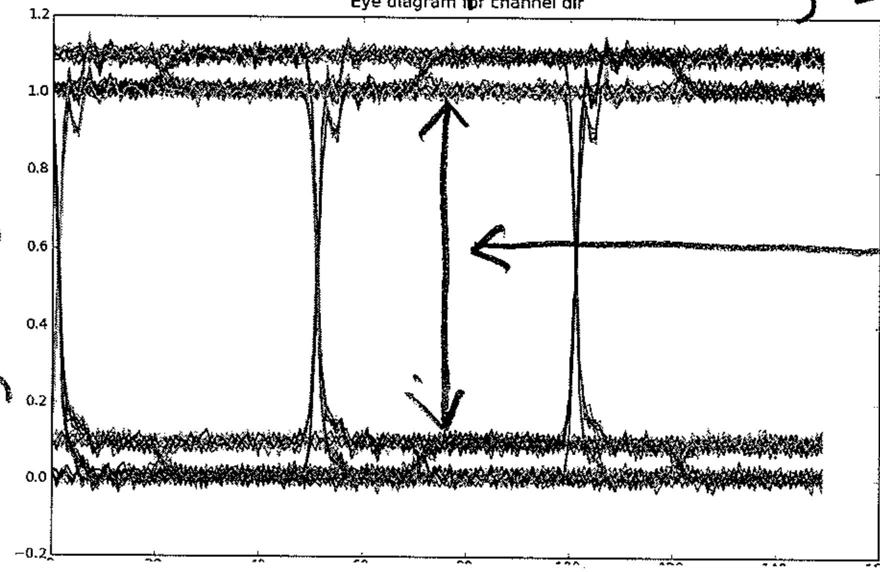
A Glimpse at IR Channel Results

IR Channel, Light On, 50 samples/bit

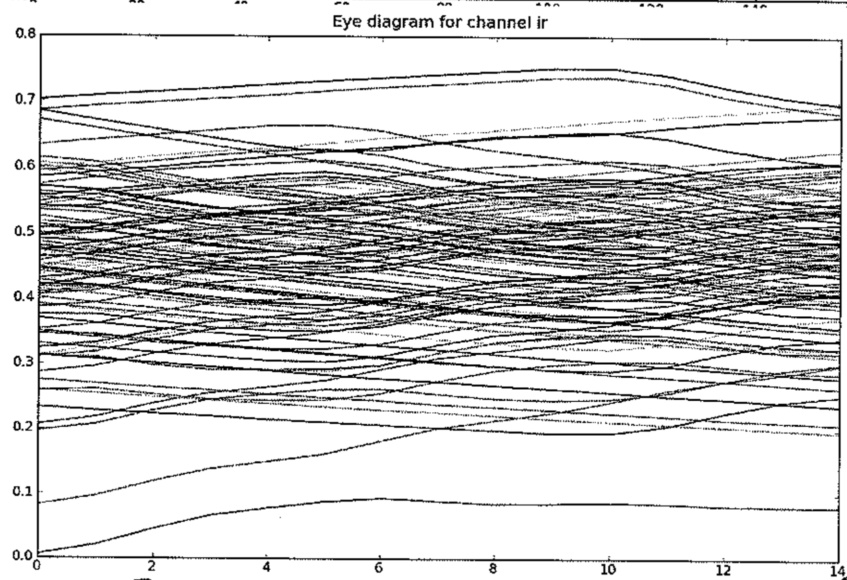


Eye Open but not wide open

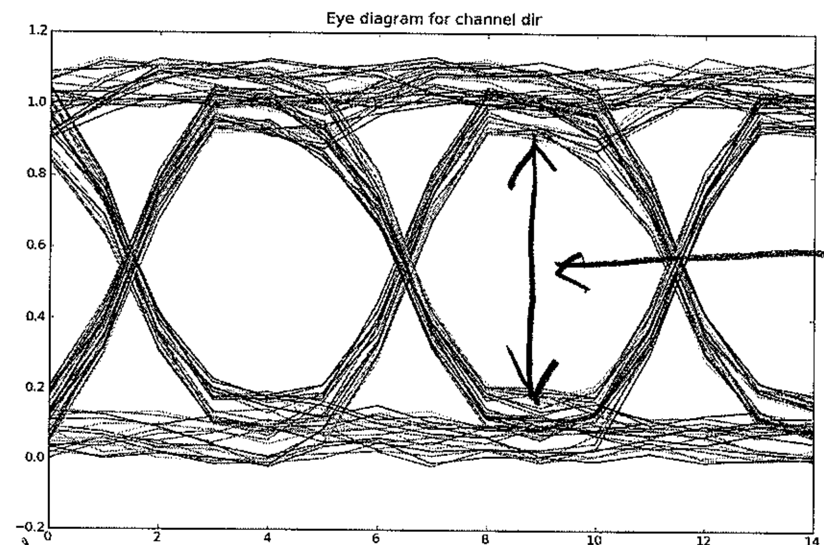
IR Channel plus Deconvolver, 50 samples/bit



Eye Wide Open



IR Channel, 5 samples/bit



Eye Wide Open 10x Faster bit rate!

Deconvolved IR Channel, 5 Samples/bit

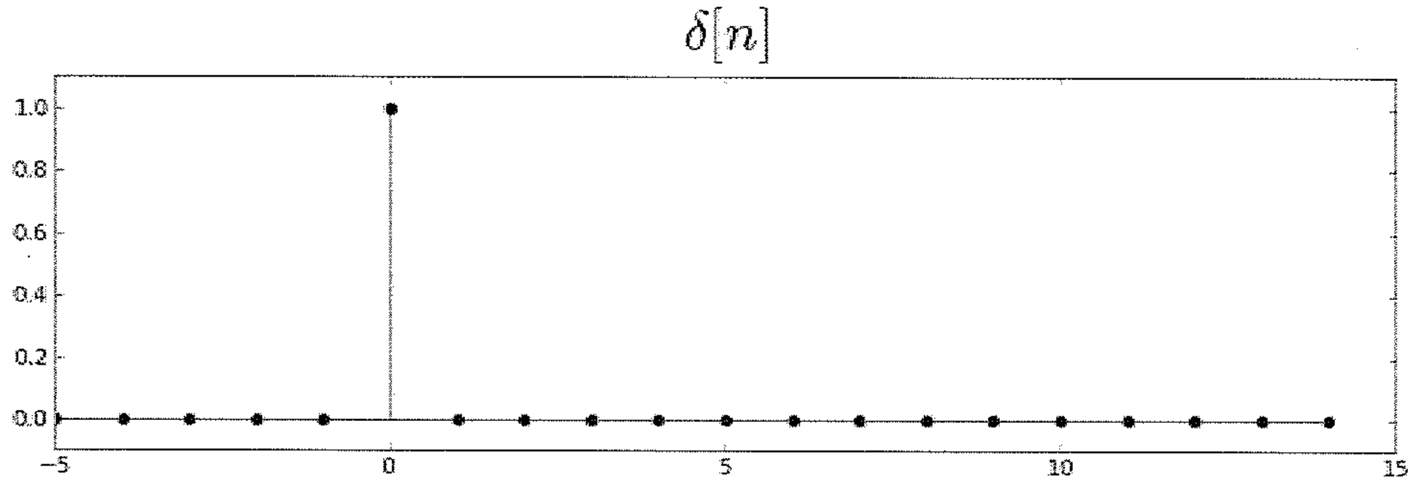
Causality, Time-Invariance and Linearity

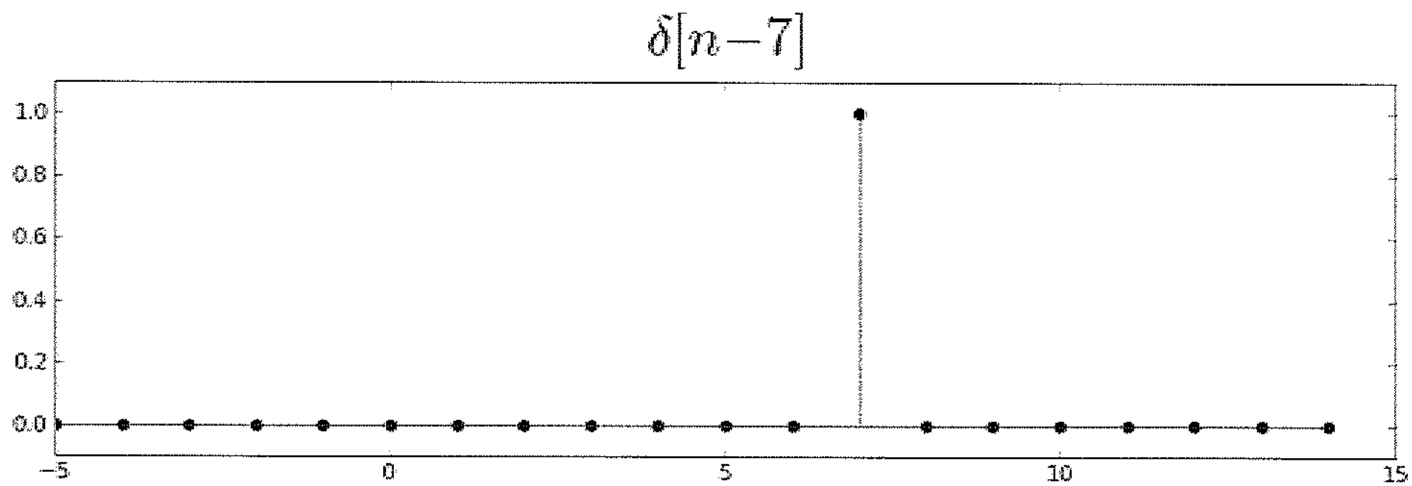
11

- Causality (out does not change before in)
  - If $x[n] = x[-\infty]$ for $n < N$
    - Then $y[n] = y[-\infty]$ for $n < N.$
- Time-Invariance (out shifts when in shifts)
  - If $in = x[n]$ → $out = y[n]$
    - Then $in = x[n-k]$ → $out = y[n-k]$
- Linearity
  - If $in = x_a[n]$ → $out = y_a[n]$, $in = x_b[n]$ → $out = y_b[n]$
    - Then $in = Ax_a[n] + Bx_b[n]$ → $out = Ay_a[n] + By_b[n]$

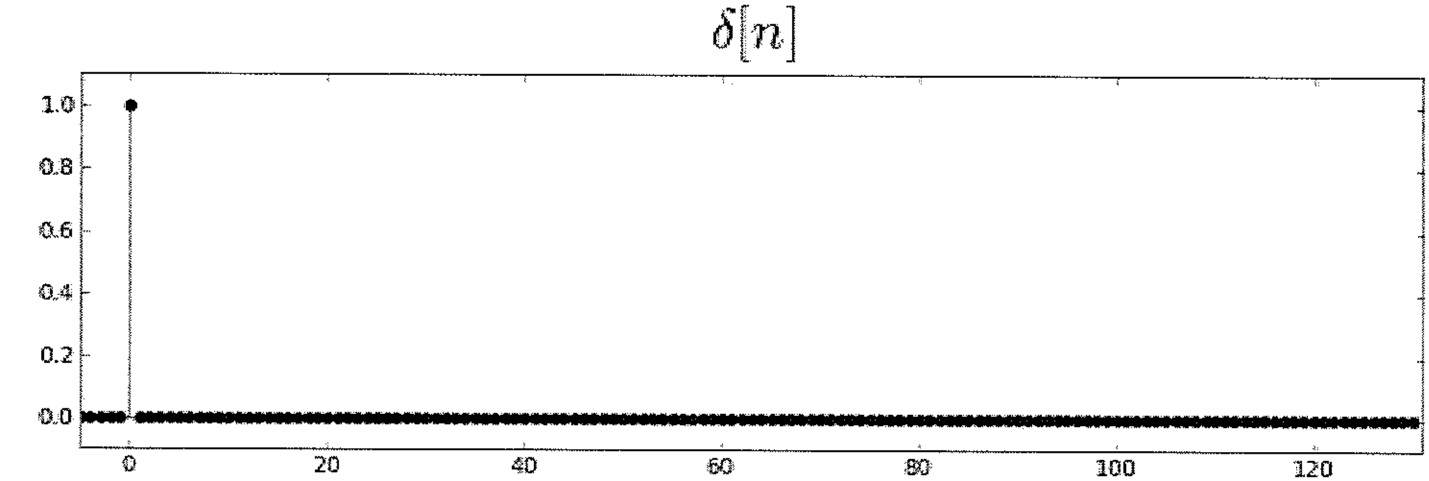
12

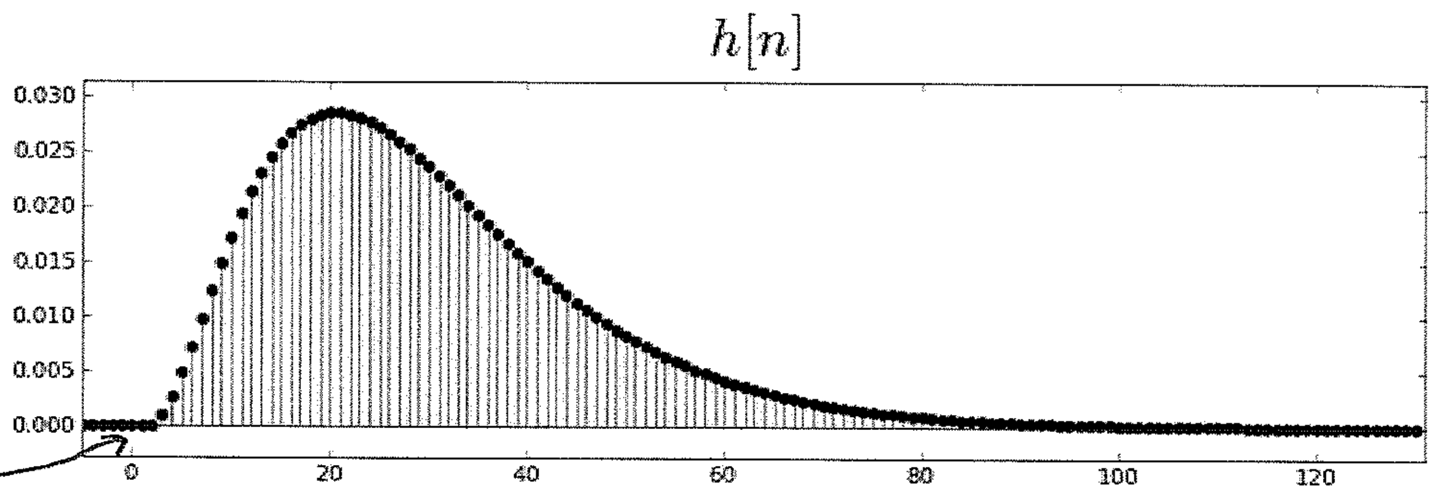
Unit Sample Reminder





Unit Sample Response (slow wire)





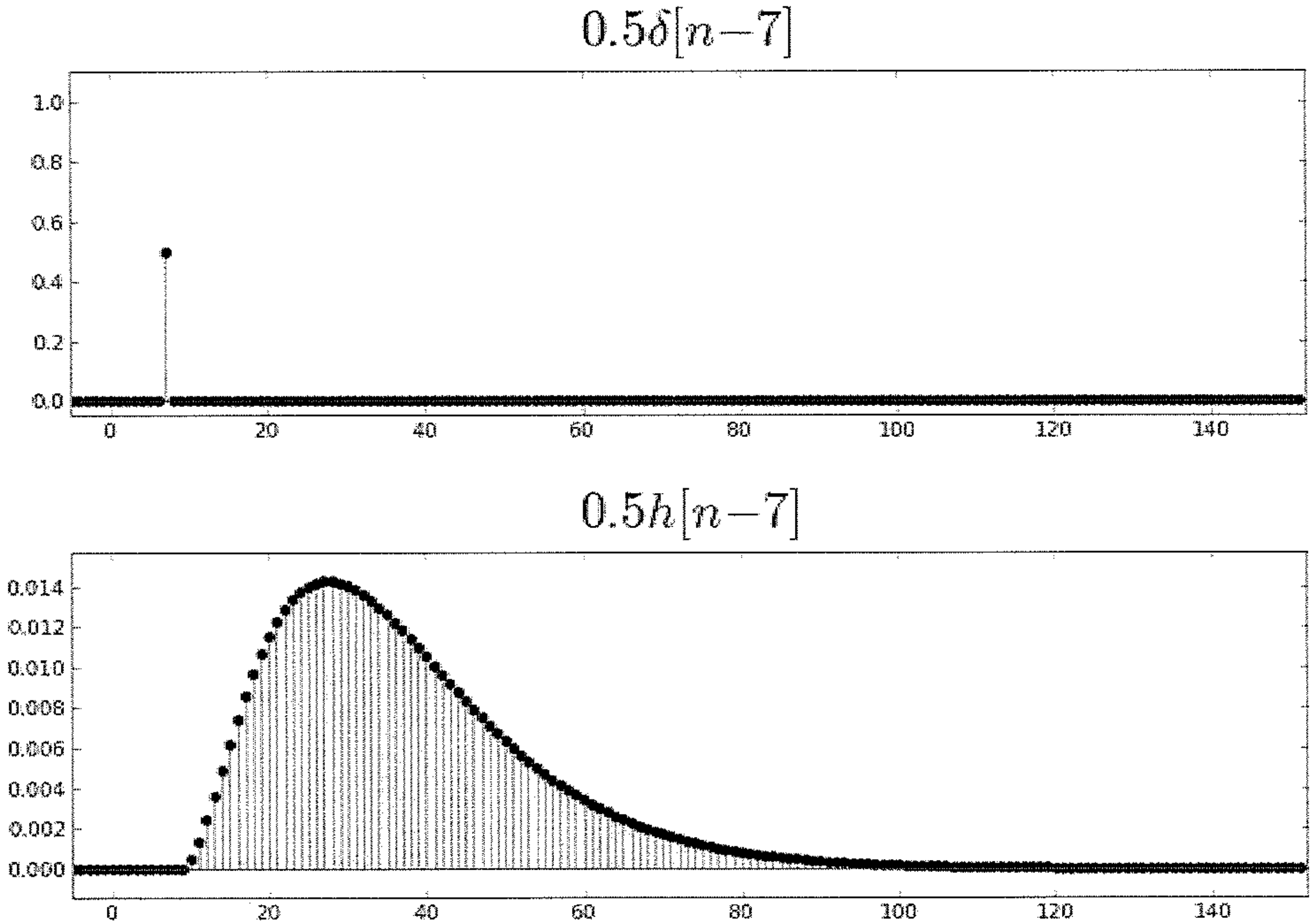
Notice
$h[0]=0$
$h[1]=0$
$h[2]=0$

Notice $h[n]$ is nonzero for ~ 80 samples (ISI for 40 sample/bit was two bits)

(That is, when $h[n]$ is nonzero for 2 bit periods, then two previous bits "interfere" with current bit)

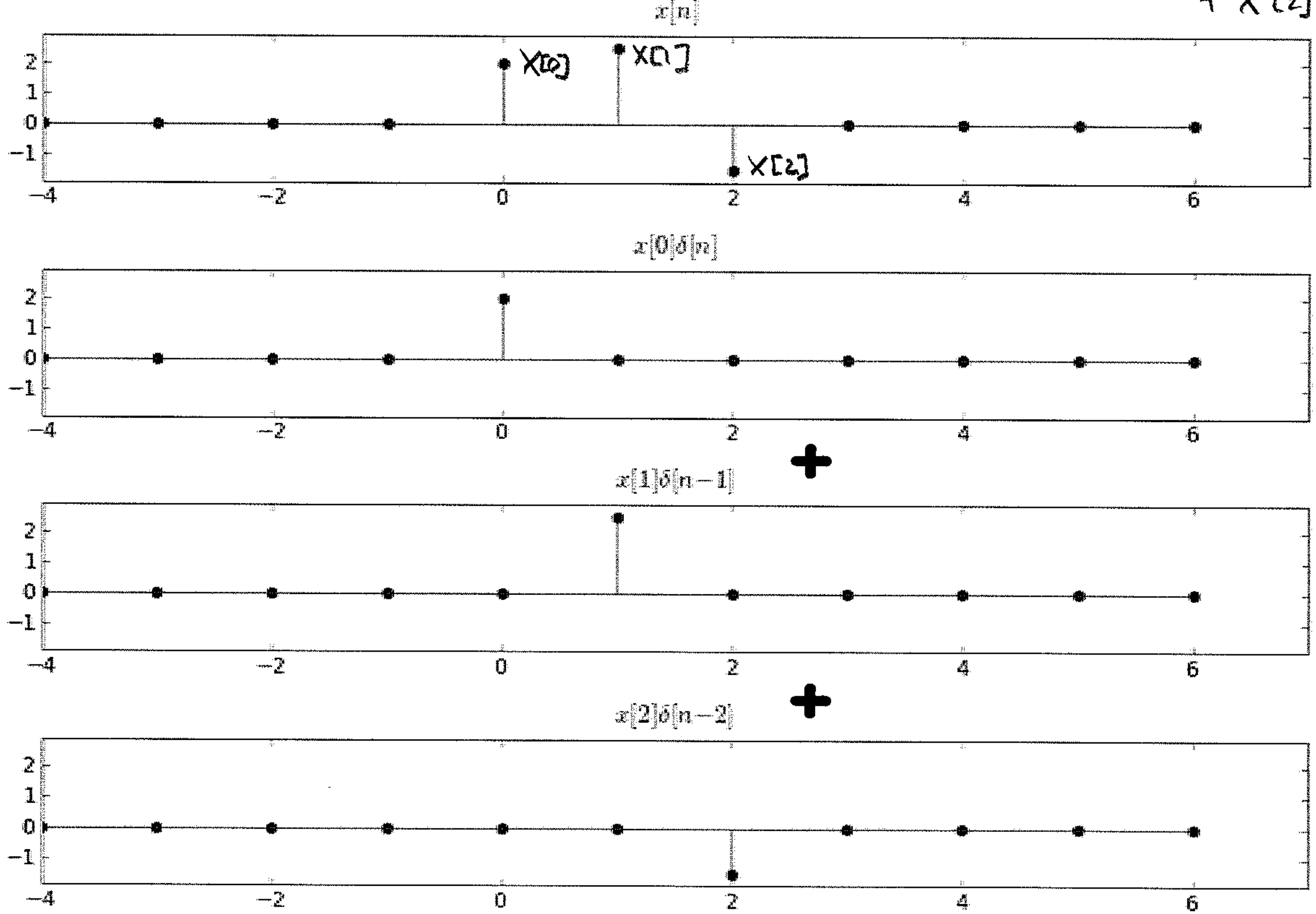
Shifted and Scaled Unit Sample Response

14



Decomposing $X[n] = X[0]\,\delta[n] + X[1]\,\delta[n-1] + X[2]\,\delta[n-2]$

(Note: $X[n]=0$ $n>2$ $n<0$)



In General for CLTI

$$x[n] = x[0]\,\delta[n] + x[1]\,\delta[n-1] + x[2]\,\delta[n-2] + \ldots$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$y[n] = x[0]\,h[n] + x[1]\,h[n-1] + x[2]\,h[n-2] + \ldots$$

In summation form:

Because $h[n-m]=0$ $(n-m)<0$

$$y[n] = \sum_{\underbrace{m=0}_{\text{Because } x[m]=0,\ m<0}}^{n} h[n-m]\,x[m]$$

Causality
Unit step response can not be nonzero before the unit step occurs.
$h[n]=0$
$n<0$

Notation:

$$\underset{\uparrow}{Y} = H \overset{\swarrow}{*} X$$

convolution symbol

sequences

Side Note Polynomial multiplication

$$(a[0] + a[1]z + a[2]z^2 + \ldots + a[n]z^n)(b[0] + b[1]z + \ldots + b[m]z^m)$$

Look like convolution?

$$= (a[0]\,b[0] + (a[0]\,b[1] + a[1]\,b[0])z + \ldots + a[n]\,b[m]z^{n+m})$$

Alternative Form

$$y[n] = \sum_{m=0}^{n} x[n-m]\, h[m]$$

$$y[n] = x[n]h[0] + x[n-1]h[1] + x[n-2]h[2] + \ldots$$

Suppose $h[n] = 0,\ n > K$

$$y[n] = h[0]x[n] + h[1]x[n-1] + \ldots + h[K]x[n-K]$$

$$\underbrace{\text{(If } n < K\text{, equation still holds as } x[n-K] = 0 \text{ because } x[n] = 0,\ n < 0)}_{\text{Difference Equation}}$$

***** Aside for the curious

Compare to 6.01 Right-shift operator $\mathcal{R}$

A S I D E

$$V = \mathcal{R}X \Rightarrow V[n] = X[n-1]$$
$$V = \mathcal{R}^2X \Rightarrow V[n] = X[n-2]$$

$$\Rightarrow Y = (h[0] + h[1]\mathcal{R} + \ldots + h[K]\mathcal{R}^K)X$$
$$= \left(\sum_{i=0}^{K} h[i]\mathcal{R}^i\right)X$$

Deriving Alternate Form

We have

$$y[n] = \sum_{m=0}^{m=n} h[n-m]\,x[m]$$

Introduce an intermediate variable

$$l \equiv n-m \underset{\text{Therefore}}{\Rightarrow} m = n-l$$

Rewriting

$$y[n] = \sum_{(n-l)=0}^{(n-l)=n} h[l]\,x[n-l]$$

$$= \sum_{l=0}^{l=n} h[l]\,x[n-l]$$

Since m and l are just used for indexing, relabel l with m

$$\underbrace{y[n] = \sum_{m=0}^{m=n} x[n-m]\,h[m]}_{\text{Alternate Form}}$$

A
S
I
D
E

$$y = \left(\sum_{i=0}^{k} h[i]\,\mathcal{Z}^i\right) x$$

w = reconstructed x

How to derive w from y:

$$w = \left(\frac{1}{\left(\sum_{i=0}^{k} h[i]\,\mathcal{Z}^i\right)}\right) y$$

or

$$\left(\sum_{i=0}^{k} h[i]\,\mathcal{Z}^i\right) w = y$$

<u>Deconvolution</u> $x \rightarrow \boxed{\text{channel}} \rightarrow y \rightarrow \boxed{\text{Decon}} \rightarrow w$

Why?

$$\left\{ h[0]\,w[n] + h[1]\,w[n-1] + \dots + h[k]\,w[n-k] = y[n] \right.$$

<u>Answer</u> Since W reconstructs X: $W \simeq X$

It should satisfy the same difference equation as X

<u>or</u>

See More General Derivation in Aside.

Solving for W

Plug & Chug

$$w[0] = \frac{1}{h[0]}\left[y[0] - \underbrace{\left(h[1]\,w[-1] + \dots + h[k]\,w[-k]\right)}_{\text{all zero as } w[n]=0\ n<0}\right]$$

$$w[1] = \frac{1}{h[0]}\left(y[1] - h[1]\,w[0]\right)$$

$$w[2] = \frac{1}{h[0]}\left(y[2] - \left(h[1]\,w[1] + h[2]\,w[0]\right)\right)$$

$$\vdots$$

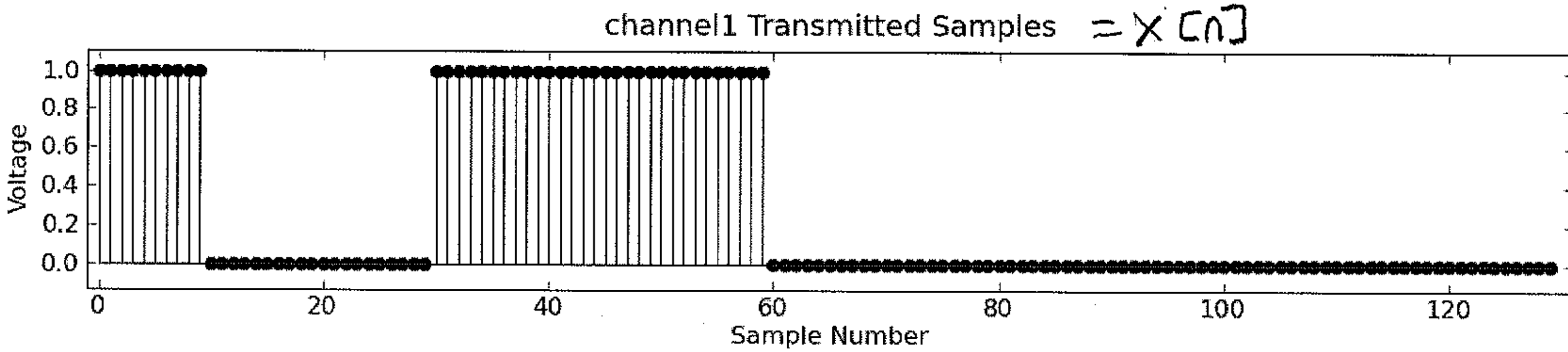
20

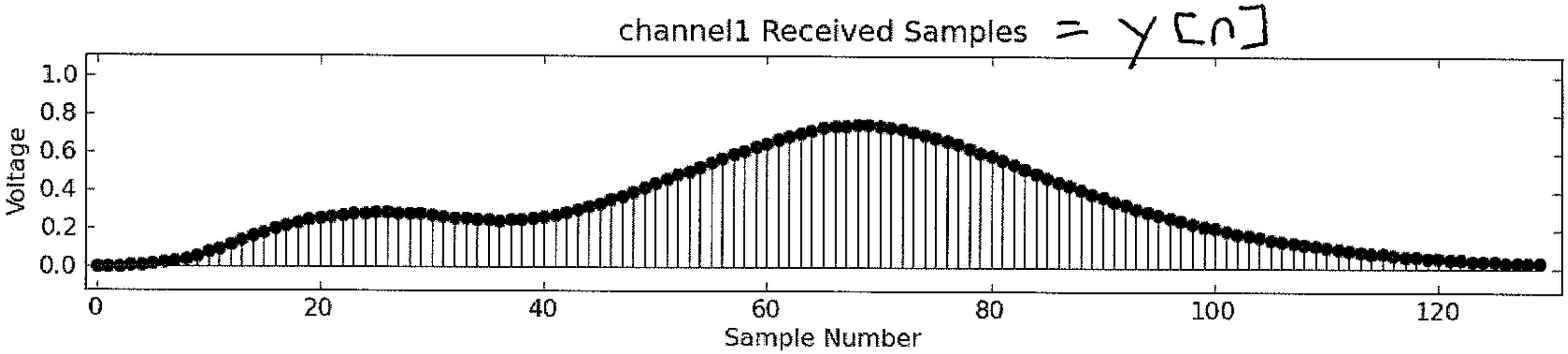
Noise-Free

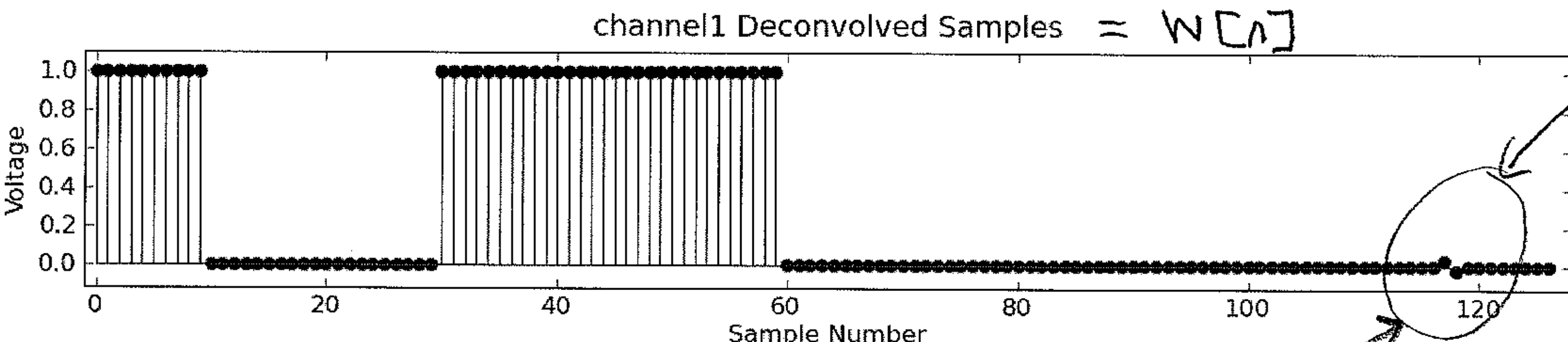
Deconvolution Result (h[n] = 0, n >= 120)

Truncated unit sample response ⇒ $W \simeq X$

Imperfect Deconvolution Because channel model Inexact!







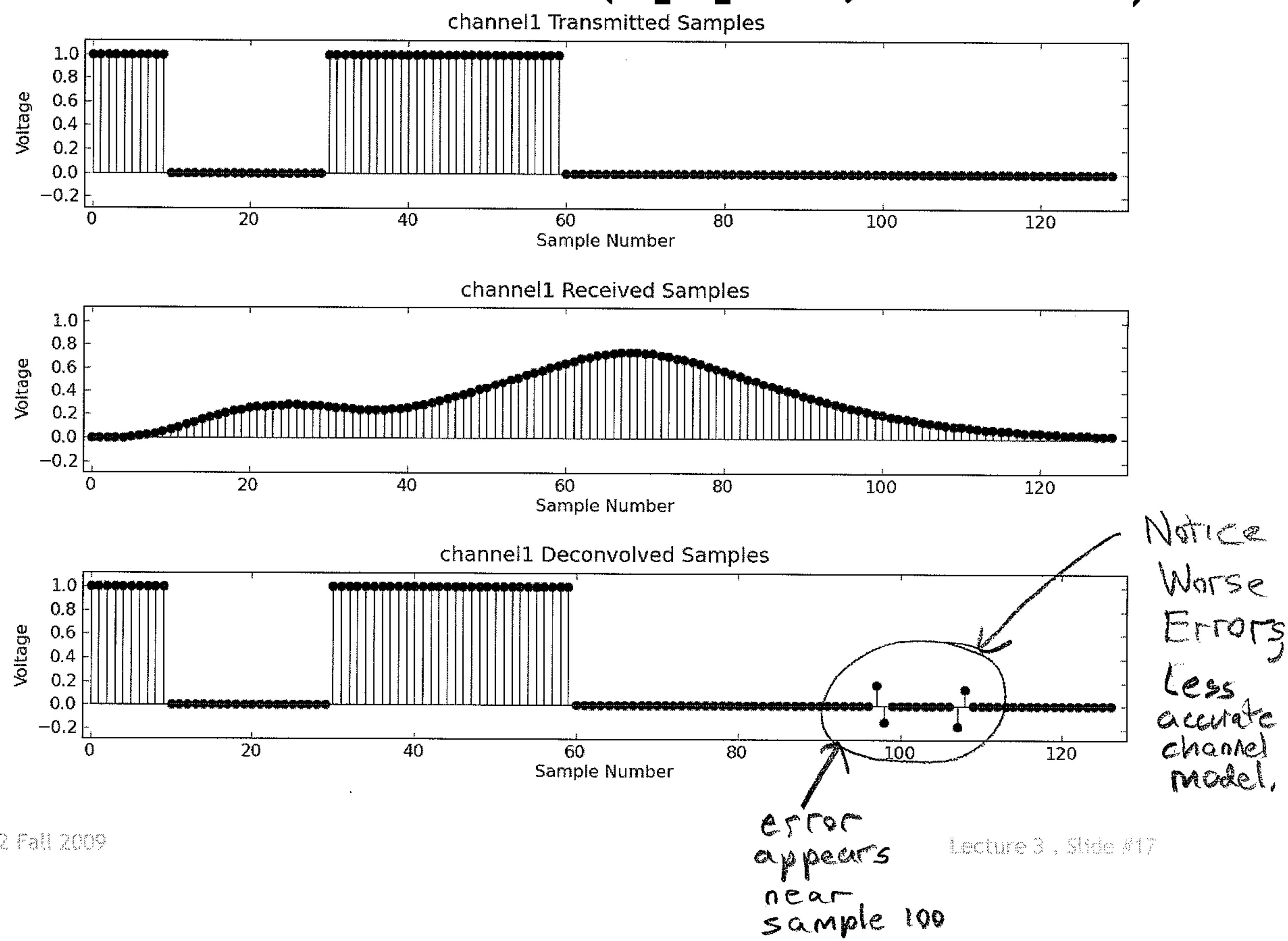
Notice small error

Error appears near sample 120

21

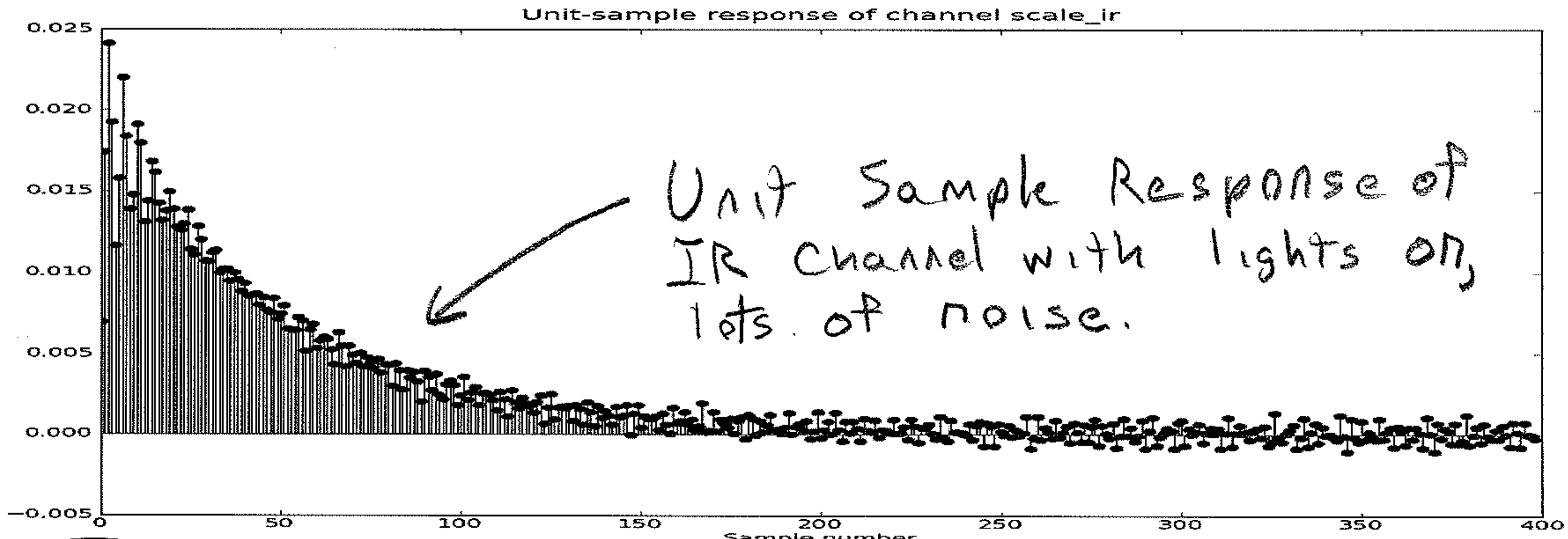
Noise-Free

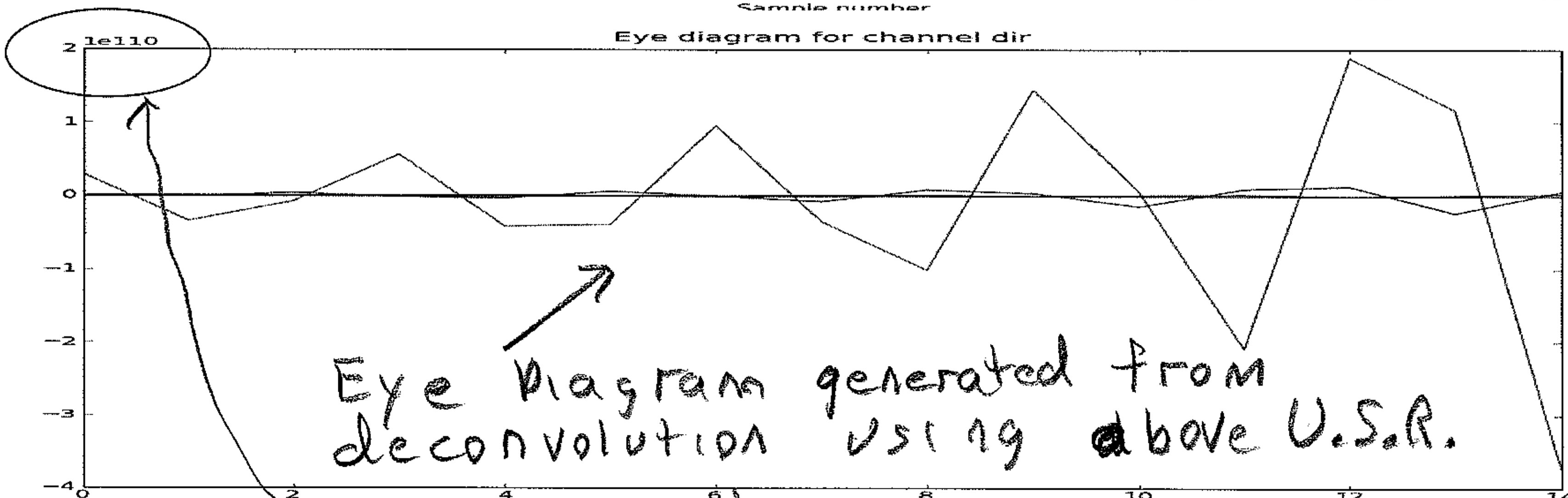
Deconvolution Result (h[n] = 0, n >= 100)



21A

How bad is noise for deconvolution?

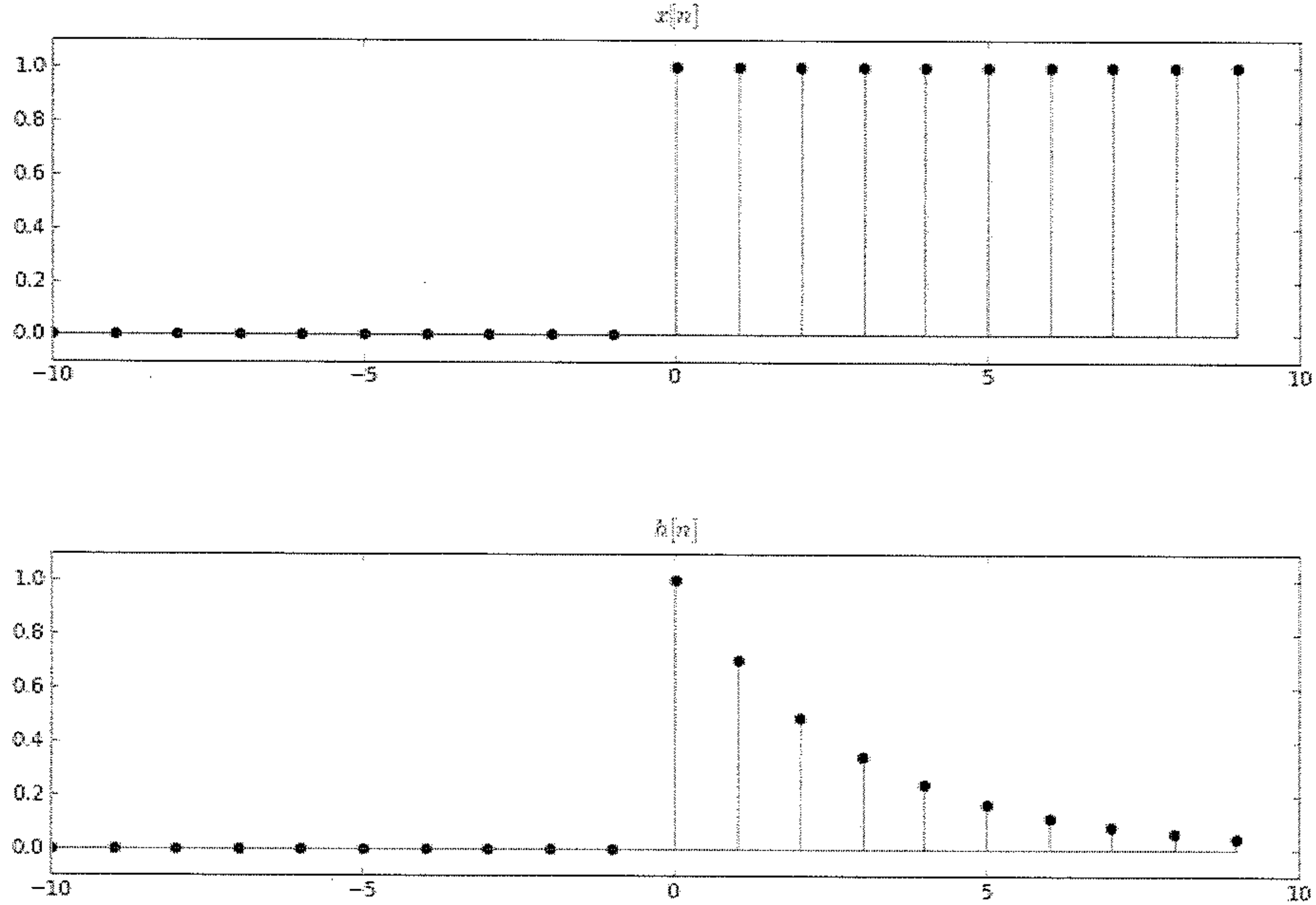




Just junk!

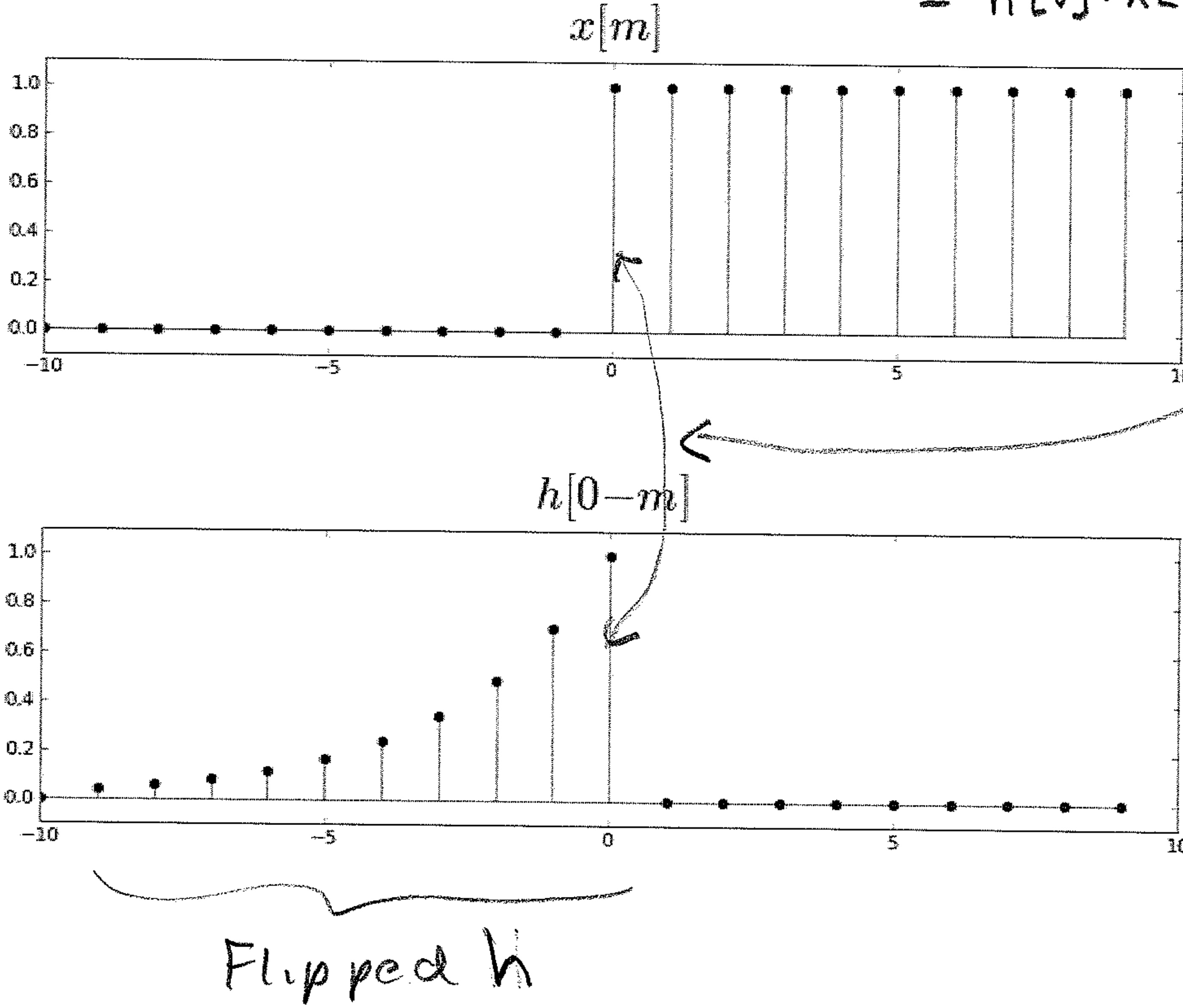
22

Example x[n] and h[n]

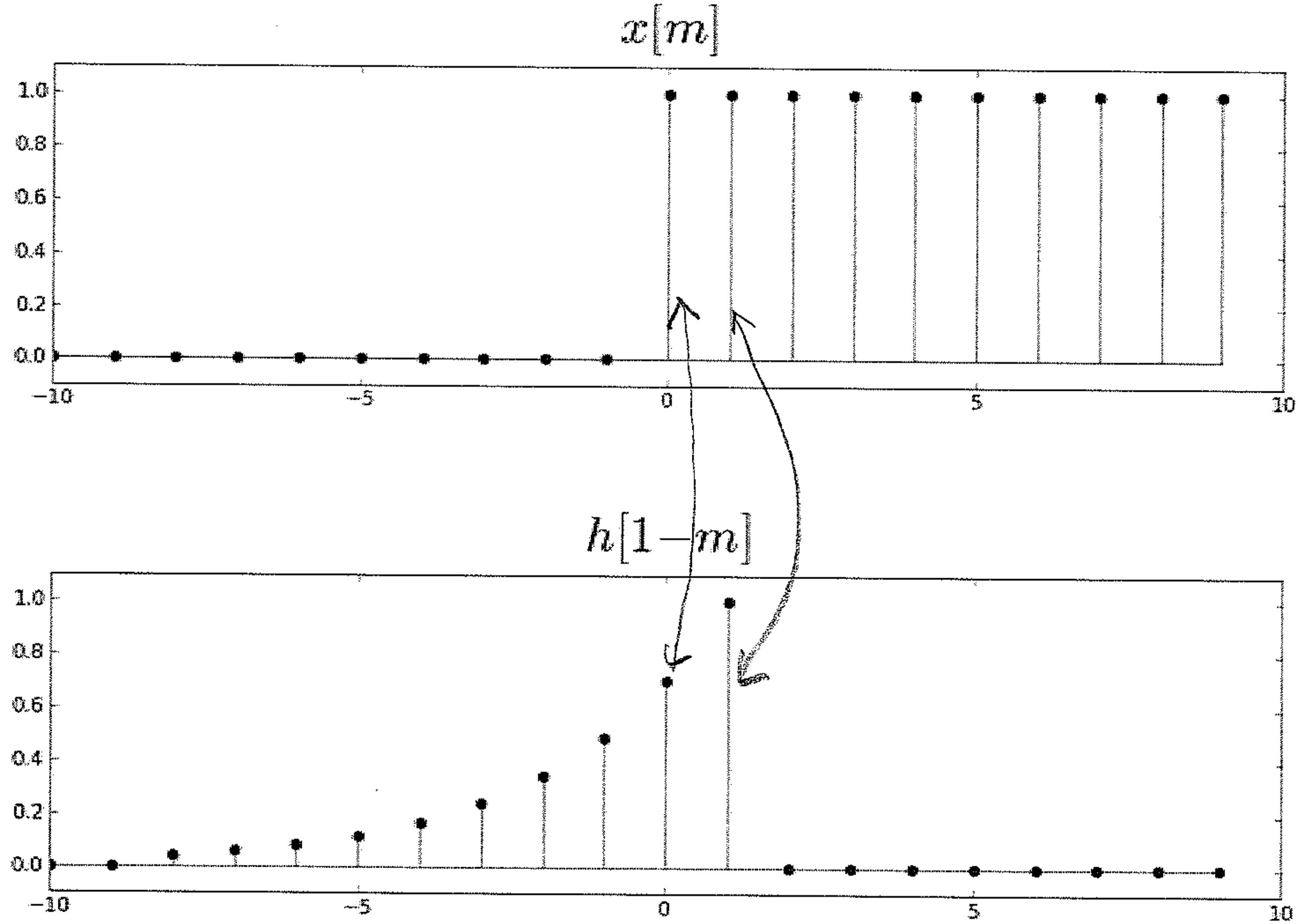


Evaluating y[0] $= \sum_{m=0}^{0} h[0-m]\, x[m]$

$= h[0] \cdot x[0]$

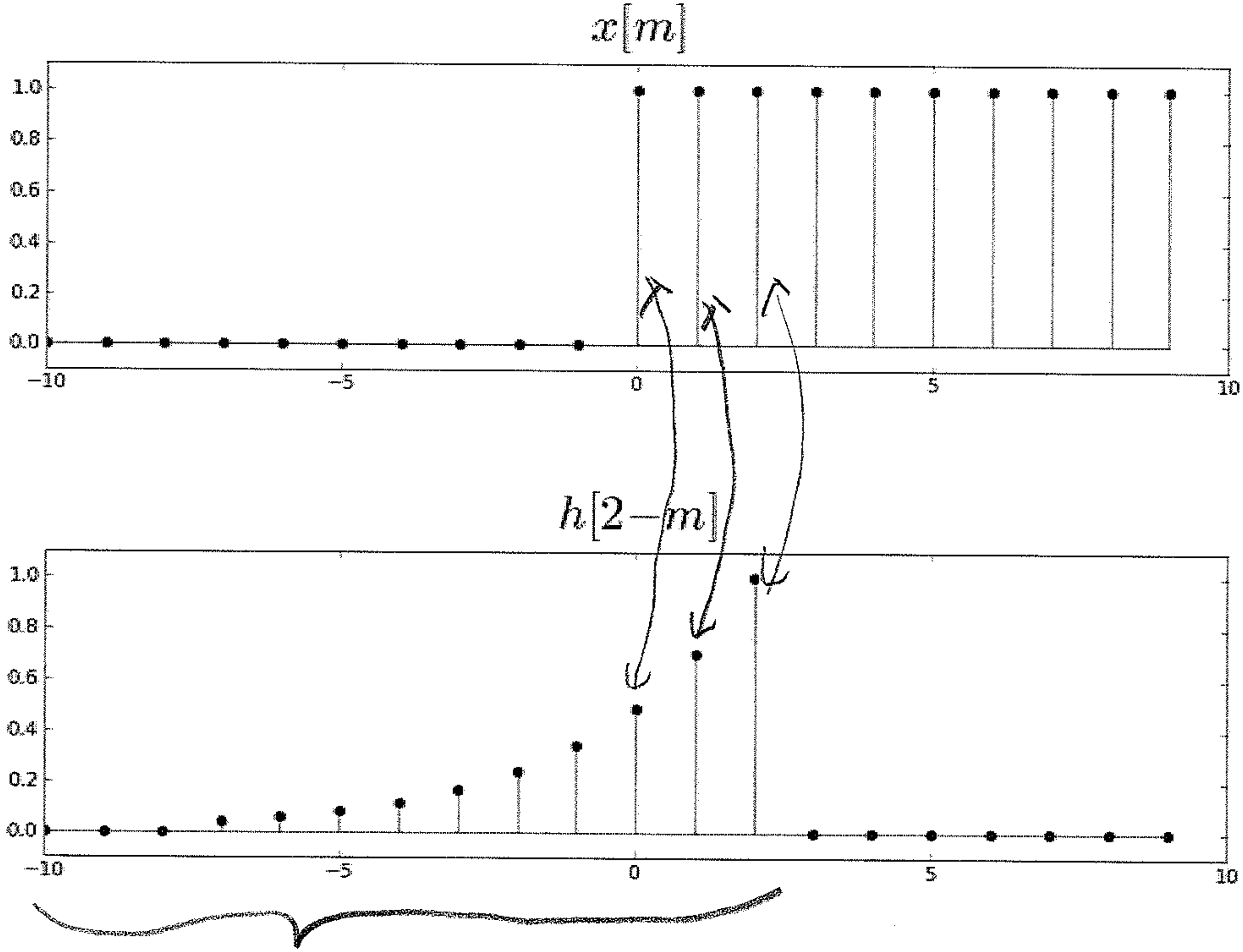


Evaluating y[1] $= h[0]x[1] + h[1]x[0]$



Flipped h, slid by 1.

Evaluating y[2] $= x[0]h[2] + x[1]h[1] + x[2]h[0]$



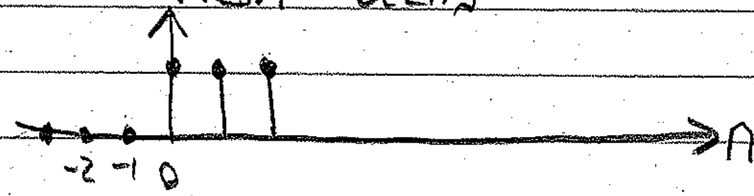
Flipped h
Slid by 2

Relating Step Response to Unit sample response

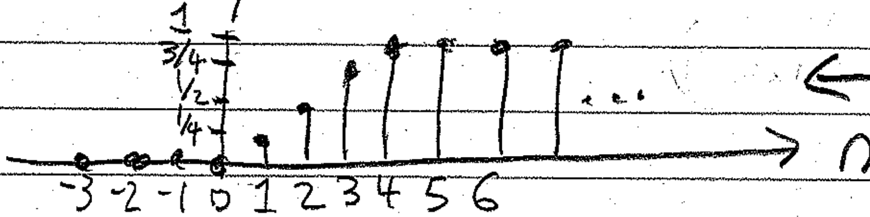
$$X \longrightarrow \boxed{\text{Channel}} \longrightarrow Y$$

If $x[n] = u[n]$

$x[n] = u[n]$



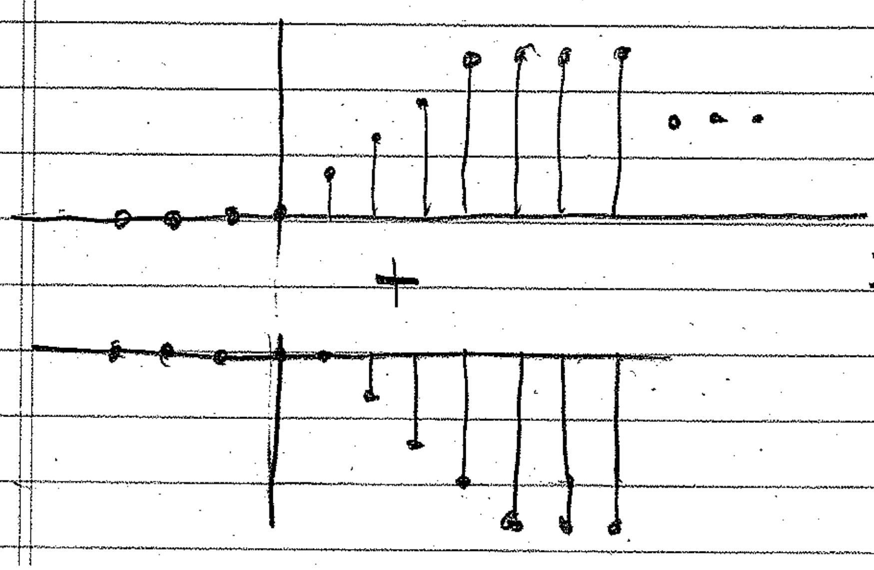
$y[n] = s[n]$ (step response)

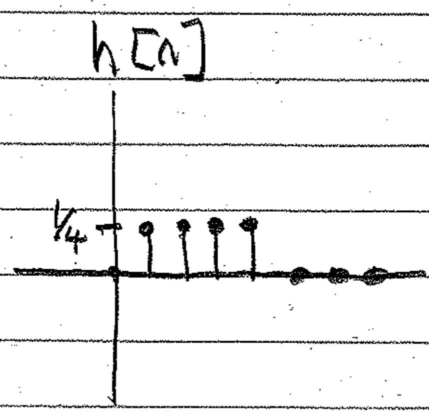


← Example

If $x[n] = \delta[n]$ then $x[n] = u[n] - u[n-1]$

$$y[n] = s[n] - s[n-1] = h[n]$$





(25B)

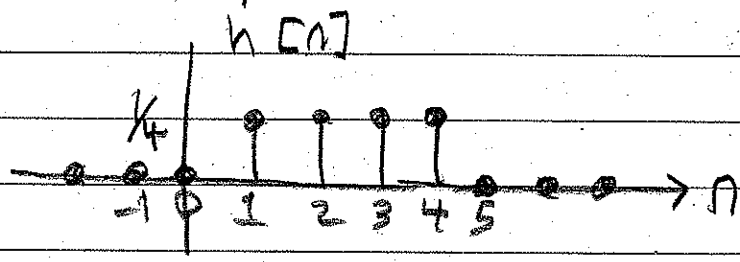
If $x[n] = u[n]$

$$y[n] = \sum_{m=0}^{n} h[m]\, x[n-m]$$

$$= \sum_{m=0}^{n} h[m]\, u[n-m] \quad = 1 \;\; n-m \geq 0$$

$$y[n] = \sum_{m=0}^{n} h[m]$$

Example



$y[0] = h[0] = 0$

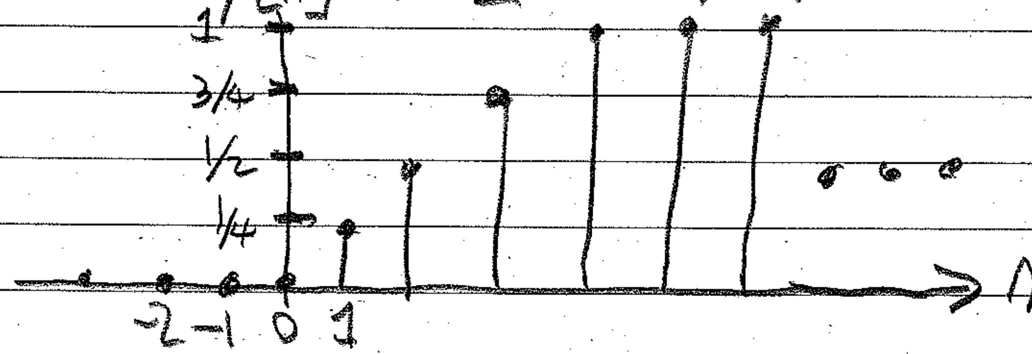
$y[1] = h[0] + h[1] = 1/4$

$y[2] = h[0] + h[1] + h[2] = 1/2$

$y[3] = \sum_{m=0}^{3} h[m] = 3/4$

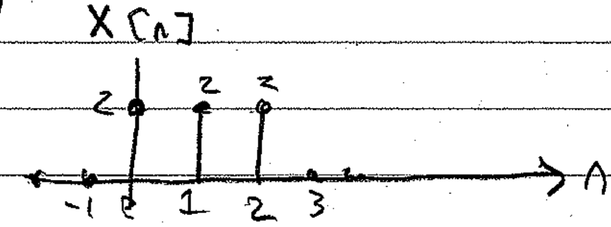
$y[4] = \sum_{m=0}^{4} h[m] = 1$

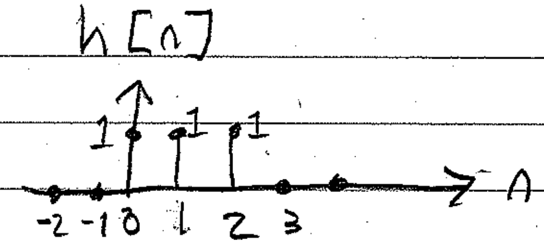
$y[n] = 1 \quad n > 4$



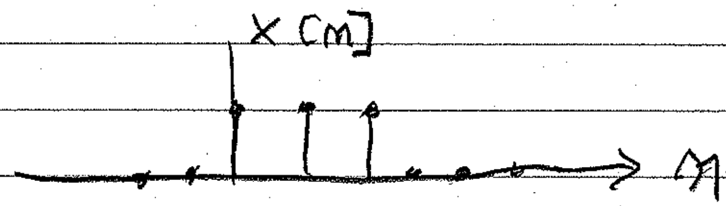
Convolution Examples

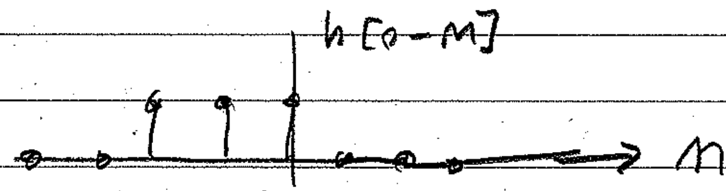
1)

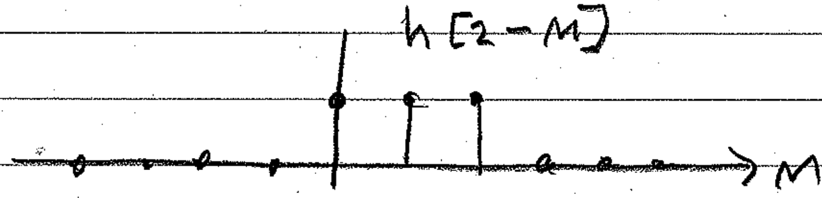


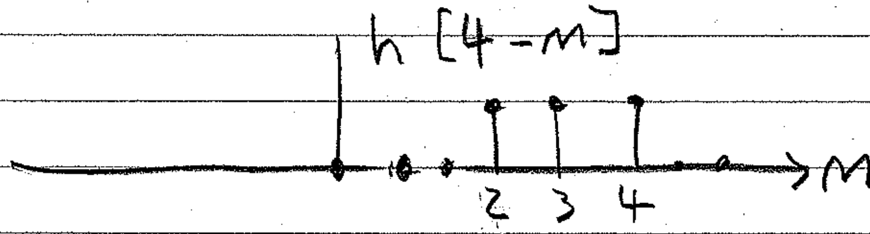


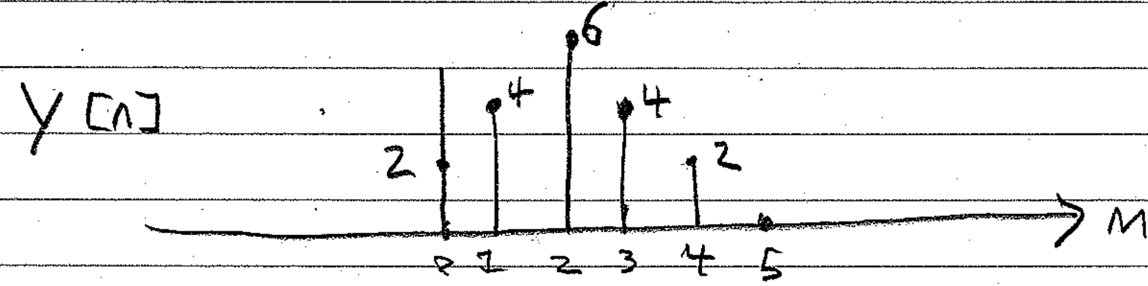
$$Y = H * X$$





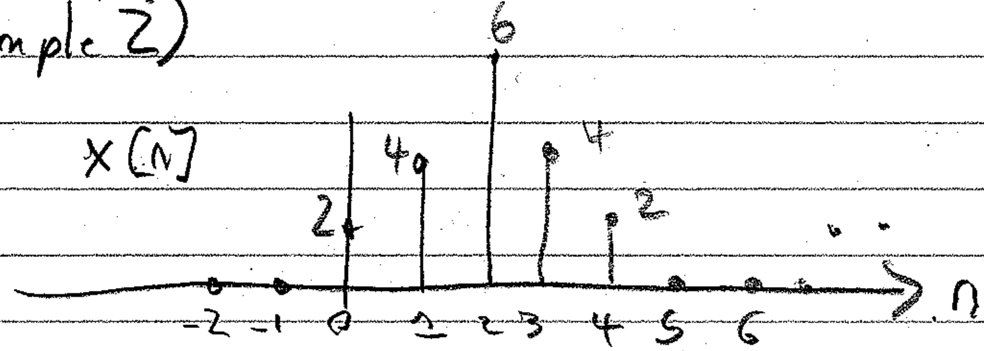


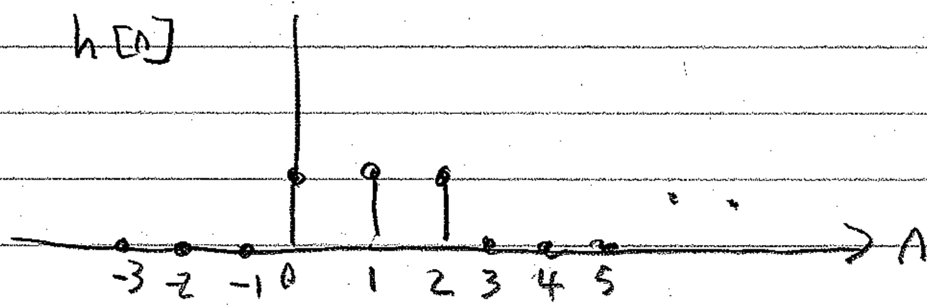




<u>Note</u> y[n]'s maximum value is 6 at n=2.
Easy to see from flip & slide

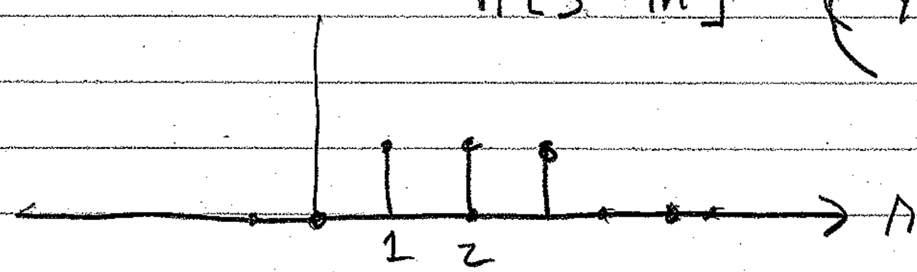
Example 2)

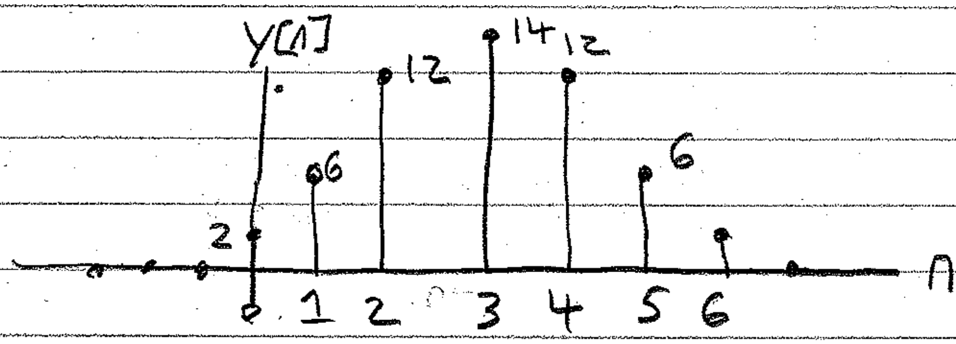




Flip & Slide

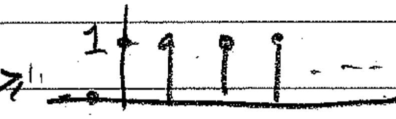
h[3 − m]   (y[n] max at n=3)

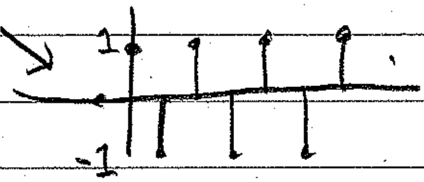


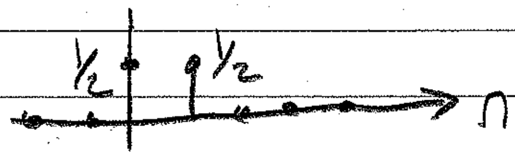


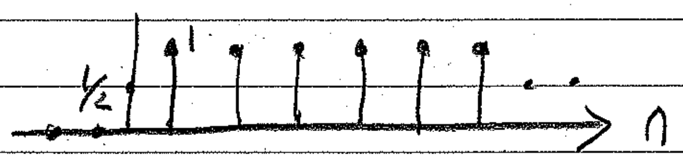
Note   Example 1 □ * □ = Δ
Example 2 Δ * □ = ⌒   } Each Convolution smooths

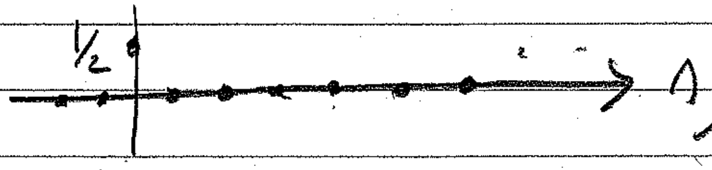
Example 3)

$x_1[n] = u[n]$


$x_2[n] = (-1)^n u[n]$


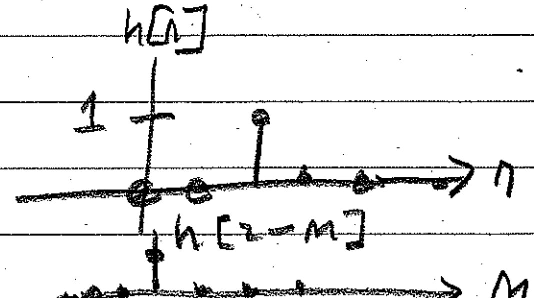
$h[n] =$


$y_1[n] =$


$y_2[n] =$


Easy to see by Flip and slide

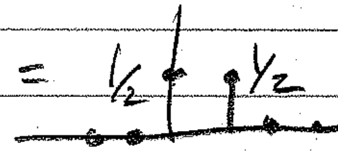
Example 4)

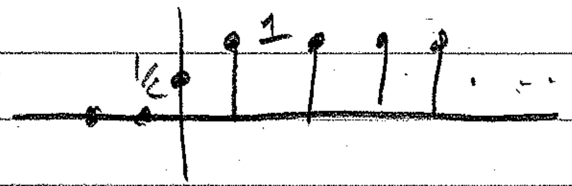
$h[n] = \delta[n-2]$


$y[n] = x[n-2]$

Deconvolution Example

Example 1)

$x[n] = u[n]$ $\qquad h[n] =$ 

$\Rightarrow y[n] =$ 

$$h[0]\,w[n] + h[1]\,w[n-1] = y[n]$$

$$w[0] = \frac{1}{\frac{1}{2}}\left(y[0] - \cancel{h[1]\,w[-1]}^{=0}\right)$$

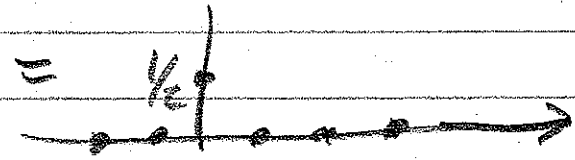
$$= 1$$

$$w[1] = \frac{1}{\left(\frac{1}{2}\right)}\left(y[1] - h[1]\,w[0]\right)$$

$$= 1$$

$$w[n] = 1 \quad n \geq 0 \Rightarrow \overline{W} = \overline{X}\ !$$

Example 2) $\qquad x[n] = (-1)^n u[n]$, same $h[n]$

$y[n] =$ 

$$w[n] = \left(\frac{1}{h[0]}\right)\left(y[n] - h[1]\,w[n-1]\right)$$

$$= 2\left(y[n] - \tfrac{1}{2}\,w[n-1]\right)$$

Example 2 Cont)

$$w[0] = 2\left(y[0] - \tfrac{1}{2}\underbrace{\cancel{w[-1]}}_{=0}\right)$$

$$= 2\left(\tfrac{1}{2} - 0\right) = 1$$

$$w[1] = 2\left(\underbrace{\cancel{y[1]}}_{=0} - \tfrac{1}{2}w[0]\right)$$

$$= -1$$

$$w[2] = 2\left(y[2] - \tfrac{1}{2}w[1]\right)$$

$$= 1$$

$$w[n] = (-1)^n u[n]$$

Example 3

$$x[n], \quad h[n] = \delta[n-2]$$

$$\Rightarrow \quad y[n] = x[n-2]$$

Deconvolution Equation:

$$\cancel{h[0]}^{=0}\, w[n] + \cancel{h[1]}^{=0}\, w[n-1] + \cancel{h[2]}^{=1}\, w[n-2] = y[n]$$

$$\Rightarrow \quad w[n-2] = y[n] = x[n-2]$$

$$\text{or} \quad w[n] = y[n+2] = x[n]$$

$$\Rightarrow \quad w[n-2] = x[n-2]$$

$$w[n] = x[n]$$

Note To compute $w[n]$, you need $y[n+2]$! The deconvolution equation does NOT describe a causal system!