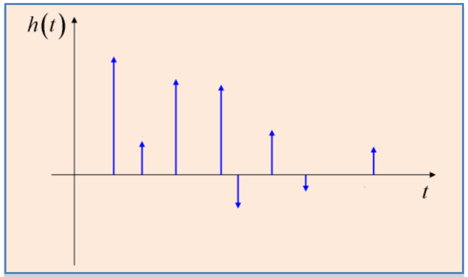
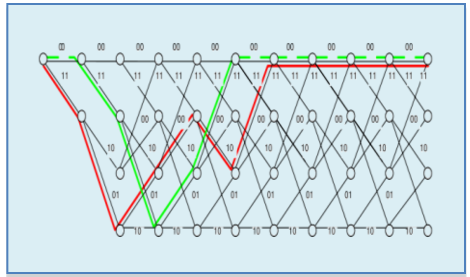
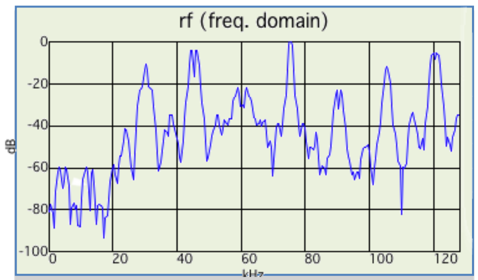
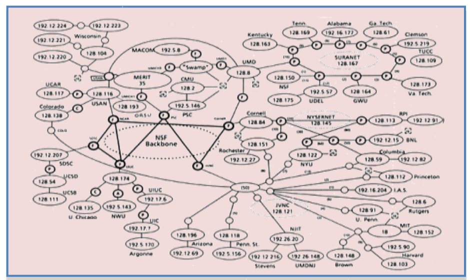








INTRODUCTION TO EECS II

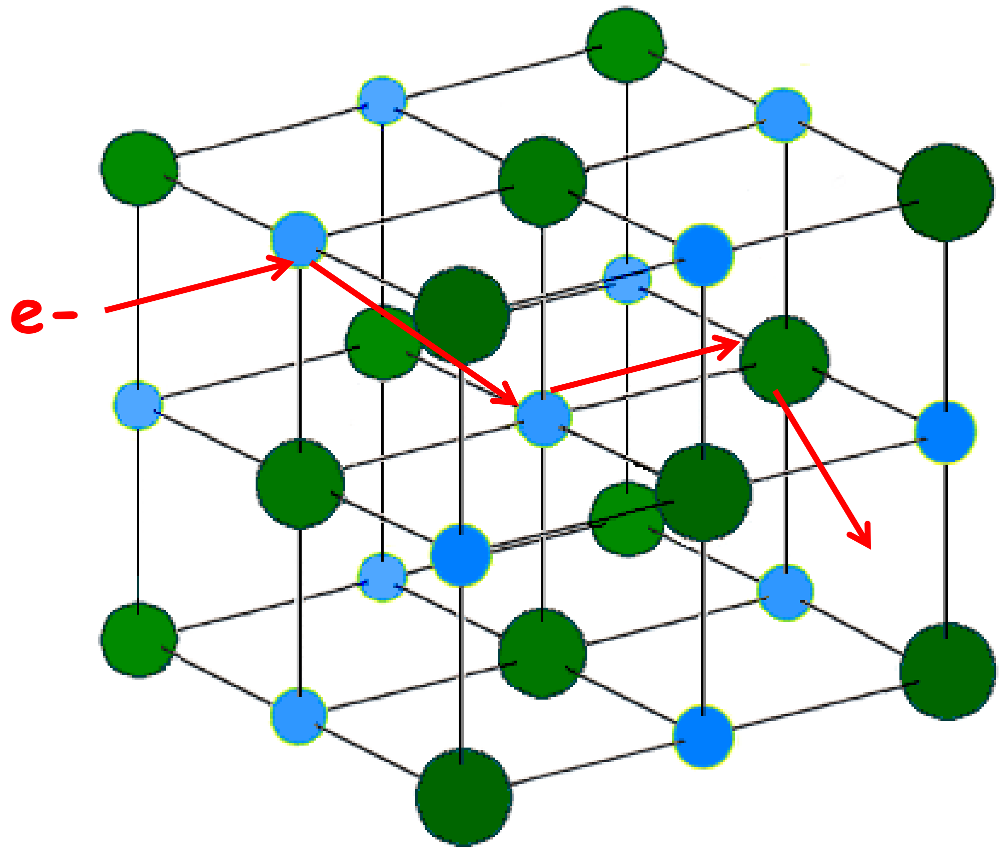
DIGITAL COMMUNICATION SYSTEMS

6.02 Fall 2010
Lecture #4

- Signals and Noise
  - Noise sources and bit errors
- Analyzing Noise:
  - PDF, CDF, and Noise + Noisefree decomposition
- Computing Bit Error Rates:
  - The No ISI case, the impact of the Noise PDF.

Noise Can Be Due to Fundamental Processes

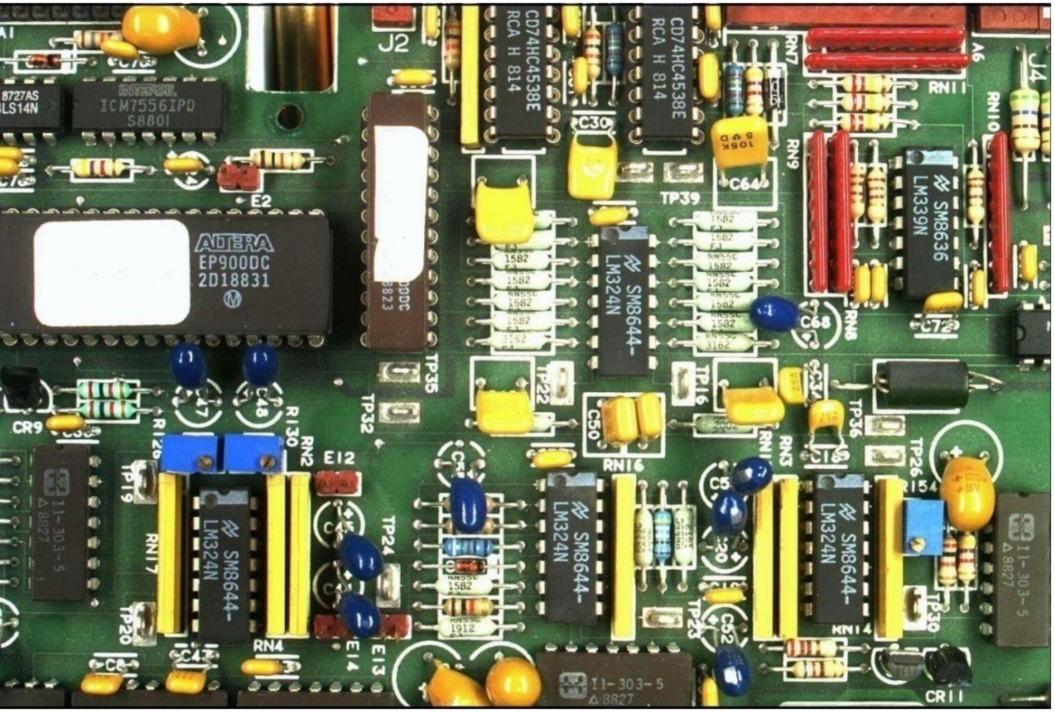
Electron Moving Through Crystal with Vibrating Atoms

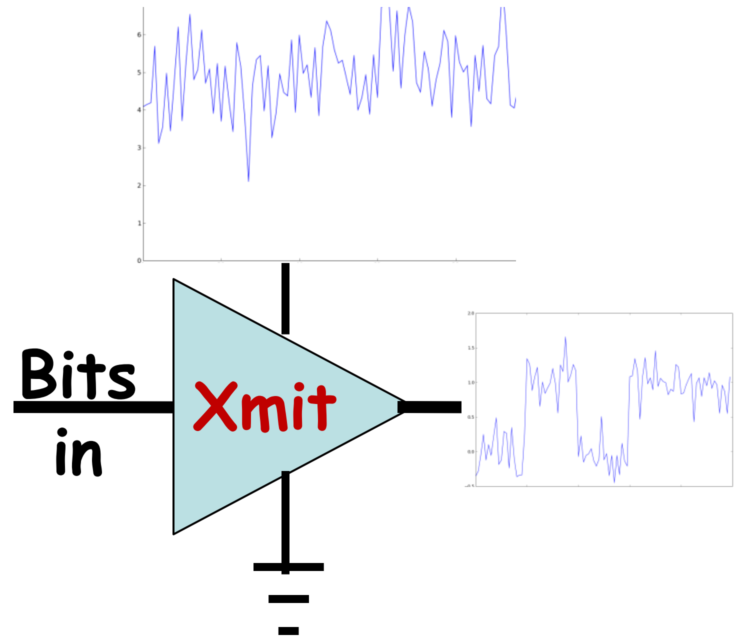


http://www4.nau.edu/meteorite/Meteorite/Images/Sodium_chloride_crystal.png

Randomized path leads to noisy current flow

Effect of Many Interactions Can Be Modeled as Noise

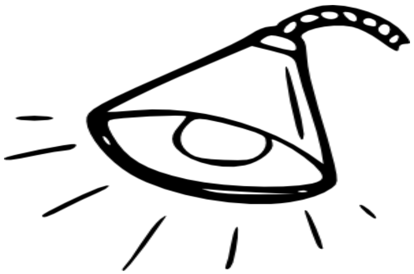




http://www.imageteck.net/PCB%20Board%203.JPG

Many components connected by thin wires (that have inductance and resistance) to single power supply – 1000's of devices switching on and off creates "noisy" power supply.

6.02 IR Transceiver Noise Source - Room Light



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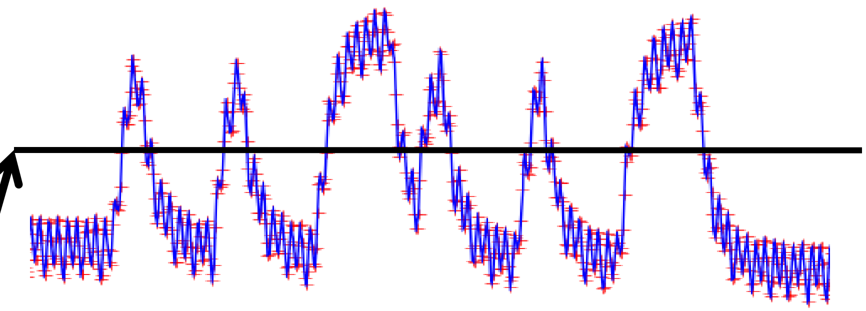
71585298002ryanlerch_lamp_outline.svg.med.png

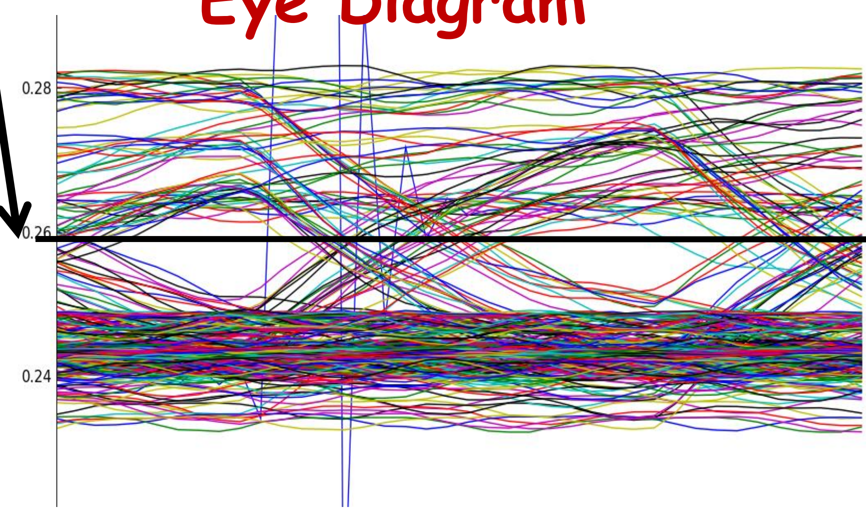
20 Samples/bit

Receiver Waveform

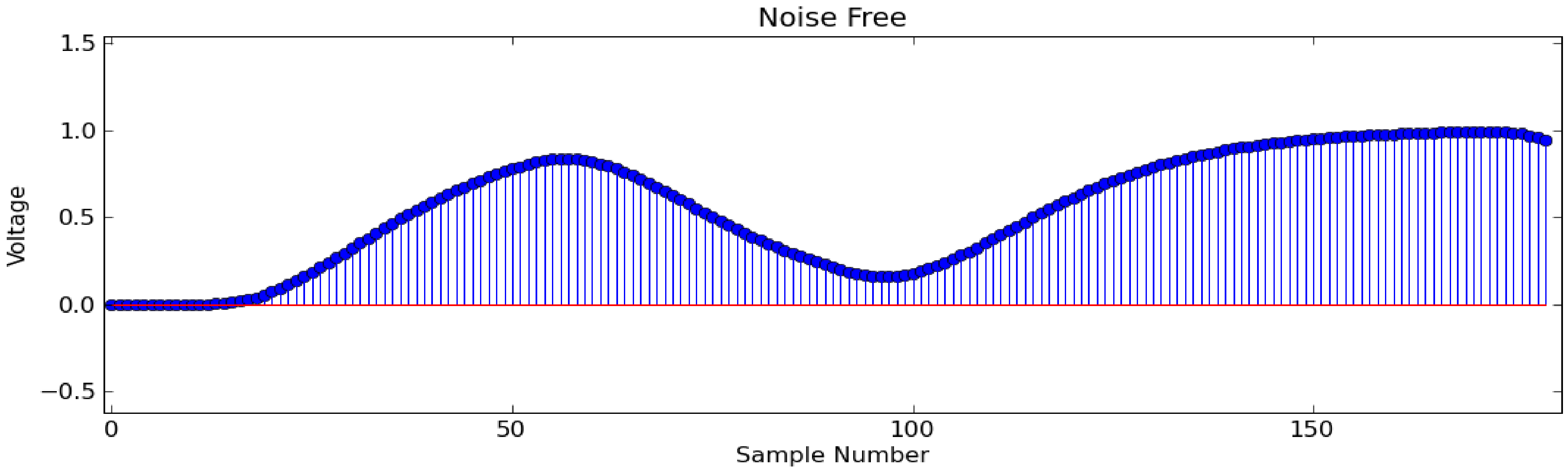


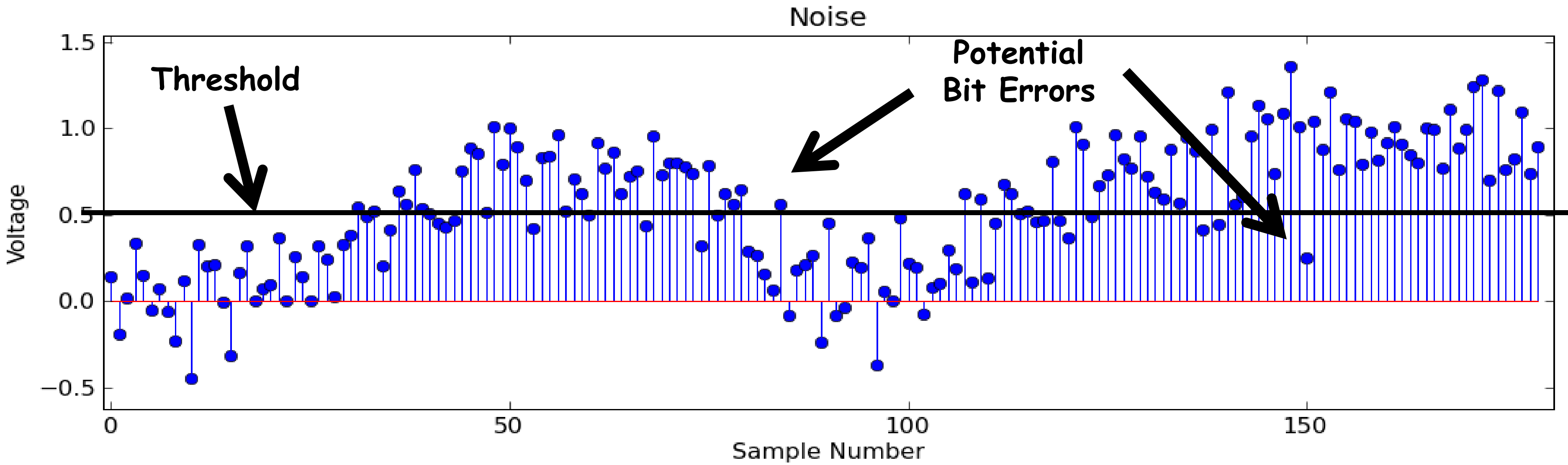
Threshold

Eye Diagram



Noisy Signal Can Cause Bit Errors





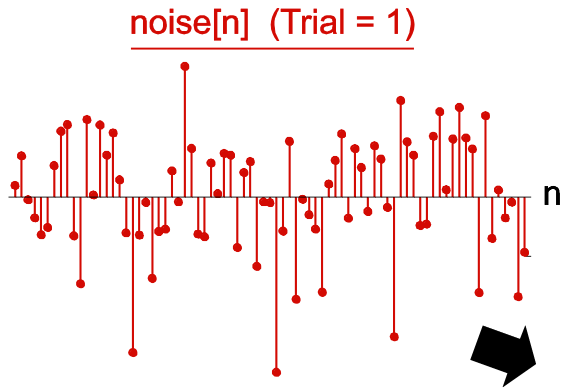
Key Noise Questions For A Channel

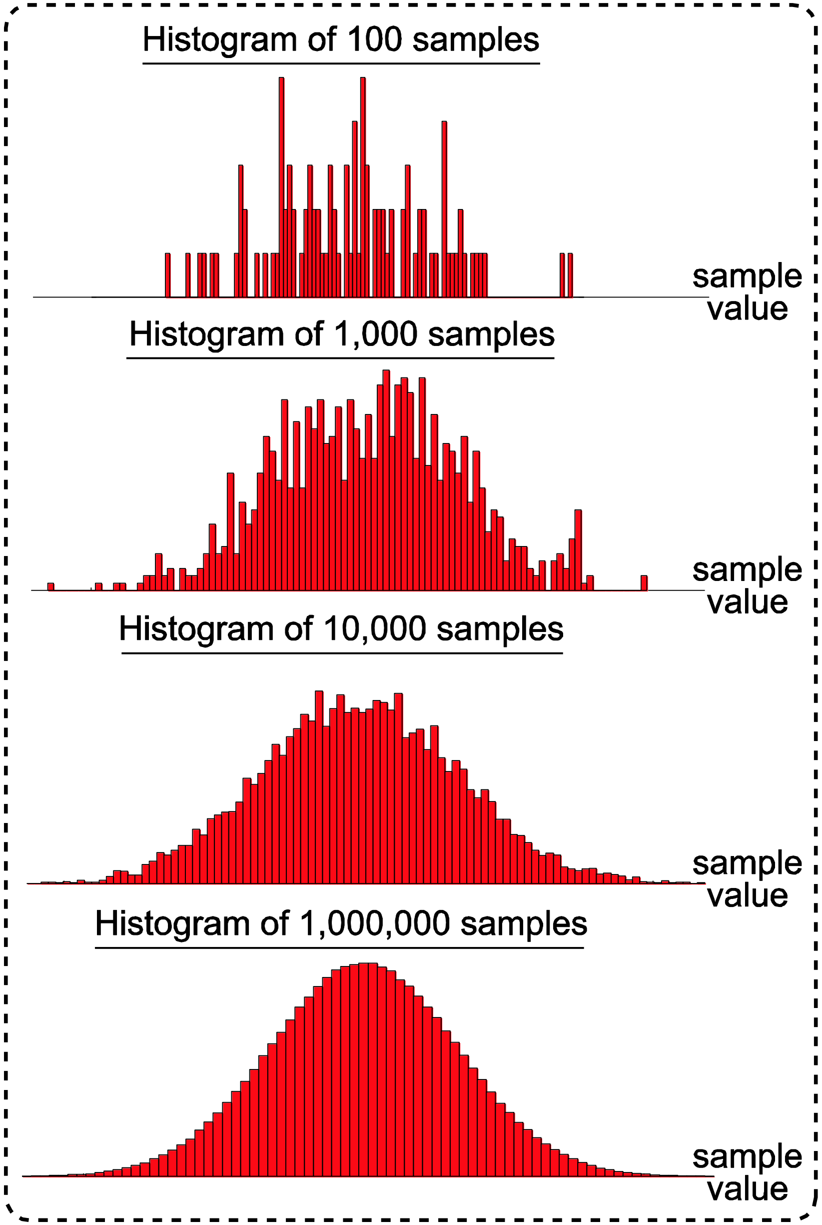
- For a given Transmission Scheme:
  - What's the Bit Error Rate (BER)?
    - BER: Fraction of erroneously received bits
- If the Signal is increased:
  - How much is BER reduced?
- If ISI is reduced:
  - Is BER reduced significantly?

To Answer these questions

- Need to Characterize the Noise
  - What "shape" (probability density function)?
  - How "big" (variance)?

Experiment to see Noise "Shape"





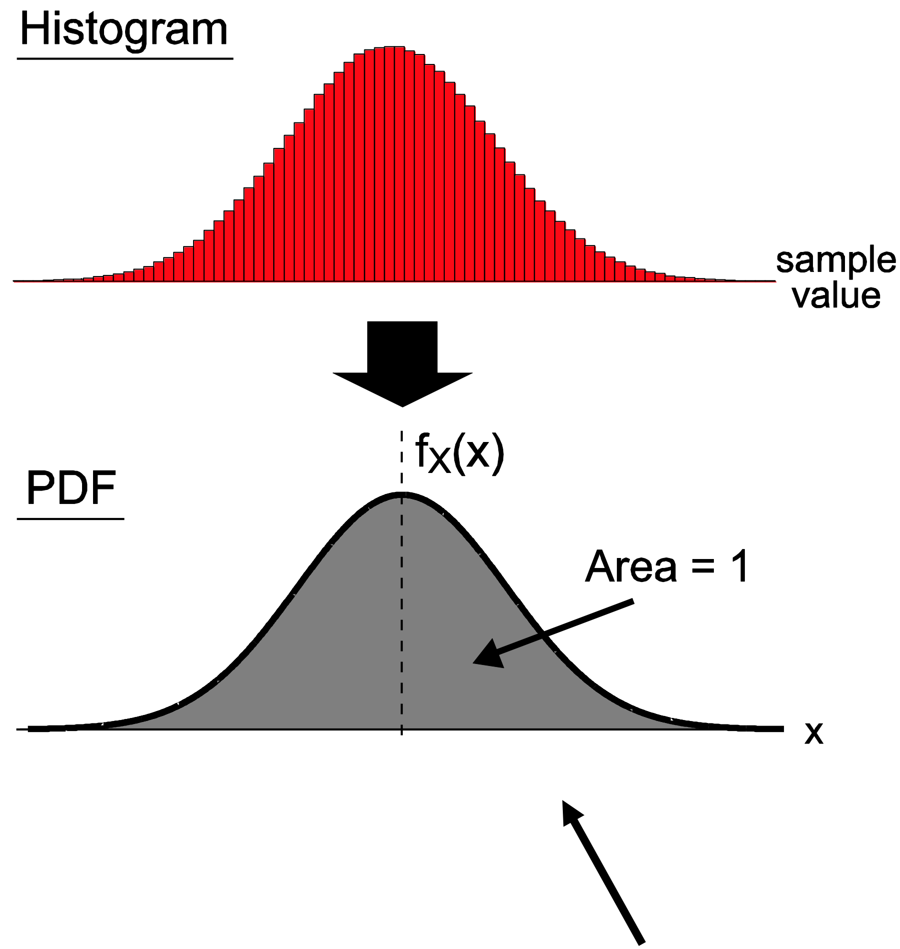
- Create histograms of sample values from trials of increasing lengths
- Assumption of independence and stationarity implies histogram should converge to a shape known as a probability density function (PDF)

Slide thanks to C. Sodini and M. Perrott

"Shape" = Probability Density Function PDF

- **Define X as a random variable whose PDF has the same shape as the histogram we just obtained**
- **Denote PDF of X as $f_X(x)$**
  - **Scale $f_X(x)$ such that its overall area is 1**

$$\Rightarrow \int_{-\infty}^{\infty} f_X(x) = 1$$



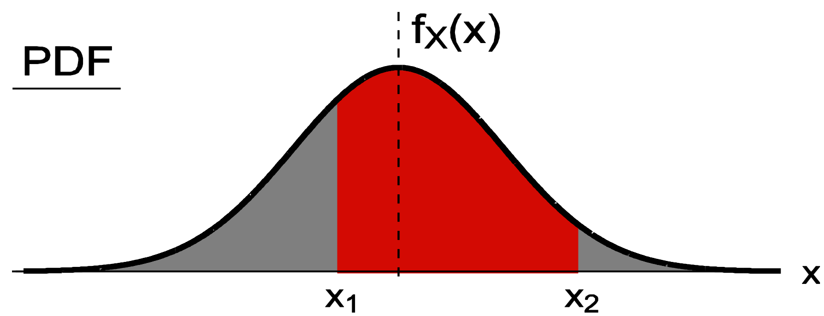
This shape is referred to as a Gaussian PDF

Slide thanks to C. Sodini and M. Perrott

PDF → Probability and PDF → CDF

- The *probability* that random variable *X* takes on a value in the range of x_1 to x_2 is calculated from the PDF of *X* as:

$$\mathrm{Prob}(x_1 \leq x \leq x_2) = \int_{x_1}^{x_2} f_X(x)dx$$

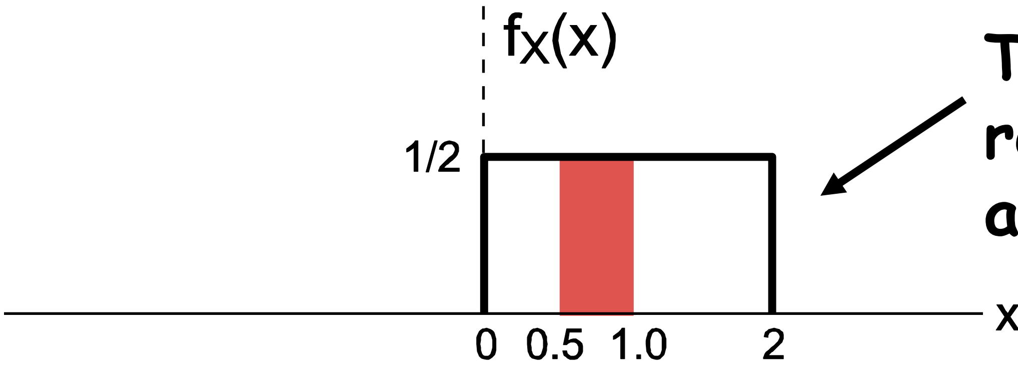


- The *probability* that random variable *X* takes on a value less than x_1 is the cumulative distribution function **CDF(x_1)**

$$\mathrm{CDF}(x_1) = \mathrm{Prob}(x \leq x_1) = \int_{-\infty}^{x_1} f_X(x)dx$$

Slide thanks to C. Sodini and M. Perrott

Example Probability Calculation



This shape is referred to as a uniform PDF

- **Verify that overall area is 1:**

$$\int_{-\infty}^{\infty} f_X(x)dx = \int_0^2 0.5\,dx \; = \boxed{1}$$

- **Probability that x takes on a value between 0.5 and 1.0:**

$$\text{Prob}(0.5 \le x \le 1.0) = \int_{0.5}^{1.0} 0.5\,dx \; = \boxed{0.25}$$

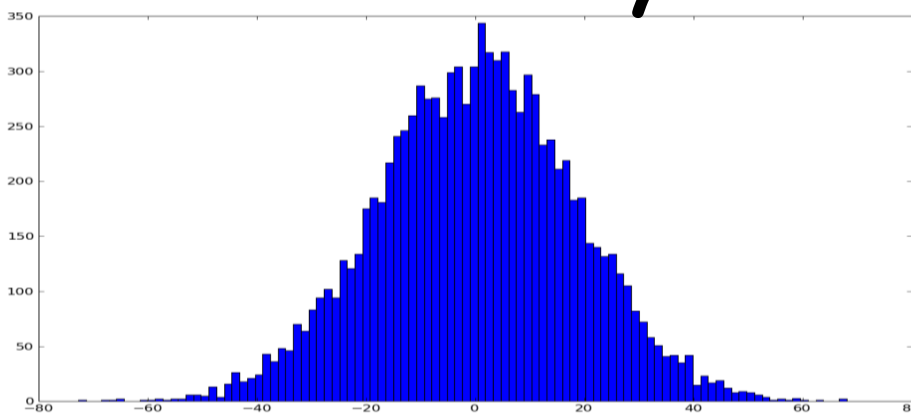
Slide thanks to C. Sodini and M. Perrott

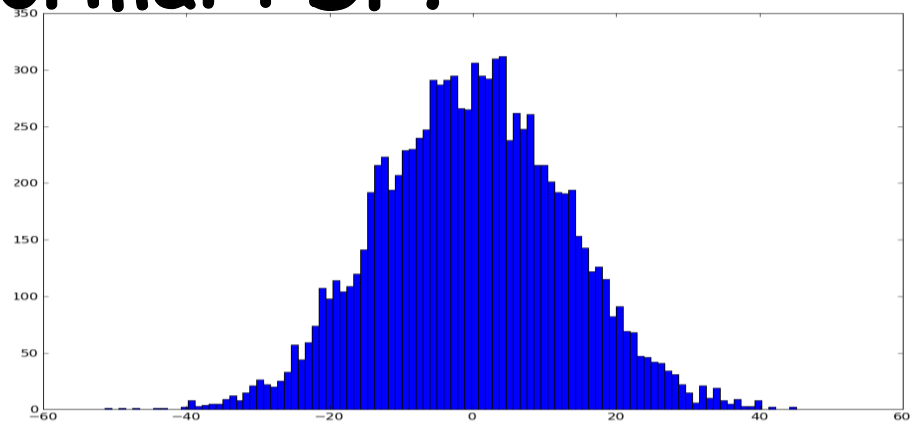
Noise Modeled Using Normal (Gaussian) PDF

$$f_X(x) = \frac{1}{\sqrt{2\pi}\ \sigma} e^{-\frac{x^2}{2\sigma^2}}$$

$$\sigma = \text{standard deviation}$$

Why the Normal PDF?

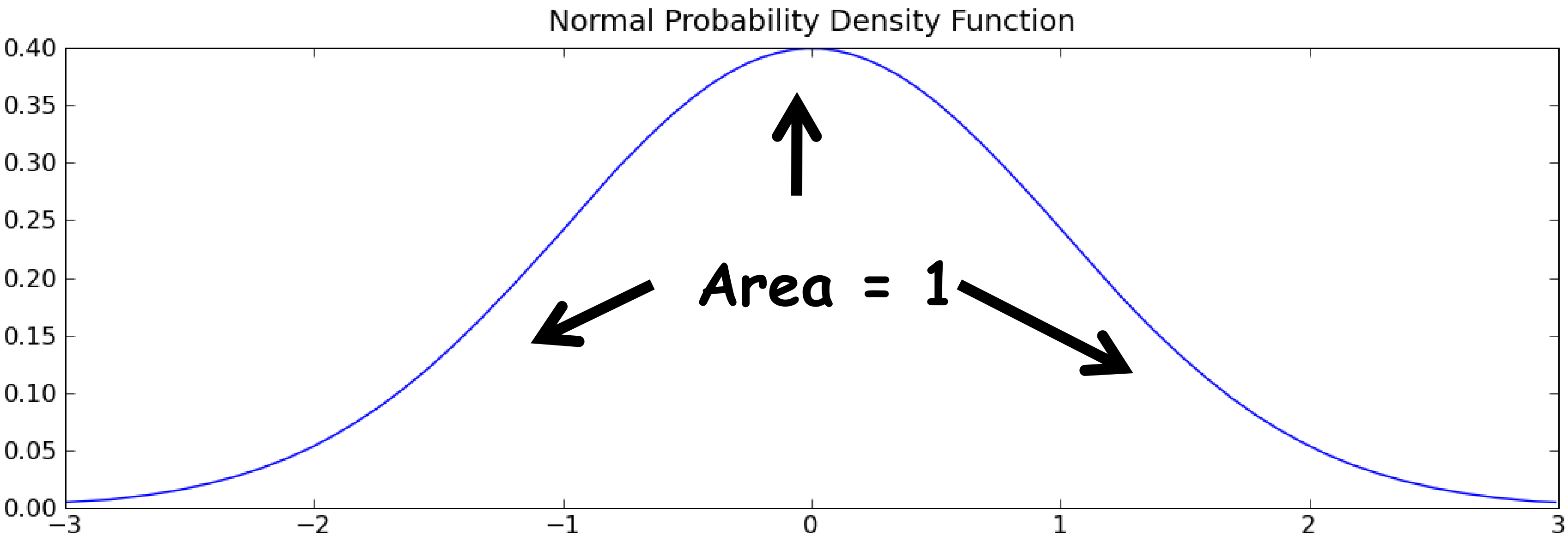


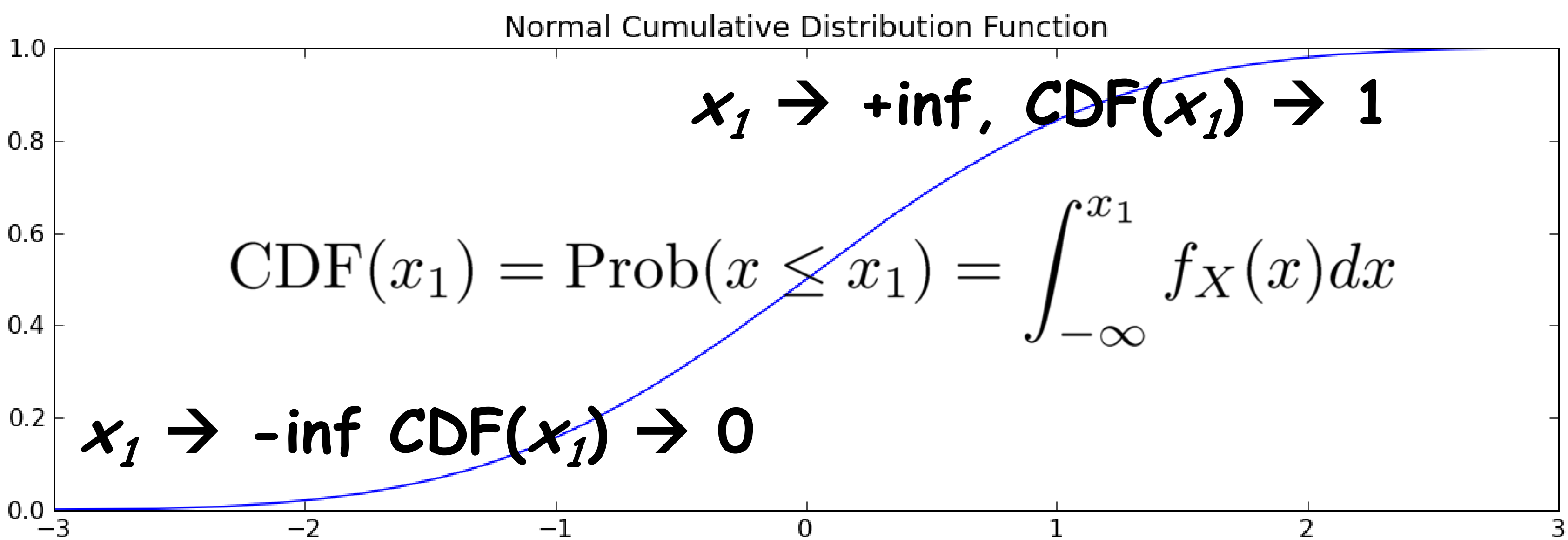


Histogram for 10,000 trials of sums of 1000 uniformly (right) or triangularly (left) distributed [-1,1] random variables

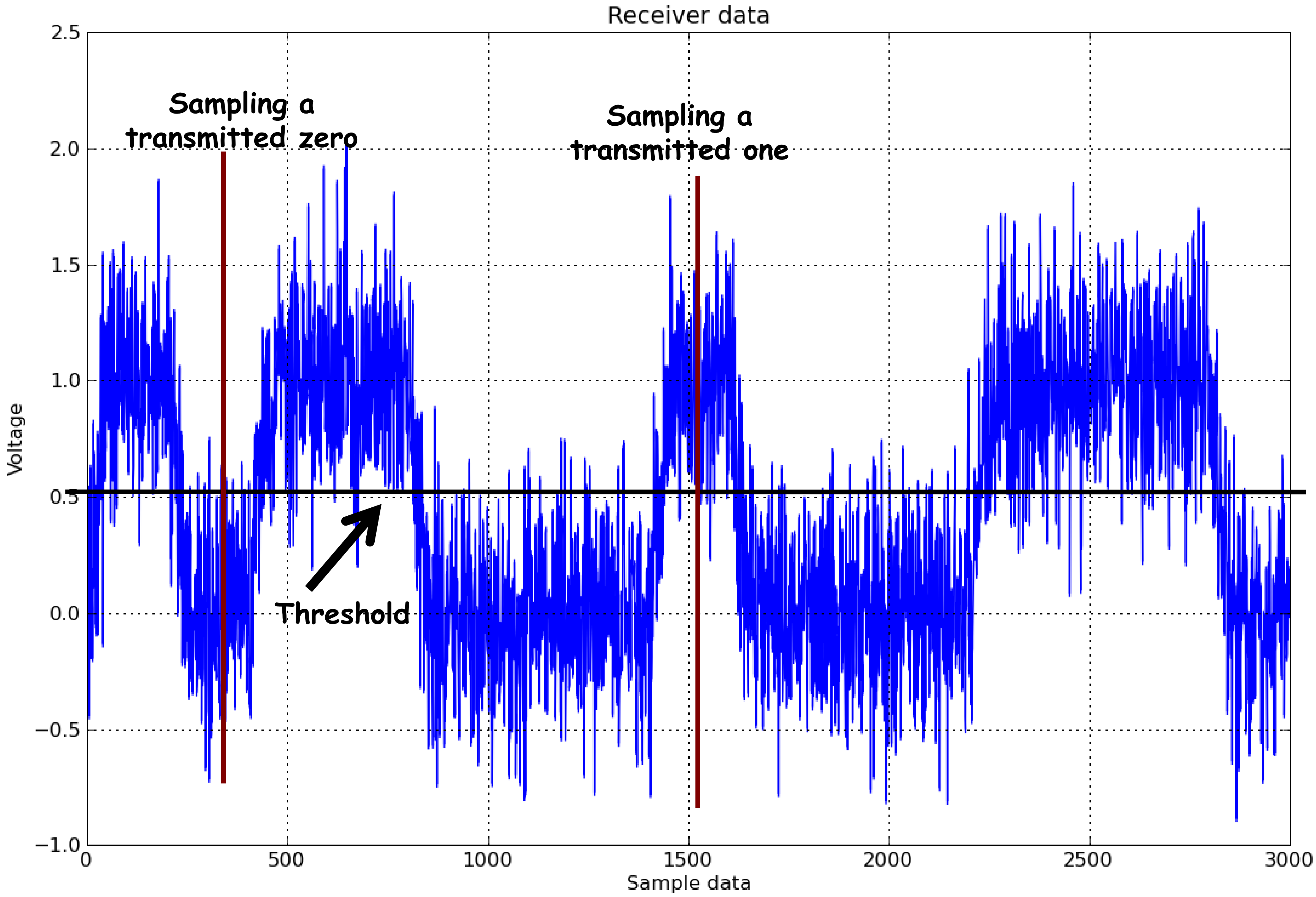
Key Point: Sums of Noise from many sources well approximated by the Normal(Gaussian) PDF

Unit Normal (Gaussian) PDF and CDF

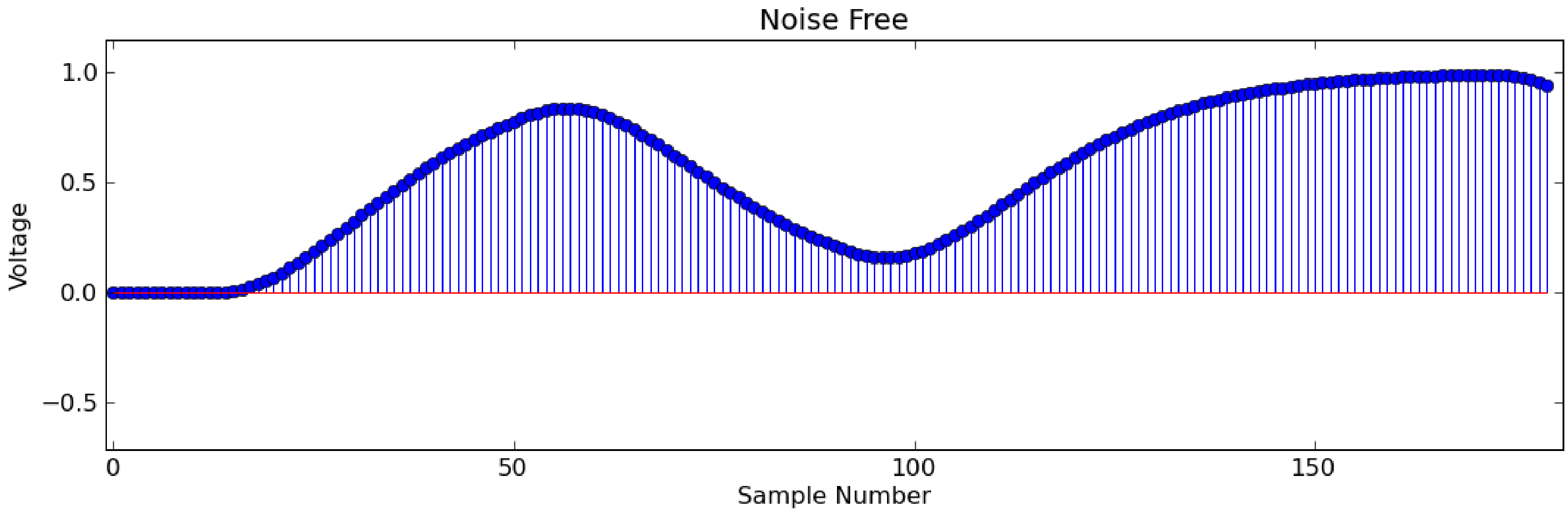


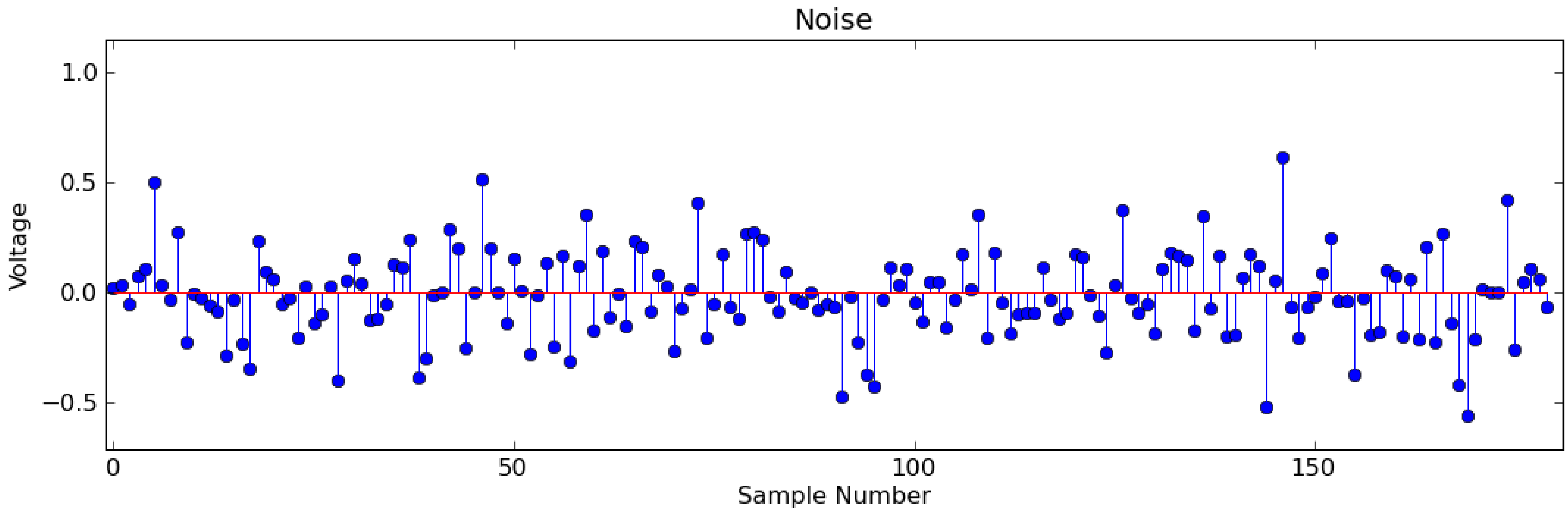


Estimating Probability of Bit Error

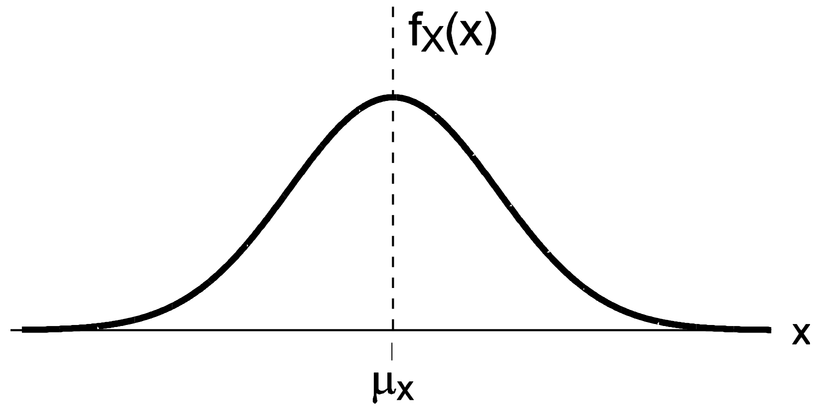


Decompose Into Noisefree + Noise





Mean and Variance



- The mean of random variable *X*, μ_x , corresponds to its average value
  - Computed as

$$\mu_X = \int_{-\infty}^{\infty} x f_X(x) dx$$

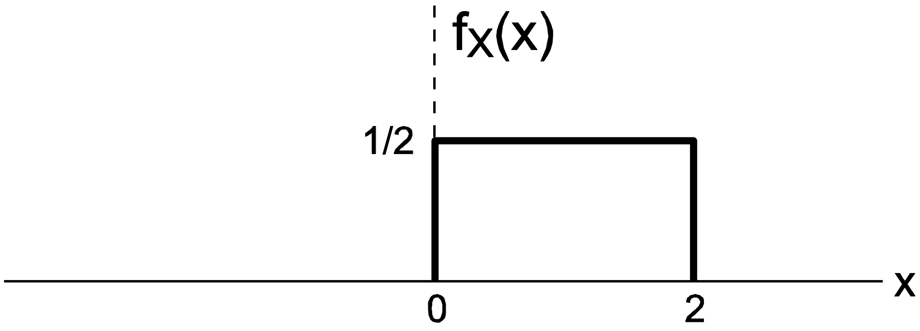
- The variance of random variable *x*, σ_x^2, gives an indication of its variability
  - Computed as

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx$$

- The standard deviation of a random variable X, is denoted σ_X

Slide thanks to C. Sodini and M. Perrott

Example Mean and Variance Calculation



- **Mean:**

$$\mu_X = \int_{-\infty}^{\infty} x f_X(x) dx = \int_0^2 x \frac{1}{2} dx = \frac{1}{4} x^2 \Big|_0^2 = \boxed{1}$$

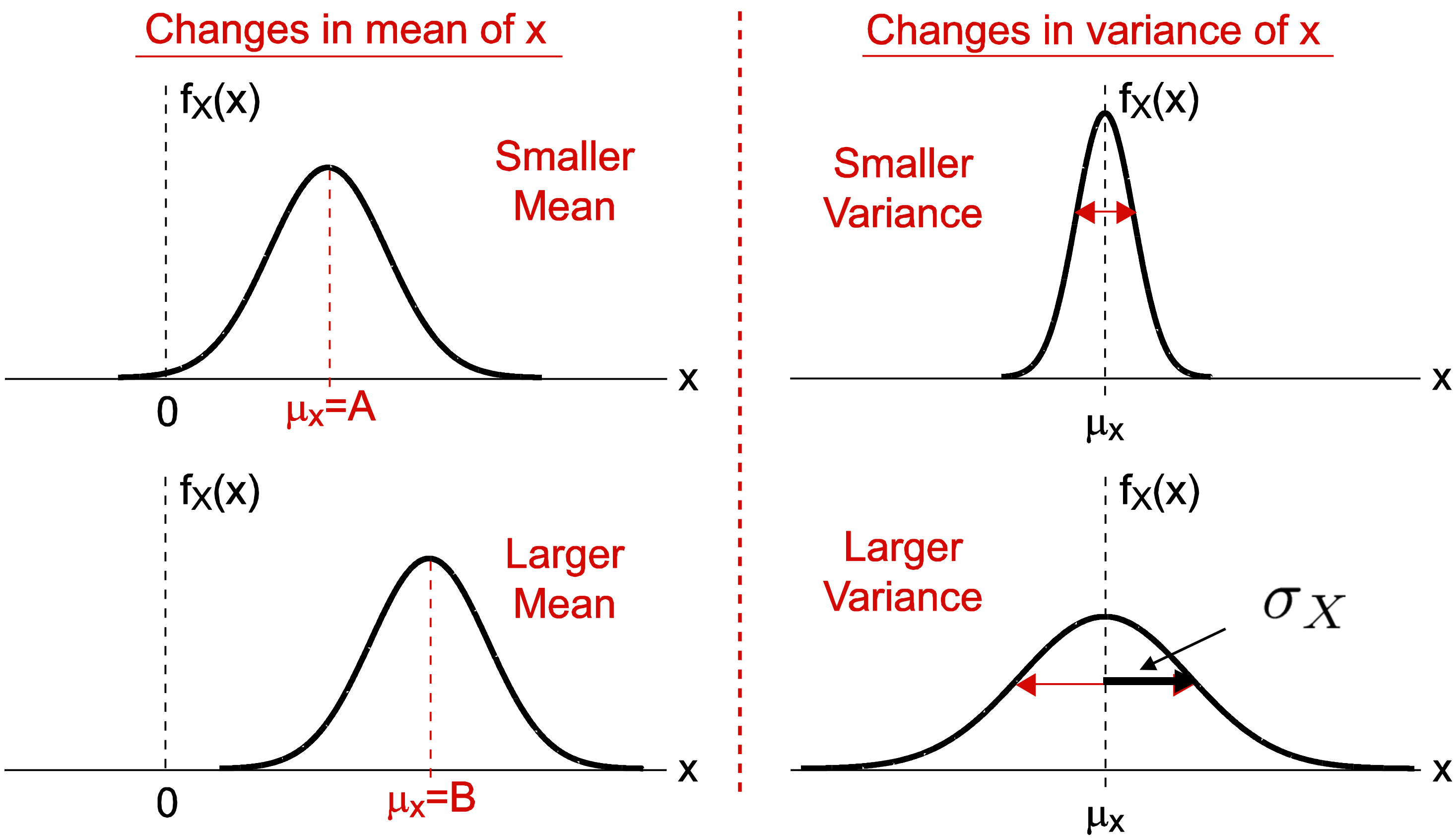
- **Variance:**

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_X(x) dx = \int_0^2 (x-1)^2 \frac{1}{2} dx$$

$$= \frac{1}{6}(x-1)^3 \Big|_0^2 = \frac{1}{6} + \frac{1}{6} = \boxed{\frac{1}{3}}$$

Slide thanks to C. Sodini and M. Perrott

Visualizing Mean and Variance from PDF

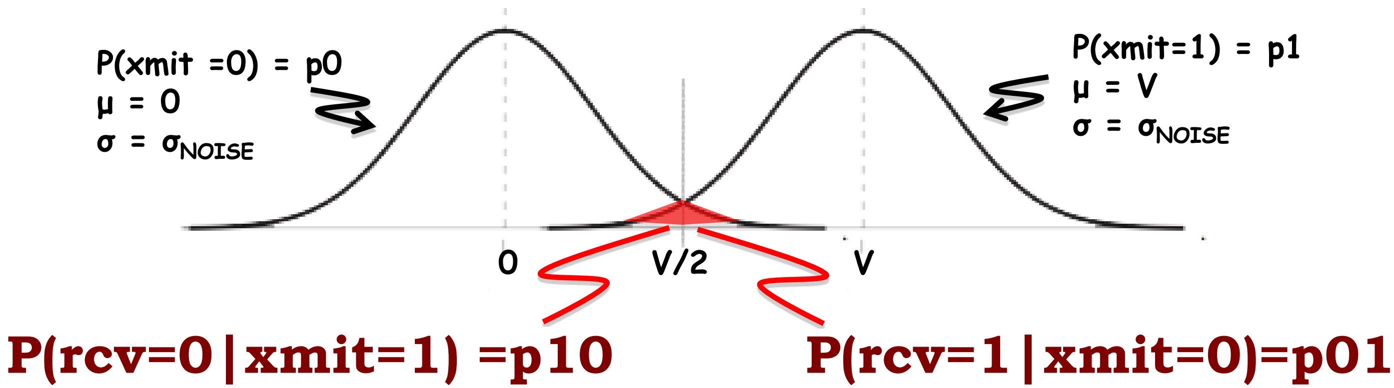


- Changes in mean shift the *center of mass* of PDF
- Changes in variance narrow or broaden the PDF
  - Variance tells us how "big" the noise is!

Slide thanks to C. Sodini and M. Perrott

Summary

- **Assume Gaussian PDF for noise and no inter-symbol interference (ISI next time) yields the following picture:**



- **We can estimate the bit-error rate (BER) as**

$$\boxed{P(bit\ error) = p0 * p01 + p1 * p10}$$