MAC PROTOCOLS REVIEW

Analytical calculations:

* models: deterministic
  probabilistic
* quantities: throughput
  utilization
  "wasted slots"
* methods: direct counting
  relating probabilities to averages

Ex. 1 Nodes: 1, 2, ... N (ALL backlogged)
Node i sends packet with probability p_i
Slotted network (1 packet = 1 slot)
CALCULATE: THROUGHPUT (TOTAL & INDIVIDUAL)
UTILIZATION
WASTE
WHATEVER ELSE ...

! THIS IS A PROBABILISTIC MODEL
! SHOULD ASSUME INDEPENDENCE OF NODE'S DECISIONS TO TRANSMIT (IN GENERAL: INDEPENDENCE AFTER CONDITIONING ON THE PAST)

WASTE = AVG. # OF SLOTS WHEN NO ONE IS TRANSMITTING

! THIS IS THE SAME AS THE PROBABILITY THAT NO-ONE IS TRANSMITTING IN A GIVEN TIME SLOT

$\mathbb{P}$(node "i" not transmitting) $= 1 - p_i$

$\Rightarrow$ (BY INDEPENDENCE)

$\mathbb{P}$(ALL NODES ARE NOT TRANSMITTING)

$$= (1-p_1)(1-p_2)\dots(1-p_N)$$

= WASTE

Ex.1 (cont.)

THROUGHPUT? = AVERAGE # OF SUCCESSFUL TRANSMISSIONS (OUT OF A LARGE NUMBER OF TIME SLOTS)

= $\mathbb{P}$(SUCCESSFUL TRANSMISSION)

= $\mathbb{P}$(EXACTLY ONE NODE TRANSMITS)

$$= \underbrace{p_1(1-p_2)\cdot\ldots(1-p_N)}_{\mathbb{P}(\text{NODE 1 TRANSMITS, OTHER DO NOT})} + \underbrace{p_2(1-p_1)(1-p_3)\cdot\ldots(1-p_N)}_{\mathbb{P}(\text{NODE 2 TRANSMITS, OTHER DO NOT})} + \ldots + \underbrace{p_N(1-p_1)\ldots(1-p_{N-1})}_{\mathbb{P}(\text{NODE } N \text{ TRANSMITS, OTHER DO NOT})}$$

$$= (1-p_1)\ldots(1-p_N)\cdot\left[\frac{p_1}{1-p_1} + \frac{p_2}{1-p_2} + \ldots + \frac{p_N}{1-p_N}\right]$$

UTILIZATION? = $\dfrac{\text{THROUGHPUT}}{\text{MAX. THROUGHPUT}}$ ← IDEAL "LUCKY" CASE

IN THIS CASE MAX. TROUGHPUT = 1 $\frac{\text{PACKET}}{\text{TIME SLOT}}$

HENCE UTILIZATION = THROUGHPUT

THROUGHPUT FOR NODE "K":

= AVG. # OF SUCCESSFUL TRANSMISSIONS FROM NODE K

= $\mathbb{P}$(SUCCESSFUL TRANSMISSION FROM "K")

$$= \frac{p_K}{1-p_K}(1-p_1)\ldots(1-p_N)$$

COLLISION RATE:

= AVG. # OF "COLLISION SLOTS"

= $\mathbb{P}$(collision)

= 1 − $\mathbb{P}$(no collision)

= 1 − ($\mathbb{P}$(WASTE) + $\mathbb{P}$(SUCCESSFUL TRANSMISSION))

$$= 1 - (1-p_1)\cdot\ldots\cdot(1-p_N)\left[1 + \frac{p_1}{1-p_1} + \frac{p_2}{1-p_2} + \ldots + \frac{p_N}{1-p_N}\right]$$

Ex. 2 A slotted Aloha Network has N backlogged nodes, $N \gg 1$

If every node sends with optimal probability $p = \frac{1}{N}$, the throughput is $\frac{1}{e}$

What will be the throughput when every node uses HALF OF THE OPTIMAL PROBABILITY?

$$N \cdot \left(\frac{1}{2N}\right)\left(1 - \frac{1}{2N}\right)^{N-1} \sim \frac{1}{2}\left[\left(1 - \frac{1}{2N}\right)^{2N}\right]^{1/2} \to \frac{1}{2\sqrt{e}}$$

Ex. 3 Token-passing TDMA MAC:

2N nodes

N backlogged

TOKEN-PASSING TIME $= \frac{1}{4}$ TIME SLOT FOR BACKLOGGEG

$= \frac{1}{2}$ TIME SLOT FOR THOSE NOT BACKLOGGED

THROUGHPUT?

UTILIZATION?

A "FULL ROUND": $N + \frac{N}{4} + \frac{N}{2}$ TIME SLOTS

FOR: N PACKETS

$$\text{THROUGHPUT} = \frac{N}{N + \frac{N}{2} + \frac{N}{4}} = \frac{1}{7/4} = \frac{4}{7}$$

UTILIZATION?

$$\text{MAX. THROUGHPUT} = \frac{2N}{2N + \frac{N}{4}} = \frac{8}{9}$$

$$\text{UTILIZATION} = \frac{4/7}{8/9} = 9/14$$