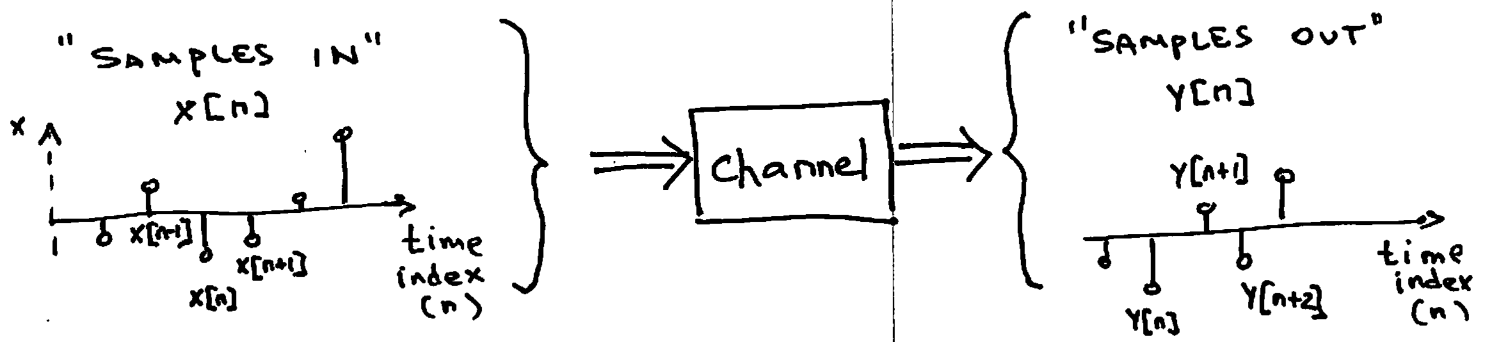
COMMUNICATION CHANNELS AS LTI SYSTEMS



Idealized Assumptions (pure math):

* what goes in defines what comes out in a unique way [? what about noise ?]
* invariance to scaling & addition

$$Ch(a x[\cdot]) = a \cdot Ch(x[\cdot])$$
$$ch(x[\cdot]+y[\cdot]) = Ch(x[\cdot]) + Ch(y[\cdot])$$

what about finite voltage source ?

* Invariance to time shift

$$ch(Delay(x[\cdot])) = Delay(Ch(x[\cdot]))$$

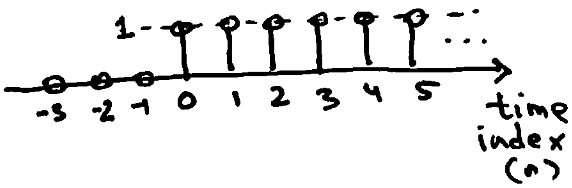
Ex.0 How does LTI system respond to zero input ($x[n]=0$ for ALL n)?

Answer: WITH $Y[n] \equiv 0$, because

$$Ch(0[\cdot]) = ch(0 \cdot x[\cdot]) \underset{\text{LINEARITY}}{=} 0 \cdot Ch(x[\cdot]) = 0[\cdot]$$

So, is the IR channel Linear ???

Unit step response:

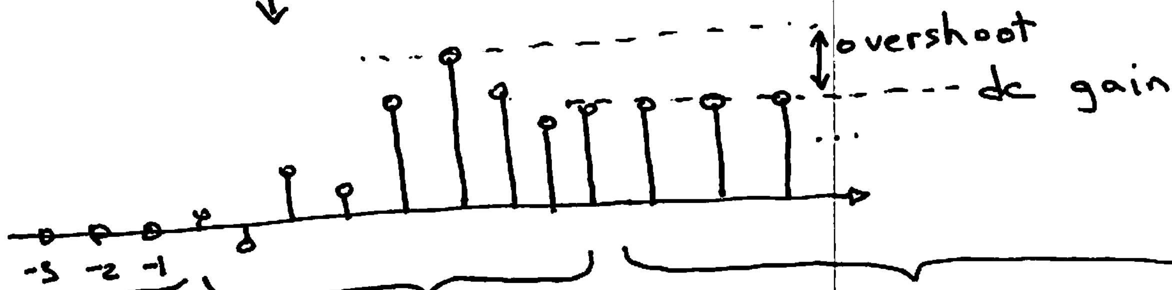


$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

↓

LTI System

↓



Zero response here because of <u>CAUSALITY</u>

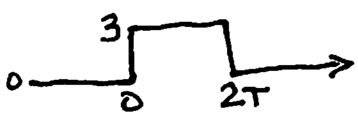
SETTLING TIME

THE RESPONSE SETTLES (AT LEAST ASYMPTOTICALLY) DUE TO <u>STABILITY</u>)

SEE 6.003 ETC. { IF YOU KNOW RESPONSE OF A CAUSAL LTI SYSTEM TO <u>ANY</u> SINGLE NON-ZERO "CAUSAL" INPUT, YOU CAN PREDICT ITS RESPONSE TO ARBITRARY "CAUSAL" INPUT.

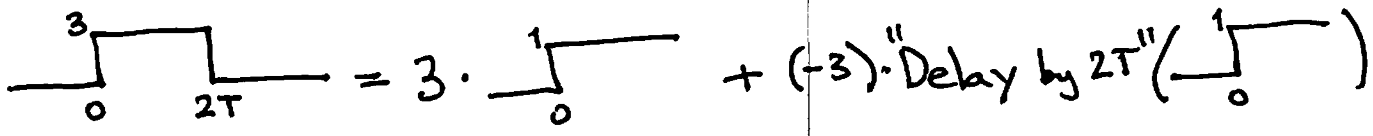
<u>Ex. 1</u> If

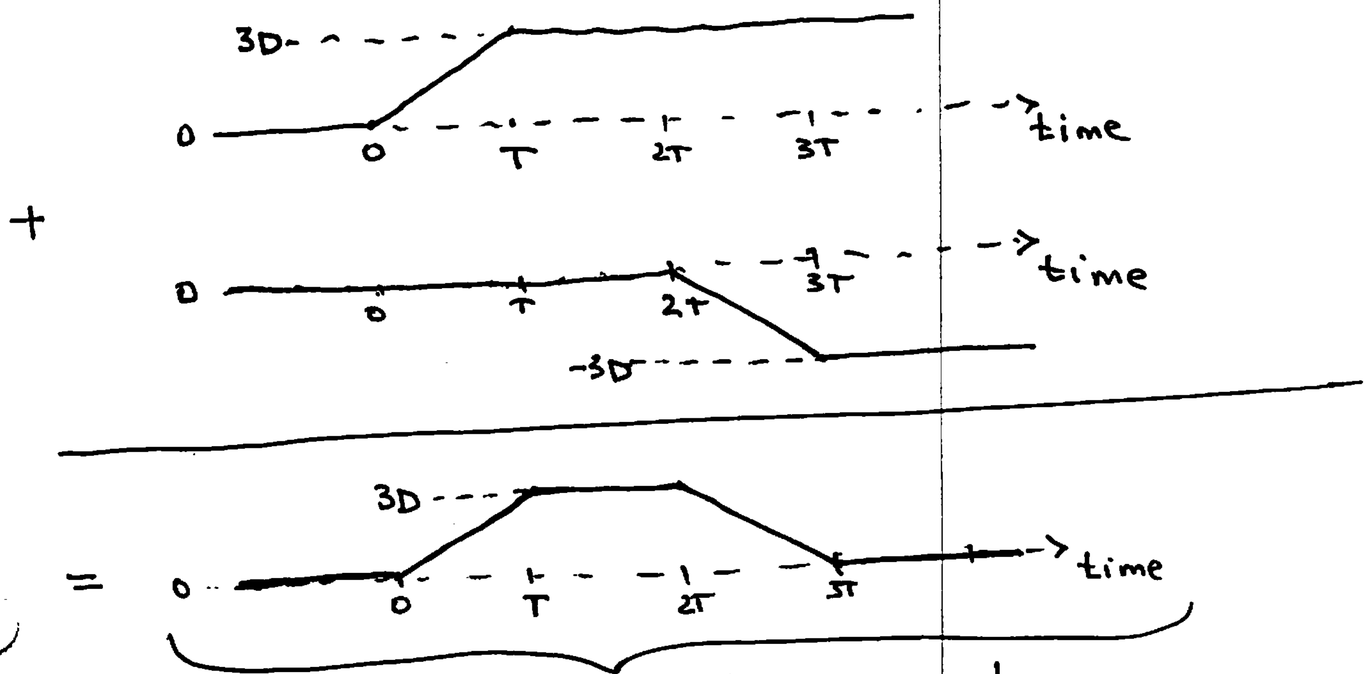
then



→ ?

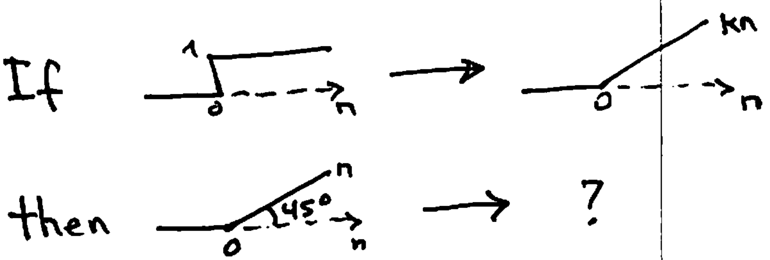
<u>SOLUTION:</u> START WITH REPRESENTING $x[\cdot]$ AS A <u>LINEAR COMBINATION</u> OF $u[\cdot]$ AND ITS "SHIFTS"



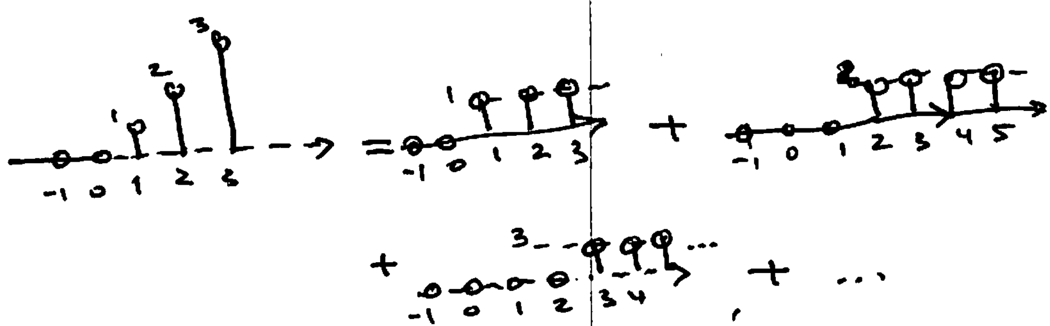


this is the answer!

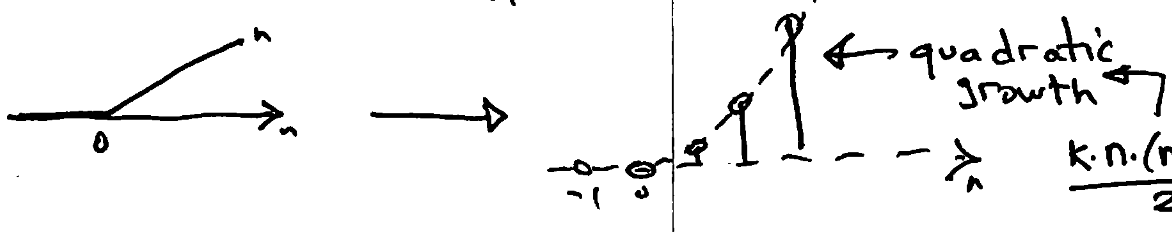
<u>**Ex. 2**</u>



<u>Solution:</u>



Hence



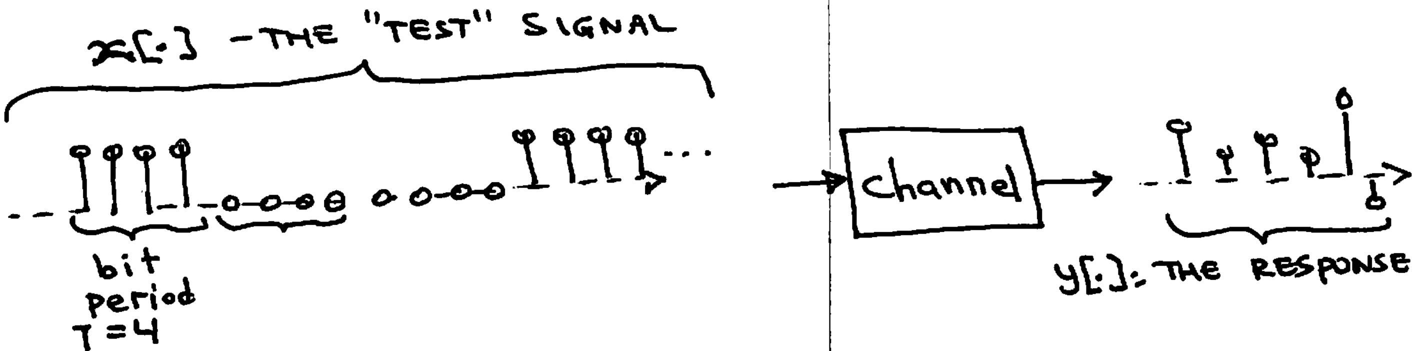
$$\frac{k \cdot n \cdot (n+1)}{2}$$

MORE ON LTI SYSTEMS IN THE NEXT lecture

EYE OPENING/DIAGRAMS

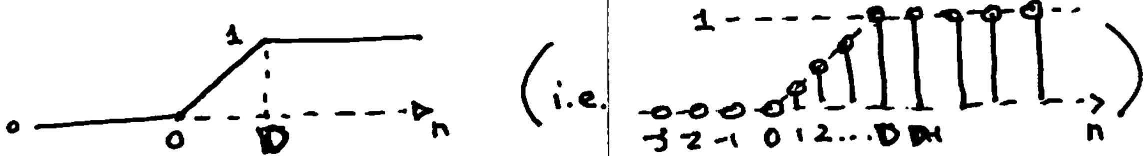
NEED: * A SYSTEM ("CHANNEL")

* A 0/1 SIGNAL CONSTANT OVER T-SAMPE PERIODS



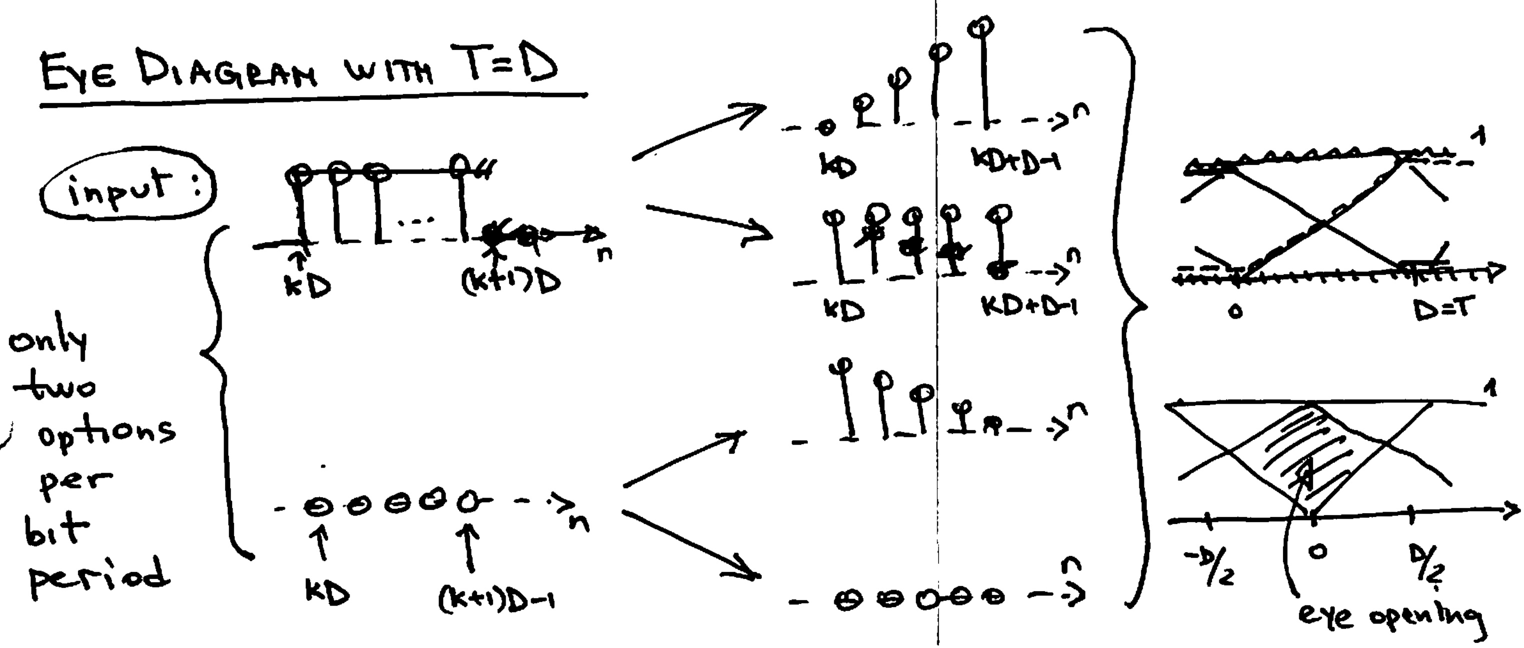
CUT y[·] into pieces (EACH SHIFED BY A MULTIPLE OF T FROM THE OTHER) AND PLOT ON THE SAME GRAPH (THE PIECES SHOULD BE AT LEAST T SAMPLES LONG (MORE THAN 3T APPEARS TO BE AN OVERKILL)

EXAMPLE 3 LTI SYSTEM WITH UNIT STEP RESPONSE

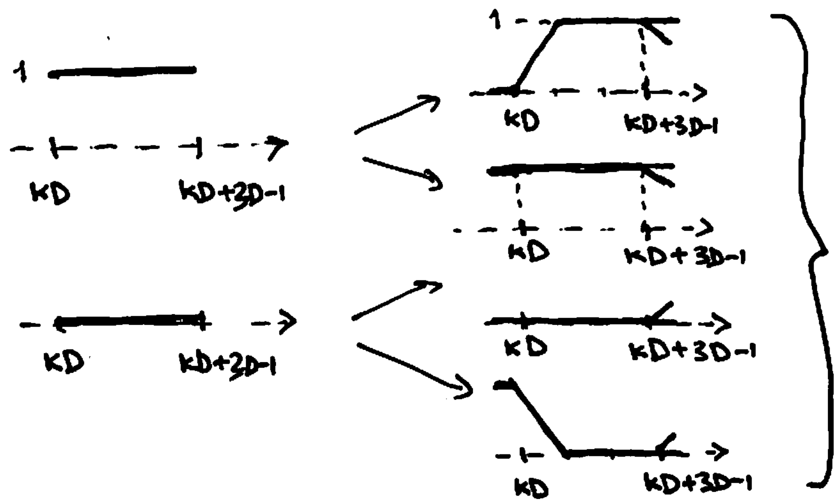


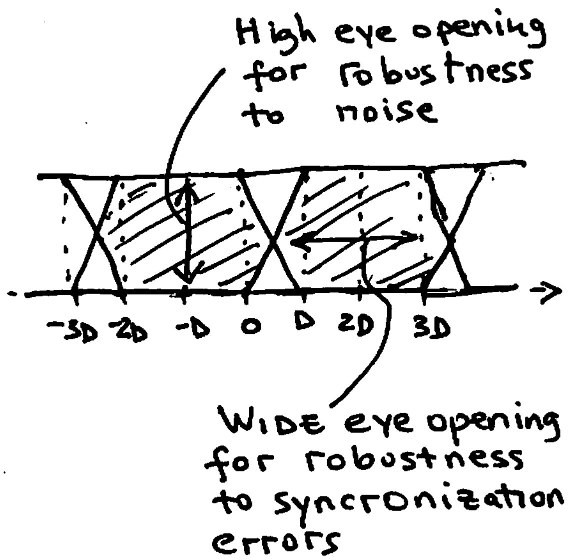
(D ≈ "TIME CONSTANT")

EYE DIAGRAM WITH T=D



Eye Diagram with T= 3D:



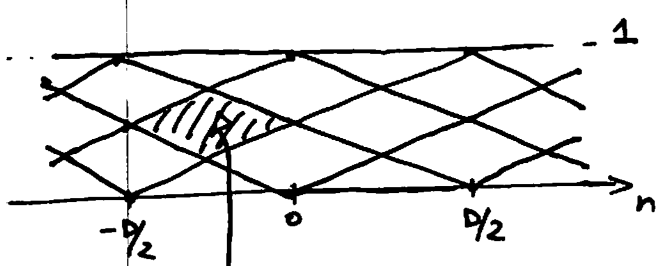


Eye Diagram with T= D/2:

POSSIBLE "STATES" AT $n = kT$: 0.0, 0.5, 1.0

POSSIBLE "TRANSITIONS" FROM $n=kT$ TO $n=(k+1)T$:

0.0 → 0.0 $(x=0)$
0.0 → 0.5 $(x=1)$
0.5 → 0.0 $(x=0)$
0.5 → 1.0 $(x=1)$
1.0 → 1.0 $(x=1)$
1.0 → 0.5 $(x=0)$



IS THIS REALLY AN "EYE OPENING" ? I DON'T THINK SO!

* IN THE CASE OF IDEAL SYNCHRONIZATION, THE EYE DIAGRAMS WOULD BE "TWO COLOR": ONE COLOR WHEN "0" IS TRANSMITTED, ANOTHER FOR "1"

* FOR T=D/2, COMPARING OUTPUT VALUE TO A THRESHOLD WOULD NOT TELL YOU WHETHER "1" OR "0"

* NOT ALL IS LOST: E.G. LOOKING AT THE SLOPE DOES DISTINGUISH 0/1. HOW ABOUT "SLOPE" WHEN "NOISE" IS THERE?