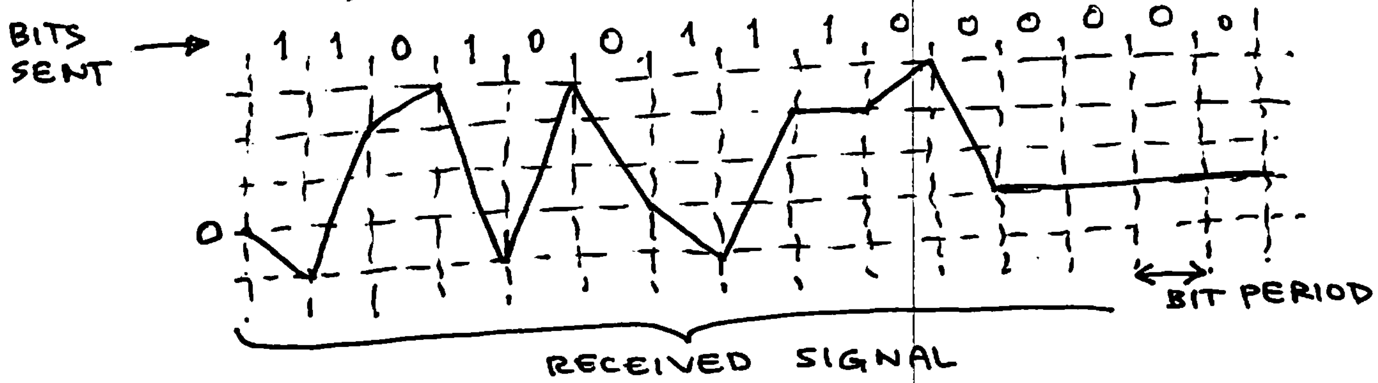
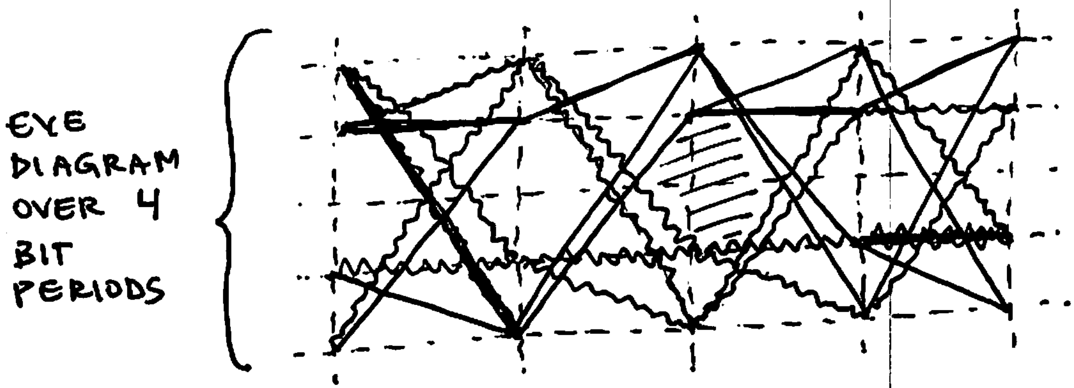
# LTI Models

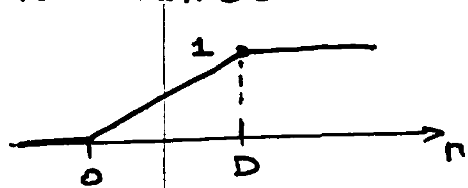
## Ex. 1 : EYE DIAGRAM FROM BIT RESPONSE + BIT PERIOD (+, POSSIBLY BITS REFERENCE)




## Ex 2 : EYE DIAGRAM FROM STEP RESPONSE (SEE THE 14.09.2010 REC. HANDOUT)

GIVEN: STEP RESPONSE




BIT PERIOD : $T = D$

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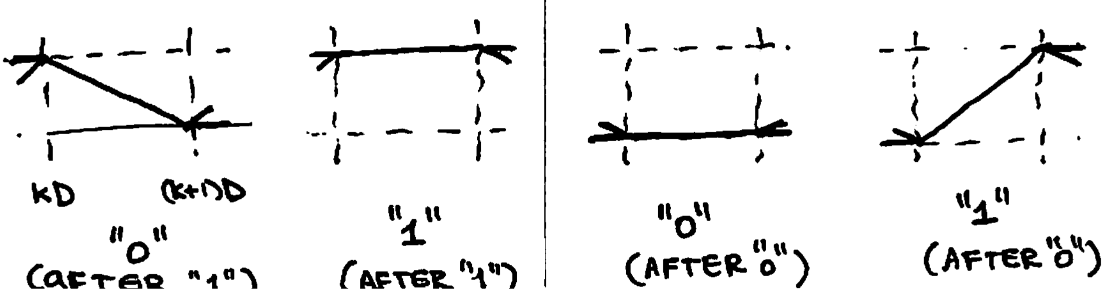
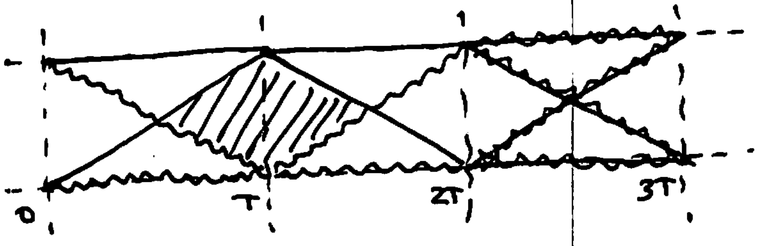
POSSIBLE RESPONSES OVER A BIT PERIOD:

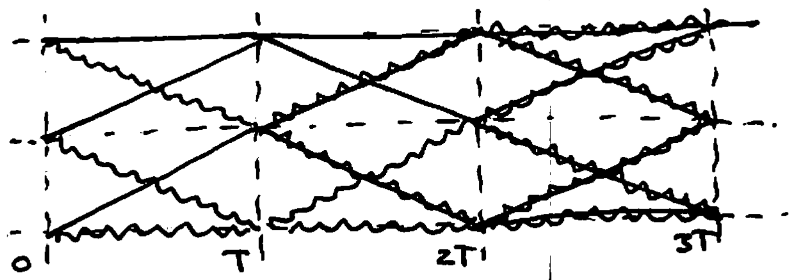


## Ex.2 (CONT.)

EYE DIAGRAM OVER 3 BIT PERIODS




## Ex.3 EYE DIAGRAM FOR STEP RESPONSE FROM Ex.2 WITH $T = D/2$ (SEE 14.09.2010)




SOME THINGS DO LOOK LIKE EYE OPENINGS, BUT THERE ARE REALLY NONE

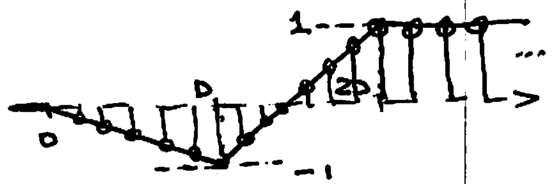
## UNIT SAMPLE RESPONSE

BY DEFINITION : RESPONSE TO $x[n] = \delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

## Ex.4

If unit step RESPONSE IS THIS:

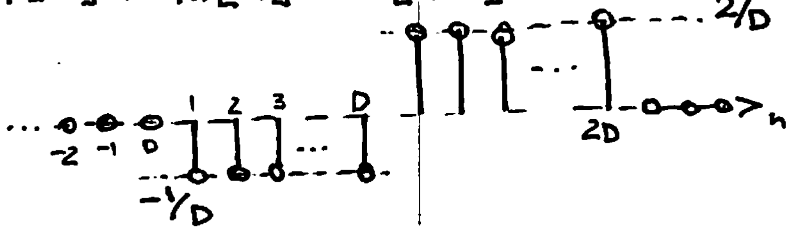
$m[n] =$




WHAT IS THE UNIT SAMLE RESPONSE $h[n] = ?$

MATH : $h[n] = m[n] - m[n-1]$

ANSWER:



# CONVOLUTION

$$y = h * x \quad \text{MEANS} \quad y[n] = \sum_i h[i]\,x[n-i] = \sum_i h[n-i]\,x[i]$$

THIS IS WHAT LTI SYSTEMS DO!

$$x[\cdot] \longrightarrow \boxed{h[\cdot]} \longrightarrow y = h * x$$

"SANITY CHECKS":

IF $\quad x[n] = 0$ outside $\{N_x, N_x+1, \dots N_x+L_x-1\}$

$\quad\quad h[n] = 0$ outside $\{N_h, N_h+1, \dots, N_h+L_h-1\}$

then $\quad y[n] = 0$ outside $\{N_x+N_h, N_x+N_h+1, \dots, N_x+N_h+L_x+L_h-2\}$

i.e. BEGINNING $(x*h)$ = BEGINNING$(x)$ + BEGINNING$(h)$

END $(x*h)$ = END$(x)$ + END$(h)$

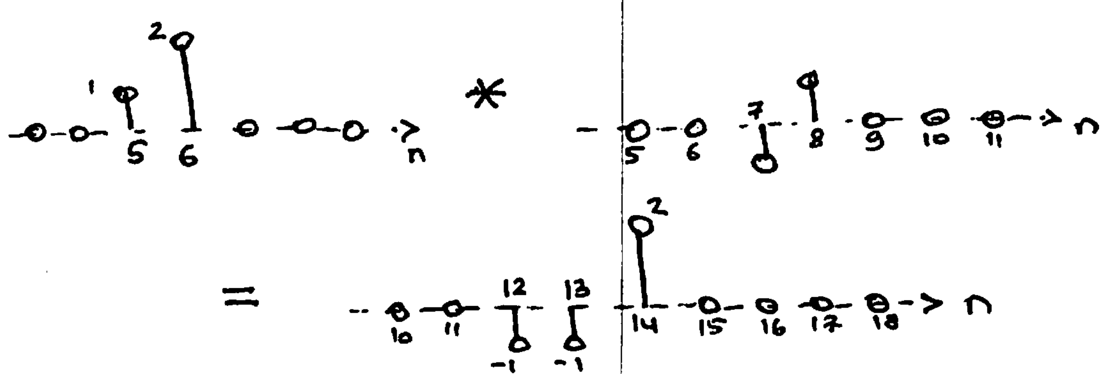
~~LENGTH~~ "WIDTH" $(x*h)$ = ~~LENGTH~~ "WIDTH" $(x)$ + ~~LENGTH~~ "WIDTH" $(h)$

**Ex. 5**

$$[1, 2]\Big|_{\text{START AT } 5} * [-1, 1]\Big|_{\text{START AT } 7} = ?$$

| | | | |
|---|---|---|---|
| $(-1)\cdot[1,2]$ = | $-1$ | $-2$ | |
| $1 \cdot [1,2]$ = | | $1$ | $2$ |
| $[-1,1]*[1,2]$ = | $-1$ | $-1$ | $2$ |

ANSWER: $[-1, -1, 2]\Big|_{\text{START AT } 12}$



**DECONVOLUTION:** GIVEN $y[\cdot]$ AND $h[\cdot]$

FIND $x[\cdot]$ SUCH THAT $y = h * x$

IF WIDTH $(h) = m$, i.e. $h[n] = 0$ FOR $n < 0$ OR $n > m$

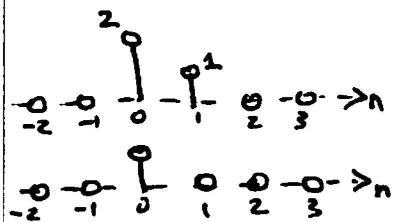
$$y[n] = h[0]x[n] + h[1]\cdot x[n-1] + \ldots + h[m]x[n-m]$$

$$x[n] = \frac{1}{h[0]}\left(y[n] - h[1]x[n-1] - h[2]x[n-2] - \ldots - h[m]x[n-m]\right)$$

**Ex. 6**

$$h[n] = 2\delta[n] + \delta[n-1]$$

$$y[n] = \delta[n]$$

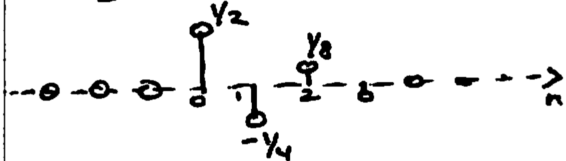



"STRAIGHTFORWARD DECONVOLUTION" WORKS WELL → IS AN LTI SYSTEM ITSELF !!!

$$x[n] = ?$$

$$\left.\begin{aligned} x[0] &= \tfrac{1}{2}(1 - 1\cdot 0) = \tfrac{1}{2} \\ x[1] &= \tfrac{1}{2}\left(0 - 1\cdot\tfrac{1}{2}\right) = -\tfrac{1}{4} \\ x[2] &= \tfrac{1}{2}\left(0 - 1\left(-\tfrac{1}{4}\right)\right) = \tfrac{1}{8} \end{aligned}\right\} \quad x[n] = \left(-\tfrac{1}{2}\right)^{n+1} \text{ FOR } n \geq 0$$

$$\ldots$$




**Ex. 7**

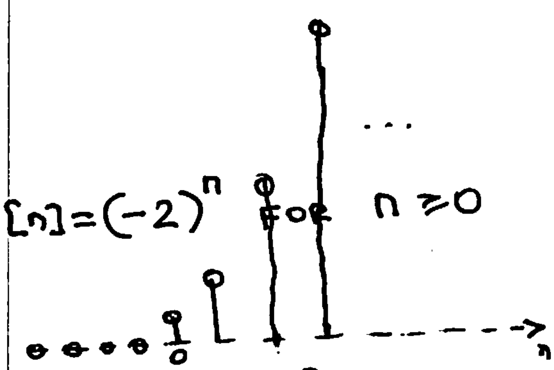
$$h[n] = \delta[n] + 2\delta[n-1]$$

$$y[n] = \delta[n]$$

FAILURE?

$$\left.\begin{aligned} x[0] &= 1 - 2\cdot 0 = 1 \\ x[1] &= 0 - 2\cdot 1 = -2 \\ x[2] &= 0 - 2\cdot(-2) = 4 \end{aligned}\right\} \quad x[n] = (-2)^n \text{ FOR } n \geq 0$$

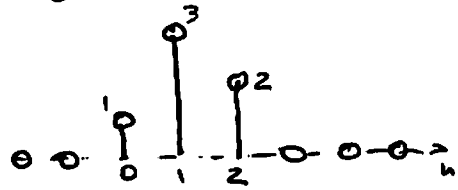
$$\ldots$$




**Ex. 8**

$$h[n] = \delta[n] + 2\delta[n-1]$$

$$y[n] = \delta[n] + 3\delta[n-1] + 2\delta[n-2]$$




↓ DECONVOLUTION

NOISE AMPLIFICATION "INSTABILITY"

$$\begin{aligned} x[0] &= 1 - 2\cdot 0 = 1 \\ x[1] &= 3 - 2\cdot 1 = 1 \\ x[2] &= 2 - 2\cdot 1 = 0 \\ x[3] &= 0 - 2\cdot 0 = 0 \end{aligned}$$

$$\ldots$$