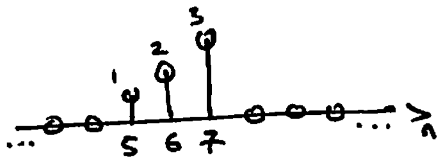
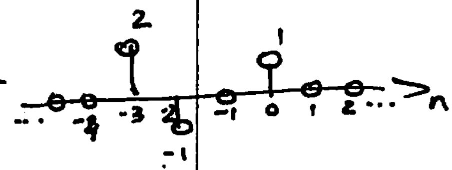
6.02 .09.2010 (1)

CONVOLUTION / DECONVOLUTION (OVERFLOW FROM 16.09.2010)

+ PROBABILISTIC MODELS

Ex. 1

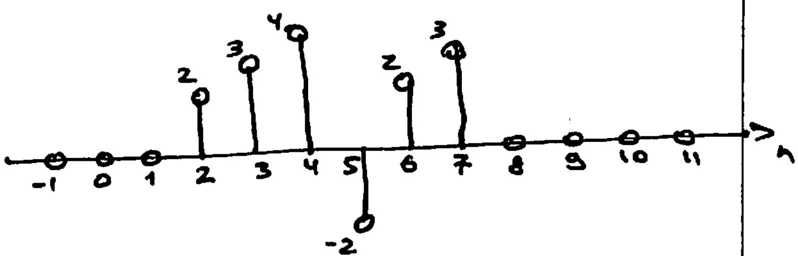
 $*$  $= ?$

[1, 2, 3] (STARTS AT 5) [2, -1, 0, 1] (STARTS AT -3)

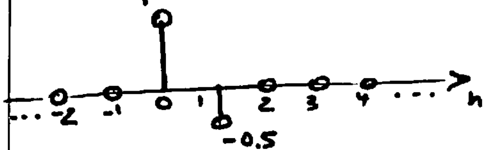
1. | 2 | -1 | 0 | 1 |
2. | 2 | -1 | 0 | 1 |
3. | 2 | -1 | 0 | 1 |

2	-1	0	1		
	4	-2	0	2	
		6	-3	0	3
2	3	4	-2	2	3

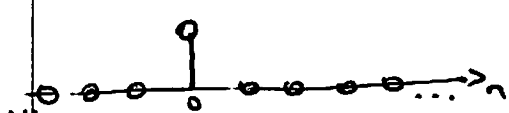
Answer: [2, 3, 4, −2, 2, 3] (STARTS AT $2 = 5 + (-3)$)



Ex. 2

$h[n] = \delta[n] - 0.5\,\delta[n-1]$ 

$h * x = y$

$y[n] = \delta[n]$ 

$x[n] = ?$

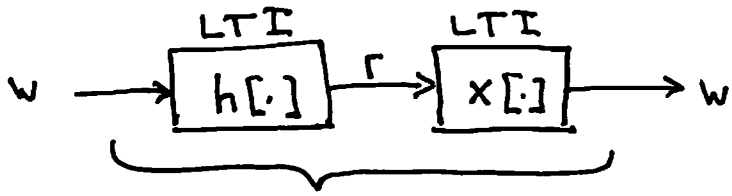
A "DECONVOLUTION" TASK!

Ex. 2 (CONT.) $\boxed{x[n] - 0.5x[n-1] = y[n]}$

ASSUMING $x[n] = 0$ FOR $n < 0$:

$$x[0] = y[0] = 1$$
$$x[1] = 0.5x[0] + y[1] = 0.5 \cdot 1 + 0 = 0.5$$
$$x[2] = 0.5x[1] + y[2] = 0.5 \cdot 0.5 + 0 = (0.5)^2$$
$$\cdots$$
$$x[n] = (0.5)^n \quad \text{FOR } n \geq 0$$

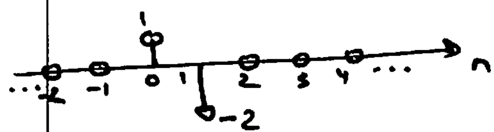
!! **Note:** $h[\cdot] * x[\cdot] = \delta[\cdot] \implies x[\cdot] * h[\cdot] = \delta[\cdot]$



IDENTITY TRANSFORMATION!

$x[\cdot]$ IS AN LTI "DECONVOLVER" FOR EVERY w

Ex. 3 $h[n] = \delta[n] - 2\delta[n-1]$



$$h * x = y$$
$$y[n] = \delta[n] \qquad \boxed{x[n] - 2x[n-1] = y[n]}$$
$$x[n] = ?$$

ASSUMING $x[n] = 0$ FOR $n < 0$:

$$\left.\begin{aligned} x[0] &= y[0] = 1 \\ x[1] &= 2x[0] + y[1] = 2 \\ x[2] &= 2x[1] + y[2] = 4 \\ &\cdots \\ x[n] &= 2^n \quad (n \geq 0) \end{aligned}\right\} \text{"UNSTABLE" DECONVOLVER}$$

ASSUMING $x[n] = 0$ FOR $n \geq 0$:

$$\left.\begin{aligned} x[-1] &= 1/2\,(x[0] - y[0]) = -1/2 \\ x[-2] &= 1/2\,(x[-1] - y[-1]) = -(1/2)^2 \\ &\cdots \\ x[-n] &= -(1/2)^n \quad (n \geq 0) \end{aligned}\right\} \text{"STABLE" BUT "UNCAUSAL" DECONVOLVER}$$

PROBABILISTIC MODELING

{FROM␣SCI-FI␣IMPORT␣TIMELINE}

A "PROBABILISTIC MODEL" ASSIGNS** PROBABILITY $\mathbb{P}(A)$ (A NUMBER BETWEEN 0 and 1)* TO EVERY $\underbrace{\text{SET OF TIMELINES}}_{\text{EVENT}}$ A

* WITH "natural" PROPERTIES

$$\mathbb{P}(\text{EVERYTHING}) = 1$$

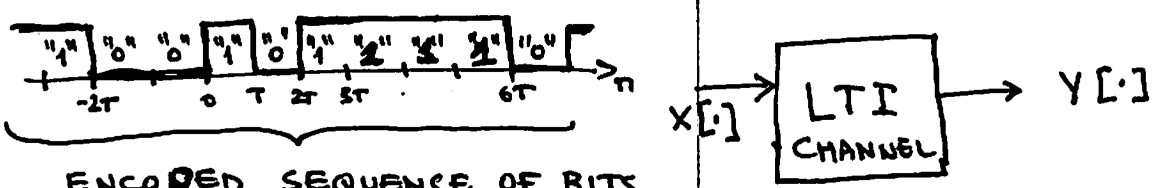
$$\mathbb{P}(A \text{ and } B) = \mathbb{P}(A) + \mathbb{P}(B) \text{ WHEN } A, B \text{ MUTUALLY EXCLUSIVE}$$

** THE "ASSIGNMENT" IS DEFINED IMPLICITLY, AS A RULE

FREQUENCY INTERPRETATION:

WHEN THE "TIMELINES" HAVE FINITE HORIZON (SAY, ONE DAY, MIDNIGHT – MIDNIGHT)
PROBABILITY OF AN EVENT ~ FREQUENCY OVER MANY DAYS
ASSUMING ONE DAY DOES NOT AFFECT THE OTHER

Ex. 4



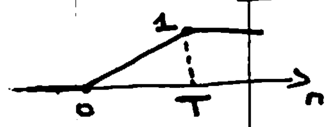
ENCODED SEQUENCE OF BITS
BIT PERIOD = T

Assume:

(a) EACH BIT HAS $\mathbb{P}(\text{"0"}) = 1/2$

(b) DIFFERENT BITS ARE INDEPENDENT

(c) $h[n]$ is



$\boxed{4.1}$

FIND $\mathbb{P}(y[T] = a)$ FOR ALL REAL a:

$y[T] = 0$ IF THE BIT SENT ON $[0, T[$ IS "0"
$y[T] = 1$ IF ——— " ——— "1"

ANSWER: $\mathbb{P}(y[T] = a) = \begin{cases} 1/2, & a = 0 \\ 1/2, & a = 1 \\ 0, & \text{OTHERWISE} \end{cases}$

Ex. 4 (CONT.)

$\boxed{4.2}$ FIND $\mathbb{P}(y[3T/2] = a)$ FOR REAL a (ASSUMING T IS EVEN)

$$y[3T/2] = \begin{cases} 0, & \text{IF THE } (0,2T) \text{ BITS ARE "00"} \\ 1/2, & \text{IF THE } (0,2T) \text{ BITS ARE "01" OR "10"} \\ 1, & \text{— '' — "11"} \end{cases}$$

BY INDEPENDENCE ASSUMPTION:

$$\mathbb{P}(\text{"00"}) = \underset{\text{at }(0,T)}{\mathbb{P}(\text{"0"})} \cdot \underset{\text{at }(T,2T)}{\mathbb{P}(\text{"1"})} = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

$$\mathbb{P}(\text{"01"}) = \mathbb{P}(\text{"0" at }(0,T)) \cdot \mathbb{P}(\text{"1" at }(T,2T)) = \frac{1}{4}$$

etc.

ANSWER:

$$\mathbb{P}(y[3T/2]=a) = \begin{cases} 1/4, & a=0 \\ 1/2, & a=1/2 \\ 1/4, & a=1 \\ 0, & \text{otherwise} \end{cases}$$

$\boxed{4.3}$ ARE $y[T]$ AND $y[2T]$ INDEPENDENT? $\boxed{\text{YES}}$

ARE $y[T]$ AND $y[3T/2]$ INDEPENDENT? $\boxed{\text{NO}}$

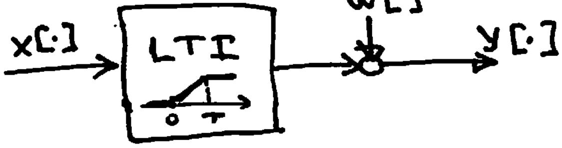
$$1/8 = \mathbb{P}(Y[T]=0)\cdot\mathbb{P}(Y[3T/2]=1) \neq \mathbb{P}(Y[T]=0, Y[3T/2]=1) = 0$$

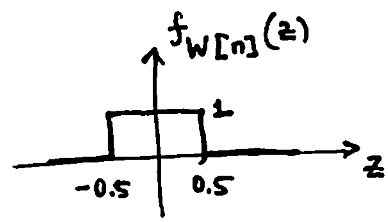
$\boxed{4.4}$ SAMPLING $y[kT+T/2]$ AT $k=1,2,3,\dots$

WHAT IS THE FREQUENCY OF GETTING $1/2$? $\rightarrow 1/2$

(FREQ.(0) = $1/4$, FREQ.(1) = $1/4$)

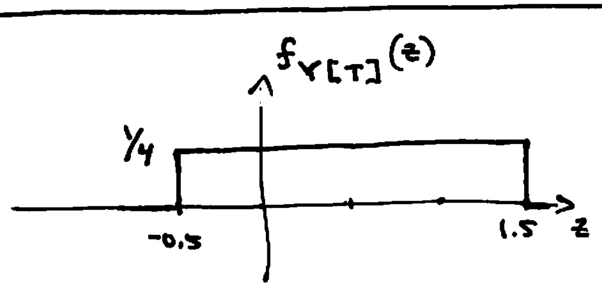
Ex. 5





(+INDEPENDENCE OF $w[n]$, $x[n]$)

"SAME" QUESTIONS AS IN Ex. 4:



THINK AS IF QUANTIZATION HAS BEEN APPLIED: