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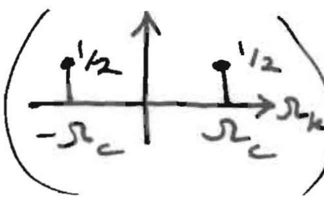
6.02
12/10/2010

Rec 16 ADDENDUM: Some figures to help you remember how modulation operates in the DTFS (i.e., spectral) domain:

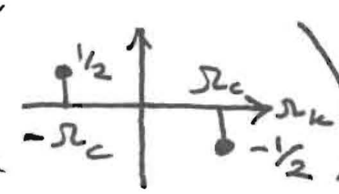
N even, $\Omega_1 = 2\pi/N$, $\Omega_c = k_c \, 2\pi/N$

'Carrier' frequency — Ω_c; k_c — integer

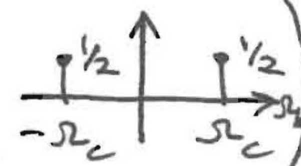
(Real) Time function $x[n]$ $\longleftrightarrow$ DTFS (or spectrum) $X[k]$; $\Omega_k = k\,2\pi/N$

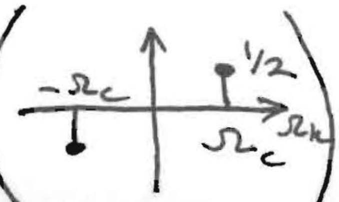
$\cos \Omega_c n \longleftrightarrow$ () $+ \, j \, (0)$

$\Omega_c = k_c \, 2\pi/N$

$\sin \Omega_c n \longleftrightarrow (0) + j$ ()

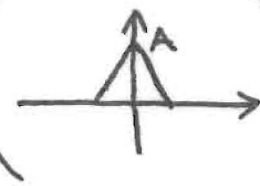
So:

$j \cos \Omega_c n \longleftrightarrow (0) + j$ ()

$j \sin \Omega_c n \longleftrightarrow$ () $+ \, j \, (0)$

These 4 cases set the pattern for determining the DTFS of a modulated signal, as we now see:

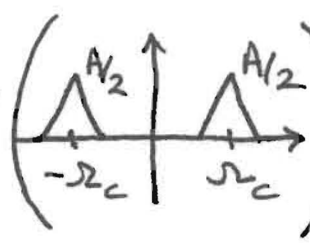
Suppose first that

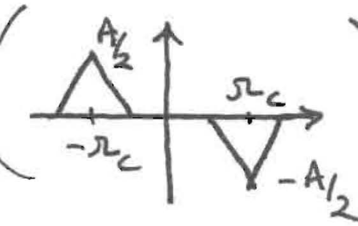
$x_R[n] \longleftrightarrow$ () $+ j(0)$

so $x_R[n]$ is an <u>even</u> function: $x_R[-n] = x_R[n]$

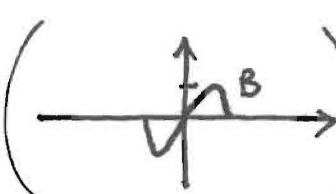
i.e., a purely real spectrum. Then it <u>turns out</u> that the DTFS's of the modulated signals $x_R[n]\cos\Omega_c n$ and $x_R[n]\sin\Omega_c n$ have a structure that parallels the <u>upper</u> two figures on p(1):

It takes a proof, not so obvious otherwise

$x_R[n]\cos\Omega_c n \longleftrightarrow$ () $+ j(0)$

$x_R[n]\sin\Omega_c n \longleftrightarrow (0) + j$ ()

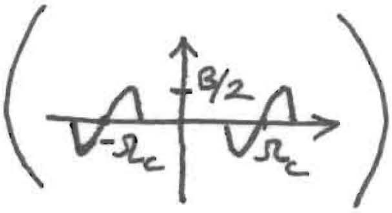
If on the other hand

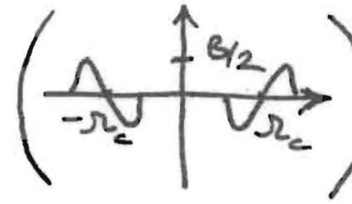
$x_I[n] \longleftrightarrow (0) + j$ ()

so $x_I[n]$ is an <u>odd</u> function, $x_I[-n] = -x_I[n]$

i.e., a purely imaginary spectrum, then <u>it's the case</u> that the DTFS's of the modulated signals $x_I[n]\cos\Omega_c n$ ~~and~~ $x_I[n]\sin\Omega_c n$ have a structure that parallels the <u>lower</u> two figures on p(1):

It takes a proof.

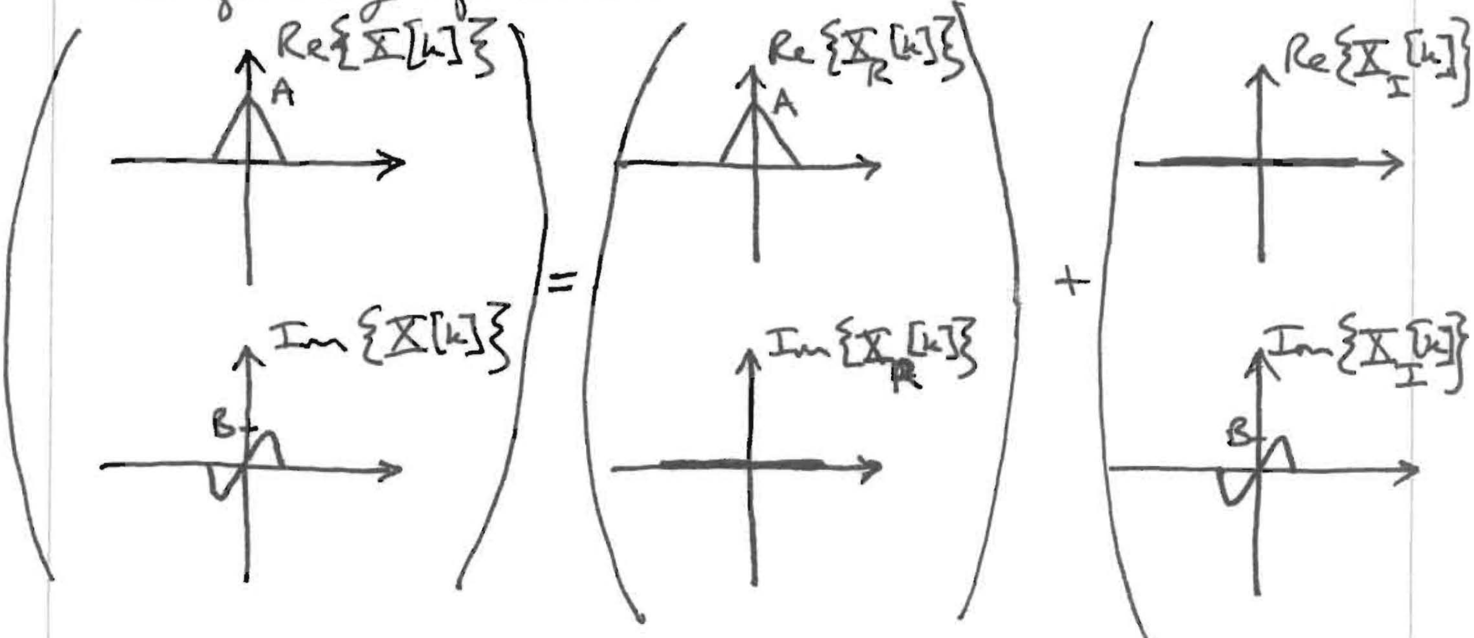
$x_I[n]\cos\Omega_c n \longleftrightarrow (0) + j\Big($  $\Big)$

$x_I[n]\sin\Omega_c n \longleftrightarrow \Big($  $\Big) + j(0)$

A general $x[n]$ can be written as

$$x[n] = x_R[n] + x_I[n]$$

for some $x_R[n]$ that has a purely real spectrum and $x_I[n]$ that has a purely imaginary spectrum:



So $x[n]\cos\Omega_c n = x_R[n]\cos\Omega_c n + x_I[n]\cos\Omega_c n$

and $x[n]\sin\Omega_c n = \underbrace{x_R[n]\sin\Omega_c n + x_I[n]\sin\Omega_c n}_{\text{and we know how to do all four of these!}}$