6.02

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Rec 13

(Much of Rec. 13 spent on left-over things from Rec 12 notes! Only Question 3 below was covered in detail)

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Frequency response

$$\boxed{H(e^{j\Omega}) = \sum_m h[m] e^{-j\Omega m}} \quad (*)$$

by definition

Why is it called this? One reason:

shown in Rec 12 notes

When $x[n] = \cos(\Omega_0 n + \theta)$ for all n,

$$y[n] = |H(e^{j\Omega_0})| \cos\left(\Omega_0 n + \theta + \angle H(e^{j\Omega_0})\right)$$

i.e. output is a sinusoid of the same frequency, but scaled in amplitude by magnitude of freq. resp. at the frequency of the cosine, and shifted in phase by the angle of the freq. resp. at this frequency.

A more compact statement that includes the above (actually equivalent to it):

see lecture notes

$x[n] = e^{j\Omega_0 n}$ for all n (the math of convolution allows complex inputs)

$$\Longrightarrow y[n] = H(e^{j\Omega_0}) e^{j\Omega_0 n}$$

Another way to write $H(e^{j\Omega})$ — just using Euler's identity in Eq. $(*)$ — is

$$H(e^{j\Omega}) = \underbrace{\left(\sum_m h[m] \cos \Omega m\right)}_{C(\Omega)} - j \underbrace{\left(\sum_m h[m] \sin \Omega m\right)}_{S(\Omega)}$$

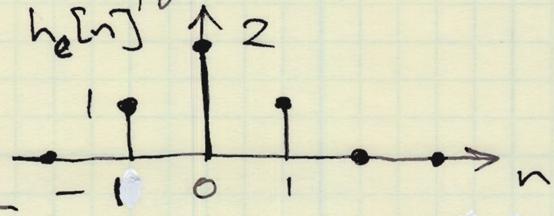
i.e., $$\boxed{H(e^{j\Omega}) = C(\Omega) - jS(\Omega)} \quad (**)$$

Note $C(-\Omega) = C(\Omega)$, $S(-\Omega) = -S(\Omega)$.

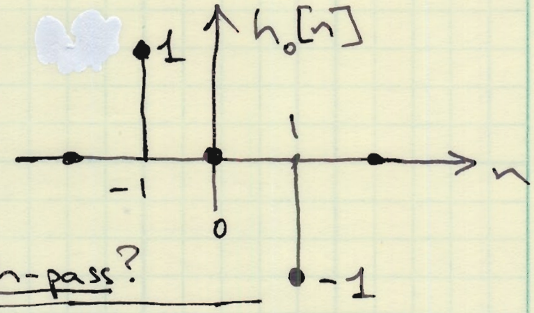
Hint: use (**)

<u>Question 1</u>: If $h[n]$ is an <u>even</u> function of time, i.e., $h[-n] = h[n]$ (so a noncausal system in general), what can you say about $H(e^{j\Omega})$? And what if $h[n]$ is <u>odd</u>, i.e., $h[-n] = -h[n]$? (<u>Ans</u>: $H(e^{j\Omega})$ is real, respectively imaginary)

Verify on



and

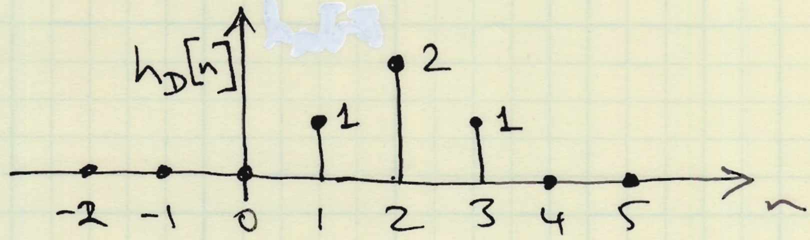


<u>Question</u>: Which of the above examples is <u>low-pass</u>? which is <u>high-pass</u>?

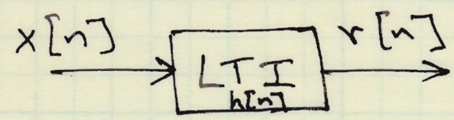
<u>Question 2</u>: Consider another LTI system with unit sample response $h_D[n] = h[n-D]$, i.e. $h[n]$ delayed by D. What is $H_D(e^{j\Omega})$ in terms of $H(e^{j\Omega})$? Also compare their respective magnitudes, as well as their respective angles (phases). (<u>Ans.</u> $H_D(e^{j\Omega}) = e^{-j\Omega D} H(e^{j\Omega})$, so magnitudes are same, while

$$\angle H_D(e^{j\Omega}) = \angle H(e^{j\Omega}) - \Omega D \;)$$

Verify on



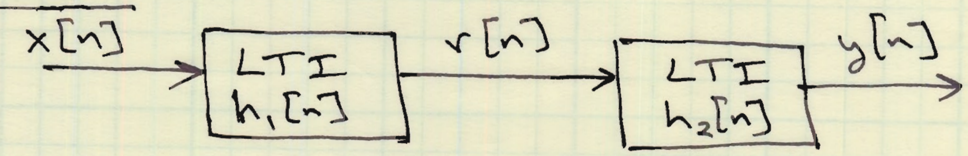
<u>Question 3</u>:



Also compare with example in Rec 12 notes

Suppose $r[n] = x[n] - A\,x[n-1] + x[n-2]$.
Find $H(e^{j\Omega})$ in two ways: (1) First finding $h[n]$, then ...; and (2) Directly using the difference equation and the fact that $x[n] = e^{j\Omega n} \Rightarrow r[n] = H(e^{j\Omega})e^{j\Omega n}$.

Question 4: Two systems in 'series' or 'cascade':

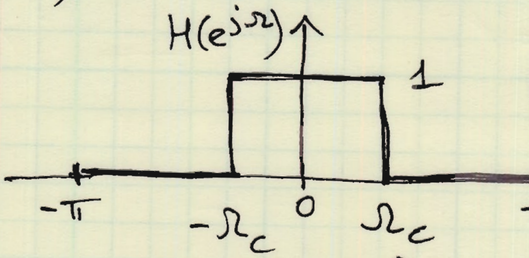


$$r[n] = x[n] - A_1 x[n-1] + x[n-2]$$

$$y[n] = r[n] - A_2 r[n-1] + r[n-2]$$

Find $H_1(e^{j\Omega})$, $H_2(e^{j\Omega})$, also the overall frequency response $H(e^{j\Omega})$ that relates $y[n]$ to $x[n]$, and the associated unit sample response $h[n]$. (Compare w. Prob 1 of Problem Set 6.)

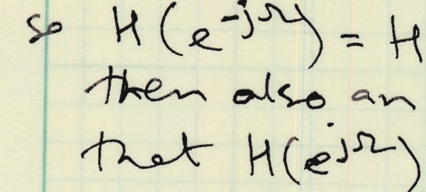
Question 5: Suppose $h[n]$ is designed such that $H(e^{j\Omega})$ is a real function of Ω [$= C(\Omega) = C(-\Omega)$ so $H(e^{-j\Omega}) = H(e^{j\Omega})$, i.e., the frequency response is then also an even function of Ω], and such that $H(e^{j\Omega})$ is an ideal low-pass frequency response:



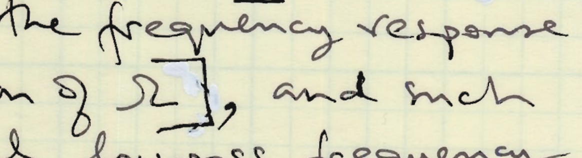
Turns out that the $h[n]$ for which $H(e^{j\Omega})$ are real/even are themselves even.

Cutoff frequency.

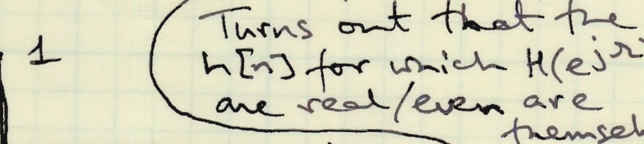
Find $H_1(e^{j\Omega})$ in each case below such that the magnitude of the overall frequency response from $x[n]$ to $y[n]$ is the magnitude of the ideal high-pass frequency response:



Case 1:



Case 2:



where $h_D[n] = h[n-D]$, i.e. $h[n]$ delayed by D

Extra question: In each case, find the phase of the freq. resp., $x[n]$ to $y[n]$