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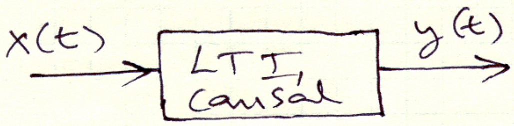
6.02
9/16/10

Rec. 3

Let's continue a little with the model from last time:

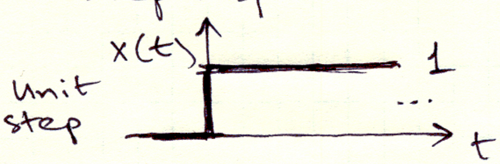


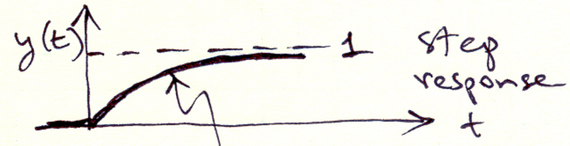
Channel model

e.g. $\frac{dy(t)}{dt} + 8y(t) = 8x(t)$ (*)

'at rest' till time 0, $y(0)=0$.

Step response:



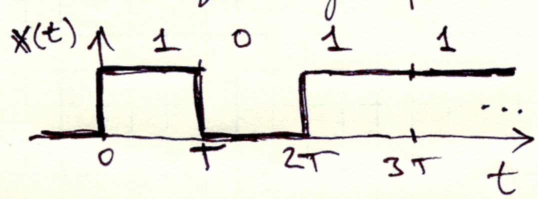


$1-e^{-8t}$ for $t \geq 0$

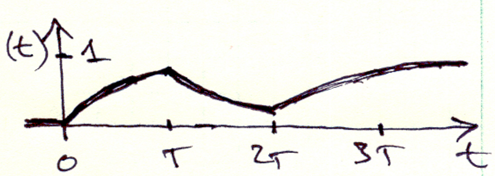
95% of the way to steady state value in three time-constants

Time-constant $= 1/8 \Longrightarrow$
at $t = 1/8$, $y(t)$ is ~~at~~ .63
at $t = 2 \times \frac{1}{8}$, $y(t)$ is at .86
at $t = 3 \times \frac{1}{8}$, $y(t)$ is at .95

0 – 1 signalling input:

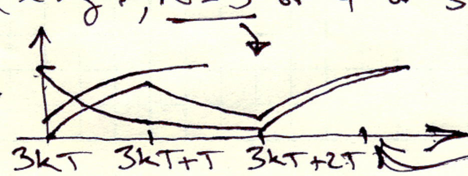


$\rightarrow$



Instead of looking at a looong trace of y(t), to see if a simple threshold rule at some well chosen set of periodic sample points will suffice to distinguish 0 from 1, we overlay each NT periods (e.g., N=3 or 4 or 5) to get eye diagram:

Overlay for $k=0,1,2,3,\ldots$



$3kT+3T = 3(k+1)T$

For this case, picking $T > 3 \times$ time-constant should lead to good (open) eye diagram.

For more general examples, pick $T >$ 'settling time' of channel.

big if!! p. 1

NOTE: <u>If</u> the channel characteristic (*) is known <u>exactly</u>, and if there is <u>no noise</u>, we can get $x(t)$ from $y(t)$, no matter how small T is. Just compute $\underbrace{\frac{dy(t)}{dt} + 8\,y(t)}$ — this is $x(t)$!!

this is equivalent to 'deconvolution' mentioned in lecture.

Computation of $\frac{dy(t)}{dt}$ is very sensitive to noise!

Parallel story in discrete-time:

$$x[n] \longrightarrow \boxed{\text{LTI, causal}} \longrightarrow y[n]$$

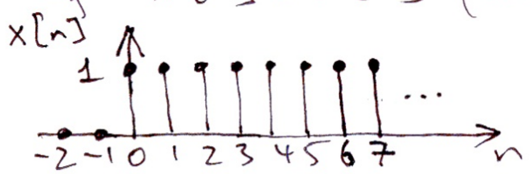
channel model

e.g. $y[n+1] = r\,y[n] + (1-r)\,x[n]$ (**)

$|r| < 1$, e.g. $r = 1 - \varepsilon$

small, > 0

If $x[n] = u[n]$ (unit step <u>function</u>)

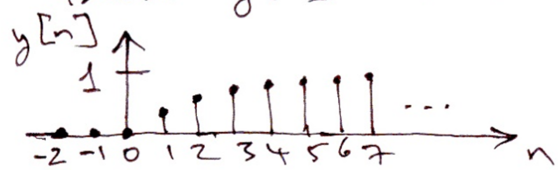


$= \begin{cases} 1 & \text{for } n \geq 0 \text{ (integer)} \\ 0 & \text{for } n < 0 \end{cases}$

and if $y[0] = 0$,

should be, for causal system with $x[n] = u[n]$

then $y[n] = 1 - r^n$ for $n \geq 0$:



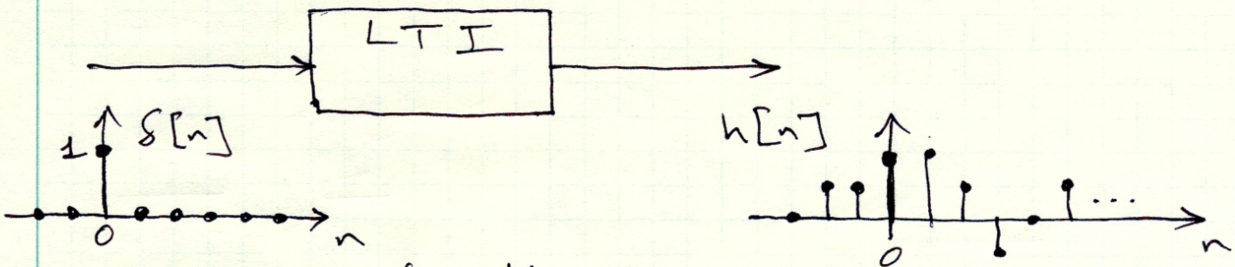
(note that r^n acts like disrete-time exponential)

step response $s[n]$. e.g. if $r = .9$, reaches 95 % level in 29 steps.

$\Rightarrow$ (<u>Continue on</u> to convolution examples, pp 25A – 28 of <u>Lec 3</u> <u>annotated slides</u>)

↑ do this!!

Unit-sample response



unit-sample function

not causal, in this case!

For our channel example:

unit step $\quad u[n] \longrightarrow s[n]$ unit-step response

$= (1-r^n)\,u[n]$ in our example

But note that $\delta[n] = u[n] - u[n-1]$ (always true)

so for an LTI system

$$\delta[n] \longrightarrow s[n] - s[n-1] = h[n]$$

note how linearity & time-invariance are used

$$= \begin{cases} (1-r^n) - (1-r^{n-1}) & \text{for } n \geq 1, \\ 0 & \text{elsewhere} \end{cases}$$

i.e. $h[n] = r^{n-1}(1-r)\,u[n-1]$ for our example

You can confirm this by checking what our causal model (**) on p. 2 does when $x[n] = \delta[n]$.

Significance of unit-sample response $h[n]$ for LTI systems:

Can write any input $x[n]$ as $\displaystyle\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$,

so response of LTI system is

$$y[n] = \sum_k x[k]\,h[n-k] = \sum_m x[n-m]\,h[m]$$

(k & m are 'dummy' indices) prove easily by change of variable