George Verghese
verghese@mit.edu

6.02
9/21/2010

Rec. 4

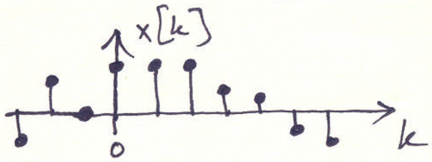
A little more on convolution:

To compute $\sum_k x[k]h[n-k] = y[n]$,

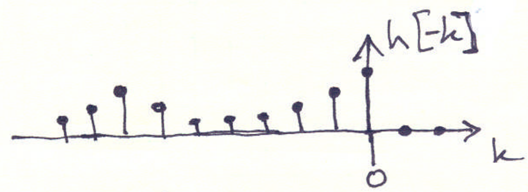
for a given n, e.g. $n=3$:

$$\sum_k x[k]h[3-k] = y[3]$$

Plot $x[k]$ and $h[3-k]$ on the k axis:

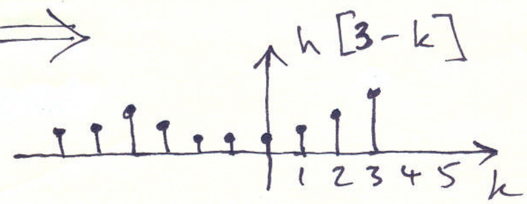


FLIP



SLIDE ⟹

(to the right by 3)



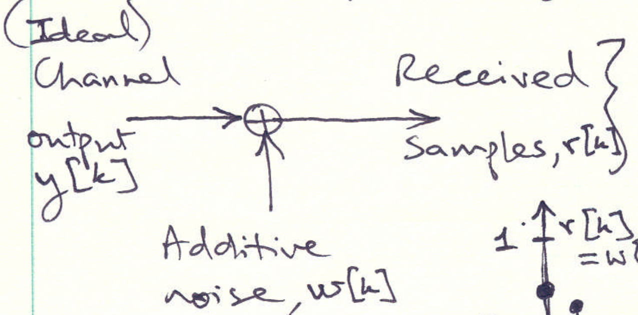
Now take the INNER PRODUCT $\sum_k x[k]h[3-k]$ to get $y[3]$.

Similarly for $y[n]$ — just shift by $|n|$

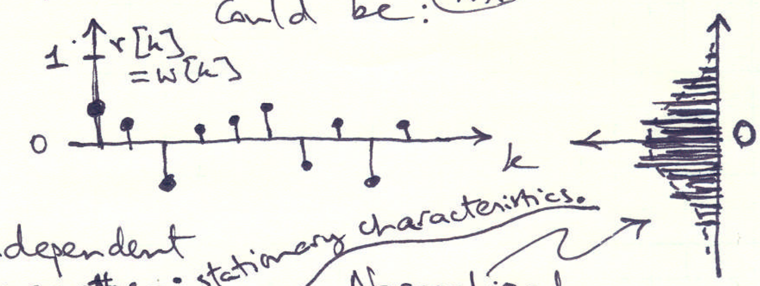
a right-shift if $n>0$, a left-shift if $n<0$.

Summary: Keep one function fixed, flip and slide the other, take the inner product.

Noise, probability models



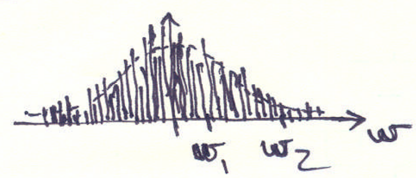
When a steady 0 is sent, received samples could be: (**)



- independent of channel output $y[k]$; independent from one time to another; stationary characteristics.

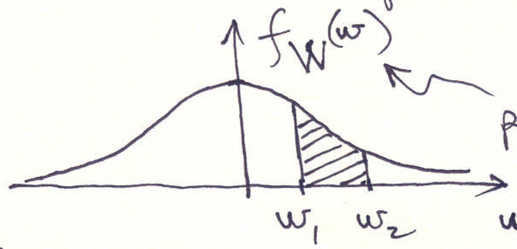
Normalized histogram (total area = 1)

Fraction of time in a long record spent between levels w_1 and w_2 is given by area between w_1 and w_2 in normalized histogram:



$\Downarrow$

Abstract this to a probability model for random variable W (in lecture notes, X)



Probability density function, pdf

$$\int_{-\infty}^{\infty} f_W(w)\,dw = 1$$

Probability that W (i.e., $w[k]$ at any particular time k) lies between w_1 and w_2 is given by

$$\int_{w_1}^{w_2} f_W(w)\,dw$$

(**) When a steady 1 is sent, similar story, but samples bounce up & down around higher level V.

Our bit-detection task:

Sent bit S is random $\begin{cases} \text{Suppose } S=0 \text{ sent with probability } p_0, \\ S=1 \quad \text{"} \quad \text{"} \quad \text{"} \quad p_1 = 1-p_0 \end{cases}$

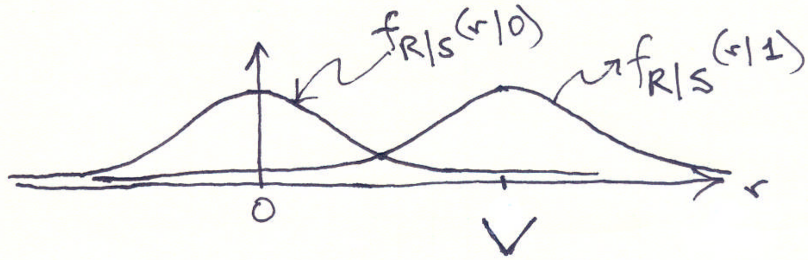
Assume no ISI issues, and that at the sampling time a '0' is received as 0 volts in the absence of noise (or on average), and a '1' is received as V volts in the absence of noise (or on average).

Our received sample is then $R = r_0$ ← specific value

$R = W$ if '0' is sent

$R = V + W$ if '1' is sent

random variable



$f_{R|S}(r|0)$ is conditional pdf of R, given that '0' was sent

$f_{R|S}(r|1)$ is conditional pdf of R, given that '1' was sent.

---

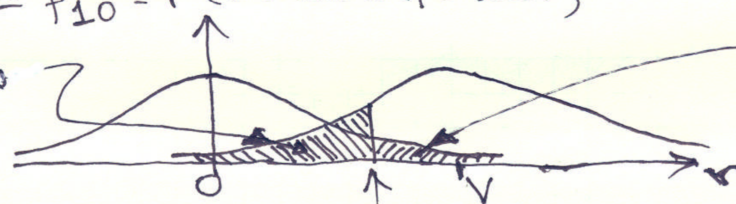
area $= P_{10} = P(0 \text{ decided} \mid 1 \text{ sent})$

$= \int_{-\infty}^{V-\lambda} f_W(w)\,dw$

area $= P_{01}$

$= P(1 \text{ decided} \mid 0 \text{ sent})$

$= \int_{\lambda}^{\infty} f_W(w)\,dw$



decide '0' ⇐ | ⇒ decide '1'

threshold λ

$$\boxed{P(\text{bit error}) = p_0 \cdot P_{01} + p_1 P_{10}}$$

(*) Look at examples on pp 22-31 of Lec 4 annotated slides.

Where did this expression for $P(\text{bit error})$ come from? $\Rightarrow$ We make an error when

0 is sent and 1 is decided at receiver

OR

1 is sent and 0 is decided at receiver

$\Big\}$ mutually exclusive possibilities, so probabilities add

$$\therefore P(\text{bit error}) = P(0 \text{ sent}, 1 \text{ decided}) + P(1 \text{ sent}, 0 \text{ decided})$$

But $P(0 \text{ sent}, 1 \text{ decided})$

$$= P(0 \text{ sent}) \cdot P(1 \text{ decided} \mid 0 \text{ sent})$$

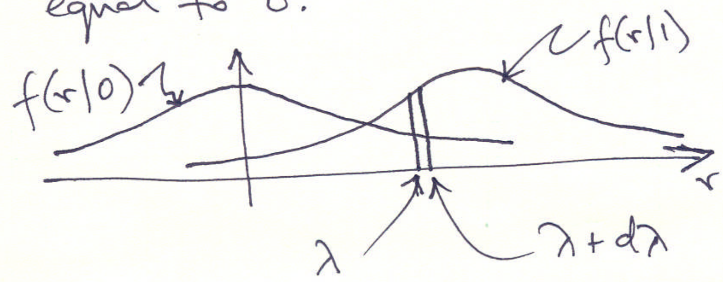
(this is just from the <u>definition</u> of conditional probability)

$$= p_0 \cdot P_{01}$$

Similarly $P(1 \text{ sent}, 0 \text{ decided}) = p_1 P_{10}$.

OPTIONAL

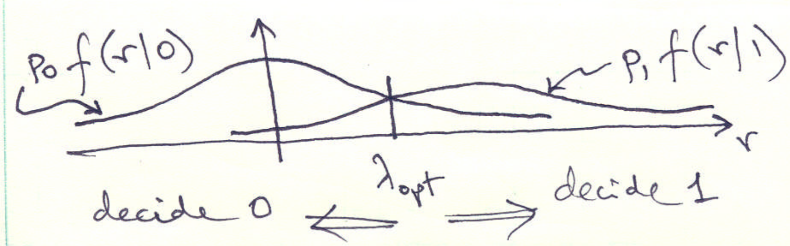
What is the <u>optimum</u> threshold for <u>minimum</u> $P(\text{bit error})$? Compute $\dfrac{d\,(P(\text{bit error}))}{d\lambda}$, set equal to 0.



P_{01} decreases by $f(\lambda|0)\,d\lambda$,

P_{10} increases by $f(\lambda|1)\,d\lambda$.

$$\Rightarrow \boxed{\text{At optimum } \lambda,\ p_0 f(\lambda|0) = p_1 f(\lambda|1)}$$



$\boxed{\text{Where } p_0 f(r|0) \text{ exceeds } p_1 f(r|1), \text{ decide 0, otherwise decide 1.}}$

More generally,