# 6.055J/2.038J (Spring 2010)

# Homework 1

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*Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).*

In the following questions, you are often asked to give your answer as a plausible range. For most of the questions, it is the exponent x in $10^x$ that you are asked for. You can specify $10^x$ as $10^{a \pm b}$ or as $10^{c \ldots d}$ (where $c = a - b$ and $d = a + b$). Think of b as the sigma ($\sigma$) measuring your uncertainty, or $c \ldots d$ as the one-$\sigma$ range. Use the format that easier for you to think about in that question.

When you choose your plausible range, remember that the goal is not to be 'right' by choosing a giant, guaranteed-safe range or, at the other extreme, to pretend to have extra confidence by choosing an overly narrow range. Rather, the goal is to choose your range such that you would be somewhat surprised if the true value falls outside your range. Numerically, choose the range so that it has a 2/3 probability of containing the true value.

That criterion explains why the range narrows after you estimate using divide and conquer. At first, you have little idea about the true value, so you would not be surprised were it to fall outside a fairly large range; after the estimate, you know more, your confidence in the estimate increases, and your plausible range shrinks.

## Warmups

1. **One or few**
   Use the 1 or few method of multiplication (and division) to estimate

   $$161 \times 294 \times 280 \times 438$$

   (a random multiplication problem generated by a short Python program).

   $10^{\boxed{\phantom{xx}} \pm \boxed{\phantom{xx}}}$   *or*   $10^{\boxed{\phantom{xx}} \ldots \boxed{\phantom{xx}}}$

   Then compare your range with the actual answer.

2. **Air mass**
   Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

   $10^{\boxed{\phantom{xx}} \pm \boxed{\phantom{xx}}}$ kg   *or*   $10^{\boxed{\phantom{xx}} \ldots \boxed{\phantom{xx}}}$ kg

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<span style="background-color:#f5c4c4">## Homework 1</span>

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I've posted Homework 1 on NB so that you can collaborate with each other. Don't post your answers here or in other ways give away the store! Rather, ask questions and make helpful suggestions.

Basically, follow the Mathcamp (and I think ESG) motto: "Be excellent to each other." Think about what comment and suggestion would most help others to learn these ideas and methods.

How do we submit the answers? I don't see any submit button or a save button.

NB does not yet have a way to do that. Once the system I'm writing works for the diagnostic, you'll be able to use it for the homeworks too. One step at a time...

Do we need to comment on here if we don't have any questions in order to show a good effort?

Not on HW 1 (or in general on the homework sets). I put it up on NB to help people collaborate if they wish to.

On the reading memos (and the upcoming solution memo): If you have no questions, you can answer other questions. As you can see, there are so many comments that I cannot answer them all (although I read them all).

How do we estimate going from "a * 10^b" to "10^c". I'm having trouble making the approximation of c from a and b.

The simplest way is to convert "a" to 1, few, or 10 – whichever is closest. If a becomes 1, then c=b. If a becomes 10, then c=b+1. If a becomes 'few', then c=b+0.5.

how fine of an estimate would you like? would you like to the closest 0.1, 0.5, 1?

In real life that depends on the purpose for which you make the estimate. If it's for an initial design, plus/minus 1 is often fine. If it's for the final stages, plus/minus 0.1 (or finer) might be appropriate. For the purposes of the problem set, which doesn't have any of that context, try plus/minus 0.5 (but don't stress too much about the accuracy).

are we supposed to estimate how much the air occupying the room weighs (i.e. volume * unit weight) or just the unit weight?

We're looking for mass, not weight, but you have the right idea of multiplying the volume * the unit per volume. Your tree will have branches of quantities you need to calculate those two things. The density of air is pretty easy to remember, and it's on the sheet of constants with the diagnostic.

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It would be much easier to do this if there was a way to draw a tree on the answer sheet.

> Sorry, I meant for you to illustrate it for yourself. I don't expect you to be able to enter a tree into the online system.

> Would you still like us to explain the idea of our tree, though?

> > Sure, no need to write a book – just one or two sentences. One way to decide what to write is to think to yourself, "What one or two sentences would have been most useful to me when I started the problem?"

how would you like us to enter trees into the "explanations" box? i had trouble even entering how i approached the last problem appropriately? could we please turn in paper psets?

> I haven't figured out how to have a simple figure-drawing program as part of a browser. So, don't worry about entering a whole tree. Just give a couple sentences describing what you did, perhaps by listing the top-level nodes in the tree (maybe with values), or something similar.

> But just because one cannot submit something does not mean it shouldn't be done! Draw a tree for yourself on paper. You can compare that against the solution set and maybe find improvements to the solution set.

I lol-ed a bit out loud.

## Problems

**3. Mass of the earth**

Estimate the mass of the earth.

$10\boxed{\phantom{00}} \pm \boxed{\phantom{00}}$ kg      *or*      $10\boxed{\phantom{00}} \cdots \boxed{\phantom{00}}$ kg

After choosing your range (in either form), check it against the measured value.

**4. Explain a UNIX pipeline**

What does this UNIX pipeline do?

```
ls -t | head | tac | head -1
```
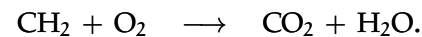
If you are not familiar with the individual UNIX commands, use the man command on Athena or on any other handy UNIX or GNU/Linux system.

**5. Atmospheric carbon dioxide**

What is the mass of $CO_2$ generated by the world annual oil consumption?

$10\boxed{\phantom{00}} \pm \boxed{\phantom{00}}$ kg/year      *or*      $10\boxed{\phantom{00}} \cdots \boxed{\phantom{00}}$ kg/year

Here is the (unbalanced!) combustion of a generic hydrocarbon (including oil, gasoline, and kerosene):

$$CH_2 + O_2 \longrightarrow CO_2 + H_2O.$$
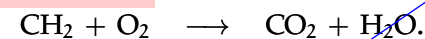
**6. Piano tuners**

Here is the classic Fermi question: Roughly how many piano tuners are there in New York City? (These questions are called Fermi questions because the physicist Enrico Fermi was an acknowledged master of inventing and solving them.)

$10\boxed{\phantom{00}} \pm \boxed{\phantom{00}}$      *or*      $10\boxed{\phantom{00}} \cdots \boxed{\phantom{00}}$

**7. Your turn to create**

Invent an estimation question that divide and conquer might help solve. You do not need to solve the question!

Particularly interesting or instructive questions might appear on the course website or as examples in lecture or the notes (let me know should you not want your name attributed in case your question gets selected).

**8. Length of the homework**

The following question is to help me calibrate the level and number of homework problems: Roughly how long did you spend on the homework?

$\boxed{\phantom{00}} \pm \boxed{\phantom{00}}$ hours      *or*      $\boxed{\phantom{00}} \cdots \boxed{\phantom{00}}$ hours

**are we just calculating the mass of just the earth or earth with people and other stuff on it**

I suspect that the mass of the other stuff is insignificant when comparing to the mass of the Earth

It would be instructive to estimate the mass of the other stuff and then see whether it is significant (I'll eat my laptop if it is significant).

**I have no idea how to solve this thing. I don't know how to code. where should I start?**

**Hypothetically, if I have no experience with UNIX (and have somehow avoided all but a few Athena commands thus far), how should I approach this problem? I have no idea how to start.**

On Athena, try the "man" command and see if you can figure out what the command does. For example, type "man ls" and then look for the description of the "-t" option.

**This command is missing on mac unix / darwin, just wondering if there is an equivalent.**

I haven't found one, but rev does something related to the problem in the book about finding the word before trivia alphabetized in reverse.

Right, rev is kind of similar. rev will reverse one line at a time (character by character), whereas tac reverse the whole file (line by line). Try "man tac" on Athena. By the way, a much faster Athena is "linux.mit.edu" (a bunch of GNU/Linux machines run by, I think, your fellow students). And "man tac" definitely works there.

**we usually see a number attached to the head command, but here we don't see any numbers, do we assume it prints the 1st 10 times by default?**

Right, 10 is the default. One use of head is to prevent lots of output from scrolling beyond the screen. A typical screen (in the old days) was 24 lines, so 10 must have been chosen as the default based on its being a significant (but not too big) fraction of 24.

**Can someone give me a hint on this? Thanks.**

Can you narrow down what aspect or aspects are most confusing? That'll make it much easier for giving useful hints.

just google it. it'll tell you what it does. there's no trick to this question, it's just knowing what the command is. speaking of which, i don't really understand the point of it actually....

The point is to analyze the effect one piece at a time (divide and conquer).

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**Does anyone have ideas on how to estimate the world annual oil consumption? I feel like that's an essential part of the problem.**

I estimated the US consumption and then assumed that we consume way more than any other nation. Hope that helps!

I would start with the stuff we did in class about the US consumption of oil and go from there.

**Why don't we call this class the Art of the Fermi Question (and Unix too!)**

that's a little snarky. while so far we've focused on unix, i'm sure there'll be more in the future.

It doesn't quite have the rhythm of "Art of Approximation in Science and Engineering". But that title isn't quite correct.

I'm thinking about naming it "Street-Fighting Methods in Science and Engineering" (I just finished a math textbook called "Street-Fighting Mathematics").