6.055J/2.038J (Spring 2010)

Homework 2

*Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 24 Feb 2010**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

On `linux.mit.edu` (the Athena GNU/Linux machine), an (American) English dictionary lives in the `/usr/share/dict/words` file.

For the (optional) question that references `decline.txt`: It is the plain-text file on the course website that contains volume 1 of Gibbon's *Decline and Fall*. It is also available – as is any other file on the course website – on any Athena machine as `/mit/6.055/data/decline.txt`

Warmups

1. **Direct practice with one or few**
   Here is another 'one or few' problem generated by my Python script:

$$985 \times 385 \times 721 \times 319 = ? \tag{1}$$

$$10^{\boxed{}} \pm \boxed{} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \ldots \boxed{}$$

2. **Land area per capita**
   Here is another problem on which to practice the 'one or few' method of multiplication and division: Estimate how much land area each person would have if people were evenly distributed on the (land) surface of the earth.

$$10^{\boxed{}} \pm \boxed{}\ \text{m}^2 \text{ per person} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \ldots \boxed{}\ \text{m}^2 \text{ per person}$$

3. **Nested square roots**
   Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \tag{2}$$

4. **Searching for ...gry words**
   What English words, other than `angry`, ends in `gry`?

Here is Homework 2 on NB, in case you want to use NB to collaborate or to ask questions. Have fun!

where do we enter answers for hw2?

Would it be possible for the future to put the questions (the text) on the online submission? I think that would make completing the homework more smooth.

can anyone tell me how to find this number? I mean in general, not this particular problem. I always have trouble finding the ranges

10ˆ0.5 = 3 10ˆ0.7 = 5 10ˆ0.9 = 8 Multiply these to your powers of 10 to increase the accuracy of your answers. hope this helps.

Mostly I just use the "Force". I tally mentally how many approximations I made (e.g. going from 385 to "few times 100") and how inaccurate they are, then guestimate how much those might combine to make the final error. In Unit 7 we'll do that formally, but for now using the force is more than enough.

The accuracy itself doesn't need to be too accurate. e.g. Are you unsure by a factor of 10 on either side (plus/minus 1 in the exponent) or a factor of 3 (plus/minus 0.5 in the exponent), etc.

I think a good idea is to know a few numbers from a table of mantissas for the common log. Then you will know that log_109 approx 1 and log_103 approx 0.5, etc.

So if we get 1/few, what does that equal? few*10ˆ(-1)?

Yes, or it could be approximated to 10ˆ-5.

* 10ˆ-.5

The online submission form for HW set 2 hasn't been posted, so I'm wondering if we're required to convert our answer to 10ˆa +- b form. The answer I got was 2ˆ(2/3), but I am unsure how to convert it, or will I not be required to?

I'll post that tonight. On that question it will ask you for the number itself (rather than for the exponent).

I had this question in an interview!

could i please get a little help on this? i really don't know how to approach it and can't seem to get started.

The method here is very similar to one that might work for the resistor ladder problem later in the pset, if that helps you.

maybe if you think of sqrt() as ˆ(1/2) that might help

6.055J/2.038J (Spring 2010)

Homework 2

*Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 24 Feb 2010**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

On `linux.mit.edu` (the Athena GNU/Linux machine), an (American) English dictionary lives in the `/usr/share/dict/words` file.

For the (optional) question that references `decline.txt`: It is the plain-text file on the course website that contains volume 1 of Gibbon's *Decline and Fall*. It is also available – as is any other file on the course website – on any Athena machine as `/mit/6.055/data/decline.txt`

Warmups

1. Direct practice with one or few
Here is another 'one or few' problem generated by my Python script:

$$985 \times 385 \times 721 \times 319 = ? \tag{1}$$

$10^{\boxed{}} \pm \boxed{}$ ***or*** $10^{\boxed{}} \ldots \boxed{}$

2. Land area per capita
Here is another problem on which to practice the 'one or few' method of multiplication and division: Estimate how much land area each person would have if people were evenly distributed on the (land) surface of the earth.

$10^{\boxed{}} \pm \boxed{}\ \mathrm{m}^2$ per person ***or*** $10^{\boxed{}} \ldots \boxed{}\ \mathrm{m}^2$ per person

3. Nested square roots
Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \tag{2}$$

4. Searching for ...gry words
What English words, other than `angry`, ends in `gry`?

Could someone provide a hint on how to approach this?

At the top of the page, there is a link to the location of a dictionary. Use a "pattern searching" function in Unix perhaps...

where can we type in unix commands again? can we just type in a terminal on athena?

That works. Or sometimes I use "linux.mit.edu" instead of regular Athena (linux.mit.edu is a network of Athena Linux machines).

I ended up learning how to ssh into Athena from Terminal on my mac. for anyone else, I don't know if this is the best way to do it, but I typed in:

ssh -l username athena.dialup.mit.edu

(username is my user name, and the -l is a dash then a lowercase L.) you then enter your password and can type in unix commands, access the dictionary, etc.

Or "ssh linux.mit.edu" (if your local username is the same as your Athena name, then you don't need the -l username, which is why my username/email is "sanjoy" on every machine I use).

I typed in the dictionary command when i ssh'ed into athena. I get a Permission Denied error.

I'm having the same problem. I don't think my local username is the same. Can I still access athena from my computer?

may i please get clear instructions on how to access/use the software/computer/server we need to solve this problem for a PC?

The easiest way would just be to visit an athena cluster. If that isn't an option, then you can try downloading and installing PuTTY and entering linux.mit.edu as the hostname.

I used secure crt in order to open linux.mit.edu. Then I just used cd in order to open the different folders in order to get to the dictionary

I checked the permissions and it seemed okay. So, can you give more details? What is the exact command, in what directory did you try it, and on what machine?

i don't understand how using Athena's commands to find the answer to this question is any different, in terms of teaching us how to figure things out for ourselves, than searching for it in google. both are just "looking up" the "correct" answer.

Problems

5. Geometric series

Use abstraction to find the sum of the infinite series

$$1 + r + r^2 + r^3 + \cdots. \qquad (3)$$

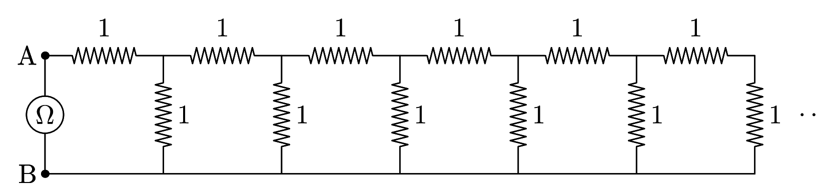
6. Pool temperature

A large outdoor swimming pool in the Arizona desert has a time constant of 4 days for exchanging heat with the air. Roughly how large are the peak-to-peak fluctuations of the water temperature caused by 30 °F (peak-to-peak) night–day fluctuations in the air temperature?

$10^{\boxed{}} \pm \boxed{}$ °F ***or*** $10^{\boxed{}} \ldots \boxed{}$ °F

7. Resistive network

In the following infinite network of 1 Ω resistors, what is the resistance between points A and B? This measurement is indicated by the ohmmeter connected between these points. (If you want to read about series and parallel resistances, a useful reference is the Wikipedia article 'Series and parallel circuits'.)



$\boxed{} \pm \boxed{}$ Ω ***or*** $\boxed{} \ldots \boxed{}$ Ω

Optional

These problems are optional in case you want more practice or want to try a (possibly large) project.

8. Email indexer

Design a set of shell scripts for doing quick keyword searches of a large database of emails. Assume that each email is stored in its own plain-text file. Perhaps one shell script generates an index, and a second script searches the index.

9. Running time

Ordinary long multiplication requires $O(n^2)$ digit-by-digit multiplications. Show that the Karatsuba multiplication method explained in lecture requires $O(n^{\log_2 3}) \approx O(n^{1.58})$ digit-by-digit multiplications.

10. Counting empires

How often does the word `Empire` (uppercase E, then all lowercase) occur in `decline.txt`? [*Hint:* Look up the `tr` command.]

Comments on page 2

It should be noted that abs(r) < 1 for this problem.

The sum should be valid for r >= 1, but perhaps not as well-defined. It's certainly not a Cauchy sequence to produce the standard definition of a real number...

> There are lots of definitions of convergence, including Cesaro and Borel, which allow one to sum otherwise divergent series. _Divergent Series_ by Hardy is the classic text.
>
> Divergent series are very useful in physics (the whole theory of quantum electrodynamics is divergent, but with clever accounting it produces experimentally very accurate results).

Could someone give me a hint on this one? Either I'm still struggling with abstraction, or just maybe recursion... I'm not even sure where to start.

Same here...

i feel like abstraction, more than anything else we've done thus far (i guess just divide and conquer...) requires a sort of "cleverness" where you either "see it" or you don't.

for what values of r? r < 1?

I think so

thanks

> Don't worry, be happy. In short, don't worry about rigor; just assume that it converges and charge ahead even where angels fear to tread.

Problems

5. Geometric series

Use abstraction to find the sum of the infinite series

$$1 + r + r^2 + r^3 + \cdots. \tag{3}$$

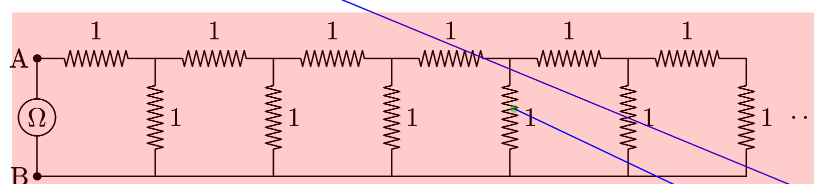
6. Pool temperature

A large outdoor swimming pool in the Arizona desert has a time constant of 4 days for exchanging heat with the air. Roughly how large are the peak-to-peak fluctuations of the water temperature caused by 30 °F (peak-to-peak) night–day fluctuations in the air temperature?

$10^{\square} \pm \square$ °F ***or*** $10^{\square} \ldots \square$ °F

7. Resistive network

In the following infinite network of 1 Ω resistors, what is the resistance between points A and B? This measurement is indicated by the ohmmeter connected between these points. (If you want to read about series and parallel resistances, a useful reference is the Wikipedia article 'Series and parallel circuits'.)



$\square \pm \square$ Ω ***or*** $\square \ldots \square$ Ω

Optional

These problems are optional in case you want more practice or want to try a (possibly large) project.

8. Email indexer

Design a set of shell scripts for doing quick keyword searches of a large database of emails. Assume that each email is stored in its own plain-text file. Perhaps one shell script generates an index, and a second script searches the index.

9. Running time

Ordinary long multiplication requires $O(n^2)$ digit-by-digit multiplications. Show that the Karatsuba multiplication method explained in lecture requires $O(n^{\log_2 3}) \approx O(n^{1.58})$ digit-by-digit multiplications.

10. Counting empires

How often does the word `Empire` (uppercase E, then all lowercase) occur in `decline.txt`? [*Hint:* Look up the `tr` command.]

I don't really understand what the question means here? also, any hints as to how to start this one?

This is a Newton's Law of Cooling problem from 18.03. But even if you don't know the math, you can take an educated guess with reasonable numbers. The concept here is that, the air temperature of the desert fluctuates between a high and a low every 24 hours. The difference (high minus low) is 30 degrees F. However, the pool doesn't instantaneously adjust to the air temperature so it lags behind. As you can imagine, by the time the pool starts heating up, the day is almost over and the air temp is going down so before the pool temp can reach the day's high temp, it is going down again.

The other thing you need to know is the rate at which the pool adjusts to the air so you can figure out how much it changes in one day. That's where the "time constant" comes in. A time constant of 4 days is defined to be like so: If there were no such thing as day/night and the air remained at a fixed temperature, T1, while the pool started at a temperature, T2, then the time constant, tau, is equal to the amount of time that it takes for the pool water to change by 63.2% of the difference, T2 minus T1,

Thus, if the air is higher, then it will take 4 days for the pool to rise by 63.2% of whatever their initial temperature difference was. But, if the air is lower, then it will take 4 days for the pool water to decrease by 63.2% of the initial temperature difference.

For a more mathematical treatment of time constant you can google that phrase.

Lastly, since we are dealing with diurnal variations in temperature, the air temperature isn't staying still. Thus, the pool water is trying to become the same temperature as something that's always changing.

Temperature highs and lows are separated by 12 hours, not 24 hours.

Right, but the period is 24 hours, menaing that the highs are separated by 24 hours (as are the lows). I have to think about this factor of 2 almost every time I think about this kind of problem, so hopefully I have it right now.

This question will make more sense after the reading on low-pass fliters (given out on Monday).

I think this question was on a 6.002 exam a few years back.

For this problem, I know I should use recursion somehow, however, I can't figure out how to break down the problem, any hints?

The way I approached this problem and the one on the coin game is to find something self-similar and use that symmetry. Sometimes you know something about the smaller part and can transform/relate it to the larger whole which you also might be able to define.

Is this "optional" thing kind of like a trick? Because wouldn't doing more work get you closer to a decent effort, and therefore actually end up counting?

No, it really is purely optional and for you to have fun. I won't even look at the responses (which is why the online system doesn't even have a spot to enter them). Although I might once in a while include solutions to them.