6.055J/2.038J (Spring 2010)

Homework 3

Do the following warmups and problems. Submit your answers, including the short explanation, online by ***10pm on Wednesday, 3 Mar 2010****.*

Open universe: *Collaboration, notes, and other sources of information are* ***encouraged.*** *However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Warmups

1. **Fuel efficiency of a 747**
   Use the cost of a plane ticket to estimate the fuel efficiency of a 747, in passenger–miles per gallon (passenger–mpg).

   $10^{\square} \pm \square$ passenger–mpg ***or*** $10^{\square} \ldots \square$ passenger–mpg

2. **High winds**
   At roughly what wind speed is the force on your body from the wind approximately equal to your weight?

   $\square \pm \square$ $\text{m}\,\text{s}^{-1}$ ***or*** $\square \ldots \square$ $\text{m}\,\text{s}^{-1}$

3. **Daunting integral**
   Evaluate

$$\int_{-\infty}^{\infty} \frac{x^3}{1 + 7x^2 + 18x^6}\,dx. \tag{1}$$

   $\square \pm \square$ ***or*** $\square \ldots \square$

Problems

4. **Solitaire**
   You start with the numbers 3, 4, and 5. At each move, you choose any two of the three numbers – call the choices a and b – and replace them with $0.8a - 0.6b$ and $0.6a + 0.8b$. The goal is to reach 4, 4, 4. Can you do it? If yes, give a move sequence; if no, show that you cannot.

Here is Homework 3 on NB in case you want to collaborate.

somehow, I don't feel that using a plane ticket is the best, or quickest way to estimate this. it seems to open up so many more divide and conquer branches, with the division of the plane tickets money. fuel efficiency seems to me more of a geometrical estimation problem. what do you guys think?

Are we supposed to use a formula?

Maybe. I think we might be covering it when we do drag. I actually ended up using a symmetry to relate it to a value I do know.

That question will be a lot easier after the reading for Wednesday's lecture (hopefully posted tomorrow).

Is there any way you can note on the homework the relevant reading(s)? Or at least the ones that will come out after the homework does?

yes, I remember after reading the homework I was like:" woah, I could have used this on the bike/drag force problem from the previous homework", and sanjoy wrote: "well... in fact, I'm going to put it on the next homework, muahahahaha" and then I was like," noo.." [in the voice of Conseula, from Family Guy]

any hints on how to start this?

There is a nice symmetry in this function. It might help you to graph it.

You won't always have access to a graphing program. I feel that part of the challenge of this class is to think of ways where you don't need too much aid to solve these sorts of problems. That being said, think about odd versus even functions, how they divide each other, and what happens when you integrate a function from -infinity to +infinity. If you do that, you'll immediately see that you don't need to graph it at all or do any "math".

Anyone know of any nice, easy to use online graphing calculator?

wolframalpha if you want to do it online. also, mathematica, matlab, and maple are all free downloads for MIT students.

wolfram alpha

I have no idea

I'm with you. I'm looking at something with even an odd functions that someone else hinted to, but I don't know enough about it. What if you set x^3 to something?

remember symmetry guys. and -infinity and infinity is symmetric about what? think about those values.

Best regards.

Has anyone managed to find an invariant?

Yes. Think about what you can do with three numbers. See if the operations keep one of those values constant.

6.055J/2.038J (Spring 2010)

Homework 3

*Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 3 Mar 2010**.*

Open universe: *Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Warmups

1. Fuel efficiency of a 747

Use the cost of a plane ticket to estimate the fuel efficiency of a 747, in passenger–miles per gallon (passenger–mpg).

$10^{\square} \pm \square$ passenger–mpg ***or*** $10^{\square} \ldots \square$ passenger–mpg

2. High winds

At roughly what wind speed is the force on your body from the wind approximately equal to your weight?

$\square \pm \square$ $\mathrm{m\,s^{-1}}$ ***or*** $\square \ldots \square$ $\mathrm{m\,s^{-1}}$

3. Daunting integral

Evaluate

$$\int_{-\infty}^{\infty} \frac{x^3}{1 + 7x^2 + 18x^8}\, dx. \tag{1}$$

$\square \pm \square$ ***or*** $\square \ldots \square$

Problems

4. Solitaire

You start with the numbers 3, 4, and 5. At each move, you choose any two of the three numbers – call the choices a and b – and replace them with $0.8a - 0.6b$ and $0.6a + 0.8b$. The goal is to reach 4, 4, 4. Can you do it? If yes, give a move sequence; if no, show that you cannot.

Do you replace them respectively and does it matter?

It doesn't matter in what order you replace them, but I think it does the order that you choose.

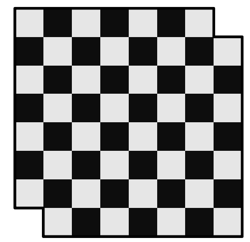
5. Maximizing a polynomial

Use symmetry to find the maximum value of $6x - x^2$.

[ ] ± [ ] *or* [ ] . . . [ ]

6. Tiling a mouse-eaten chessboard

An 8×8 chessboard gets two diagonally opposite corners eaten away by a mouse. You have dominoes, each 2×1 in shape – i.e. each covers two adjacent squares. Can you tile the mouse-eaten chessboard with these dominoes? In other words, can you lay down the dominoes to cover every square exactly once (no empty squares and no overlaps)?



- [ ] yes
- [ ] no

Optional!

7. Symmetry for second-order systems

This problem analyzes the frequency of maximum gain for an LRC circuit or, equivalently, for a damped spring–mass system. The gain of such a system is the ratio of the input amplitude to the output amplitude as a function of frequency.

If the output voltage is measured across the resistor, and you drive the circuit with a voltage oscillating at frequency ω, the gain is (in a suitable system of units):

$$G(\omega) = \frac{j\omega}{1 + j\omega/Q - \omega^2},$$

where $j = \sqrt{-1}$ and Q is quality factor, a dimensionless measure of the damping. Do not worry if you do not know where that gain formula comes from. The purpose of this problem is not its origin, but rather using symmetry to maximize its magnitude.

The magnitude of the gain is

$$|G(\omega)| = \frac{\omega}{\sqrt{\left(1-\omega^2\right)^2 + \omega^2/Q^2}}.$$

Find a variable substitution (a symmetry operation) $\omega_{new} = f(\omega)$ that turns $|G(\omega)|$ into $|H(\omega_{new})|$ such that G and H are the same function (i.e. they have the same structure but with ω in G replaced by ω_{new} in H). Use the form of that symmetry operation to maximize $|G(\omega)|$ without using calculus.

8. Inertia tensor

[For those who know about inertia tensors.] Here is the inertia tensor (the generalization of moment of inertia) of a particular object, calculated in a lousy coordinate system:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 5 \end{pmatrix}$$

Change coordinate systems to a set of principal axes. In other words, write the inertia tensor as

Comments on page 2

I'm not sure how to do this by symmetry, explicitly. My tendency is to just want to do it by calculus...

Me too. I really just want to take a derivative and then find the maximum of the original equation.

start by figuring out the zeroes, then think about parabolas and try to use their symmetry to figure out where the maximum should be...

I'm pretty sure this can't be done - has anyone figured out how to go about proving that?

I would also like a hint.

I would think about what it means to cover "adjacent squares."

There is a symmetry that is broken. Think about what the symmetry is. Also, there is a symmetry (kind of) with the domino that you will need in conjunction.

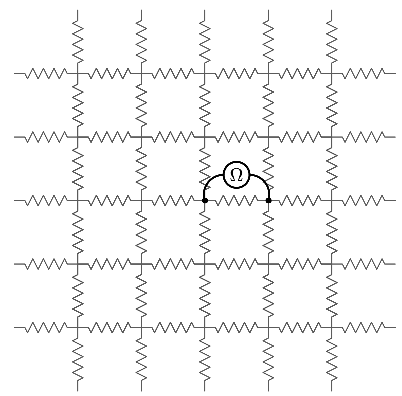
Is it implied you have black and white dominos?

$$\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

and give the values of I_{xx}, I_{yy}, and I_{zz}. *Hint:* What properties of a matrix are invariant when changing coordinate systems?

9. Resistive grid

In an infinite grid of 1-ohm resistors, what is the resistance measured across one resistor?



To measure resistance, an ohmmeter injects a current I at one terminal (for simplicity, say $I = 1\,\text{Å}$), removes the same current from the other terminal, and measures the resulting voltage difference V between the terminals. The resistance is $R = V/I$.

Hint: Use symmetry. But it's still a hard problem!

Comments on page 3

Hurray for the spectral theorem!

This one is still hard for me...