6.055J/2.038J (Spring 2010)

Homework 7

*Submit your answers and explanations online by **10pm on Friday, 23 Apr 2010**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Problem 1 No need to remember lots of constants

Many atomic problems, such as the size or binding energy of hydrogen, end up in expressions with $\hbar$, the electron mass m_e, and $e^2/4\pi\epsilon_0$. You can avoid remembering those constants by instead remembering the following values:

$$\begin{aligned} \hbar c &\approx 200\,\text{eV nm} = 2000\,\text{eV Å} \\ m_e c^2 &\sim 0.5 \cdot 10^6\,\text{eV} \\ \frac{e^2/4\pi\epsilon_0}{\hbar c} &\equiv \alpha \approx \frac{1}{137} \quad \text{(fine-structure constant).} \end{aligned}$$

Use those values to evaluate the energy of a visible-light photon. Note: The photon energy is $E = hf$, where h is Planck's constant and f is the frequency of the radiation; equivalently, $E = \hbar\omega$, where $\hbar = h/2\pi$ and ω is the angular frequency of the radiation.

$10^{\square} \pm \square$ eV ***or*** $10^{\square} \ldots \square$ eV

Problem 2 Boundary-layer thickness

How thick is the boundary layer on a golf ball traveling at, say, $v \sim 40\,\text{m}\,\text{s}^{-1}$?

$10^{\square} \pm \square$ m ***or*** $10^{\square} \ldots \square$ m

COMMENTS ON PAGE 1

Here is the next pset – note that it is due on Friday (not Wed). There are eight problems, but many are short (some of which are subtle). Don't spend too long on any one problem, and enjoy.

Problem 3 Viscous versus form drag

The form drag (drag due to moving fluid aside) is

$$F_d \sim \rho v^2 A. \quad (1)$$

The viscous (skin-friction) drag is

$$F_v \sim \rho\nu \times \text{surface area} \times \text{velocity gradient}, \quad (2)$$

where $\rho\nu$ is the dynamic viscosity η. The velocity gradient is v/δ, where v is the flow speed, and δ is the boundary-layer thickness.

The ratio F_d/F_v is dimensionless, and must therefore be a function of the only dimensionless measure of the flow, namely the Reynolds number Re. In fact, the function is a power law:

$$\frac{F_d}{F_v} \sim \mathrm{Re}^n, \quad (3)$$

where n is the scaling exponent. What is n?

☐ ± ☐ *or* ☐ ... ☐

Problem 4 Viscous versus form drag while walking

Use the result of **Problem 3** to estimate the ratio

$$\frac{\text{form drag}}{\text{viscous drag}} = \frac{F_d}{F_v} \quad (4)$$

for a person walking.

$10^{☐}$ ± ☐ *or* $10^{☐}$ ... ☐

Problem 5 Rolling down the plane

Four objects, made of identical steel, roll down an 30-degree inclined plane without slipping. The large objects have three times the radius of the small objects. Which object has the greatest acceleration?

- ☐ a large spherical shell
- ☐ a large disc
- ☐ a small solid sphere
- ☐ a small ring

Comments on page 2

Ah! That isn't velocity. That's the kinematic viscosity...

Wow good catch! I totally missed that.

Yeah, sorry about that. I've fixed it for all subsequent readings and homeworks by changing the font to Palatino for math and text (instead of Euler for math).

Equation 6.5 from the notes does not have density here. Are you sure this isn't a typo?

Not a typo, but it was sloppy. Equation 6.5 uses "viscosity" by which I meant there "dynamic viscosity", which is rho * kinematic viscosity.

help? I have no clue where to start here...

Their dimensionless-ness is really confusing me. How do we start this?

To start, work out the skin-friction drag using Equation 2 and what you know about the boundary-layer thickness delta (compared to r, the size of the object). The reading on boundary layers will give you that last bit.

this means that they have the same mass, right?

I am confused about this as well... do they have the same mass or the same density?

It doesn't matter. Consider dropping objects of different density or masses in a vacuum...

It does, though, since there's rolling motion involved and that depends on mass.

i think this means they have the same density.

The same density (e.g. all are made of stainless steel or all are made of high-carbon steel).

Problem 6 Hydrogen binding energy

In lecture and readings we analyzed hydrogen (`r26-lumping-hydrogen.pdf` on NB), which is one electron bound to one proton. Using those results, one can show that the binding energy is

$$E \sim \frac{1}{2} m_e (\alpha c)^2, \tag{5}$$

where α is the fine-structure constant, c is the speed of light, and m_e is the mass of the electron.

Use the methods of **Problem 1** to calculate the binding energy in electron–volts.

$10^{\square} \pm \square$ eV ***or*** $10^{\square} \ldots \square$ eV

Problem 7 Heavy nuclei

In this problem you study the innermost electron in an atom with many protons (i.e. with a heavy nucleus). So, imagine a nucleus with Z protons around which orbits just one electron. Let $E(Z)$ be the binding energy. The case $Z = 1$ (**Problem 6**) is hydrogen.

Find how $E(Z)$ depends on Z. Namely, what is the scaling exponent n in

$$E(Z) \propto Z^n \tag{6}$$

or, equivalently, in

$$\frac{E(Z)}{E(1)} = Z^n\,? \tag{7}$$

$\square \pm \square$ ***or*** $\square \ldots \square$

Problem 8 Heaviest nuclei

Consider again the system of **Problem 7**: a nucleus with Z protons surrounded by one electron.

When the binding energy $E(Z)$ is comparable to $m_e c^2$ – the rest energy of the electron – then the electron has enough kinetic energy to produce, out of nowhere, a positron (an anti-electron). As a result of this process, which is known as pair creation, the positron leaves the nucleus, turning one proton into a neutron. That makes the atomic number Z drop by one. The consequence is that, for large-enough Z, the nucleus is unstable! Relativity sets an upper limit for Z.

Use the results of **Problem 7** to estimate this maximum Z set by relativity (feel free to ignore factors of $1/2$ in $E(1)$).

$\square \pm \square$ ***or*** $\square \ldots \square$

To include in the explanation box: Compare your estimate with the Z for the heaviest stable nucleus (uranium).

Comments on page 3

I've read the notes but still I have no idea how to do this or where to start.