6.055J/2.038J (Spring 2010)

Homework 8

*Submit your answers and explanations online by **10pm on Wednesday, 05 May 2010**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Problem 1 Should you be worried?

Assume that 1 in 10^4 bridges in the United States are in danger of collapse unless repaired soon, and that a new test for bridge integrity has been devised. This test is 90% accurate: It always detects an unsafe bridge (no false negatives); and 10% of the time it says that a safe bridge is unsafe (10% false positives).

You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

$$10^{\boxed{} \pm \boxed{}} \quad \textit{or} \quad 10^{\boxed{} \ldots \boxed{}}$$

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

Problem 2 Reusing plausible-range combinations

In lecture, we saw that if the width of an object has a plausible range $w = 1 \ldots 10\,\text{m}$ and the length has a plausible range $l = 1 \ldots 10\,\text{m}$, then the area $A = lw$ has the range $2 \ldots 50\,\text{m}^2$.

If instead the plausible ranges are $w = 2 \ldots 20\,\text{m}$ and $l = 5 \ldots 50\,\text{m}$, what is the plausible range for the area?

$$\boxed{} \pm \boxed{}\ \text{m}^2 \quad \textit{or} \quad \boxed{} \ldots \boxed{}\ \text{m}^2$$

Problem 3 Singing a logarithm

Estimate 1.5^{40} using the singing-logarithms method from lecture (a copy of the handout is on the course website).

$$10^{\boxed{} \pm \boxed{}} \quad \textit{or} \quad 10^{\boxed{} \ldots \boxed{}}$$

COMMENTS ON PAGE 1

Here is the new homework – due Wednesday. It has 9 problems, but most are very short. The last one is a bit longer but hopefully that is compensated for by the explanation of an everyday phenomenon that depends on viscosity.

So what happens when you test it again, and again? Let's say, you test it 4 times and it comes up positive every time. I think I know the answer, but just food for thought I guess.

Problem 7 Singing logarithms to combine plausible ranges

You are trying to esimate the plausible range for the volume of an object. You have assigned the length, width, and height the plausible ranges

$$l = 1 \ldots 10\,\mathrm{m}$$
$$w = 1 \ldots 10\,\mathrm{m} \qquad (2)$$
$$h = 1 \ldots 10\,\mathrm{m}.$$

In other words, each range is a factor of 10 wide (the 'width' is the ratio of the upper to lower endpoints). Convince yourself that the plausible range for the volume $V = lwh$ is a factor of $10^{\sqrt{3}}$ wide and is centered on $10^{1.5}\,\mathrm{m}^3$.

For the answer box, use $\sqrt{3} \approx 1.7$ or $\sqrt{3} \approx 1.73$ and the singing-logarithm method from lecture (a copy of the handout is on the course website) to estimate $10^{\sqrt{3}}$.

$\boxed{} \pm \boxed{}$ ***or*** $\boxed{} \ldots \boxed{}$

Problem 8 Perfume

If the diffusion constant (in air) for small perfume molecules is $10^{-6}\,\mathrm{m}^2\,\mathrm{s}^{-1}$, estimate the time for perfume molecules to diffuse across the lecture room.

$10^{\boxed{} \pm \boxed{}}\,\mathrm{s}$ ***or*** $10^{\boxed{} \ldots \boxed{}}\,\mathrm{s}$

To include in the explanation box: Now try the experiment, at least mentally. How long does it actually take to smell the perfume from across the room? Explain the large discrepancy between the theoretical estimate and the experimental value.

Comments on page 3

Where in the lectures are these range problems

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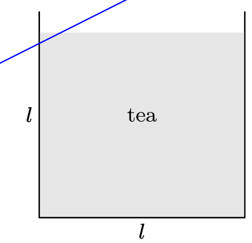
Problem 9 Teacup spindown

You stir your afternoon tea to mix the milk (and sugar if you have a sweet tooth). Once you remove the stirring spoon, the rotation starts to slow. In this problem you'll estimate the spindown time τ: the time for the angular velocity of the tea to drop by a significant fraction.

To estimate τ, consider first a physicist's idea of a teacup: a cylinder with height l and diameter l, filled with a water-like liquid. Tea near the edge of the teacup – and near the base, but for simplicity we'll neglect the effect of the base – is slowed by the presence of the edge.



Because of the no-slip boundary condition, the edge creates a velocity gradient. Because of the tea's viscosity, the velocity gradient produces a force along the edge. This force tries to accelerate each piece of the edge along the direction of the tea's motion. The piece in return exerts an equal and opposite force on the tea. That is how the edge slows the rotation. Now analyze this model quantitatively using the following steps. Keep the results in symbolic form until the final step (**Step e**) when you get a numerical value for τ.

a. Convince yourself that the spindown time τ is given by

$$\tau \sim \frac{\rho l^5 \omega}{\sigma l^3} = \frac{\rho l^2 \omega}{\sigma}, \tag{3}$$

where ρ is the density of tea, σ is the viscous stress (the viscous tangential force per unit area), and ω is the initial angular velocity. *Hint:* Consider the torque on and the angular momentum of the rotating blob of tea. In addition, drop all dimensionless constants like π and 2 by invoking the Estimation Theorem 1 = 2.

b. Now estimate the viscous stress σ by using the idea that

$$\text{viscous stress} \sim \rho\nu \times \text{velocity gradient}. \tag{4}$$

The velocity gradient is determined by the thickness of the region over which the the edge significantly affects the flow; this region is the boundary layer. Let δ be its thickness (you'll find δ in **Step d**). In terms of δ, estimate the velocity gradient near the edge. Then estimate the viscous stress σ.

c. Insert your expression for the viscous stress σ into the earlier estimate for the spindown time τ. Your new expression for τ should contain only the boundary-layer thickness δ, the cup's size l, and the viscosity ν.

d. Now estimate the boundary-layer thickness δ using your knowledge of random walks. The boundary layer is a result of momentum diffusion – and ν is the momentum-diffusion coefficient. In a given time t, how far can momentum diffuse? This distance is δ. Estimate a reasonable t for the rotating blob of tea. [*Hint:* After rotating by 1 radian, the fluid is moving in a significantly different direction than before, so the momentum fluxes no longer add.] Use that time to estimate δ.

e. Now put it all together. For a typical teacup stirred with a typical stirring motion, what is the predicted spindown time τ? [Tea is roughly water, and $\nu_{\text{water}} \sim 10^{-6}\,\text{m}^2\,\text{s}^{-1}$.]

$10^{\boxed{}} \pm \boxed{}$ s ***or*** $10^{\boxed{}} \ldots \boxed{}$ s

To include in the explanation box: Estimate τ_{exp} experimentally by stirring tea. Compare the experimental time with the predicted time.

Comments on page 4

I don't really know how to convince myself of this...