6.055J/2.038J (Spring 2010)

Solution set 1

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That criterion explains why the range narrows after you estimate using divide and conquer. At first, you have little idea about the true value, so you would not be surprised were it to fall outside a fairly large range; after the estimate, you know more, your confidence in the estimate increases, and your plausible range shrinks.

Warmups

1. **One or few**
   Use the 1 or few method of multiplication (and division) to estimate

   $$161 \times 294 \times 280 \times 438$$

   (a random multiplication problem generated by a short Python program).

   $10^{\square \pm \square}$ ***or*** $10^{\square \ldots \square}$

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   The first step is to convert each factor in the product to the nearest power of ten, perhaps also including a factor of a few. For example, 161 contains two factors of 10 and a factor of 1.61; and 1.61 is closer, on a log scale, to 1 than it is to few ($\sqrt{10}$). So 161 becomes simply 100 or 10^2. Here are the conversions for all four factors:

   $$161 \to 10^2$$
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These answers were very well explained and helped me to see step by step where I could have improved my answer- thanks!

man, it's really satisfying to have all your answers come so close to the solution set. I guess divide&conquer works pretty well!!

Looking through this solution set, I realize that using divide and conquer not only is a quick and clever way to simplify problems, but it also makes error analysis much easier. when you look at the final answer, and you're off, by say a factor of 10ˆ2 (which was the case in my estimation of mass of CO2, you look back at your tree. the only value with a large enough order of magnitude that I could have estimated off by 10ˆ2 was the world oil consumption; all the other values are small, and I most likely wouldnt under/over estimate them by 2 orders of magnitude. Indeed, looking through the solution and explanation, I was off in my original estimate of world oil consumption by 10ˆ2

One thing that I struggled with was how small/large my error was... It seems here we have fairly strong confidence in our answers so would the error be +/-0.5?

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Comments on page 1

Here is solution set 1. Submit your memo on it by 10pm on Thursday. Maybe you'll find mistakes (I didn't put it any on purpose) or suggest new solutions or spot confusing places.

should this be a+b? [later edited:] sorry i just zoomed in more. i couldn't see it until i got to 181%...

This was definitely a good starting problem. Took maybe 30 seconds. I think we could have maybe 3-4 of them , very short, very easy to turn into 1 and few, and get us into good shape for doing it. Maybe 1-2 per PSET from now on just to get really quick.

I agree, these are kinda fun.

I also think these problems are good to warm up on and aren't too strenuous. Maybe you could also add problems that help us get better at listening to our gut or that help us learn nice numbers that are important to know (numbers like number of seconds in a year)

I like the idea of having just simple estimation problems for things we don't often think about. How many meters in a roll of toilet paper? How many trees on campus? Just something to give us a feel for very rough estimation

I like having these warm-up type questions. Basic questions that test our speed with fundamental ideas.

I ended up doing the multiplication by splitting the numbers up, similar to the abstraction 3 memo, instead of using the 1 or few method to find the exact answer. Going back now, I see that I read the problem wrong...

It feels almost like cheating because we are estimating so much, but it works out! I feel like these are good for forcing me to be ok with the lack of accuracy.

I'm still a little confused as to when we "borrow" an overestimate from another number. Since 438 is a little over a "few" and we rounded down, I thought we should round the 161 up to a "few", but this put my answer off by a larger factor. How close must the estimation differences be before we're allowed to "borrow"?

You actually borrowed or compensated well. Think of it in multiplicative terms. Rounding 161 up to 300 is almost a factor of +2X, and rounding 438 down to 300 is -1.3X. These actually offset each other nicely – instead of rounding BOTH down by factors of -1.6X and -1.3X – so I think your answer (10^10?) is actually closer to the correct answer in terms of the ratio of correct/estimate. (You OVEREstimate, but by LESS than a factor of 2, but the solution underestimates and is off by a factor of 2.)

i did the same thing, and when you compare it to the actual value, it becomes apparent that it's a good decision

I did something pretty similar to this rather than using the "few" idea and feel like both are pretty good estimates with one being over by a little less than the solutions' is under.

I also just saw I would be overestimating by calling 2.9 and 2.8 a few, so it seemed all right to call 1.61 a 1.

I estimated 1.61 as a few because of the overestimation of the 4.38. How does this work out on a log scale?

This balancing error in either direction is easy to see when we know what numbers we are dealing with from the start, but it bothers me more when we do it in problems such as divide in conquer when you don't know what you might have to over or under estimate later along in the problem.

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I did exactly this but I was wondering what the limit to a few would be...would 500 and 600 still be considered a few. I tried to round up for some and round down for others to compensate

When I solved this I took few to equal 3 instead of 10ˆ.5 but it still ended up the same. :)

i think the point is that 3 10ˆ.5, since 3ˆ2=9, which is close to 10.

the deviation of 161 and 438 roughly cancel each other out

Woohoo - this is exactly the same way I did this problem, with the same answer. Good to know that we're learning...

Me too! The class time dedicated to "few" really helped me with the concept- which I even used in other problems besides this one! :)

I agree. This concept of few is really handy and useful in lots of situations. Also, using this few vs. 10 method, it's easy to see where we are overestimating and underestimating, so we can relatively easily have a sense of which direction and around how much we might be off.

I agree as well - that was very obvious after having gone through the class (before I would have done a lot of tedious math to estimate it)

Yeah, I thought the method of "few" has been the strongest methodology taught. I actually use in day to day approximations.

I agree, I also think it was important to understand when you are approximating a little bit lower then the value and a little bit higher in order to make up for this compensation later in the problem.

Is this the preferred format for this number or is it equally acceptable to express it only in powers of few?

Yes, the preferred format is a power of 10 (may be 0) times nothing or few.

This was the one I wasn't sure about – whether 4.38 can be approximated as a few, or closer to 10ˆ.7 (like 6 is). When should I consider a number big enough to no longer approximate as a few? (i.e., what about 5?)

Now the product is easy to do mentally. There are eight factors of 10 and three factors of a few. Since $(\text{few})^2 = 10$, three factors of a few becomes $10 \times \text{few}$. So

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2. Air mass

Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

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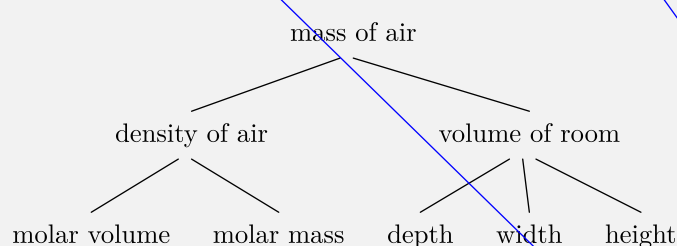
One way to estimate the mass is to subdivide into the volume of the room and the density of air. The volume of the room subdivides into its length, height, and width. I remember that the density of air is roughly $1\,\text{g}\,\ell^{-1}$ (or $1\,\text{kg}\,\text{m}^{-3}$) because I have used the value often in estimation problems. Alternatively, you can use a useful fact from chemistry, that one mole of an ideal gas at standard temperature and pressure occupies 22 liters, and combine that fact with the molar mass of air. Using that method, the tree is

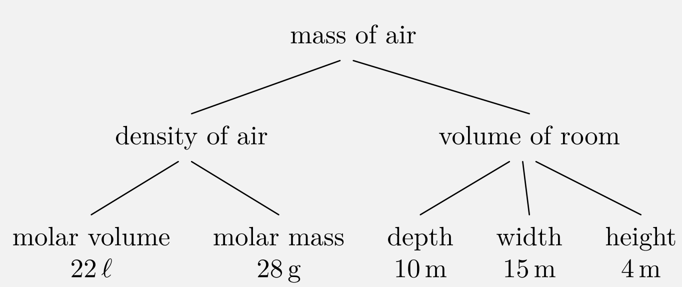


Now put values at the leaves.

For the room dimensions, the MIT schedules office webpage gives the room area, but let's estimate the dimensions by eye. Most rooms are 8 or 9 feet high but our classroom (4-163) has high ceilings, so let's say 12 feet high or 4 m. The room has about 10 rows, spaced around 1 m apart. So the depth is about 10 m. The room is perhaps 1.5 times as wide as it is deep, so the width is roughly 15 m. As a check, these estimates mean the area is $150\,\text{m}^2$ or about $1600\,\text{ft}^2$; the MIT classroom-inventory page (linked from the course website) says that the area is 1303 square feet, so our estimate of the area is accurate to 25%.

The molar volume for air (like any ideal gas) is 22 liters. The molar mass is, roughly, the molar mass of nitrogen, which is 14 g. But nitrogen is diatomic, so the actual molar mass is 28 g.

The tree with values is:



Comments on page 2

i wrote my answer as fewE9, should i instead have written 3E9?

Either is fine.

Something that also helps is knowing that approximately: 10^0.5 = 3 10^0.7 = 5 10^0.9 = 8

Yah, he covered this in class more recently, and it actually seems rather cool. At least for our +/- correction factors

is there a reason we have to enter all our data in the form of 10^x and not as __ *10^x. because I always just drop the first number and I don't think that provides the best estimate

How would you have calculated your error for this?

i don't think that it's so much a "calculating" error as thinking about where error might have come from and estimating how much that would be off...for this one, you know that each of your "estimates" are not off by much, so the final value should be quite close.

regardless, i'd also like to know what a reasonable error would have been to understand how far off an estimation like this could/should end up being

For a problem of this scale, you could even have a sense of the direction and magnitude of the error. The middle two are really close to 300, and the other two are rounded down, so we'll be underestimating, and we know it'll be by a factor of about 1.6 x 1.33 = 2. This is one way to start refining our estimates, since people have been concerned about how accurate to be.

I agree. I'd be kind of embarrassed to get a problem like this wrong to a factor of 2... Especially since the answer can be determined quite precisely. I can really only see this accuracy being useful for sense-checking another result.

So when our answer is fewx10^(5) we should actually input our answer as 10^(5.5)? I had previously only been taking the exponent and not factoring the few into it.

sanjoy replied to someone else who asked this question above and said either is fine

So long as its within the confidence interval, i think its ok

I would have expected the actual number to be less because 438 is the only number above 3 in the mantissa, while the rest our below 3.

When we do the range of answers? Do we use the actual answer to figure out the range or just do a random estimate? I have just been adding 0.5 because I figure that's a reasonable factor.

I really liked this problem, it was very clear how to use divide and conquer to solve it, and I had a lot of fun doing it!

I used a similar divide and conquer method on the homework, however I had trouble conveying it because I was limited to writing in text. Perhaps in the future you may want to add an option to attach a jpeg

On a similar note, I wish the text box for inputting explanations on the homework were larger so I could see my whole answer without scrolling. Or maybe I write to much for the explanations...

What does that variable represent?

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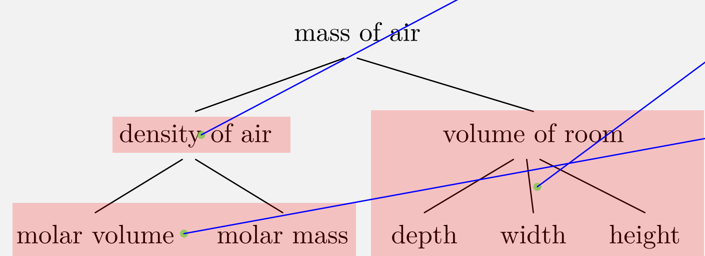

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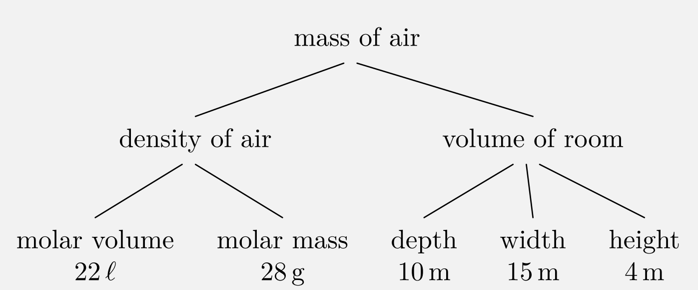

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This variable represents gram per liter. The liter is raised to the negative one because it is in the denominator.

I think that the most tricky part of these problems are the units.

there is no "molar mass of air". you'd make the assumption that it's mostly nitrogen, i suppose

Thankfully, this is also on your sheet of constants :)

I took into account the space taken up by the stadium seating and assumed it reduced the volume by about 1/5 or so. But it probably was a small enough compensation were error somewhere else probably overshadowed it.

I did the same, but I also thought I'd overestimated the height, so I kept the adjustment.

Yah, same here with the keeping the final amount because the ceiling is really high

I didn't think of accounting for the furniture in the room, but that is a good way to approach it. I do agree though since we are doing a lot of assumption making, it doesn't matter in the end.

I don't really see how this could help get the density any more accurately than just using intuition, since the numbers involved are so big/small and could have large errors.

I think the purpose of the tree is also to help organize our thoughts better.

MIT floorplans are also available on the facilities page. I used these to decide which dorm rooms I wanted during in-house lotteries.

also, just fyi, these are usually only available with certificates, which shouldn't be a problem for any of us.

This was hard for me to recall, since I rarely look up.

You could probably figure it out by looking up at the ceiling of the room you are in when working on this problem, then estimate from there.

I broke this down by estimating the size of my room, thinking about how many people could fit in my room, and multiplying until I had enough room for all my classmates.

wouldn't this be basically the same as saying 10 ft high or few meters high, since we're just doing an approximation anyway?

That I did not realize, i think calculating depth was the hardest part for me

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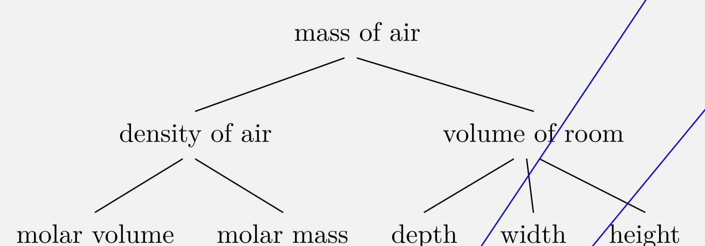

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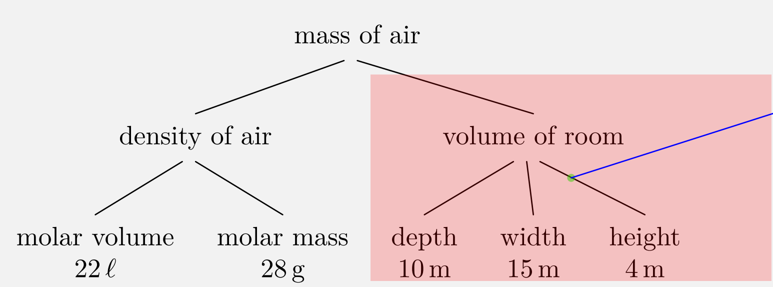

i overestimated here thinking that it was twice as wide, but upon further inspection, this makes more sense.

Me too... but I had the height at "few" meters high... I ended up with 10^3 kg in the end so it all worked out.

I overestimated even more, the room seemed wider to me.

I guessed 20m, personally

I had no idea that we had a classroom-inventory page.

Out of curiosity, are we allowed to consult answers to subproblems before we move onto solving the main problem? As in, can we use other resources to make sure that our values at the leaves are correct before computing the root of the tree?

That would seem to defeat the purpose of the process. It might make you feel better, but trust yourself all the way through and just keep your confidence limits in mind. On a real problem, you might not be able to check partway through, although things like building early prototypes or making quick toy models might be ways to check your prediction before you go on.

Ideally, or at least randomly, your errors might cancel out, and you can end up in the same place anyway

I agree. Checking your answers would be nice, but the point here is to estimate this final value, not to check a bunch of answers to subquestions that will eventually give you the exact value. Plus, in this case, even if you were terribly off–if you guessed that two of the dimensions of the room were twice as big (huge overestimate), your final answer would only be a factor of 4 off, and you'd probably have a sense that you overestimated.

My biggest problem on this homework was figuring out when to try and think of a way to estimate a quantity and when to just give up and look for the answer somewhere else. For instance, is there a good way to estimate the density of air if you don't know the molar volume and molar mass off the top of your head?

I think it's better to estimate from experience then wasting time looking for answers if you don't know where to look. Also it depends on how actuarate you're trying to be.

I had used the table on constants on the p-set from the pre-test. should I have estimated the values a different way?

I didn't know this piece of information. So i likened air to water, which i know the density of, and then pulled the oxygen out of the water and fudged for the new spacing.

That's an interesting approach. The volume of 1 mole of air is pretty easy to estimate if you remember PV=nRT and that R=.0821Latm/molK, which I think is pretty easy to remember since we use it so often in a lot of different classes.

Wow, those are both good methods. I actually went to wikipedia for this... I had no idea how to estimate molar volume.

I simply assumed that there are 100 people in the class and that each person takes up 1 m^3 and that the classroom is 3 m high. That gave me 10^2.5 kg, which was on the lower range

My volume was much smaller than displayed, yet my answer (and range of confidence) was nearly the same as this one. That's interesting.

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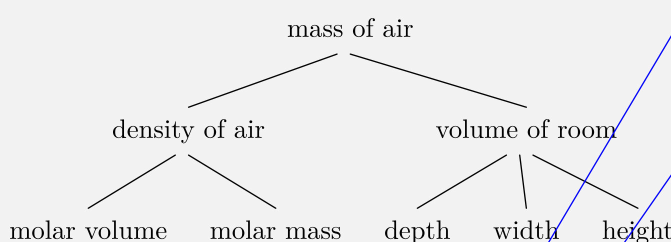
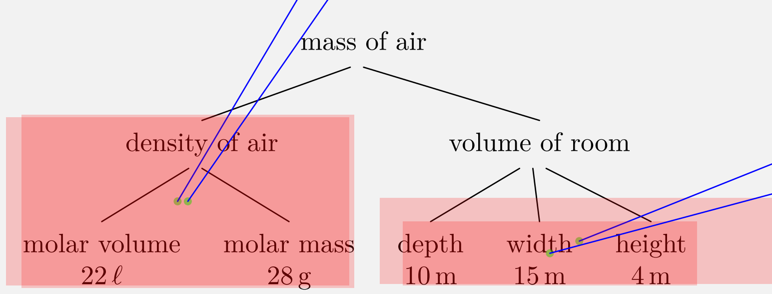


Now put values at the leaves.

For the room dimensions, the MIT schedules office webpage gives the room area, but let's estimate the dimensions by eye. Most rooms are 8 or 9 feet high but our classroom (4-163) has high ceilings, so let's say 12 feet high or 4 m. The room has about 10 rows, spaced around 1 m apart. So the depth is about 10 m. The room is perhaps 1.5 times as wide as it is deep, so the width is roughly 15 m. As a check, these estimates mean the area is $150\,\text{m}^2$ or about $1600\,\text{ft}^2$; the MIT classroom-inventory page (linked from the course website) says that the area is 1303 square feet, so our estimate of the area is accurate to 25%.

The molar volume for air (like any ideal gas) is 22 liters. The molar mass is, roughly, the molar mass of nitrogen, which is 14 g. But nitrogen is diatomic, so the actual molar mass is 28 g.

The tree with values is:



oh i just used the value from the table on the diagnostic. was i not supposed to?

There's no real 'supposed to' in estimation. For a first estimate, feel free to use the table values. Then, to learn more about estimation, see if you can figure out ways to estimate the values that you used. Myself I try to estimate everything totally closed book, just to stay in training.

Since the density of air was given on our "useful equations" sheet for the pre-test, is it alright if we used that value instead of breaking it down into volume and mass (although I understand that this way is more fit to the tree approach)?

I have the same question. In my opinion, the point of breaking things down is to get to a point where you have numbers that you can be reasonably sure are accurate. If you have the value already, why not use it??

Agreed, if you know it, you should use it. Don't pretend not to just to fulfill some sense of duty to break down the problem to an arbitrary level. There is always some deeper branching possible, but it may not add any value to your estimate.

I would amend that slightly to say: "If you know it, use it in your first estimate. Then if you have time and interest, and it seems fun, see how you might derive any of the numbers that you took as a given."

I used the 1 kg/m^3 value instead of bothering with the molar volumes but I still think it is a useful exercise in case one forgets these values.

I guess I just did some multiplication wrong, because I over estimated the size of the room but underestimated molar mass, but forgot that N is diatomic.

I also took this value from the equations sheet from the pretest. In the future, is it acceptable for us to use these numbers, or should we stick with only numbers we know off the top of our heads?

I think "about 1" is such an easy number (or abstraction?) that there wouldn't be any reason to just plug it in.

One reason is to stay in training. The practice in estimating quantities you already know (and can therefore easily check whether your method was sensible) helps you when you get to quantities that you do not know.

I haven't played baseball for eons, but I remember as a child practicing swinging a bit with a heavy ring at the end. One could say, what's the point, since in real life you won't swing such a bat. But it helped develop our arm strength and control for the harder problem that was about to come (with a pitcher throwing fast baseballs right near you).

Another reason to break it down is that it helps you explain the estimate to someone who doesn't have the same set of known values in his/her head.

my estimates were really similar, which, though a minor accomplishment, is exciting.

you choose very even round numbers, how do you decided between trying to estimate the best and fudging the numbers to make your calculations easier

Now the product is easy to do mentally. There are eight factors of 10 and three factors of a few. Since $(\text{few})^2 = 10$, three factors of a few becomes $10 \times \text{few}$. So

$$161 \times 294 \times 280 \times 438 \approx 10^8 \times 10 \times \text{few} \approx 3 \cdot 10^9.$$

In the form 10^x, the estimate is $10^{9.5}$ because 3 (or few) is one-half of a power of 10. The estimate is only a factor of 2 smaller than the actual value of 5805041760 or roughly $6 \cdot 10^9$.

2. Air mass

Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

$10^{\square} \pm \square$ kg *or* $10^{\square} \ldots \square$ kg

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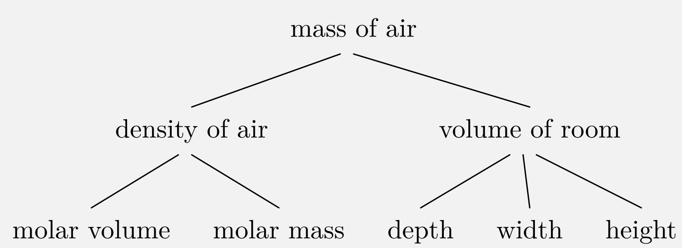

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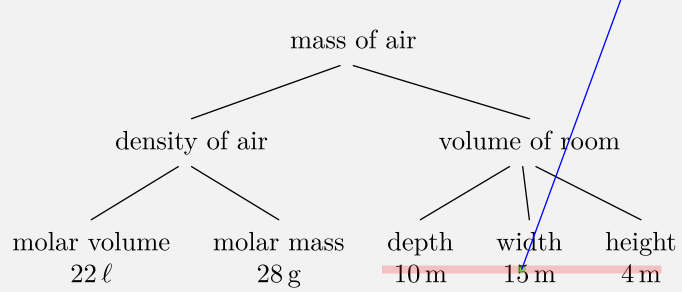

I totally overestimated all of these! I was off most in the height...i thought the room was about as tall as it was deep...I need to get better at estimating distances!

Now propagate values upward. The volume of the room is $600\,m^3$. The density of air is roughly $28/22\,g\,\ell^{-1}$, or roughly $1\,kg\,m^{-3}$. Therefore, the mass of air in the room is roughly 600 kg. In the form 10^x kg, it is halfway between (on a log scale) few × 10^2 kg and 10^3 kg. Because few is one-half of a power of 10, the mass is in the middle of the range $10^{2.5}..10^3$ kg. So let's call it $10^{2.75}$ kg. Either $10^{2.5}$ kg or 10^3 kg would also be a reasonable estimate if you are rounding the exponent to the nearest 0.5.

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$$m \sim \frac{4}{3}\pi r^3 \rho, \quad (1)$$

where r is the radius of the earth, and ρ is the density of the earth. Note that even in this first step we have already approximated by assuming that the earth is spherical and that it has a uniform density.

To estimate r, I remember that California is about 3000 mi away from Boston (a typical flight at 500 mph takes about 6 hr) and it is also 3 time zones away. So each time zone is about 1000 mi, meaning that the circumference of the earth (24 time zones) is $C \sim 2.4 \cdot 10^4$ mi giving a radius of $r = C/2\pi \sim 4000$ mi. In metric units, that is $6.4 \cdot 10^6$ m.

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Comments on page 3

I used the normal sized ceilings and a little but of an undersized room so I was off by a little but still within about 100kg

I got this value. I used a 10*5*4, and a 1kg/m density.

ended up missing a decimal in my estimation of volume, I was really confused how I ended up so far off.

why not just use the cube method, as we did with the CD and a square? Radius 6000m, so cube approx w/ side length 10,000m?

why not? we know the formula and since pi = 3, you even cancel out the fraction. then it's just a few times the rest.

That's a good idea. Let's see how it works out: 10^7 m (you meant 10,000 km) gives 10^21 m^3. Then with the density you get 5*10^24 kg.

So it works exactly the same

I didn't incorporate density into my approximation... I just did F=ma=GMm/r^2. The m cancels out and we know acceleration due to gravity, G and the radius of the earth from the numbers sheet.

I did the same thing and got an answer of 10^24.7. I'm positive I would not have been anywhere as close using the density/volume approximation (which actually seemed like the most inituitive way to do it but I was at a total loss for approximating the density of rock and iron...)

yeah I ended up doing the same thing because earth's density not very constant, and least dense on the surface.

It is really easy to sometimes forget all these other approximations that we make, I completely forgot that the earth is probably not completely spherical.

i also used the 3000 number, but estimated how many US's it would take to go around the earth.

To get r, I used a value from the useful numbers sheet. Are we allowed to use this sheet as a reference when doing these problem sets or should we work from scratch?

I used the back of the envelope numbers from the diagnostic test to find the radius of the earth. Is that unacceptable, should I have approximated the radius using some method like you did here?

I also used the number from the sheet.. This is impressive logic but I don't know how I would have thought of this.

I just use the constant from the sheet that you provided us

That's useful to know!

I really like this approach to finding the radius, For some reason I have the number 24000 miles stuck in my head as the earth's circumference, so I could figure it out, but now when I think about it, if I had not known that I would have been at a loss as to how to find the earth's radius.

I agree, I never would have thought of estimating the radius, i just looked it up online

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I wouldn't have thought of using time zones.

again, for these values i used the table from the diagnostic.

I found this method of using the timezones to gauge the earth's circumference to be rather clever. A good trick to remember.

Yeah, I would never have thought to use the time zones. I guess I was just lucky to know trip distance to China.

I feel like random trivia is a big part of being able to approximate on the fly. Will we be learning lots of just that in class? The stuff about the pianos and cochlea was pretty neat.

I agree. This is a really clever and quick way to get a value for the radius of the earth. I used the value given in the constants, but Idefinitely would not have come up with this, and whatever method I used probably would have taken much longer.

Agreed, this is very clever. I feel like this is a value similar to the 300 million people in the US, something we should have memorized.

This was very clever. I had no idea where to start so I had to look up the radius.

I agree, the radius of the earth comes up in a lot of physics problems in the gravitational unit

This is actually a pretty good way of measuring distances. Its easier to remember about how long a flight was as opposed to the distance traveled and planes always fly around 500mph.

Are we supposed to estimate everything for the homework? I just used the values from the list of useful numbers you provided on the course website.

Iron and magnesium... peridotite mainly. But that information is useless if you aren't studying Geo.

in my hw, I just used the density of water, because I thought the earth was composed of 70% of water. I think it is why my answer was off by a factor

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I approached this problem very differently. I equated the familiar F=mg with F=GmM/r^2 and solved for M. This worked out extremely well.

The density for rock, as listed in the constants page, is 5 * 10^3. I always figured that the materials below the surface of the earth grow more and more dense as they get closer to the core, making the density much higher.

It cannot get too much higher: Solids, even under very high pressure, have almost the same density as at standard pressure: the atoms don't have any room between them. So the core, which is iron at very high pressure, has the same density as iron at the surface (about 7 g/cm^3).

I too did this and when I checked my final answer I was much closer than when I attempted the density*volume method.

I also used that method, of a = Gm/r^2 and solved for m since I knew 9.8 m/s^2 (aka 10)

using the force of gravity is definitely an interesting approach. I wish I had thought of that.

That's what I did too, I thought it was much easier than trying to approximate the average density of the earth.

Yeah I started out doing this problem as a density volume problem but got lost in the numbers and then realized how easy it would be using GmM/r^2

I remember when I did this problem I did it both ways and got the same answer (within some error, obviously)

Are we supposed to be able to justify our numbers like this? I just looked them up..

The density for rock, as listed in the constants page, is 5 * 10^3. I always figured that the materials below the surface of the earth grow more and more dense as they get closer to the core, making the density much higher.

I also found this factor of 5 very unintuitive.

I figured if it was on our constants sheet it was fair game and did not require justification.

I was really confused on how to estimate the density of Earth. I instead used the F=mg type equations.

I still don't understand your use of a factor of 5.

Are we supposed to be able to justify our numbers like this? I just looked them up..

Should we not use the numbers table on homeworks? I didn't on the pretest but everyone else said they had, I felt at a disadvantage.

I never would have thought of this. Very good.

I estimated this first to 3x10^6. It's probably why I got 23.5 instead of 24.5.

There's a general principle in that point, which we'll see often: When quantities are raised to a high exponent (e.g. 3), a moderate inaccuracy in the quantity turns into a large inaccuracy in the result. For example, a factor of 2 in the radius turns into a factor of 8 (almost an order of magnitude) in the volume and mass.

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I rounded 4 down to a few an 6^3 as few*10*10 (round 2 of the 6s up and 1 down). This still gives me the same answer.

missed superscript (10^24), but clear from context

> Thanks. A frequent bug in my use of TeX. I have a short TeX program that typesets scientific notation. So 5 edot5 turns into 5*10^5. And 5 edot24 turns into 5*10^24. But, because of how TeX parses procedure arguments, 5 edot24 turns into 5*10^2 followed by the 4.
>
> I had caught one mistake just like that (the 5*10^24 earlier on the same line originally read 5*10^2 followed by the 4).
>
> Having made it a second time on the same line, it's time to make an abstraction. Here is a grep invocation that will catch the same kind of error:
>
> grep '   edot[0-9][0-9]' hw01.tex
>
> Before I fix the mistake that you caught, it finds this line:
>
> is \$6 edot24 kg\$.)
>
> Which I will now change to
>
> is \$6 edot24 kg\$.)
>
> Thanks for giving me the chance to illustrate another use of UNIX. (As a further abstraction, I've now filed that grep line as one line in my "check TeX files for common errors" script.)

I definitely missed the answer by a lot (a factor of 10^12!!) because I used the density of rock as the density for the entire volume, using the number provided on the constants page. I guess instead of assuming it was equal to that number, I should have thought more about the actual composition of the Earth's mass.

I really like this problem as its the only one in the class so far that is directly related to my area of expertise. I never thought about file systems as an abstraction before this term.

This was a good problem. Unfortunately I got it wrong. Never learned how to code

I understand how the development of unix was a method of abstraction, but I don't understand how us simply using it really helps us learn anything about estimating. It just teaches us how to use Unix. I don't find this very helpful or instructive for this class.

> The class is partly about estimating, but only partly. The broader theme is "solving hard problems", and to that end divide and conquer is useful, as is making good abstractions (divide and conquer can be considered as using good abstractions).

I totally switched this around! arg. i was thinking ls -t(head(tac(head -1))) in stead of one after the other...i should have know it wouldn't be a "trick" questions!

unfortunately tac isnt in mac os's darwin. Rather straight forward though. I enjoy learning these.

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The mass breaks into density times volume:

$$m \sim \frac{4}{3}\pi r^3 \rho, \quad (1)$$

where r is the radius of the earth, and ρ is the density of the earth. Note that even in this first step we have already approximated by assuming that the earth is spherical and that it has a uniform density.

To estimate r, I remember that California is about 3000 mi away from Boston (a typical flight at 500 mph takes about 6 hr) and it is also 3 time zones away. So each time zone is about 1000 mi, meaning that the circumference of the earth (24 time zones) is $C \sim 2.4 \cdot 10^4$ mi giving a radius of $r = C/2\pi \sim 4000$ mi. In metric units, that is $6.4 \cdot 10^6$ m.

To estimate ρ, I start the density of water: $10^3\,\text{kg}\,\text{m}^{-3}$. The earth is made up mostly of iron and dense rock, both much denser than water – maybe by a factor of 5. Why a factor of 5? A factor of 3 would be too low, since that is the density of typical surface rocks, and they are the material that floated to the top when the earth was cooling, so they are less dense than the rest of the earth. A factor of 10, on the other hand, sounds way too dense. So I'll choose a factor of 5, making $\rho \sim 5 \cdot 10^3\,\text{kg}\,\text{m}^{-3}$.

Then the mass is, using $\pi \sim 3$,

$$m \sim 4 \times (6.4 \cdot 10^6\,\text{m})^3 \times 5 \cdot 10^3\,\text{kg}\,\text{m}^{-3}. \quad (2)$$

Do the arithmetic by divide and conquer. The powers of 10 total to 21: 18 from the cubed radius and 3 from the density. Then there's the factor of 4, a factor of 6.4^3, and a factor of 5. If the 6.4^3 were 6^3, it would be 216, so let's pretend that 6.4^3 is 250. Then the factors are $4 \times 250 \times 5 = 5 \cdot 10^3$. The result is a mass of $5 \cdot 10^{24}$ kg or $10^{24.7}$ kg. (The true value is $6 \cdot 10^24$ kg.)

4. Explain a UNIX pipeline

What does this UNIX pipeline do?

```
ls -t | head | tac | head -1
```

If you are not familiar with the individual UNIX commands, use the `man` command on Athena or on any other handy UNIX or GNU/Linux system.

Does having UNIX syntax pose relevance to abstraction? I'm all for learning UNIX, I wowuld just like to know if we should start making a point of getting to know it

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The overall calculation is then:

$$3 \cdot 10^9\ \text{barrels/yr} \times 4 \times \frac{1.6 \cdot 10^2\ \ell}{1\ \text{barrel}} \times \frac{1\ \text{kg oil}}{1\ \ell} \times \frac{3\ \text{kg } CO_2}{3\ \text{kg oil}}. \quad (3)$$

Now do the numbers. There are 11 powers of 10 and then the following factors:

$$3 \times 4 \times 1.6 \times 3 \sim 60. \quad (4)$$

So the estimate is $6 \cdot 10^{12}$ kg per year.

Out of curiosity, I wanted to compare this number to the actual world production of carbon dioxide. It's hard to find the carbon-dioxide production due just to oil. But oil might be one-third of the world energy consumption (there's also natural gas, hydroelectric, coal, etc.). Multiplying the above estimate by 3 gives an estimate for the world production of carbon dioxide: $18 \cdot 10^{12}$ kg per year or roughly $2 \cdot 10^{13}$ kg per year. The actual total in 2006 was $3 \cdot 10^{13}$ kg.

Comments on page 4

were we really supposed to know this, and is it really that important to know?

You weren't supposed to know it off hand, but I wanted people to look it up using the "man" command, because it is important to learn how to use the "man" command. "Teach a (wo)man a UNIX command, and you show him/her how to solve one problem. Teach a (wo)man the UNIX 'man' command, and you help him/her solve any problem."

i didn't have easy access to a place where i could use the "man" command, but wikipedia actually had articles on each of the commands and that's what i ended up using to work this problem :)

You could argue that most things in this class aren't "really important" to know, but they are all pretty interesting and quite useful to know, and the same thing here since Unix is the basis of file systems and computers have such an important role in everyday transactions nowadays.

i didn't know it automatically took the first 10 lines when you didn't specify a number

I still don't really understand what it does after reading this.

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So I totally botched this example - I wonder if other people had trouble with the UNIX pipelines as well? I thought that the rest of the problems were quite fun, but this one really did confuse me.

I liked this. I didn't understand at first, so I looked up the commands. Got it; even the summary at the end.

It took me about 2 minutes to look up the commands on Wikipedia and another 2 minutes to figure out what was happening, so I didn't find it that difficult (and I don't have much programming experience). That being said, I did feel that this problem was not very connected to the other estimation problems.

I messed this up because I didn't realize each line was one of the files. I thought it would be the content of each. But now that I think more about it, this makes way more sense.

It does involve breaking a non-obvious, if arbitrary, problem (display the 10th most recent file) into discrete parts, like d&c.

Yeah I was kind of confused for a little bit because I actually typed in the code into my athena. I only got the name of some random file, so I was pretty confused.

Actually Athena was a great tool for answering this question. Knowing that each | denotes a new command, if you apply each command separately, you will find that the commands act upon a list of file names, thus the result of MacData.

someone mentioned that this problem was kind of random but isn't the point seeing a bunch of different problems to get a better idea of the flexibility of this concept?

I agree, Athena was an excellent resource for this question since it was easy to man each command and checking the final result was as simple as running a few commands in succession.

With no programming experience I was happy to be able to figure this out. After reading these comments I no longer feel bad about looking up some commands.

I was able to figure out all the meanings but for some reason I couldn't tie than together to derive a meaning.

I liked this question - taught me something new. A friend showed me.

I kind of got this problem. I had to look up all the commands and go through everything in my head but I still arrived at the wrong answer. I think for those of us who have no UNIX experience these problems tend to be much more difficult. With some practical experience I'm sure these would become cake.

Whoops. For some reason my computer has no idea was tac is so I looked it up and misunderstood it to mean that it reversed file names letter by letter...

That would be a kinda cool program, but I thought we went over that one in class to find the ending letters?

I understood the combine part as them combining the files together, not just the files name. So I thought i printed the last line of the 10th most recent file.

In the man page, it uses the "file" to mean input - when you pipe in the results of the previous commands, those become the input "file"

i just don't know why it took the top 10 lines – aren't you supposed to specify? i didn't see it define the number of lines. for the first instance of "head"

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do you have a directory in mind when you give UNIX examples? I think you should have given an overview of UNIX in general before jumping in to specific pipeline examples. I asked some of my course 6 friends about this and they were confused as well, especially as to why we were learning pipelines in this class...

Isn't it technically the tenth most recently modified file/folder?

You are right, that is a more accurate way to describe it.

oops. i didn't read world. i guess i was just thinking US again.

When I initially read the problem, I had trouble figuring out how I would use the equation. Once I started thinking about it more and one I had figured out the mass of oil consumed in kilograms, I figured out that I was supposed to look at the ratio between CH2 and CO2.

I was really surprised that this showed up with this examples. Had you not included this reaction, I would never have thought to approach this problem using chemistry.

Yea, I really enjoyed incorporating chemistry into the solution. It really is breaking it down into the smallest bits we can.

We derived this in class, right? Otherwise I would have had no idea how to do this problem.

When I was doing this problem I took the fact that barrels are crude oil into consideration which had a big impact on my answer, how can we just ignore this?

I think the barrels should refer to the amount of crude oil. Are you trying to distinguish between crude oil and refined oil? Because if that's the case, I have read that a barrel of crude oil actually translates to about a barrel of refined products. Therefore, your answer should not be largely impacted.

How did you approximate this number?

I believe we went over this in class. But one way to do it is divide and conquer. We estimate how many cars there are and how much gas each one uses, etc.

Is this a leaf from one of the examples that we did in class? (I know we had a similar problem, but don't recall our exact answer)

I didn't remember that number off the top of my head from class either, so maybe it would be beneficial to have the calculation for that also posted along with the rest of the solution.

It is a leaf from the oil imports example we used - however, this was the number we found for number of barrels per year that the US imports, not the amount of barrels that the US consumes. That number would be roughly 6*10^9, but since that's only a factor of 2, it probably won't change the answer too much.

Ah, that factor of 2 is probably where a lot of the underestimate in my total world CO2 production/year comes from.

I messed up this factor, I thought it was larger, I said 7

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For those who don't know about this, wouldn't it make more sense to say that US probably uses a significant portion of the world's oil (I don't think many people would dispute that it's > 10%) so you can multiply this number by "few" instead of 4?

Both ways seem fine. I also remembered the 25% figure, but your approach is smart and logical.

yeah I multiplied it by few also.

I took a combined approach, remembering that it was about 25% but also realizing that using the "few" estimation would make the end product easier to manage. They both make sense and "few" is very close to 4

For this problem, I could not remember the number of barrels of oil the US (or world) consumed (though I guess I should start) - so I used a maximum of the earth's mass, and then assumed some fraction of it that could be oil (I think I guessed billionth), and assume 200 years of use...which I think gave me the right ball park numbers.

That's an awesome way to look at it! Thanks for sharing, it's really interesting.

I was able to estimate the number of barrels correctly but I ended up botching the rest of the problem. I got 2 moles of CO2 and never broke down the barrels into masses themselves

I totally messed up the chemistry stuff..

It's been so long since intro chem so I've forgotten about the moles, etc and I think there would be others in the same position (I used mass directly instead of converting to moles). I guess the factor of a "few" isn't too significant in the final answer though.

I definitely screwed this part up.

I also completely forgot how to do this portion of the problem. Wikipedia really helped though!

Wow, I actually did this right! It was nice to have this equation though, all the elements had masses that I actually remembered from chemistry.

Is Wikipedia fair game? Doing all of these problems I thought we couldn't look anything up or even use a calculator. (Except for comparing my estimation to the actual for #1.)

I did not think about this ratio at all. Oops.

Are we supposed to divide by 3 here? I thought we were going to multiply by 3 because the mass of CO2 is 3 times that of oil?

yeah, isn't the ratio 1 kg of oil gives 3 kg of CO2?

If you look at the next line, it says 3 x 4 x 1.6 x 3. I think the 3kg oil is just a typo, since in the next line it's treated as 1kg oil.

Yes, that's my mistake.

This seems like a bit of a large number. I had about 10ˆ8 barrels per day

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So I feel a little guilty because my estimate was pretty close but I couldn't think up a way to calculate this. After reading though it makes sense.

So when I entered my answer, I estimated 10ˆ12.5, which was a little off, but my question is when we enter our answers, since we can't put 6*10ˆ12, should we put 10ˆ12.8 instead?

Wow, I have to say the biggest advantage of these homework problems (for myself) is finding out how close my estimates are to reality, because it gives me this confidence that I'm doing something right; and it definitely helps with the forcing myself to "listen to my gut" because I'm realizing I'm close to right (this was the question I was most unsure about).

I agree, I'm still having a bit of trouble understanding how close to the correct answer I should be trying to get.

6. Piano tuners

Here is the classic Fermi question: Roughly how many piano tuners are there in New York City? (These questions are called Fermi questions because the physicist Enrico Fermi was an acknowledged master of inventing and solving them.)

$10^{\square} \pm \square$ *or* $10^{\square} \ldots \square$

I'll break this one into several pieces:

1. The number of families in NYC. It is a big city, so maybe there are 10^7 people and therefore $10^7/4$ families.
2. The fraction of families that have a piano. Having a piano is not common – often people say, "Oh, you have a piano!" when they come to our apartment – but it's not so uncommon that I am amazed when I see a house with a piano. So I'll estimate this fraction as 1/10 (i.e. 1 family in 10 has a piano).
3. How often a piano needs to be tuned. Judging by our own piano, it needs to be tuned every year, but we somehow don't arrange it that often; maybe once every 2 years is more realistic.
4. How long it takes to tune a piano. Piano tuning looks like an intricate task investigating all the strings, etc.; maybe it takes half a day. I'll estimate 3 hours for it.
5. How many hours of work a piano tuner needs to stay afloat. A regular work week of 40 hr times 50 weeks gives 2000 hr in the year. Perhaps piano tuning involves lots of traveling; plus it's hard work. So maybe a fulltime piano tuner spends 1500 hours per year tuning pianos.

Now I use convenient forms of unity to find the number of piano tuners:

$$10^7\,\text{people} \times \frac{1\,\text{family}}{4\,\text{people}} \times \frac{1\,\text{piano}}{10\,\text{families}} \times \frac{1\,\text{tuning/piano}}{2\,\text{yr}} \times \frac{3\,\text{hr}}{1\,\text{tuning}} \times \frac{1\,\text{yr of work}}{1500\,\text{hr tuning}}. \qquad (5)$$

There are a total of 3 powers of 10: 7 from the 10^7 and 4 in the denominators (10 families and 1500 hours of work). What's left is

$$\frac{1}{4} \times \frac{1}{2} \times 3 \times \frac{1}{1.5}. \qquad (6)$$

The 3 and the 2×1.5 cancel leaving 1/4. The number of tuners is therefore $10^3/4$ or 300. In the form 10^x, it is roughly $10^{2.5}$.

7. Your turn to create

Invent an estimation question that divide and conquer might help solve. You do not need to solve the question!

Particularly interesting or instructive questions might appear on the course website or as examples in lecture or the notes (let me know should you not want your name attributed in case your question gets selected).

Comments on page 5

I really liked how this problem utilized d/c. I think it would be a great example to walk through in the readings for d/c.

This was the hardest problem in the pset for me.because I had no knowledge of any of the information need to get the answer. after looking at the solution, even if I had had the numbers, I would not know what percentage I would have used of each to calculate the numer of piano tuners in New York

To be honest, I'm not too familiar with the maintenance of a piano, so I had to pause for a moment to consider if this was a person or some sort of device.

Would Fermi come up with random questions for fun? Also, what sort of techniques did he use? Did he ever write an essay about solving these problems systematically (like George Polya did for solving problems)?

I foolishly forgot to split people into families.

I didn't know that number but I figured that US population is 300mill and NY must have at least 1/6 of that and so NYC must have like 1/10 of USpop. I got 3e7 that way

i estimated that nyc was 1% of the us's pop and got 3e6 and am pleasantly surprised to see how close that was.

i didn't use a fraction of families, but rather just a fraction of the population

Me too... and I put it as 1:1000

I also used this method, but I said 1/50 people

Me too, because I figured there were pianos in public places, not just homes.

I am curious what the ratio of single people to families is; I thought that there were many more single people in NYC than families and used 1 piano per 100 people.

OP: although I still ended up with the exact same answer you did, sweet!

Ha, cool to use personal experience like that to generalize to the number of families with pianos in all of NY.

But I feel like the story is different in NYC. That city is not really family oriented. I would think young working people.

I feel like 1/10 is too large of a fraction to estimate how many people have pianos in NYC. In fact, the apartments in New York are really small, so I feel like the fraction is more close to 1 in 30.

I thought this too, but I made the same estimation as well and somehow my numbers worked out to give me a pretty accurate answer

Given the income level of most of New York, the size of the housing stock, and building height, I think this is where this estimate goes most wrong. The Westchester suburbs might give you a higher fraction like 1/10.

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There are a total of 3 powers of 10: 7 from the 10^7 and 4 in the denominators (10 families and 1500 hours of work). What's left is

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I guess that seems fairly high to me, given how densely packed NYC is (I assumed most people in apartments do not have pianos)

I agree - 1/10 seems high, as growing up, I was the only family with a piano I could think of. Then when you add all the homeless people in and others who could not own one, I estimated 1/100.

I grossly over estimated this number, though, I agree 1/10 seems too high still.

but I think the fraction should be higher because you need to account for the pianos in schools, especially universities and music schools, so I would increase the fraction by a bit to account for these factors?

These two numbers were the hardest for me to estimate since I have no idea what goes into maintaining a piano and how many people actually have one. ended up having to ask others to get a reasonable estimate on this one.

I used 1/yr. I'm sure some pianos are turned way more; like many of the pianos at music schools which NY has a bunch of. I'd say 1 each 2 years is too low, but that's just me.

Is there another way to reason this out if we don't have any experience with owning/maintaining a piano?

Perhaps (and this method also supports the 1/yr estimate): Pianos need tuning because of temperature and humidity changes (those change the tension in the strings, the responsiveness of the sounding board, and maybe a lot else). So after a whole year of all the seasonal fluctuations in both temperature and humidity, the tuning probably gets quite a bit out of whack.

Also in support of the 1/yr estimate: Our piano needs tuning once a year (it's now quite a bit out of tune). But maybe people don't always do what they need to, so 0.5 tunings/yr is closer to what really happens (basically, until you really cannot stand the mistuning any more).

And overall, it's only a factor of 2. So don't worry. There are probably other factor of 2 errors, e.g. in the number of families with pianos. I'm not at all confident about the 1/10 number.

If you don't know anything about pianos (like me) you might read this question as simply how many pianos are there... I didn't know a piano tuner meant a person who tunes them.

I don't think I used a time component when using divide-and-conquer. Having never owned (let alone played) a piano, there's no way I could have gotten a reasonable answer with this reasoning.

I didn't consider this aspect, just the number of pianos a tuner needs to tune per year based on the cost of living in NYC.

I thought another easy way to tackle this problem would be to relate it to something you actually know. For instance, I live in a close suburb of Boston and know my towns population and stores pretty well. I can guess as to the number of tuners in my town, and adjusted the population to match that of NYC. The estimate came out almost exactly correct (a few * 10ˆ2)

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For this I just estimated a ratio of tuners to pianos. The solution seems much more detailed though and makes more sense.

I did the same thing. Also I'm not sure how fair it is to assume that most piano tuners work only one job. It seems like it might be hard to stay afloat just by tuning pianos.

I also did tuners to pianos, it seems too difficult to know how long tuners spend at their job. It seems like it would be the second job of a lot of people. I try to move toward information I'm more certain about when doing these problems.

I thought it was an interesting way to look at the problem. Estimating how often people need a tuner and how much they need to work to make a living.

Something kind of funny: I sort of miss read this problem. I thought tuners, meant little hand-held devices. But my estimate was still close since I assumed only professional who would tune would own them. But perhaps you could reword.

I took a piano tuner as something like a tuner for a guitar... not an occupation. This put me off by one order of magnitude

I just guessed that piano tuners did NOT spend all of their time tuning, because that made no sense to me. I ended up way off as a result!

I made a serious over estimate...I wish I knew more about tuners. Is this something you also estimate or is it fair to look some of this up?

i thought this part was interesting- I wouldn't have thought to do that this way

I fee like this information is unnecessary. What I did was simply estimate about how many piano tuners there are for a given population or given area then multiplied for the area or population of NYC.

It's really satisfying to see the same answer come up using a different way. I got a really close answer through totally different methods.

I feel kinda silly because I guessed 1000. Maybe that's just because I think being a piano tuner is an alright backup job??

I did something similar - I figured that not all piano tuners are full time piano tuners, and as such the number would be higher since some people would be part time teachers or something

I totally thought that 'a few hundred' just made sense...however, I think that I totally goofed and wrote down 10ˆ3.5 instead of 10ˆ2.5 ... I really should have paid more attention when i moved my stuff from the paper i did all my work on to the online submission thing...

I got the same answer. Awesome

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I got the same exact value. It's really interesting because while I used the same general strategy of figuring out how many people are in NYC and how often pianos are tuned, etc., I think the estimates I made at each step were a little off from all the estimates made here, and yet my answer ended up exactly the same.

This is probably because all your overestimates and underestimates basically cancelled out in the end. I used a slightly different approach as well–I didn't divide it into families, I just directly said that about 1 in 20 people might own a piano, since not everyone who plays piano owns one. I also didn't divide a year of work for a piano tuner into hours–I just said 200 days, and about 3 pianos a day.

The method I used which produced the same answer was organizing NYC into a grid of avenues and streets. There are 12 avenues and 150 streets, so 1800 blocks. Then I assumed that there is a piano tuner on 1/5 blocks, so 1800/5 = 360 = 3.6 * 10^2 = 10^2.5. Thoughts?

That's a very interesting and different way to look at it - I guess I'm wondering, how did you reason that there was a tuner every 1/5 blocks? My personal guess would have been 1/10 or 1/20 since I personally think piano tuners aren't particularly common.

I guess if you assume an equal distribution of people with pianos and a fairly dense area (such as NYC) where the number of pianos would be high enough, the 1/5 number works. I still think an approach based on the population is slightly better.

I origianlly had 10^3, thinking there'd be about 1 tuner per 100 pianos (I went by a method figuring # of pianos in the population of New York), but felt that this was a bit too low for a population near 10 million. Is it actually that few?,

NYC is more than just Manhattan (10X factor of population) and 1 every 5 blocks seems like a big overestimate. There isn't even a Starbucks every 5 blocks, even if it might seem that way.

As to the actual answer here, I think it's considerably lower, maybe even <100. I couldn't quickly find a reliable count, but this Yellow Pages listing gives a pretty good indication. http://bit.ly/be1PGE Piano tuners aren't big operations, as you can tell by the fact that most of the listings are just men's names.

I got about a factor of 10 less when I did it. It came from my assuming 1 piano/ 100 families, instead of 10. Do that many people in NY actually drag pianos all the way up their apartment buildings?

So I simplified even more. I said one in ten thousand people is a piano tuner, since one in thousand seems far too many and one in one hundred thousand seems too few. My estimate turned out to be 10^3 and I only made one assumption, which seems very reasonable.

How far off is "too" far off to be reasonable? My answer was 10^1.5, a factor of 10 off from yours. Is this totally unreasonable? It was based on 10^7 people in New York, 1/1000 have pianos (10^4 pianos), 250 work days/year*2pianos/day to 500 pianos per year. This leads to 20 piano tuners... I guess it seems low...

I could not even begin to estimate the key parameters such as pianos/families and the times associated with tuning. I merely guessed that there needed to be at least 10 but not more than 1000.

I honestly have no comments on the solutions. They all seem clear to me. The only question I don't think I quite got was the Piano tuners one, and that's because I used more questionable values. All yours make more sense to me.

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i didn't really understand the point of this question. didn't help me learn anything.

I think this question was a good one to include. While it doesn't serve any real educational purpose, it really shows you that divide and conquer can take you a long way. I was thinking of a bunch of estimation quesitons, and when you really break a question down, some seemingly complex questions start to look a lot easier.

i didn't think it mattered whether we learned from it. it gives sanjoy an idea of what we want to know and might give him good ideas for examples later on.

It was pretty useful for me to have to sit down and try to think of a "cool or interesting" problem that could be solved using divide and conquer - I wanted to find a problem that wasnt just an interview question I'd heard of but something hopefully original

It would be nice to see our problems come up in problem sets and lecture.

Are we going to go over any of these questions?

I liked doing this

I agree. This problem works like reverse engineering. It presented a new way of looking at divide-and-conquer.

Where are all the responses to this question? I would be interested in knowing.

How many questions have you gotten from the class/from this problem?

You should post all the questions!

I agree, it would be kinda fun to see what my classmates came up with at the end of the semester!

Maybe they'll make it into our final.

I would definitely like to see some of these questions in one way or another, I am sure there are some great ones.

Has anything happened here? I'd be really excited to see some of the old questions... i had to go back to the homework to remember what I had written.