6.055J/2.038J (Spring 2010)

Solution set 2

*Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 24 Feb 2010**.*

Open universe: *Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

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For the (optional) question that references `decline.txt`: It is the plain-text file on the course website that contains volume 1 of Gibbon's *Decline and Fall*. It is also available – as is any other file on the course website – on any Athena machine as `/mit/6.055/data/decline.txt`

Warmups

1. Direct practice with one or few

Here is another 'one or few' problem generated by my Python script:

$$985 \times 385 \times 721 \times 319 = ? \qquad (1)$$

$$10^{\square} \pm \square \quad \textbf{or} \quad 10^{\square} \ldots \square$$

Here are the approximations for each number:

$$\begin{aligned} 985 &\to 10^3, \\ 385 &\to \text{few} \cdot 10^2, \\ 721 &\to 10^3, \\ 319 &\to \text{few} \cdot 10^2. \end{aligned} \qquad (2)$$

The approximate product has 10 powers of 10 and two factors of a few, giving 10^{11}. The exact value is $87,221,370,775$ or roughly $0.9 \cdot 10^{11}$.

2. Land area per capita

Here is another problem on which to practice the 'one or few' method of multiplication and division: Estimate how much land area each person would have if people were evenly distributed on the (land) surface of the earth.

$$10^{\square} \pm \square \ \text{m}^2 \text{ per person} \quad \textbf{or} \quad 10^{\square} \ldots \square \ \text{m}^2 \text{ per person}$$

Comments on page 1

Read through Solution Set 2 and do your memo by Thursday at 10pm.

Are we supposed to comment on both sol. set 2 and the reading by Thurs or is commenting on the sol. set optional?

Comment on both by Thursday. I try to keep the Thursday reading short so that you are not overloaded.

I always forget to comment on the solution set. Not sure what I can do to remind myself...

I actually didn't know until today that comments on the solution set were even necessary!

I've often been breaking coefficients down further and temporarily leaving in numbers such as 5 or 2, since they often cancel out later down the line (or I get rid of them later if they don't). Is that generally an extraneous step?

nice. got it

I thought this was in between few and rounding to 10, but I guess not. In the future I will round up.

For me personally, I rounded this up because I thought that 7ˆ2 = 49, which is half of 10ˆ2, but 5 times 3ˆ2. Since we work with squaring stuff a lot, I figured it was closer to the upper bound. Also, we round down both 385 and 319, so it doesn't really hurt to round the 721 up.

I also rounded this up but because of that I rounded 319 down to 10ˆ2. Don't you need to compensate for the large round up somewhere?

How do we know at what point to round up and when we should round down? What if it had been 621 or 521? I rounded down but gave a large error to account for it.

My intuition was that it was closer to a few than 10.... in retrospect i can see how this is 10 by thinking about how we deal with powers here (from the comments above).

Like someone said previously, I figured since I rounded down on 385 and 319, I could round up on 721 as it was half of 10ˆ2.

yay!! i did this right!

These problems really help me crunch large number together in the real world much faster.

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$10^{\square} \pm \square$ m^2 per person ***or*** $10^{\square} \ldots \square$ m^2 per person

you know that you overestimated, but how can you tell by how much you overestimated (aka what to put in the other box)?

> What you put in the box depends on how closely you look over the calculation.
>
> For example, here is a quick and crude estimate. The 985 going to 1000 is almost exact, within a couple percent. The 3.85 going to "few" is maybe a 20% error. The 721 to 10ˆ3 is maybe 30%. And the 3.19 to "few" is nearly exact. So at worst, the percentage errors conspire, meaning that they add, giving about 50% or a factor of 1.5. That's plus/minus 0.2 in the exponent because log_10 (1.5) = 0.2.
>
> Upon a closer inspection, the errors almost cancel: The 3.85 to "few" is an underestimate, and the 7.21 to 10 is an overestimate. So the total error is maybe 10%, namely a factor of 1.1 (or plus/minus 0.05 in the exponent).
>
> Does that help? Perhaps all of the above belongs in the solution set.

That is a really good explanation. I thought it was clear that the 7 should go to 10, but some people didn't. When introduced in this way it makes perfect sense, and even explains how this rule should be used in the future

I forgot to distinguish land from sea.

Oh man, I totally forgot to take out the water too.

Me too–I totally forgot about that

I estimated that 60% was water...guess I was a little off.

I remembered the land/water distinction, but I was wondering: how does Antarctica, Greenland, et al figure (mostly ice)?

Haha that's a good point...I just assumed people could live on anything 'solid', including ice

The surface area of the earth is $4\pi r^2$, where r is the radius of the earth. The land area is some fraction f of the total surface area. I half remember that f is about 0.25, which seems plausible: There is a lot of land, but the oceans are giant. The radius of the earth (from Homework 1) is $r \sim 6.4 \cdot 10^6$ m. The world population is about $5 \cdot 10^9$.

So, the per-capita area A is

$$A \sim \frac{4\pi \times (6.4 \cdot 10^6\,\text{m})^2 \times 0.25}{5 \cdot 10^9}. \quad (3)$$

There are three powers of 10. In the remaining factors, the 4 and the 0.25 cancel each other. The $\pi \times 6.4^2/5$ is (using $6.4^2 \sim$ few $\times$ 10) few $\times$ few $\times$ 10/few or few $\times$ 10. The final per-capita area is few $\cdot 10^4$ m^2.

3. Nested square roots

Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \quad (4)$$

The computation is recursive in that it contains a copy of itself. To see that, define

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The solution to this equation is $P = 2$.

4. Searching for ...gry words

What English words, other than `angry`, ends in `gry`?

Humans are much worse than computers at this question, because we store words not by their endings but more by their beginnings and meanings. For a computer, it's all bit strings, and computers don't care whether the bit string happens at the beginning or end of the word (and there's no meaning).

The regular expression that matches words ending in `gry` is `gry$`. In the following pipeline, the first `grep` finds all those words, and the second `grep` excludes `angry` from the list:

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Comments on page 2

Instead of working with pi and earth being round, I thought of the Earth as a cube. Turns out I shouldn't have, so how would I know when to change circular shapes into rectangular?

did you underestimate the radius when you did this? my guess is that is the only way that it actually works, ever

Yeah, you'd want to underestimate the radius to make it work. The surface area of a sphere is pi*d^2 where d is the diameter. For a cube it is 6*d^2, which is about a factor of 2 larger than the sphere's area. So, underestimate the radius by 30% and you'll be home free (0.7^2 = 0.5). Here, try something like 5000 km.

yeah, I also approximated this as a cube since last time we approximated Earth as a cube.

For this i looked at the people on earth and estimated their weight...as well as adding a couple of factors of 10 for all the large animals. Everything else I kept negliblie.

I too also remembered this, but I remembered it as water being 75%. It's not a very rare piece of information.

I think I estimated something slightly different: that land takes up 1/3 of the earth's surface.

this is the key. because "the oceans are giant" isn't a good reason.

While I agree that this is key, a vague mental recollection of a map would surely lead one to pick a number between .1 and .5 - which would give fairly reasonable answers.

I thought it was 2/3 water? Not a huge difference, but this is what I used and have always heard.

I have also always heard 2/3rds water.

Also on the constants sheet. Does looking at the "back of the envelopes" sheet count as looking stuff up or is that acceptable to look at?... I've been using it all semester.

I just realized I should have mentioned something about how I am assuming that the earth is a perfect sphere...

I didn't realize there was that much of a difference, its almost a sphere right?

I wasn't sure if we were allowed to use previously established numbers- that requires reference. I guess we could just calculate it again though...

I thought there were 6 billion?

Me as well, in fact I believe it's halfway to 7 by now. Perhaps this figure should be updated?

All things considered, this isn't actually important for solving the problem since 5 6.

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The result is just one line: 'hungry'.

I didn't know this value off the top of my head, and I ended up having to look it up. Is there an easy way that we could have derived an estimate for this value if we didn't know it and we couldn't look it up (in an interview, etc.)?

The simplest method would probably be to just memorize the number (world population is somewhere in the 5-7 billion range).

The only methods I can think of that would allow you to derive it is remembering vague numbers for certain countries (US 400m, China 1bill, etc), which seem like a lot more work.

Hopefully this number will continue to come up so that it will be easier to recollect by the end of the term.

I remember this being quite a bit bigger (not enough to change the estimation of course..), but closer to 7 billion. Maybe this example is slightly outdated? Also, wait a few more years and 'f' might drop a lot, too :)

The world population isn't changing that fast and a billion is a lot. I think a good way to remember this is to think in terms of upper limits. Human population is somewhere in the low billions and you will at least be correct to an order of magnitude.

I just guessed this as 10^10.. I guess that's a big error.

Same here, I approximated upwards to 10^10 since I figured population was around 7*10^9. I guess 5 makes a little more sense though.

Should we attempt to put these numbers in terms of just 10 to a power or few x 10 to a power?

Funny, I actually used 10^8 here and still got 10^4 as my answer.

Yeah I rounded up too, since I knew it was 7*10^9...I thought we were supposed to only use 1, few or 10 so I decided to round up to 10...should I have kept it at 7?

I'm pretty sure the number is closer to 7 billion, but given that we just end up calling it a few billion anyway, it's not much of an issue. But for the sake of correctness in the solutions maybe this should be changed.

Wow, you are right! I think I must be remembering the number from back when I first learnt it. I have a clear memory of learning it in 6th grade. Then I must have slightly increased it to get 5 billion.

This is exacty what I did, except that I assumed f=1/3 instead of 1/4

I used 0.33 here to make the math simpler (the pi in the numerator cancels with the 3 in the denominator). Not much of a difference, but slightly easier to compute.

i used the same method. nice

That is a lot of land per capita!

Very suprising!

this is a really confusing sentence to read. you could have used numbers.

i got 10^2, but I approached it in the exact same way

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You would also get the same answer (10^4.5) if you assumed that land is a 1/3 of the total surface area and that the world population is 6 x 10^9.

I tried both methods and got this answer - I think its nice to be able to confirm your answer with a different estimation

Hoo boy, I got that one completely wrong... unfortunately, I can't find the paper where I wrote by calculations, but my method seemed the same, so hopefully I just messed up the math.

Did anyone else just enjoy this answer a little bit? I found it incredibly surprisingly and gave me a new perception on overpopulation.

Yah, its a bit scary to think about. It was one of those problems where I sat there going "that can't be right..."

So I got that and actually almost changed my answer because I thought it was so absurdly high. I guess my gut just totally lied to me.

I had a very similar response - I immediately thought of all the places like china that are overcrowded. I wonder how much this would be if we took out uninhabitable places...

I think I solved this problem without recursion, although looking back, that is definitively the best way to solve it. I just started working it out, and quickly realized that it could never rise above square root of 4.

It took me a little bit to get how this one worked. But once I realized it was a recursion problem, this as well as the other geometric series problems became a lot clearer.

After looking at this for a long enough time, I realized it was just the multiplicative equivalent of 1 + 1/2 + 1/4.....and it all made sense!

That's how I first approached the problem.

This problem is a famous problem in mathematical analysis

That was soooo much easier than I thought. Damn, I feel stupid.

Agreed, i took forever trying to figure out a way of estimating this until I realized it was recursive. Recursion is definitely my favorite method, it's so simple and neat!

I always have trouble figuring out how to start problems...

I would have never thought or known about this- can we do a few more similar examples

I'm pretty sure the sum of the series 1/2^n is 2 so the answer should be 2^2 which is 4....?

I think that's totally right except the sum of 1/(2^n) is one, not two. Even if you just try the first 4 terms or so it appears to be converging at one (.9375).

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I did this a little differently. I changed the radicals to powers, so it was 2^1/2 * 2^1/4 * 2^1/8 ... or 2^(1/2+1/4+1/8...) and I noticed that the power was a geometric series, which I used abstraction to get the sum of.

I accidentally said P = sqrt(2xsqrt(P)) and thus got a different answer. Hopefully no one else made the same mistake.

Wow, this now seems like a very obvious abstraction. I used the radical into power method described above, but this simple abstraction is so much more straight foward.

I understood that this is what the problem was asking but writing it with the use of P helps make the solutiona lot more clear.

I kept on telling myself that it was just root2 times the whole thing over again, but never actually made the step in my head to write out p=sqrt(2*p). I wrote out the terms and summed the geometric series.

Using a variable definitely helps a lot. I had the same problem with trying to think of it as "oh, it's just the original thing all over again."

Really cool way to use abstraction..and it helped a lot to have done this problem in order to solve the resistor problem after.

I used the power method mentioned above. I feel like these abstractions are very useful but at times very difficult to make or see, especially when dealing with mathematics and such. They may be easier in another arena however, i still feel that I would not be able to make these extremely simplifying abstractions like those above, in math problems.

You hit the nail on the head when you say, "Using a variable definitely helps a lot." Indeed, a variable is an abstraction, so it's another example of the usefulness of abstraction.

looking back, it would have been helpful to know invariants here...!

I didn't realize the series; I just tried to figure it out as is. It seems a lot easier when figured out with powers.

Nice. I tried rewriting it with exponents but wasn't really sure of how it would look.

I definitely was not great at seeing abstractions before but I'm starting to know how to look at these problems in a more useful way

This only works for things that repeat infinitely, right?

oh that makes a lot of sense and now i feel like a ditz

Don't feel bad, thats the point of a problem set! So now you learn to recognize these patterns in the future and apply the things you learned

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The result is just one line: 'hungry'.

For some reason, it took me awhile to realize this, but I felt a certain level of satisfaction that something so complex at first reduces to something so simple.

I too had to work a little at this, but it really helped put me in the recursive mindset for the later problems. Like you said, it's satisfying seeing the simple answer in the end.

Agreed - this question made me feel really good about myself, and was very helpful for the other recursive problems.

yeah I was able to do the other ones after finishing this problem.

For me, this was one of those that I stared at for hours and could never even get started. Now the answer makes perfect sense once I see it, it's just kind of frustrating.

so i knew it converged to 2 but i didn't know what i would put as the error...since it should be 2. is it then 2 +/- 0?

I just put 0 for the error since I was able to solve it. In this case, I didn't do any approximation per se so there isn't any error margin.

Is there a reason for all of the UNIX stuff in this class? I find it mostly annoying because i'm not interested in learning more about it.

There is. For one, UNIX is an incredibly useful tool, and someday you might be able to use it to solve a problem. See for example one of the comments on Problem 8, about running a website and needing to convert mailing-list data from one format to another.

Second, UNIX illustrates the generality of divide and conquer and of choosing good abstractions.

So, try not to be too annoyed.

I think this question was poorly worded, I had no idea how you wanted us to go about this- actually list out all the words? But then I remembered the previous problem using UNIX

Apparently there are many riddles surrounding this.

This is also a really famous riddle, often starting with "there are 3 words ending in -gry".

Yah, I realized after the due date that he wanted a UNIX answer, not just an answer to the riddle!

Yeah I did not realize that we were supposed to come up with a unix pipeline for this problem until just now, but I guess that makes more sense

I didn't really understand the reason for this question. Just looking something up?

more practice using UNIX commands?

I used google... was this a bad idea?

This is something you can't just look up yourself the way you usually would though. It requires figuring out how to tell the computer what to do in general terms (not necessarily UNIX), which is a useful problem-solving skill.

It was intended to be a UNIX example we could try out ourselves. Turned out to be pretty fun.

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The computation is recursive in that it contains a copy of itself. To see that, define

$$P \equiv \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}}. \quad (5)$$

Notice that P is repeated inside the square root:

$$P = \sqrt{2 \times P}. \quad (6)$$

The solution to this equation is $P = 2$.

4. Searching for . . . gry words

What English words, other than angry, ends in gry?

Humans are much worse than computers at this question, because we store words not by their endings but more by their beginnings and meanings. For a computer, it's all bit strings, and computers don't care whether the bit string happens at the beginning or end of the word (and there's no meaning).

The regular expression that matches words ending in gry is `gry$`. In the following pipeline, the first `grep` finds all those words, and the second `grep` excludes `angry` from the list:

```
grep 'gry$' /usr/share/dict/words | grep -v '^angry$'
```

The result is just one line: 'hungry'.

The subject-verb mismatch of "words" and "ends" made me suspicious that hungry was the only one when I thought about it before running grep.

but if he had only said 'word', and you thought of 'hungry', then you wouldn't have bothered doing the problem, right?

Well, I would have still done it out of curiosity since I don't have a complete mentai dictionary. I just think the problem would be better stated vaguely as, What English word(s) end in gry?

I automatically started thinking of proper nouns, like Pingry. But I laos thought of hungry immediately. I'm not sure I felt this was the best problem; but it did force the use of linux shells.

Good catch!

My brain handled it OK – though I didn't have the certainty that I hadn't missed anything. Maybe if it were harder I would see the necessity of this problem.

good to know, this will be helpful for later, although I ended up using a different solution to get the same result

I used a different solution as well: ".gry" The solution gry$ works here because the file we were searching was organized so every word was on a new line, but if we were searching any text document I think the .gry pattern is more robust at finding what we want.

I never would have thought of this method for answering this question.

why does 'gry$' work? i thought that the $ allowed all endings, but gry is our ending here.

$ in a regular expression means "end of line", so it forces the word to end in "gry".

I don't know UNIX and I totally forgot about this $ operator. I inverted the dictionary and then found words starting in yrg. Is this valid?

I did the same thing. If it works then I don't think you can say it's not valid. It's just not the easiest way to do it.

Ha yeah I'm glad other people did it that way too. The embarassing part is I am familiar with unix and I still di it this way...

I looked this up: '-v' stands for 'invert' in the sense of matching. so find everything except for 'angry'

I was actually thinking about reversing the words, until I just looked for grep 'gry' and got only hungry and angry as results.

I did reverse the words. I got the answer but took up more computing power I guess. Is that bad?

the method in the reading seemed a lot longer

I didn't see that in the reading..

were we supposed to use code to do this, I just brainstormed for a long time- any other ideas to help that do not require a computer

The surface area of the earth is $4\pi r^2$, where r is the radius of the earth. The land area is some fraction f of the total surface area. I half remember that f is about 0.25, which seems plausible: There is a lot of land, but the oceans are giant. The radius of the earth (from Homework 1) is $r \sim 6.4 \cdot 10^6$ m. The world population is about $5 \cdot 10^9$.

So, the per-capita area A is

$$A \sim \frac{4\pi \times (6.4 \cdot 10^6\,\text{m})^2 \times 0.25}{5 \cdot 10^9}. \quad (3)$$

There are three powers of 10. In the remaining factors, the 4 and the 0.25 cancel each other. The $\pi \times 6.4^2/5$ is (using $6.4^2 \sim$ few $\times$ 10) few $\times$ few $\times$ 10/few or few $\times$ 10. The final per-capita area is few $\cdot 10^4\,\text{m}^2$.

3. Nested square roots

Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \quad (4)$$

The computation is recursive in that it contains a copy of itself. To see that, define

$$P \equiv \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}}. \quad (5)$$

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The regular expression that matches words ending in `gry` is `gry$`. In the following pipeline, the first `grep` finds all those words, and the second `grep` excludes `angry` from the list:

```
grep 'gry$' /usr/share/dict/words | grep -v '^angry$'
```

The result is just one line: 'hungry'.

Not true for my dictionary. I got a whole lot of weird words I've never heard of before.

I think wikipedia has a list of those words but they are considered to be "archaic" that no one uses them anymore. So they dont include them in modern dictionoaries

That sounds fun. What were the words?

I wrote a pipeline to find all words ending in -gry, and since it only spit out 2 words, angry and hungry, I just stopped there and said the other word was hungry. Is that acceptable? (I figured it was, since the question doesn't specify writing pipelines that give a certain result.)

I did this as well. But I suppose the problem did specifically ask for solutions other than angry, so we should not let angry show up at all in our output.

I also used grep to return all words ending in -gry. I think the second grep would have been a more obvious command if the output had been a lot more than two; it would have been the time saver of deleting the word out of the list.

For the purposes of practicing using grep this might be true. But I don't see any other reason why one shoudn't just find all the words ending in "gry" and then subtract one from that number. Especially since it's probably faster to just subtract one in your head than type out the additional command to remove angry from your output.

For this, I just downloaded a text file of a dictionary and searched for the key 'gry'; does this qualify as a valid intelligently redundant method?

I think it does. The given solution is a mini-program, and it's always a good idea to check whether a program is giving nonsense – which your method accomplishes.

Problems

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$$1 + r + r^2 + r^3 + \cdots. \quad (7)$$

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A large outdoor swimming pool in the Arizona desert has a time constant of 4 days for exchanging heat with the air. Roughly how large are the peak-to-peak fluctuations of the water temperature caused by 30 °F (peak-to-peak) night–day fluctuations in the air temperature?

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$$\omega = 2\pi f = \frac{2\pi}{1\ \text{day}}. \quad (11)$$

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The practical consequence for swimming is as follows. These fluctuations happen around the average daily temperature (the DC or zero-frequency input signal). In the Arizona winter, the daytime temperature is often 70 °F, but the nighttime temperature can be only 40 °F (the 30 °F variation). Therefore, the pool will sit mostly at 55 °F. It is far too cold for swimming. The small fluctuation of 1 °F around 55 °F does not make the pool comfortable even at its peak temperature.

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i felt like this problem here was what made me finally understand recursion. it might be nice to have this problem in the text/readings to illustrate another way of using abstraction

Yeah, I sort of agree. I wasn't entirely getting recursion, and got help with one of the recursive problems in this assignment, and suddenly the other two made sense. The coin example wasn't quite as useful, possibly because the symbolic layout of these homework problems really helped it hit home.

this was brought up in the comments on the HW, but should there be a requirement that 0<r<1?

Depends on how you want to define convergence.

Could you be more specific?

I think there is still some confusion...doesn't r have to be 0<r<1?

I didn't know the formula for this so I thought the result would be infinity- this explanation of the solution clarifies how I should have been thinking

I think this is a very intuitive guess...it does seem like it would keep increasing; it is an infinite sum after all, and the numbers keep getting bigger!

I think I finally got recursion. Neat.

pretty cool–just finding a pattern that is inside itself...

I agree. This is the most helpful example of recursion.

Thanks to whoever posted this method in the reading memo :)

I couldn't get the equation I learned in high school out of my head when I was doing this.

this is kind of hard to see because it requires factoring r into 1*r. you can also multiply both sides by r in order to realize that it's recursive.

This is exactly what i did.

I did the problem this way also. I subtracted S*r from S and solved for S.

looks like recursion

I don't really understand why the "formula" for the geometric sequence is always taught to people in high school, but it seems very few people learn how simple it is to derive it.

I agree. I never remember this even though I've taken plenty of classes that cover the material. I also don't get why its called "geometric" although I'm sure there's a good reason for it.

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This makes sense. For some reason I thought he wanted a concrete number though. I guess because everything else is approximating?

I think the difference here is that there is an actual variable (r) in the problem without given a value for it.

Yeah I'm sure there's no way to get an exact number without a defined value for r.

I didn't expect that we would be able to solve these problems with an answer that puts no weight on the size of the series. This is a lot more helpful then I thought.

Ooh, this makes sense. It reminds me of the formula for the sum of geometric series that I learned in high school but have since forgotten.

This is the geometric sum from high school. I thought against using this at first, thinking that the answer couldn't be a simple formula, but I couldn't think of anything else.

I still think that unless a person has seen this before it would be very difficult to make the jump to factoring out the r. This seems like something that just comes to you in a stroke of genius and that there is no real systematic way to approach it. Basically if it doesn't just come to you then you are out of luck.

It's true, that I saw this and remembered doing it before, so it wasn't much of a challenge, however this concept is still a good abstraction, as recognizing this sort of pattern (recursion) is very useful, and can help solve many otherwise challenging problems.

For some reason I found this equation harder to come by than the other recursion problems.

I feel like this type of thing is related to discount rates (finance theory). There could probably be a nice application to be shown regarding bonds and coupons.

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The solution should be dependent on the value of r. Everybody seems to forget that.

Right, but the only way us non-mathy people could remember that is through remembering the reading or what he did explicitly in class. Otherwise, it seems more logical to look at what was done above with the recursive squares and try and find an actual "answer," as in a number.

This made more sense after lecture Wednesday

So what would have been the best inputted answer in the form asked for.

I think the 1 F variation is reasonably reliable, perhaps even more accurate than a factor of 3 (which would be plus/minus 0.5 in the exponent). So maybe

10^(0 plus/minus 0.2) F

how do you know it relates to the angular frequency? this is kinda abstract to me

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I didn't understand the real answer until now, but my estimation was right.

i got confused by the 2pi*f*tau equation in the notes. there's no tau in this particular equation.

tau is 4days. w*tau=2pi*f*tau is the dimensionless parameter that appears in the transfer function.

This is just the equation for converting our period of one day into an angular frequency for our input before we consider the time constant (tau) of the system.

I forgot the 2*pi and thus I ended up with 6 degree F.

parameter

This is a nice example of the modeling of physical system using circuit model/transfer functions.

What does this mean?

i'd like to know too. i'm not sure what "limit" refers to.

Sorry for the jargon. (By the end of the book and course, it'll be very familiar.) It means just that the oscillations are very fast, as if omega*tau were tending to infinity (hence the use of "limit").

I'm not sure about the cutoff for >> here. Is it, in general, when the lesser part is < 1% of the greater part?

This problem makes sense after seeing the solution, but I sure I no idea how to do this. My answer was somewhat close, but then again it's obvious that the temp doesn't change by much.

Yeah my answer was higher than this and I assumed instant changes in temperature which might have thrown my numbers off by more than i expected.

This kind of problems always eluded me a little bit. We did them in some classes before and I never really quite got it... seeing it done here is a lot nicer .

Agreed, I had to look at the solutions in order to solve this problem, and even then it took me quite some time

this is kinda confusing- not how I approached it

This seems so simple now after the fact but before I was just not able to make any connections with what I knew to this problem. I started writing a book of physics equation and that and your notes have been really helping me with the psets.

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Wow, I can't believe I got this exact answer. I was pretty confused when I read this section of the heat flow notes, and then an answer of 1 degree seemed a bit low to me. It's an interesting result.

Is this just 30/abs(25) = 1?

yes

yeah, I got a pretty low number also (3). however, this make sense because if you have ever been in a swimming pool in a hot climate, the water never really seems to change temperature that much.

That's right. What prompted the question is the swimming pool in my parents backyard. They live in Tempe, Arizona. When I visit, I often think how nice it would be to go swimming. But my visits are often during winter break (summer break is baking hot in Arizona). The water is always cold even when it is 70 degrees during the day. And that puzzled me, until I thought about it as an RC circuit.

I forgot about this part–I got omega*tau=about 25, but completely forgot about this–that's why my answer was so high...

This answer makes sense after reading it, but I don't think I would have been able to come up with it my own.

Wow, I did not do this.... My answer is totally wrong... :(But it helps to look at the solution so we know how to do it next time we face a problem like this

So if the pool was somehow artificially heated up to 70 deg one day, would it take the full time constant (4 days) to settle at its 55 deg oscillation?

You'd see an exponential decay toward 55 degrees (and the decay would have a 4-day time constant). The 1-deg oscillations would be superimposed on that exponential decay.

Linearity means that you can work out the responses to each input separately, then add them to get the response to the total input. Another example of divide and conquer...

wet suit in a swimming pool?

I got an answer very close to this but didn't understand the "correct" logic until after reading this

This is 1 degree peak to peak, right? So it should be about 0.5 degrees around 55 degrees?

I was a bit confused about how to enter this with such a small variation

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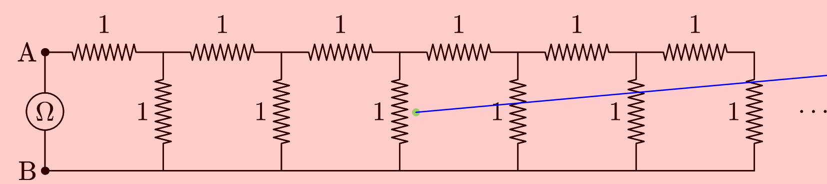
This seems like a good problem to solve using symmetry (and that's actually what I tried to do for my own solution). It might be a worthwhile exercise to do.

Would you mind explaining your symmetry argument? I'm having a little trouble understanding the process and it might help to see it on a problem we've already thought through.

The answer shown uses symmetry. The network is size-invariant. If you increase the size by an additional "ladder rung" the whole thing looks the same and has the same resistance.

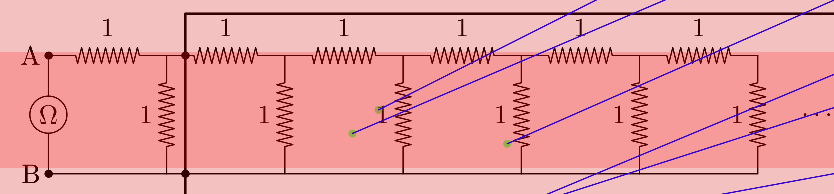
I think you're confusing recursion (or self-similarity) with symmetry.

Why be stingy with methods? It uses both. For finite systems, the symmetry is broken, so you're using just recursion. For infinite systems like this one the symmetry is not broken, so you have recursion and symmetry.

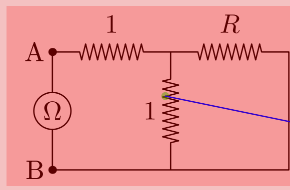


☐ ± ☐ Ω *or* ☐ … ☐ Ω

This resistive network contains a copy of itself (enclosed in the box):



Call R the resistance of the network inside the box, measured between the two dots as the terminals. Then the original network, which also has resistance R, is



It is a 1 Ω resistance in series with the parallel combination of 1 Ω and R. So

$$R = 1 + \underbrace{\frac{R}{1+R}}_{1\,\Omega \| R}, \qquad (12)$$

or $R^2 - R - 1 = 0$. The positive solution is

$$R = \frac{1+\sqrt{5}}{2} \approx 1.618, \qquad (13)$$

which is the Golden Ratio.

An alternative, direct method is the following continued fraction that accounts for the infinite cascase of series and parallel resistors:

$$R = 1 + \frac{1}{1 + \frac{1}{1+\cdots}}. \qquad (14)$$

This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)

Comments on page 4

Now that I understand recursion, I feel I could get this problem.

This is so classic 8.022...

This problem took up a whole recitation in 8.022 back when I took it. It's nice to have the same argument we use to solve infinite series also solve resistors. They seem so different.

Does it matter what we define as the box?

I think you need to be more explicit here about why the the resistance of the original network is R also.

self-symmetry. Its the same principle used in several of the other problems on this pset.

How is the resistance inside the box equal to the total resistance? I don't understand this abstraction.

its not the total resistance, it's just "R" a variable for right now.

well, it is in fact equal to the total resistance. The idea is that the pattern inside the box is identical to the pattern of all of the resistors, since they stretch on to infinity. So the resistance between the vertex near the top-left of the box and point B is equal to the resistance between A and B. This is ok because of the way resistances add in series and in parallel.

Imagine the diagram is like the series in problems 3 and 5. These infinite series contain themselves.

It's because you're still measure from point A to b. Look at the box he's drawn. He's drawn new dots that are analogous to the dots at A and B. So the resistance of the box is the same as the resistance of the whole circuit.

The trouble I had with this question was going from the original diagram to this simplified one. This was the key. However, once I saw this diagram, I was able to do the problem easily and got the same answer as the rest of the solution.

The same was true for me - the reduced diagram does make the problem much easier, and it was great that the previous problems put us into the recursive mindset.

I thought the resulting box would be in parallel and not in series with the left side. Then suprisingly I got 1/2 of the compliment of the golden ratio

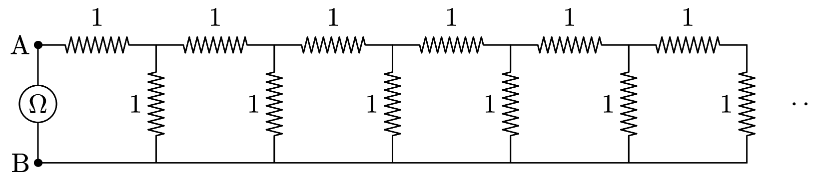
I think I did this completely wrong...oh well

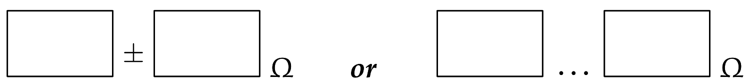
I understood it much better now! always look for symmetry! this is indeed similar to the square root of 2 problem above, but just in a different format/setting

I understood the recursion in this problem, and the idea in this simplified diagram. But I didn't get the next step–noticing that it's one ohm + the parallel resistance of 1 and R...(I wrote down the formula for parallel resistance, but totally blanked out on the concept)

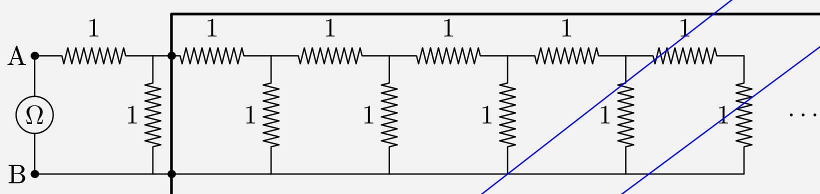
can we please go over this one in class

I need a little help with the resistive networks I think

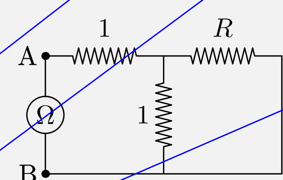




This resistive network contains a copy of itself (enclosed in the box):



Call R the resistance of the network inside the box, measured between the two dots as the terminals. Then the original network, which also has resistance R, is



It is a 1 Ω resistance in series with the parallel combination of 1 Ω and R. So

$$R = 1 + \underbrace{\frac{R}{1+R}}_{1\,\Omega \| R}, \tag{12}$$

or $R^2 - R - 1 = 0$. The positive solution is

$$R = \frac{1+\sqrt{5}}{2} \approx 1.618, \tag{13}$$

which is the Golden Ratio.

An alternative, direct method is the following continued fraction that accounts for the infinite cascase of series and parallel resistors:

$$R = 1 + \frac{1}{1 + \frac{1}{1+\cdots}}. \tag{14}$$

This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)

Here it seems you are finding the total resistance. However, if so you denote the total resistance as R, the same as the resistance in the box. How does this work?

What was confusing for me in this problem was the equivalence of R...its first in series, then in parallel, etc...and I didn't get how to actually add those up correctly. The last operation is the in parallel resistors, but how does this account for all the other resistors, as the R on the right hand side is a different R than the left hand side?

One way to look at it is to take the network and add another repitition of the series parallel 1 ohm resistors. Does the overall resistance change? How does the the addition of these elements affect the overall resistance if you write it out in symbolic form?

You didn't do a great job of explaining this one in class, but I think I get it now.

They aren't different R, they are the same, just like S for the sum in the earlier problem is the same S on both sides of the equation.

I don't really understand how this got simplified down so quickly.

I'm a little confused on this problem. Why wouldn't the current just take the path of least resistance? and shouldn't the answer be like 1.4 (from the first resistor and then the 1 and R)?

this is close to what i got, but by totally different logic.

I didn't really know how to do this mathematically, but I did guess the range correctly!

This is pretty cool. I didn't notice when I did the problem myself.

Me neither, but it seems to make sense. The ability to look at problems and break down a series into a definite answer is so useful and something I never really looked for.

Yeah, I agree–this is very interesting that an infinite series can converge to something like this, and that we can solve it out using this quite simple method (if you see the pattern)

Yeah wow. I'm kind of mad at myself for not seeing this before.

I remembered that it was the Golden Ratio, which I thought was really cool also

Yep, I too liked this a lot.

I did not realize this, definitely cool

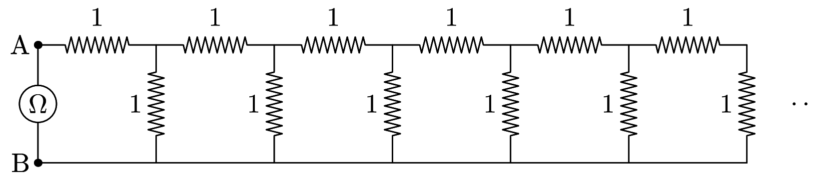
i wouldn't have seen this. the previous example was what i thought though, because it's similar to the other recursion problems.

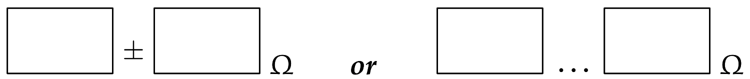
this is what i did! yay!

After starting from here I had a fun time solving it...forgot how algebra actually gets tricky at times.

Seeing the previous examples helped me get to this infinite sum

This is what I ended up doing but had a hard time spotting the exact recursion. ended up adding an extra 1 in somewhere from the beginning of the recursion





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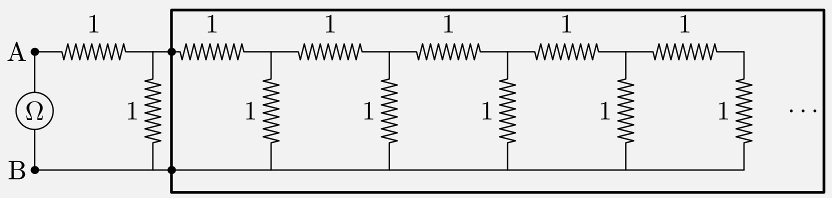

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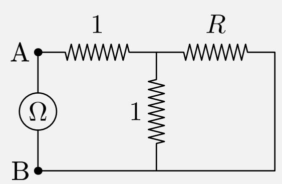

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Why is this called the Golden Ratio?

This number suprisingly appears in a large amount of places, including Da vinci paitings and flowers and nature itself. There is a book on it and its quite good called suprisingly "The Golden Ratio"

Thanks for the note. I'll look for the book later.

The golden ratio is incredibly fascinating, It would be great to hear more about it in class and how it applies to this sort of thing.

It pretty much all seems clear and concise.

Optional

These problems are optional in case you want more practice or want to try a (possibly large) project.

8. Email indexer
Design a set of shell scripts for doing quick keyword searches of a large database of emails. Assume that each email is stored in its own plain-text file. Perhaps one shell script generates an index, and a second script searches the index.

9. Running time
Ordinary long multiplication requires $O(n^2)$ digit-by-digit multiplications. Show that the Karatsuba multiplication method explained in lecture requires $O(n^{\log_2 3}) \approx O(n^{1.58})$ digit-by-digit multiplications.

10. Counting empires
How often does the word `Empire` (uppercase E, then all lowercase) occur in `decline.txt`? [*Hint:* Look up the `tr` command.]

Divide and conquer! First turn all non-letters into newlines (squeezing out repeated newlines); second, look for lines that exactly match 'Empire'; and third, count the lines. Those three stages are the three stages of the following pipeline:

```
tr -cs 'a-zA-Z' '\012' < ./data/decline.txt | grep '^Empire$' | wc -l
```

It produces '37'.

Comments on page 5

This is an interesting problem. I manage the website for my lab, and I have just been given the task of converting an Excel file with 75+ contacts into a list that can be printed in HTML. I thought about brute-forcing it, but was too lazy and am now writing a script for this.

In such situations, I often use gnumeric (nonproprietary software) to read the excel-format file, then ask it to export the file in tsv (tab separated value) format. Now I finally have a text file!

Then I might use awk (the -F option is useful here) and sed to generate an HTML table. Or if it gets too messy I'd use Python.

Another idea is to simply grep for all the lines with Empire. This is almost right except it doesn't account for lines with multiple 'Empire's. You can use sed to replace "Empire" with " nEmpire" (newline + Empire). grep for "Empire" again and count the lines.