6.055J/2.038J (Spring 2010)

Solution set 4

*Do the following problems. Submit your answers and explanations online by **10pm on Wednesday, 10 Mar 2010**.*

***Open universe:** Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Global comments

I'm glad problem 1 was on here...I was starting to forget past concepts.

Problems 2 and 3 were awesome because I remember that from being a physics TA.

For problem 4, I don't really understand the E equation. Isnt E just proportional to vˆ2, not 1/vˆ2?

First is drag energy, the second is lift energy. We've often been taking them as comparable and then just using the drag energy as our indicator since we know the rest of the terms (A, rho).

After reading today's memo, this makes much more sense since I have a clearer understanding of drag

I am wondering if we could use the mass of the planets to solve this problem. Is gravitational field strength something you would have in your back pocket of constants?

They're proportional, so that's what I did–not using the g. I thought about the size order and assumed densities were near constant.

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Here is the solution set, for the memo due Thursday at 10pm.

Is there a way to increase the time window that we can do this assignment? 24 hours just seems pretty short.

This hw set was very time consuming.

I agree. Although each problem took less time, the overall effect was a lot longer.

Well, this problem set had more problems on it. Unless there's an inverse relationship between time and number of problems, I guess it should take longer...

This problem set took me a lot longer than the other ones too.

It took longer but it fit in with the lectures really well which was useful in finding formulas and such.

Actually, for me, each problem seemed to take longer than usual. So combined with the fact that there were more problems, this pset took me a lot longer to do.

You're right, it did take a while, but it allowed us to use/test some of the more impressive skills we've learned recently. Who doesn't like a good challenge? I mean come on... we do go to MIT.

i don't think any of us are denying whether it was useful or good for us. the fact remains, it took a long time. i lost motivation by the end. usually i'm really excited by the findings i find but this time i just wanted to finish and didn't seem to care any more

That is useful feedback. I was thinking about having no homework set for this coming week (i.e. no homework due this coming Wednesday). We are at a good spot in the material for that pause.

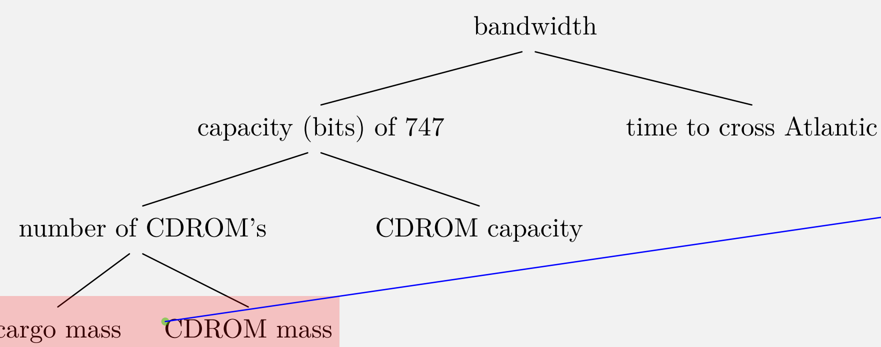
Given that this past set seemed to take longer than others, that confirms it. So, no new homework this Friday.

Problem 1 Bandwidth

To keep your divide-and-conquer muscles strong, here is an exercise from lecture: Estimate the bandwidth of a 747 crossing the Atlantic filled with CDROMs.

$10^{\square} \pm \square$ bits/s *or* $10^{\square} \ldots \square$ bits/s

Divide and conquer! Here's a tree on which to fill values:



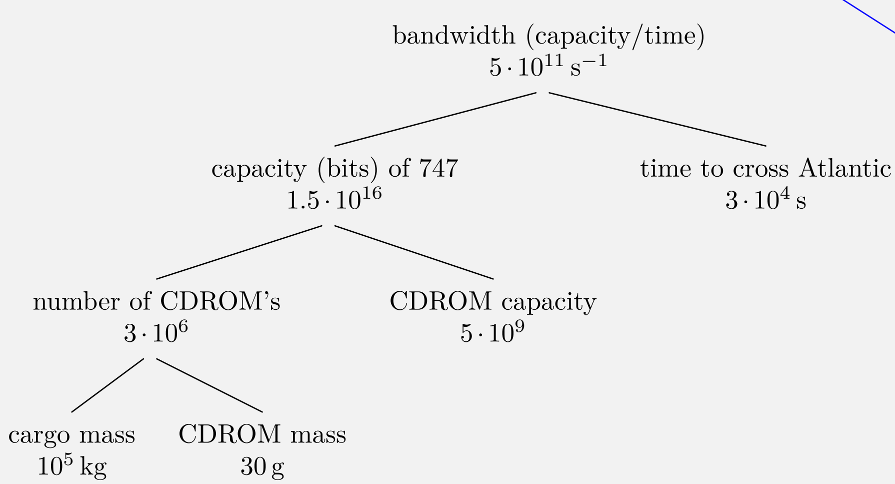
First I estimate the cargo mass. A 747 can easily carry about 400 people, each person having a mass (with luggage) of, say 140 kg. The total mass is

$$m \sim 400 \times 140\,\text{kg} \sim 6 \cdot 10^4\,\text{kg}.$$

A special cargo plane, with no seats or other frills for passengers, probably can carry 10^5 kg.

Here are the other estimates. A CDROM's mass is perhaps one ounce or 30 g. So the number of CDROM's is $3 \cdot 10^6$. The capacity of a CDROM is 600 MB or about $5 \cdot 10^9$ bits. The time to cross the Atlantic is about 8 hours or $3 \cdot 10^4$ s.

Now propagate the values toward the root of the tree:



The bandwidth is 0.5 terabits per second or $10^{11.5}$ bits/second.

Despite the large bandwidth offered by a 747 carrying CDROM's (not to mention DVDROM's), trans-Atlantic Internet connections go instead via undersea fiber-optic cables. Low latency is important!

Comments on page 2

Did anyone else think the title of this was funny? bandwidth like a link has bandwidth but here we are just stuffing a bunch of CDs with bits of information into a plane and flying them to their destination...

I had to read over the lectures again to figure out bandwidth, not very familiar with that

It would be interesting for future versions to change the media to DVDs or even maybe bluray. It's the same thing and scales well, but the new media types are interesting.

I was a little frustrated with this problem because I felt like it was an exercise in remembering lecture. I'm unsure if getting a similar answer here is a sign that I'm learning the process or just storing the bits of data.

While I agree that this was quite reminiscent of lecture, even remembering the processes from lecture is good practice. And, I think, helps solidify potential shortcuts for future problems.

i didn't use mass, but instead used volume as estimated by amount of luggage that can be carried. i feel like both are valid estimates. it's easier for me to estimate volume than weight.

We discussed this in lecture, and determined that a plane probably couldn't take off if its volume were full. You need to verify what, exactly, is the limiting factor.

I went with volume just because we went with mass in lecture and I wanted to see the other side of it all. The numbers werent too incredibly different IIRC

I also used volume and even though I knew that mass would be more accurate, volume was easier for me to work with. I figured that since we are doing first order approximations, it wouldn't be that bad.

However I first tried to quantify how much thrust the engines could provide to determine lift as opposed to how many people a plane could carry. Using people would have made the mass method easier.

Anyone who knows anything about aviation will tell you go by mass. if you filled a passenger aircraft up to maximum volume with CD's it would not be able to leave the ground.

I had no idea a 747 could carry that many people

Yeah, when I was trying to imagine how many people, I was thinking 50 rows with 6 people across was a big overestimate.

Also it might be harder to estimate because its not uniform, the people in first class are all spread out and everyone else is crammed.

I think a lot of my estimates were inaccurate. I thought way fewer than 400 people could fit on a 747, and I would have never guessed that a special cargo plane could carry 10^5 kg. That number seems very high.

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Divide and conquer! Here's a tree on which to fill values:

- bandwidth
  - capacity (bits) of 747
    - number of CDROM's
      - cargo mass
      - CDROM mass
    - CDROM capacity
  - time to cross Atlantic

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Now propagate the values toward the root of the tree:

- bandwidth (capacity/time) $5 \cdot 10^{11}\,\text{s}^{-1}$
  - capacity (bits) of 747 $1.5 \cdot 10^{16}$
    - number of CDROM's $3 \cdot 10^6$
      - cargo mass 10^5 kg
      - CDROM mass 30 g
    - CDROM capacity $5 \cdot 10^9$
  - time to cross Atlantic $3 \cdot 10^4$ s

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I made about the same estimate. I think this comes from "talking to your gut." Some parts in an estimation just can't be derived. I can now see how the "gut-talking" is being developed from exposure to all these problems.

yeah, I did this too. After doing so many problems, these methods just come to you, which is awesoem

Same here - good to know we're learning :-).

Yeah, I had no idea how much all the seats and luggage would weigh, so I just added an order of magnitude and it felt kind of right.

I went with volume and came out with 10ˆ7 CDs, so off by a factor of 3, which turned into an order of magnitude in the final answer with other error in time. I feel like I did better on this problem earlier in the course. Since my standards have gotten more stringent, an order of magnitude seems like too much to be off in this case.

I thought we estimated this to be about 700MB in class. Though I understand how that difference is insignificant within our answer.

I had to lookup to remember that 1 byte = 8 bits...I guess us MIT students should remember this before we graduate

I remember in class having a debate about mass or volume... and I went with volume. I got a slightly different answer here but not hugely different in the end

I went with mass and got a little different answer as well. My passengers and weights were different.

i also went with volume. i said: a plane is like 6mx3mx50m ish and A cd is 1/9m"x1/9m"x1/100m". So # cds 1e7. not sure if my assumptions were right but it got me a similar #

I remembered the approach from the beginning, but had forgotten a lot of the numbers.

I didn't realize this was capacity/time...

I definitely did not estimate this time right. What distance did you consider for this time? It would vary by country of destination / origin.

Since the 747 was traveling across the Atlantic, I just used the time from Boston to London, but the time didn't vary too much if I went from DC to London.

I feel like a general "time to cross Atlantic" or "max range/flight time of a 747" number would be useful, as we seem to come across it often.

I liked re-doing this problem to make sure that I still remembered how to do divide and conquer.

I forgot to account for there being more than one CD on board...but I also multiplied speed times distance to find the time instead of dividing, so my answer was closer than it could have been.

I was looking at the example done in class were we used distance travelled by 747 instead of time and arrived at the same solution. I am not sure if I am clear on the differences between the method. My notes weren't as coherent as I would have liked. Can you post that solution

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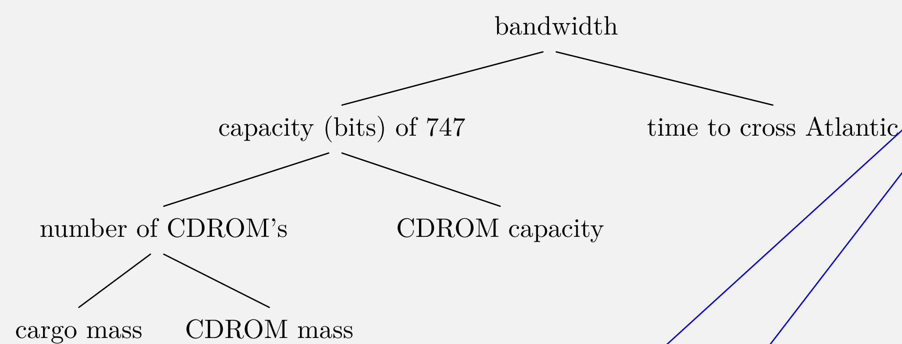

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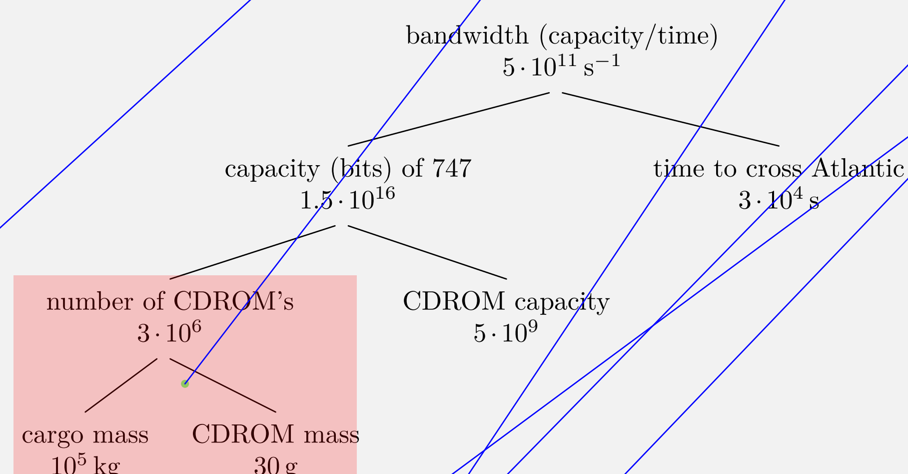

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It looks like the conversion of 30g to 3*10^-2 kg was never made... if the conversion had been made, the number of CDroms should be 3*10^3, not 10^6.

I like the continuity from earlier in class, it was easy to double check my work by looking back at what we did, especially the "gut feeling" number estimates!

How do you estimate the error? I had the same process but got a different answer by a few orders of magnitude.

What would have been an acceptable range for this answer. I got 10^12.5 which I was happy with. Your thoughts?

What was your range? did you say 10^12.5 +/- 1 ? this answer would fall within your range at that point.

However, I agree, the solutions should also have the +/- errors for these estimations.

This just occurred to me as well. The part of the problem sets I often have the least idea about actually comes down to estimating a reasonable uncertainty in my answer. It would be instructive to see some estimate of the uncertainty in the solutions.

Hm, so after I did this problem I went and looked at the notes I took in class from when we did this problem. My result was closer to 10^14, but when we did it in class we got 10^12.5

I got an answer a lot bigger than this. In retrospect I see that you would divide the amount of data by the total time for the trip. I originally thought of it as how much data you could receive in one second if it were a continuous line of planes. This way you get about 5 planes worth of data in one second.

I often find these comments at the end more fascinating than the actual answer.

I had no idea that there are undersea fiber-optic cables

surely this is more because of cost than low latency? it seems like it would rather expensive to be burning disks and shooting planes back and forth.

while it would be relatively expensive to keep doing that, the cables to setup undersea connection were not cheap when built either. His point is time critical information would take much longer for it to be received, especially initially. It would be interesting to compare the throughput of the cables and the plane.

ooh. I thought it was a bit of a joke. I couldn't imagine anyone actually transmitting information in a plane when you could just use the internet!

I loved your analogy in lecture to this..."Imagine clicking a link on a website...and then a day later, receiving ALL the information, pictures, and videos on the site!"

Problem 2 Gravity versus radius

Assume that planets are uniform spheres. How does g, the gravitational acceleration at the surface, depend on the planet's radius R? In other words, what is the exponent n in

$$g \propto R^n? \tag{1}$$

[ ] + [ ] *or* [ ] ... [ ]

The gravitational force (the weight) on an object of mass m is GMm/R^2, where G is Newton's constant, and M is the moon's mass. Thus the gravitational acceleration g is GM/R^2. But the mass M is proportional to R^3, so $g \propto R^1$. In other words, $n = 1$.

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The ratio g_{moon}/g_{earth} should be proportional to the ratio of radii R_{moon}/R_{earth}, namely 1/4. The actual ratio is lower because of an effect neglected in the analysis of **Problem 2**: the differing density. When that effect is included, then the mass M is ρR^3 (except for a constant), so

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If ρ_{moon}/ρ_{earth} is 2/3, that reduction in concert with the radii ratio would explain the factor of 6 difference in g.

Moon rock, which is less dense than the average earth rock, is comparable in density to rock in the earth's crust. This equivalence suggests that the moon was once a piece of the earth's crust that got scooped out probably by a large meteor impact.

Comments on page 3

I was unsure in this problem if we looked specifically at the radius 1/R^2 and ignored the mass's dependence on the radius or not.

you just have to think about what is constant when you're looking only at proporitonality. here, we're looking at the effect of different planet sizes on their gravity. so mass of the planet is changing. so you have to take that into account.

I thought the in-class discussion made it nice and intuitive. Thanks!

This is a good simple problem that really illustrates the point of proportionality very well.

While I think this problem is useful in science, it is not challenging for MIT students who have had this drilled into their head since high school.

The in-class explanation of this was incredibly helpful- Thanks!

Yeah, I was totally lost on this question until we went over it in class.

This sort of problem seems weird to have a +/- for, when we determine the answer exactly (or solve for the answer, basically).

I agree. Though I suppose if you were really stuck and thought your answer was way off you could add an arbitrary +/- to indicate you were unsure.

It's good to still be able to express your confidence. If +/- is zero, great, but if as above, people were wrong, then they were overly confident.

I can't believe I didn't think of this formula. I've used it a thousand times! I feel quite silly.

I definitely did not remember this formula, and thus did not get an answer for this question or the next....

I completely forgot about this equation. I think the problem for me with the proportional questions is that it requires some prior knowledge of some base equation beforehand.

I approached this problem in a mathematical way, so I took the log of both sides to solve for the exponent, using Earth's radius and g. It didn't work too well–I am embarrassed that I forgot middle school physics.

Now I know to consult and index of equations before losing my mind trying to do some weird math manipulation.

Yeah, I never know any equations to use.. It's really frustrating, I usually have no idea where to start.

I usually just fix this by looking at the equation sheet we got with our desert-island test

I always feel like all the course 2 people in the class must know these equations by heart and I'm the only one who has to look them up... guess I'm not all alone though.

I thought this was pretty tricky. At first I completely forgot to consider how the mass was proportional to R. However, the answer seemed too trivial, and I finally figured it out. Having figured out this trick for this question, was helpful in finding the relationship between L and M in question 8.

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oops I forgot to take that into account

I did the same thing, thinking about it now I realize why it affects the formula.

Oh I did this too...this makes me sad.

It might be useful to include these formulas on the back of the envelope sheet. Just in case people forget the volume of a sphere, the gravitational force of an object in circular motion, etc.

Forgot. That's one of those things that I just need to REMEMBER. Figure everything into the equation!

Tricky little question.

i forgot it too, just remembering that gravitation force was based on r^2. however, the idea is to not look at equations and be able to do these things on the fly.

Chalk me up as one more that answered -2 because of forgetting about the M. I should have trusted my intuition that larger objects should have larger gravitational acceleration.

I made this mistake as well. The way the question was worded made me think that we could take the mass of the planet as a constant of the problem.

The reading this week really cemented the fact that M is proportional to l^3 so maybe we should do that reading before this question.

I think that this point needs to be stressed more. We should break up each variable in class more often to include all the things it depends on.

The idea is that for many planets (e.g. Mercury, Venus, Earth, Mars), they are similar in density but their radius varies a lot, and that variation accounts for most of the variation in gravitational field strength. Problem 3 is then a follow-up question on that idea. I do this kind of reasoning so often now that I forget how subtle it is.

I'll think how to reword the question to clarify that density is constant across the planets (but without giving away the whole idea in the rewording).

I definitely did not take this into account either. I sometimes have a hard time with proportional reasoning and isolation the one variable of interest and disecting equations to get the information we are looking for

I screwed up here too. This was tricky.

I noticed that when we have an equation I often plow head first trying to use it to solve the problem and forget about using proportions. I think adding a comment in the reading about when it is ok to use proportions and when it would be cumbersome. I myself am not completely sure when it is ok to use them (i am afraid of using them out of context).

I also forgot that equation and ended up screwing up a little. This was a lot easier than I made it out to be...

I feel as though my discrepancy can be explained by my getting it wrong, but this seems reasonable, too.

This problem was unusual since I didn't feel like it used any of the approximation methods that we learned

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I'm pretty impressed that even though I had the wrong answer in 2, I explained the correct reason for why it was wrong in problem 3.

I also did.

I found this somewhat intuitive though, as there aren't too many other things that could be the reason.

also the fact that #2 asked you to assume that planets are uniform spheres kind of hinted to it for me – "uniform" kind of makes you not take density into account

I agree with the uniform comment- it made me feel like I didn't need to account for anything but G, M, or R. I guess that was the point though, otherwise I'm sure some people would have accounted for density and then this problem would have been trivial.

I also agree. Not much else came to mind that could effect the gravity that we didn't account for besides the density.

If you had the wrong answer in 2, wouldn't you have a crazy answer for 3, like the gravity of the moon should be 16x greater than on Earth? Even if you could guess that density was the reason for the discrepancy, would you think density would be 100x different? This is when reality checks are helpful.

I may have been off by an order of magnitude on problem 2, but I totally got this one! yippie

i guessed density initially and then postulated that G might have some sort of assumptions that is unique to earth. is this wrong? i have no idea what G actually entails, is it entirely independent of the body of mass with which it is being calculated on?

G is a universal constant or fudge factor for the gravity equation. It applies to all objects and not just earth.

This is definitely a subtle detail that is way beyond the estimate, but I'm just curious.... wouldn't the density increase with Rˆ3 just because gravity would make things more compact. I guess my question is do bigger things made of the same material have a higher density?

isn't the density of the rock on the moon the same? I thought that the moon is from the earth so it would be really close

isn't this also because there is less mass, which causes less pressure thus less dense rocks being formed. it seems that smaller planets/satellites should be less dense.

how big is the moon relative to the earth? can that mass really be scooped out?

The moon is about 30% of the size of the earth, with a gravity that is about 1/8th.

Seems like a rather large piece. But I suppose when comparing mass, the moon is 1% of the Earth. So I suppose the theory is plausible...

it's big enough to affect the tides. Remember reading it somewhere. Some say it happened when the supposed asteroid that killed the dinosaurs hit the earth.

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Interesting fact!

Agreed - I was very curious as to why there was such a large discrepancy.

the reason why the total density of the moon differs is that the core is actually much smaller. It does have a core, but it's only 1-3% of the total mass, while Earth's core is like 33% of the total mass – meaning that the density is more like a silicate rock rather than iron (which is what the core is made of). Interestingly enough, dynamical models for the moon's origin show that the moon is more likely made of the impactor rather than from part of the Earth – which is quite problematic! (aka...the moon probably did not come from the core, even though it formed from an impact)

why is what you said problematic? also, it wouldn't have come from the core no matter what. it would have come from the crust. either way, it does make sense that because it is much smaller, it would have a smaller core, which is the massive part of the planet.

ah I should have been more clear, it's just a generally problematic because the isotopes of the Earth match those of the moon – which was originally explained by the Moon being made from the Earth. It just means that things just aren't like we though :) And the impact would have melted the entire Earth, meaning "scouping" doesn't really work (i think this is the right Canup model: http://www.flickr.com/photos/thane/3134316459/)

Hmm, I actually thought moon rock would be more dense due to its chemical composition. On the other hand, I guess due to graivty, atmosphere, etc it is more like a light powder?

I also thought that it might be the opposite based on my results from the first calculation which were off. This makes much more sense. The origin of the moon comment is interesting, does that mean that is is a piece of something that collided with the earth? Does that mean it bounced off somehow and was held in orbit?

does that mean the layers of the earth and their compositions haven't changed in the millions of years since the moon was formed?

I found this interesting as well.

Problem 4 Minimum power

In the readings we estimated the flight speed that minimizes energy consumption. Call that speed v_E. We could also have estimated v_P, the speed that minimizes power consumption. What is the ratio v_P/v_E?

[ ] ± [ ] *or* [ ] ... [ ]

The zillions of constants (such as ρ) clutter the analysis without changing the result. So I'll simplify the problem by using a system of units where all the constants are 1. Then the energy is

$$E \sim v^2 + \frac{1}{v^2},$$

where the first term is from drag and the second term is from lift. The power is energy per time, and time is inversely proportional to v, so $P \propto Ev$ and

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The first term is the steep v^3 dependence of drag power on velocity (which we used to estimate the world-record cycling and swimming speeds). The energy expression is unchanged when $v \to 1/v$, so it has a minimum at $v_E = 1$.

To minimize the power, use calculus (ask me if you are curious about calculus-free ways to minimize it):

$$\frac{dP}{dv} \sim 3v^2 - \frac{1}{v^2} = 0,$$

therefore $v_P = 3^{-1/4}$ (roughly 3/4), which is also the ratio v_P/v_E.

So the minimum-power speed is about 25% less than the minimum-energy speed. That result makes sense. Drag power grows very fast as v increases – much faster than lift power decreases – so it's worth reducing the speed a little to reduce the drag a lot.

If you don't believe the simplification that I used of setting all constants to 1 – and it is not immediately obvious that it should work – then try using this general form:

$$E \sim Av^2 + \frac{B}{v^2},$$

where A and B are constants. You'll find that v_E and v_P each contain the same function of A and B and that this function disappears from the ratio v_P/v_E.

Comments on page 4

I found this problem to be very confusing and difficult. Not that the work was difficult, but figuring out what I was supposed to do was very difficult. If you use this problem again in the future, I would suggest giving a few more hints for this, because I definitely wasted a few hours on this problem.

yeah i agree... i had no idea what the question was really asking for.

isn't the point of this class to be able to figure out how to solve a problem? (and if you're really stuck, asking a question on the homework page itself?) it basically consisted of doing a derivation almost identical to the one that was in the reading...

I'm not saying it wasn't a hard problem, but it certainly wasn't unreasonably hard. It's good to have to think about what tools to use. I think problems like this are the more interesting ones.

But this problem wasn't about trying to figure out the "trick" and it wasn't like we had to figure out whether to use divide and conquer, etc. The only point to this problem was, let's see if you can sift through pages of notes to find an obscure formula, with no mention of what formula we should be using. That's not useful or helping me learn any techniques, it's just a waste of my time.

But with this class you should be able to derive that formula on your own – that's the point. Either remembering or rederiving (which would have been faster than a few hours) the lift and drag proportionalities that have been some of the main points of this unit (F.D v^2 and F.L 1/v^2) and then combining them to get E v^2+1/v^2 isn't at all out of the realm of questions to ask MIT students.

I like that we aren't being handed answers in this class as much as in most classes. Knowing how to plug-and-chug isn't engineering.

it took me a while to find exactly in which reading we covered this...it would have been useful to reference it back

Why would you want to minimize power instead of energy?

We want to minimize both, and look at how they compare.

If you have plenty of energy, but can't deliver it fast enough. Maybe this is a poor example, but I can walk a lot farther than I can run.

Why couldn't we just find the speed, why do we need to bother making a ratio? This ratio must have some significance that you will cleverly explain later.

I did this problem completely different- I just looked at the readings where it states that power is proportional to l^-1 and energy also is proportional to l^-1 so I just assumed that since P/E=1, that v_p/v_e also would equal 1. Probably too much of a simplification but it came out with the same answer..

I actuallyy had no idea on how to do it

I did not get rid of the constants and I think that hindered my ability to solve this problem

I really like this method of analysis, it seems like in class you do this in your head a lot - look at only the variables that will change and examine their relation. It is something that i never thought of before this class and is a great trick to seeing trends when a problem looks too intimidating.

can we go over this in class. i still don't follow

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Didn't we determine that this is simplification removed many different important aspects of the problem, and could be off significantly, If so, would this make further analysis even more likely to be off?

why Ev, how did you get that

Power = Energy/time = Energy*(velocity/distance). (because distance/velocity=time, so you flip this to get the reciprocal).

what isn't P proportional to Force * velocity? according to the explanation above, power = Energy * velocity / distance... where'd the distance go?

I'd like to see how you'd arrive at this a bit more rigorously. I'll grant you that P is proportional to E*v (as this allows you to ignore distance as a constant in this situation), but I don't see how, given this ignorance, you can justify jumping to a more rigid assertion that P v**3 + 1/v? This seems very hand wavy to me.

If you look at the Energy equation that was used to minimize energy for flight (it had a drag and a lift term) and divide it by time, (power is energy over time) then you will get an extra velocity term (s/t=v). These V's multiply/cancel with the other V's and you get the (1/v)+v^3.

This was the catch I couldn't figure out. Makes sense now

me too! I didn't think about this...i just took the derivative of the Ve, but ended up getting the same answer...which was totally wrong. with this simplification, it totally works out!

Yay! I actually got one. Took me a while of just messing with units on a sheet of paper

yeah i missed the setup. i wasn't using paper and pencil though, and would have felt better writing the equations out rather than just in the submit boxes.

I was using a sheet of paper and still botched it up. arg

Why can't the two parts simply be set equal as they were to minimize the energy?

I think the point of the problem is that the minimum velocity for energy consumption is not the minimum energy for power consumption. So we use find the proportions of energy consumption and power consumption in terms of velocity, respectively, and take the derivatives using calculus to find the minimum velocities, respectively. Setting them equal as you mentioned would just give us the velocity at which energy consumption and power consumption is the same, but that is not what the question asks for.

I was thinking about doing it that way also. Hmm

It's very helpful when you explain the answers along with describing where their derivations are in the notes.

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This explanation is great. It's very easy to follow. I like that you discussed what each component of the equation is.

I agree. In many explanations, I am confused because I don't know where some variables come from.

Instead of taking the derivative I set it equal to the Power used to keep the plane afloat. Why is that wrong - it's the same thing we did for energy.

This is probably one of the clearest explanations so far and I'm glad it goes with one of the problems that confused me the most.

Yes, it was simpler than I expected it to be.

This was definitely a very involved problem, and it was explained really well. Before this problem I didn't realize there wwere two different velocities that minimized energy vs power consumption

I don't quite understand how you got this V_e=1. everything else was cool though.

Please explain, I definitely did this part incorrectly. I thought one of the points of the pset was to steer away from needing to us things like derivations?

I think it's harmless to differentiate a simple and nice function like what we have here. If it were a harder equation, then we might want to use symmetry, etc. Estimation to me is all about getting the best answer in the least amount of time, and just doing the derivative seems to be a good way of going about that.

I would be interested in seeing the non-calculus way. I got to this point and spent a while looking for the symmetry for power (since we had already seen the one for energy), but gave up after a while and used calculus as done here.

Ahh... This makes so much sense! I was completely lost, and because we've been ignoring a lot of more standard math procedures, I didn't even think of differentiating.

At first I didn't understand why this velocity also equaled the ratio of velocities, might be worth mentioning again that vE = 1 even though it's also listed above.

Hm, I did this wrong, but I'm not sure why because I used the same process, I thought? Maybe I miscalculated something, or related Power to Energy incorrectly. It's helpful to see these solutions though so I can walk through step by step.

I got this far into the problem, except for figuring out that Ve=1. Without that I couldn't get a good answer...

i think that most people realize this. i'm not sure it's worth mentioning here.

I disagree, I thought it was nice to see that A and B could be left in there, since people are often worried about dropping away constants.

I remembered your graph from class where the two plots intersected at a given point, but didn't remember how to apply that correctly.

Problem 5 Highway vs city driving

Here is a measure of the importance of drag for a car moving at speed v for a distance d:

$$\frac{E_{drag}}{E_{kinetic}} \sim \frac{\rho v^2 A d}{m_{car} v^2}$$

This ratio is equivalent to the ratio

$$\frac{\text{mass of the air displaced}}{\text{mass of the car}}$$

and to the ratio

$$\frac{\rho_{air}}{\rho_{car}} \times \frac{d}{l_{car}},$$

where ρ_{car} is the density of the car (its mass divided by its volume) and l_{car} is the length of the car.

Make estimates for a typical car and find the distance d at which the ratio becomes significant (say, roughly 1).

$10^{\square \pm \square}$ m *or* $10^{\square \ldots \square}$ m

To include in the explanation box: How does the distance compare with the distance between exits on the highway and between stop signs or stoplights on city streets? What therefore are the main mechanisms of energy loss in city and in highway driving?

A typical car has mass $m_{car} \sim 10^3$ kg, cross-sectional area $A \sim 2\,\text{m} \times 1.5\,\text{m} = 3\,\text{m}^2$, and length $l_{car} \sim 4$ m. So

$$\rho_{car} \sim \frac{m_{car}}{A l_{car}} \sim \frac{10^3\,\text{kg}}{3\,\text{m}^2 \times 4\,\text{m}} \sim 10^2\,\text{kg}\,\text{m}^{-3}.$$

Since $\rho_{car}/\rho_{air} \sim 100$, the ratio

$$\frac{\rho_{air}}{\rho_{car}} \frac{d}{l_{car}}$$

becomes 1 when $d/l_{car} \sim 100$, so $d \sim 400$ m.

This distance d is significantly farther than the distance between stop signs or stoplights on city streets. In Manhattan, for example, 20 east–west blocks are one mile, giving a spacing of approximately 80 m. So air resistance is not a significant loss in city driving. Instead the loss comes from engine friction, rolling resistance, and (mostly) braking.

However, the distance d is comparable to the exit spacing on urban highways. So when you drive on the highway for even a few exit distances, air resistance is a significant loss.

Interestingly, highway fuel efficiencies are higher than city fuel efficiencies, even though drag gets worse at the higher, highway speeds, and presumably engine friction and rolling resistance also get worse at higher speeds. Only one loss mechanism, braking, is less prevalent in highway than in city driving. Therefore, braking must be a significant loss in city driving. Regenerative braking, used in some hybrid or electric cars, would therefore significantly improve fuel efficiency in city driving.

the title confused me because i couldn't figure out what it had to do with our hardcore calculations. it makes sense now but it took me way too long to figure it out.

Yeah, I was a little confused too, especially because all we were actually calculation was a distance and then answering the highway vs/ city driving qualitatively.

This was a lot less frustrating than some of the other problems because I didn't have to search for miles to figure out what equations I should use. Once I had the tools it was fun to do this problem!

I thought this question was asking for a qualitative answer, not a mathematical one. I'll try to apply equations to more of my explanations.

i don't think he's expecting the same level of detail in our answers. he just wants really detailed answers for us to read and learn from.

How are we able to assume the answer to this?

I thought it just kind of made sense, the problem led to something to do with the distance at which drag becomes significant. The difference in distances on the highway vs city is the distances between stops- exits on the highway and stop signs on the highway. I liked this application.

I used the 3m * 3m we used in class. Why change the estimation this time?

i'm pretty sure he used the same #s last time. i recall something about lying down in a car = 2m and when you stand it's almost your height but not really

I'd used 2m (wide) x 1.5 m (high) = 3 m^2 as the area. It is easy to confuse 3 square meters (i.e. 3 m^2) with 3 meters squared (i.e. (3 m)^2).

Mine was much higher than this, as I underestimated the volume.

I find it very useful to double check all of my estimations when working in metric. For me metric still doesn't come intuitively.

I agree with this - my answer was off by a lot because I messed up the car volume estimation.

How did you get this number? This is where my estimate was really off... and my final answer ended up off by about a factor of 10

He did the calculation in the above line. he shows how he got rho car = 100 kg/m^3. Air has density of 1 kg/m^3. their ratio is 100.

This is confusing to me because you say the car/air ratio is 100 but the formula asks for the air/car ratio...

That's true - I had to do a double-take, might be helpful to flip one of the ratios.

I think the reason he wrote it that way is because he asked us to set the ratio equal to one, and when you solve for d, you get the equation d/l = car/air.

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I think my answer was around there? I went about it similarly, but didn't use your density ratio. Retrospectively, I don't see why I didn't.

I really liked this problem, it was fairly straight forward and an interesting result that I think we can all relate to. I know that when I was driving to Rhode Island this past weekend every time I had to stop I thought about how much energy I was losing.

This problem was interesting to see how drag affects the car when stopping, a common occurrence (when you're not a student in the city)

i was just in nyc. i hate that place and the traffic is ridiculous. i don't know how anyone drives there at all.

I just grouped this all into kinetic energy. Is that okay?

as we saw in class as well

didn't realize there were so many components that went into stop-go driving. my answer only took into account the obvious braking component, though looking back, I guess the other components are pretty obvious too...

I thought this was really amazing!

This is kind of interesting, because as someone who grew up in the country I have a completely different sense of scale. For me an average distance between highway exits on average 5-7 miles... and much much larger in sparsely populated regions. I don't have a good sense of scale for a city block really.

What is the significance of this?

how does this create more air resistance again?

it's the point of the problem–at distances of about 400m, the air resistance becomes important, so on highways, this is a big factor. (not so much in cities). remember, one way to look at air resistance is the mass of air displaced.

So the idea is when we drive longer, we end up displacing more air, right?

Is it also related to the fact that car engines are more efficient at the rpms/gear ratios that are typically used when driving at 55 mph

yeah, I'm assuming this plays a role...when doing this problem I psyched myself out because I did say that on highways there is much more drag force, but I also knew that highway mpg is typically more than city...

I wish they had mpg rates for different speeds, and highway vs city driving

I was really confused at first with this – I didn't really pay attention to what the results were giving me, and I assumed that drag was more important for shorter distances (Because I remembered that city mpg was much worse than highway mpg), but I quickly realized I was wrong. It's amazing to realize how inefficient braking really is!

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What is this?

Quoting wikipedia: "A regenerative brake is an energy recovery mechanism that reduces vehicle speed by converting some of its kinetic energy and/or potential energy (due to elevation) into a useful form of energy instead of dissipating it as heat as with a conventional brake. The converted kinetic energy is stored for future use or fed back into a power system for use by other vehicles."

An MIT lab recently produced a prototype for a bike which uses this type of braking system to store energy in order to help propel the bike forward for hills or just to give the rider a break.

So electric cars would not help much if at all in highway driving? Hrm.

that's not entirely true...the fuel efficiency for highway driving can be greatly improved in an area with hills/mountains. The downside of it is that the cruse control on hybrids _sucks_...it doesn't take slopes into account at all.

Hybrids can still take the electricity stored up from braking and from using the gas engine and use just the electric motor to power the car at constant highway speeds. It's rapid acceleration that tends to require the gas engine.

no, most of them can't (from my experience). if on a flat road, it will use both to maintain a constant speed...rapid acceleration also uses both.

going down a gentle grade on the freeway will frequently run on just the electric motor, but otherwise it still requires gas.

*Point of Interest: Our Hybrid SUV [not a contradiction, we need the 4x4] has more get up & go than most V8 engines.

Problem 6 Mountains

Here are the heights of the tallest mountains on Mars and Earth.

Mars	27 km	(Mount Olympus)
Earth	9 km	(Mount Everest)

Predict the height of the tallest mountain on Venus.

$10^{\square} \pm \square$ km *or* $10^{\square} \ldots \square$ km

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

One pattern is that the larger planet (earth) has the smaller mountain. Large planets presumably have stronger gravitational fields at their surface, which keeps the mountains closer to the ground. The derivation in lecture on mountain heights dropped the dependence on g because we looked only at mountains on earth where all mountains share the value of g.

The same derivation can be repeated but retaining g. The weight of a mountain of size l is $W \propto gl^3$, so the pressure at the base is $p \propto gl^3/l^2 \sim gl$. When the pressure p exceeds the maximum pressure that rock can support, the mountain can no longer grow upward. This criterion is equivalent to holding gl constant. Therefore,

$$l \propto g^{-1}.$$

Here are the gravitational field strengths on the three planets:

a. Mars: $3.7\,\mathrm{m\,s^{-2}}$

b. earth: $10\,\mathrm{m\,s^{-2}}$

c. Venus: $8.9\,\mathrm{m\,s^{-2}}$

The product gl for each planet should be the same. That hypothesis works for Mars and earth:

a. Mars: $10^5\,\mathrm{m^2\,s^{-2}}$

b. earth: $0.9 \cdot 10^5\,\mathrm{m^2\,s^{-2}}$

If Venus follows the predicted scaling, then gl should be roughly $10^5\,\mathrm{m^2\,s^{-2}}$ with $g \sim 8.9\,\mathrm{m\,s^{-2}}$. Therefore l should be roughly 11 km. Indeed, the tallest mountain on Venus, which is Maxwell Montes, has just that height. Scaling triumphs!

Here is a fun question: Why aren't mountains on the moon 60 km tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in Problem 3)?

Comments on page 6

What about Mauna kea? That is around 40,000 ft high, right (if you count the distance from the base in the sea)?

actually, it's 9.100 km above the ocean floor...but the baseline that people use is sea level because that's where the center of gravity for a mountain tends to be (yes, they go under ground about as far as they are above "ground")

...yeah but the forces that keep mountains smaller on earth discussed here occur from mass differences above sea level.

Wouldn't the water weight make it even harder on the mountains with undersea parts?

I don't like the fact that weather, is not taken into account here, but what I really don't like is the fact that the definition of mountain height used in the answers is the height from the ocean, while those mountains can sit upon giant plateaus that can change the height by a factor of 2 or more. I can see the value in the analysis, but I think that when comparing the actual answers with the analysis the fact that they are so close is simply luck and not a factor of the analysis working.

I got the proportionality right, but I didn't think about the cause of it. That's pretty interesting reasoning.

What else did you think was holding the mountains down? Seems pretty obvious that gravity plays a big role...

I don't pick up on these things either..

But this is only true assuming that the planet densities are all equal.

I completely ignored g... damn

When I did this problem I implicitly used gravitational fields or do the math, but I just reasoned that since the matter on Mars is less dense than that on earth, the mountains on Mars wouldn't be under as much pressure so they could get taller.

is it better to blindly estimate things or to look up a few background things, especially for the psets

I thought about it in terms of gravity and still managed to get it wrong. I feel like I made a lot of silly mistakes on this pset.

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Predict the height of the tallest mountain on Venus.

$10^{\square} \pm \square$ km *or* $10^{\square} \ldots \square$ km

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

One pattern is that the larger planet (earth) has the smaller mountain. Large planets presumably have stronger gravitational fields at their surface, which keeps the mountains closer to the ground. The derivation in lecture on mountain heights dropped the dependence on g because we looked only at mountains on earth where all mountains share the value of g.

The same derivation can be repeated but retaining g. The weight of a mountain of size l is $W \propto gl^3$, so the pressure at the base is $p \propto gl^3/l^2 \sim gl$. When the pressure p exceeds the maximum pressure that rock can support, the mountain can no longer grow upward. This criterion is equivalent to holding gl constant. Therefore,

$$l \propto g^{-1}.$$

Here are the gravitational field strengths on the three planets:

a. Mars: $3.7\,\mathrm{m\,s^{-2}}$
b. earth: $10\,\mathrm{m\,s^{-2}}$
c. Venus: $8.9\,\mathrm{m\,s^{-2}}$

The product gl for each planet should be the same. That hypothesis works for Mars and earth:

a. Mars: $10^5\,\mathrm{m^2\,s^{-2}}$
b. earth: $0.9 \cdot 10^5\,\mathrm{m^2\,s^{-2}}$

If Venus follows the predicted scaling, then gl should be roughly $10^5\,\mathrm{m^2\,s^{-2}}$ with $g \sim 8.9\,\mathrm{m\,s^{-2}}$. Therefore l should be roughly 11 km. Indeed, the tallest mountain on Venus, which is Maxwell Montes, has just that height. Scaling triumphs!

Here is a fun question: Why aren't mountains on the moon 60 km tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in **Problem 3**)?

Should we have looked these up? I didn't bother. I just assumed that since venus was slightly less than the size of earth, it would have a slightly smaller g (from the earlier problem) and picked .9g because this made the math easier.

I guessed, and then looked it up to make sure. It makes sense, but I guess is NOT one of the constants we are required to know without looking up.

Yeah, I had no idea, so i just assumed it was around the Earth's size and therefore has around the earth's gravity...

I assumed it was somewhere in between but the answer still wasn't off by much.

I also didn't have these numbers but based my answer more on size and location to the sun. I got a similar answer but I'm not sure I got it for a good reason.

I went with a few other people and guessed that the gravity was slightly less so that the height would be a little larger than earth.

Yeah I did the same thing... I figured Venus is closer in size to earth than to mars but is still smaller than earth and made my estimation based on that.

We did an earlier problem relating radius to gravitational force – so I just used that proportional scaling to guess at g (since I knew the ratio of the radii was somewhere less than one but greater than 3/4)

I had to look it up, and when I did I was reminded of something I vaguely recall from middle school science about how Venus is sorta similar to Earth based on gravities.

Yeah I just assumed that the gravity on Venus was similar to that of Earth's, just a little smaller. I actually got quite close to the right answer.

Yeah. I had no intuition about the relative size of planets, so I just used 1.

I think it would be good to show the math here i.e. the mountain heights

I agree. A sentence or two explaining the main computation would make these two values a bit quicker to understand.

Um, I think the more important concept is that they are very similar, so they will scale similarly. I don't know why you'd need more specific math than what's given.

so if i remember, venus and earth were always the same-ish size but there was a lot more pressure on venus than earth. why? wouldn't this affect mountain height?

I would imagine because there is little geological activity for mountains to form or that the types rocks cannot support such large mountains

Problem 6 Mountains

Here are the heights of the tallest mountains on Mars and Earth.

Mars	27 km	(Mount Olympus)
Earth	9 km	(Mount Everest)

Predict the height of the tallest mountain on Venus.

$10^{\square} \pm \square$ km *or* $10^{\square} \ldots \square$ km

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

One pattern is that the larger planet (earth) has the smaller mountain. Large planets presumably have stronger gravitational fields at their surface, which keeps the mountains closer to the ground. The derivation in lecture on mountain heights dropped the dependence on g because we looked only at mountains on earth where all mountains share the value of g.

The same derivation can be repeated but retaining g. The weight of a mountain of size l is $W \propto gl^3$, so the pressure at the base is $p \propto gl^3/l^2 \sim gl$. When the pressure p exceeds the maximum pressure that rock can support, the mountain can no longer grow upward. This criterion is equivalent to holding gl constant. Therefore,

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Here is a fun question: Why aren't mountains on the moon 60 km tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in **Problem 3**)?

Because meteors hit any "mountains" that form, due to the moons lack of an atmosphere.

There is no real history on the moon to create mountains. Mountains form from tectonics....and on the young moon, well, tectonics are minimal. While the moon is quite interesting (and dynamic - a liquid thin outer core!) it is not too active, nor has had an active history (like Mars) - it doesn't undergo the same processes that make volcanoes on other planets. The moon cooled from a magma ocean and produced a rather interesting crust – decoupling it from the crazy stuff going on below the crust. Mare on the moon (topography) were produced from large impacts – otherwise, the topography is quite flat.

I'm sure spacial collisions matter, but I think the bigger culprit is the lack of a strong magnetic field or internal shifting techtonics. As such, there is a pretty stable surface and no real impetus to have formed these mountains in the primordial era.

Great response. I didn't even think about that, but it makes sense.

Problem 7 Raindrop speed

Use the drag-force results from the readings to estimate the terminal speed of a typical raindrop (diameter of about 0.5 cm).

$10^{\square} \pm \square \ \mathrm{m\,s^{-1}}$ *or* $10^{\square} \ldots \square \ \mathrm{m\,s^{-1}}$

To include in the explanation box: How could you check this result?

The weight of the raindrop is the density times the volume times g:

$$W \sim \rho r^3 g,$$

where I neglect dimensionless factors such as $4\pi/3$.

At terminal velocity, the weight equals the drag. The drag is

$$F \sim \rho_{air} v^2 A \sim \rho_{air} v^2 r^2.$$

Equating the weight to the drag gives an equation for v and r:

$$\rho_{air} v^2 r^2 \sim \rho r^3 g,$$

so $v \propto r^{1/2}$.

Bigger raindrops fall faster but – because of the square root – not much faster.

With the g and the densities, the terminal velocity is

$$v \sim \sqrt{\frac{\rho}{\rho_{air}} gr}.$$

A typical raindrop has a diameter of maybe 5 or 6 mm, so $r \sim 3$ mm. Since the density ratio between water and air is roughly 1000,

$$v \sim \sqrt{1000 \times 10\,\mathrm{m\,s^{-2}} \times 3 \cdot 10^{-3}\,\mathrm{m}} \sim 5\,\mathrm{m\,s^{-1}}.$$

First convert the speed into a more familiar value: 11 mph (miles per hour). If one drives at a speed v_{car}, then raindrops appear to move at an angle $\arctan(v_{car}/v)$. When $v_{car} = v$, the drops come at a 45° angle. So one way to measure the terminal speed is to drive in a rainstorm, slowly accelerating while the passenger (not the driver!) says when the drops hit at a 45° angle.

You could also run in a rainstorm and note the speed at which a small umbrella has to held at 45° to keep you perfectly dry.

Comments on page 7

i just put a weight but I gues a volume times density makes more sense

I left this factor in my calculation, and got about 14 for my answer

I kept it as well, i thought it conveniently came out to a few.

How do you decide when a dimensionless factor should be neglected?

we've been using a cubed diameter to approximate the volume of a sphere quite frequently.

Are you allowed to do that, without re-incorporating it again at a later point in the problem?

...oh

Hardest part of this question was estimating the raindrop dimensions.

I hoped to minimize that problem by putting the dimensions in the problem statement ("diameter of about 0.5 cm")!

I liked this problem but I don't see which approximation method we are using here.

i looked this up, and for a 5 or 6mm raindrop, it's actually about 10 m/s. I think the discrepancy is from neglecting the 4pi/3.

maybe the raindrop is not a perfect sphere. Maybe it forms into a more aerodynamic shape

like a droplet shape? but if we can estimate a sphere to a cube we can surely make the smaller jump from a sphere to a droplet.

Not sure where this angle number comes from

I thought the measurement would have something to do with an umbrella but had no idea what to do with it!

Do all raindrops reach this terminal velocity? Or might some clouds form too low for this to happen?

How would you ever come up with this?

It actually varies by size, but its a cool result!

To 1:03 am:

This method works because to fall at 45 degrees, your x and y velocity should be the same. Therefore, if you know your x velocity via a car, the moment you find that a raindrop hits at a 45 degree angle, you know it's y velocity was the same as your x velocity at that moment.

I notice this on days when I'm late for class and it's raining. I find holding my umbrella vertical doesn't keep me dry, so I have to adjust it

except for raindrops forming a couple feet off the ground, you can assume that all raindrops are falling at terminal velocity.

If anyone tries this, don't do it by the green building. pick somewhere with no wind.

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You could also run in a rainstorm and note the speed at which a small umbrella has to held at $45°$ to keep you perfectly dry.

Phrasing is weird. I understand what you mean but this sentence is not very clear.

Problem 8 Cruising speed versus air density

For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on air density ρ? In other words, what is the exponent β in $v \propto \rho^{\beta}$?

[ ] ± [ ] *or* [ ] . . . [ ]

From the lecture notes,

$$Mg \sim C^{1/2} \rho v^2 L^2,$$

where C is the modified drag coefficient. So

$$v \sim \left(\frac{Mg}{C^{1/2}\rho L^2}\right)^{1/2}.$$

The only dependence on ρ is the ρ itself in the denominator, leaving

$$v \propto \rho^{-1/2}$$

and $\beta = 1/2$.

The inverse relationship between the speed and density explains why planes fly at a high altitude. The energy consumption at the minimum-energy speed is proportional to the drag force, which is proportional to ρv^2. Because $v \propto \rho^{-1/2}$, the powers of ρ cancel in the energy consumption; in other words, the energy consumption (at the minimum-energy speed for that ρ) is independent of ρ. By flying high, where ρ is low, planes can fly faster without increasing their energy consumption.

Comments on page 8

I totally did not get any of these problems...they were very frustrating to me.

Yeah, these last 3 were especially hard. They made the pset way too long.

This phrase was kind of hard to understand, maybe add a clarification to it?

This problem was pretty straightforward once you found the section in the lecture notes.

based on this part, the relation seems fairly straight forward, so i was wondering if i was missing something.

I didn't realize at first that this was balancing the force and drag again.

What force does this represent?

And what are those variables?

I never know where to find the equations for these types of problems, and this p-set was full of them.

I used the relationship between lift energy and drag. It was easier to see the relationship.

I did too but I got my proportion for mass all wrong. I'm a little confused why you can just use that equation, wouldn't you want to take the derivative of the energy equation?

should I commit stuff like this to memory, it seems important and like we have used it alot

For some reason, the specifics of your approaches in lecture aren't sticking. I get the general idea, but your specific examples don't always form a cohesive picture. Perhaps this is my own mental limitation, but I thought I'd mention it.

this really clears it up- thanks

Should beta=-1/2? Otherwise I do not see how these two lines follow

probably a typo...

I think it's a typo too - I definitely got -1/2 for my answer on the pset.

I wonder how often he reads these comments to correct!

I'm not sure, I'm just happy to see I did this right and got the same answer.

I'm pretty sure that's a typo

I wish this were explained in more detail. It seems like your substituting back into the same equation which will always cancel terms

I don't see the purpose in saying that energy consumption is independent of p, when you write in parentheses that this only happens when something that depends on p is satisfied.

This is like saying, "Given something that depends on p, then energy consumption becomes independent of p." It seems a little counter-intuitive.

Would a better way of phrasing it be, "Regardless of the density, p, there is always a speed v, that uses the same minimum energy consumption."? Or would that be saying something else entirely?

Problem 8 Cruising speed versus air density
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☐ ± ☐ *or* ☐ ... ☐

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I wonder what the tradeoff is since flying at a higher altitude increases the distance traveled to get to the destination.

This point is very evident if you take a flight from Boston to New York or even Philadelphia. The entire flight you're able to see the ground and don't get that high because the plane doesn't go too high whereas a flight across the Atlantic requires more speed so you must go higher.

I don't think the trade off is that significant. I think planes normally go up to about 35,000 feet (about 6.5 miles) which isn't really adding that much distance when you are traveling hundreds of miles

That's really cool

Problem 9 Cruising speed versus mass

For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on mass M? In other words, what is the exponent β in $v \propto M^{\beta}$?

☐ ± ☐ *or* ☐ . . . ☐

Again from the lecture notes,

$$Mg \sim C^{1/2}\rho v^2 L^2,$$

where C is the modified drag coefficient. So

$$v \sim \left(\frac{Mg}{C^{1/2}\rho L^2}\right)^{1/2}.$$

For geometrically similar animals, g is independent of size (they all fight the same gravity) and C is also independent of size (because the drag coefficient depends only on shape). But M depends on L according to $M \propto L^3$ or $L \propto M^{1/3}$. Because L^2 is proportional to $M^{2/3}$, the denominator contains $M^{2/3}$. The numerator contains M^1, so the ratio of numerator to denominator is $M^{1/3}$. After taking the square root, we find the scaling

$$v \propto M^{1/6}.$$

In other words, $\beta = 1/6$.

Large birds (and planes) fly slightly faster than small birds and planes. The design of the 737 was affected by this fact. The 737 is for medium-range flights and carries fewer passengers than a 747. However, if the 737 were merely a geometrically scaled 747 – retaining the shape but reducing M by, say, a factor of 3 – then it would have a cruising speed roughly 20% lower than a 747 (because $3^{1/6} \approx 1.2$). That reduction would be fine if the 737 were the only plane traveling the skies. But planes are directed along fixed flight paths where it is dangerous to have planes overtaking one another. Therefore, the 737 was designed not to be geometrically similar to the 747 but instead to have the same cruising speed as the 747. Scaling matters!

Comments on page 9

Just like in problem 2 where M was proportional to R^3, I didn't do this and got a totally wrong exponent. Thus my answer to problem 10 was skewed too.

Yeah, I forgot the hidden M's too.

Wow I'm glad others did...reading these solutions was a nice wake up call for me to be more attentive in the future.

Aye, I missed them as well. I knew I had done something wrong based on my answer to #10, but was unsure where I had messed up.

i dont really get this statement. i guess i need to think through this a little more in order to be able to get it right the first time on the homeworks.

So I 'spose this is a rhetorical question, but: how do you know all these things? I know many of them from my great love of machines and mechanics, but Lex Luther is to Superman as biology is to me – I simply hate learning it. I guess I'm saying I envy your ability not to deeply resent other sciences in the unreasonable way I do.

This really seems like physics of animals more than biology. Think of them as robots if you like. Although I'd say biology is pretty amazing too.

I really like these comments at the end of the solutions. They really give meaning and matter to the problems. Gives me new insight on the world.

It's definitely nice to see these explanations. I feel like this is one of the only classes I've taken at MIT where I actually learn something practical and interesting each class that may be entirely unrelated to what I learned the last class.

Yes, and the airplane examples definitely back up the solutions by making intuitive sense!

Interesting, I wasn't sure where this question was going: if it was just comparing birds to themselves or perhaps airplanes

did not know that. I wonder how the new 787 will affect this. Isn't it supposed to fly slower but more efficiently?

Problem 10 Speed of a bar-tailed godwit

Use the results of Problem 8 and Problem 9 to write the ratio $v_{747}/v_{\text{godwit}}$ as a product of dimensionless factors, where v_{747} is the minimum-energy speed of a 747, and v_{godwit} is the minimum-energy speed of a bar-tailed godwit (i.e. its cruising speed). By estimating the dimensionless factors and their product, estimate the cruising speed of a bar-tailed godwit. [Useful information: $m_{\text{godwit}} \sim 0.4\,\text{kg}$; $v_{747} \sim 600\,\text{mph}$.]

$$10^{\square} \pm \square \ \text{m s}^{-1} \quad \textit{or} \quad 10^{\square} \ldots \square \ \text{m s}^{-1}$$

To include in the explanation box: Compare your result with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8.5 days.

Assuming that the animals and planes fly at the minimum-energy speed,

$$\frac{v_{747}}{v_{\text{godwit}}} = \left(\frac{\rho_{\text{high}}}{\rho_{\text{sea level}}}\right)^{-1/2} \times \left(\frac{m_{747}}{m_{\text{godwit}}}\right)^{1/6}.$$

A plane flies at around 10 km where the density is roughly one-third of the sea-level density. The mass of a 747 is roughly $4 \cdot 10^5$ kg, so the mass ratio between a 747 and a godwit is 10^6. Therefore, the speed ratio is roughly

$$\frac{v_{747}}{v_{\text{godwit}}} \sim (1/3)^{-1/2} \times (10^6)^{1/6} = \sqrt{3} \times 10 \sim 17.$$

A 747 flies at around 550 mph so the godwit should fly around 550/17 mph ∼ 32 mph. The actual speed of record-setting godwit is almost identical:

$$v_{\text{actual}} \sim \frac{11,570\,\text{km}}{8.5\,\text{days}} \times \frac{0.6\,\text{mi}}{1\,\text{km}} \times \frac{1\,\text{day}}{24\,\text{hours}} \sim 35\,\text{mph}.$$

is there a way to not use the results of previous problems that we may have gotten wrong, or didnt understand.

Why do we not need to include L^2in here?

I think L^2 was already taken into account when we calculated the exponent for the mass relationship (since we're assuming these flying things are geometrically similar).

I still think this is cool that a bird could fly so far without stopping...

Is there a way to format the psets to get around doing a question wrong or not understanding it? i.e. I knew my answers to the last two were wrong, so I kind of figured whatever I got here would be a lost cause, too.

That would seem to be part of the learning process. Questions that build on simpler ones are standard. Add more bounds to your confidence interval if you're not sure. Turn it back into an approximation more like we were doing earlier in the course.

What if the simple questions aren't simple to me? uh oh

I really liked this problem. I actually figure out these proportions!

Why do they fly a minimal height?

Yea, is the godwit flying at sea level? I thought migratory birds also took advantage of high-altitudes, where they can use strong tail-winds to minimize their energy expenditure.

Maybe because there isn't enough oxygen higher up in the atmosphere.

Where does this come from? I'm not following this at all...

See problems 8 and 9, in those problems we found how the cruising speed is proportional to the density of air and the mass of the flying object.

this comes from the previous parts–you just find the ratios of the velocities, and in the first two parts, we found how velocity depends on density and mass

My mistake in 9 killed me here when I used 1/2 instead of 1/6 as the exponent. I wound up with something under a meter per second, which I knew didn't make sense (although the godwit speed seems amazingly fast). I just couldn't figure out where my mistake was–it didn't occur to me that it might be in the proportionality.

I forgot to consider that the air density at different altitudes is different, so I just assumed the two density terms canceled out. Although I still got an answer in the same order of magnitude as the solution, it made a huge difference at the end when I estimated the speed of a godwit based on the ratio and the speed of the 747.

Air density differences at different altitudes is something I would just never have thought to consider if not for this class.

So I neglected the Lengths in problem 9, but incorporated Area here, as well... I was confused at first as to how my answer still came out right.

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this last bit of math doesn't seem like "approximation" to me.

I think root three is commonly used enough that a practiced approximator should know it.

Why not? sqrt(3) is 1.67, which is approximately 1.7. Are you saying you don't see how we got from (1/3)^-1/2 to sqrt(3)?

I completely forgot this factor and still got a reasonable answer.

that factor comes out to 1.73, so it wouldn't make a huge difference

Just curious, why is it you used 550 mph here instead of the 600 mph mentioned in the problem?

I wanted to use the speed that I used in lecture, and had forgotten that I had given a different speed in the problem statement. But what's a 10% speed difference among friends?!

Why use 550 here when the intro to this problem says v_747= 600 mph?

He probably just took the numbers from two sources, or forgot that he had included a number in the problem.

It's great that we can get approximations that are so close to the real world answers!

I especially liked this question because we used our results from the two previous questions to then do a calculation which was very close to the actual answer.

this is amazingly close- cool

i didn't get this answer because i got the previous one wrong.

....but if you got the process right it doesn't really matter at this point.