6.055J/2.038J (Spring 2010)

Solution set 5

Submit your answers and explanations online by ***10pm on Wednesday, 07 Apr 2010****.*

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Problem 1 9V battery

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I'll estimate it by working out the energy in my laptop battery, and then adjusting the estimate to compensate for the smaller size of a 9V battery. The energy in my laptop battery is the ideal candidate for divide and conquer: the power drawn by the laptop times the battery life. I know the battery life well: about 4 or 5 hours (I have the slightly larger 9-cell Thinkpad battery).

The power draw is harder to estimate. The screen, the CPU, and the disk drive probably use comparable amounts of power, since reducing the power consumption of each item seems to be comparably important in extending the battery's life. The methods include using a lower screen brightness, putting the CPU into idle (technically, C2 and C3 states), or spinning down the disk. The screen is an LCD screen, which is much more efficient than an incandescent (standard) light bulb. So, although it is bright like a (weak) light bulb, say a 30-watt bulb, it may draw only 5 or 10 W.

Three such items – which includes the disk drive and the CPU – add up to perhaps 20 W. (As a check, the Powertop utility that comes with my Debian GNU/Linux installation says that the laptop is using 16.6 W.)

The product of power and time is energy stored in the battery:

$$E_{\text{laptop}} \sim 20\,\text{W} \times 4.5\,\text{hours} \times \frac{3600\,\text{s}}{1\,\text{hour}} \sim 3 \cdot 10^5\,\text{J}. \tag{1}$$

Now let's shrink that energy to account for the smaller size of a 9V battery. As a simple method, I'll assume that all batteries have a comparable energy density (energy stored per mass). In mass, my laptop battery feels like about 15 or maybe 20 9V batteries. So I'll divide $3 \cdot 10^5$ J by 15 or 20:

$$E_{9\text{V}} \sim \frac{E_{\text{laptop}}}{15 \text{ or } 20} \sim 15\,\text{kJ}. \tag{2}$$

For a rough comparison with actual values, Wikipedia quotes 20 kJ as the energy stored in a 9V battery.

Global comments

did you go over what number density was in class?

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Here is solution set 5 (memo due Thu at 10pm).

The amount of time we have to do the annotations on these is irritatingly short. Sorry this one is a bit late.

I spent quite a while just sitting and trying to figure out a way to approach this question, are there any other ways that people can think of? The way that I chose failed miserably (Off by several orders of magnitude).

This is a much more intuitive, no sources needed, method of going about this problem.

I agree! I didn't think of using my laptop battery...

What's the simplest object you can use for this calculation?

Quite clever. I was at a bit of a loss as to how to do this one, as I couldn't quite recall any similar scenarios that would have been of help.

Thus, I relied on looking up values for battery life to fill that missing parameter.

I used my laptop as well, but mine only last a few hours...time to get a new one I gues...

Also, I have the larger of the two Thinkpad laptop batteries (the 9-cell rather than the 6-cell). The 6-cell lasted only about 3 hours.

I didn't want to look anything up so I avoided using actual durations/power consumptions.

So the only method I could think of was to estimate the number of atoms in the battery, assuming an Angstrom is about the order of dimension for an atom, and then use 1 electron per atom as the total charge stored. (1/2 Q * V) yielded an upper bound on the energy of 10ˆ7. I knew this was high, so I intuitively centered my range around 10ˆ4.

I used a flashlight instead to do the calculation. Sometimes it's easier to choose small appliances.

Yeah...I based my calculations of my old gameboy color.

I assumed it was equal to 2 AA's. This gave me 10kJ, only 30% below this estimate's result. Fairly consistent.

I tried a similar approach using an alarm clock. It had the number of watts it ran on (4) printed on the bottom. I overestimated how long it would last on the backup battery enough to throw my answer off by a factor of 100.

This is a really good idea, I don't know why I didn't think of using something we know the life of so well.

Also, I know personally that my 4-year-old laptop only has a battery life of around an hour, although it used to be closer to 3 or 4. so i wouldn't know which value to use..1hr would feel more natural but i'm pretty sure it would give me the wrong answer

I feel like this is a serious issue with using the laptop battery time...rechargeable batteries ware out.

I did not think to use my laptop battery either.

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I didn't think of comparing to another kind of battery, but this nicely brings scaling and divide and conquer into play.

agreed. i had pretty mich no idea how to solve this one

This is a good way to do this - I didn't realize that all batteries had similar energy density. I used and RC car and looked at how far and fast itgoes

I tried comparing it to a smaller device that used AA batteries, but I at least ended up within a order of magnitude of the answer.

We used a similar method except using something that actually used a 9v battery, a smoke detector. The box of a smoke detector tells how much power it uses and we know the batteries are changed once a year.

i thought i smoke detectors we're powered by some sort of radiation?

Perhaps this is just me, and while this is a perfectly reasonable way to address the problem, what does it have to do with dimensional analysis/easy cases (what we've been doing in class these past few weeks)?

I think he's just making sure we stay up on our divide and conquer skills.

I agree, I find myself using totally different approaches to solve these problems from what we've learned recently in class.

yeah, I think it might be helpful to indicate what approach is "easiest" to use when attacking the problems. If your goal is for us to remember all the approaches and pick which one ourselves, then maybe label a section as "all approaches" and then for those that are using the most recent material you can label it as such

I dunno, I found it most intuitive to use divide and conquer, but the comments on the homework almost mislead me to think that it was "wrong" and I should be using dimensional analysis. I think we all fall into the trap of assuming we're supposed to do the problem in a set way.

Wow this seems so intuitive! I was scrambling around my desk for batteries..

Something you can try on a non-Mac laptop is to pop out the battery and check the mAh rating on the battery.

I used a lightbulb, which i think is a lot easier to calculate with than a laptop- i definitely did not know a laptop uses about 20W

How would you know how long a 9V can power a light bulb?

I think estimating how long the light bulb would last is about as difficult as estimating how much power is used up by the laptop. Does anyone know of any devices that they have a good intuition for how long they last and also for how much power they output?

Since my laptop battery isn't very good anymore, I used my cell phone battery. Seemed like the most logical choice. Hopefully people didn't have trouble coming up with an example to get this problem rolling.

between choosing to work with a lightbulb or a laptop, it seems that you're trading ease in estimating time (but not power) for ease in estimating power (but not time).

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This was an interesting way to approach the problem. I don't know enough about the power consumption of my computer to have used it myself, but it was fun to see the problem worked out this way.

I agree. the way i solved this was much shakier than this solution. If i had known all these details about my computer i wouldve been so straightforward

do you need to go through all of this thought to figure it out? isn't there an easier method that requires less guessing?

I was also surprised by the amount of calculations that was done for this problem. I feel like the memos cover much easier problems (or they're in smaller chunks), but the homeworks are incredibly difficult to do.

So I know we've used really rough estimates like this before, but I feel like this one wasn't really supported by a really strong argument. The fact that it added up to about the right, power just seems like a coincidence.

I agree with this statement. I tried to estimate how much power a smoke detector uses, and I had no idea what was a good estimate. This resulted in a final answer that was way off.

Perhaps it would be helpful just to let us know what 1W can power...

I had to look up the fact that most small batteries draw about 10 W

My guess is that smoke detectors have a very low power consumption. They have no moving parts and almost no lights (the only one in our apartment's smoke detector is a small red LED), and probably just use a very low-power infrared beam as part of the detector. Also, you want them to be designed to last a long time on one battery, because otherwise people get annoyed and just turn them off completely, which is dangerous. What is the actual power consumption?

I have a really hard time having a feel for power consumption. even light bulbs have such a wide range.

How does the 9V manifest itself in this equation?

He compares the mass of his battery in his laptop to that of a 9V battery, assuming it is about 15-20 times the size.

I tried to use the time that I thought a 9V battery could power a 100 watt light bulb. Which from the numbers looks like maybe half an hour.

I forgot to do this.

I tried thinking of it as like a parallel plate capacitor with a separation the length of the battery, but when i divided V/L, I got a much smaller answer. Why is this so far off?

I don't think that's the right equation for the energy stored in a capacitor. Also, it's hard to know what the internal geometry might be. Similarly sized batteries can have different capacities.

A battery is sort of like an accordian of capacitors, I believe. Having an accordian gives all of the capacitors a very small distance between the plates, allowing a higher capacitance. This would be useful if you knew how many folded capacitors there were inside the battery.

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It's amazing how well this works out, definitely an interesting way to approach the problem

how can you just say - oh this feels like 20 9V batteries?

Yeah, this seemed like the most "gut-following" problem yet.

I'm confused why we can divide by volts when that unit wasn't in the original estimation. Or does 20W = 15*9V or something like that?

We're not dividing by volts, but by a number. I think he's assuming that voltage is proportional to energy, so the energy for 20 9V batteries divided by 20 will produce the energy for 1 9V battery.

Interesting idea! I used my cell phone's battery life as the comparative benchmark instead.

I ended up with 2x this by using the same ideas, I guess it highlights the idea of approximations to me :)

15kJ seems much more reasonable – I tried to do some estimates based on basic chemistry and ended up being off by about 10^4

I actually got this almost exactly by approximating 6 AAA batteries.

For this problem I got about the same answer but used what I know about physics equations to derive an equation that combined what I know- I ended up using that the change in energy= voltage*area

Cool, that's what I got

I found something that said 9V batteries used 560 mAh, and I just used that. Is there a way to really consistently estimate this one? I've done it before, but I always use a device an try to guess; I think it's hard ot guess that laptop battery uses a certain number of watts (I think mine uses 60 actually, but does much faster), and also hard to guess how many 9V in a laptop battery.

I also used mAh...however, I remembered (from some 9V work that i did earlier this semester) finding that a standard 9V has about 600mAh. then I messed up the math. whoops.

Funny that Wikipedia is mentioned here... I am always told Wikipedia is not a reliable source.

Never question the all powerful might of Wikipedia. It's second to Google in knowing random information.

while i'm personally okay that wikipedia is mentioned here, id on't know if it'd be a good idea if you're planning to publish this book. especially since wikipedia changes all the time.

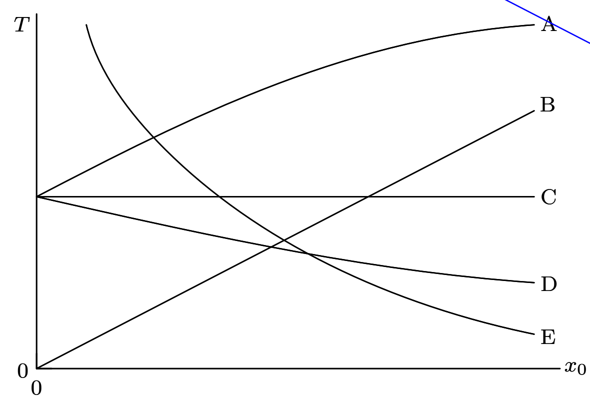
I was close to this, but I think I underestimated what it was capable of because the object I was considering was not one I was familiar with.

This problem wasn't really associated with what we were doing in lecture. Was this here to brush up our memory?

Right, it was a reminder of everyday estimations and divide-and-conquer reasoning.

Problem 2 Non-Hooke's law spring

Imagine a mass connected to a spring with force law $F = Cx^3$ (instead of the usual Hooke's law behavior $F = kx$) and therefore potential energy $V \sim Cx^4$ (where C is a constant). Which curve shows how the system's oscillation period T depends on the amplitude x_0?



☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

☐ Curve E

Dimensional analysis:

T	period	T
x_0	amplitude	L
C	spring constant	$ML^{-2}T^{-2}$
m	mass	M

The trickiest entry in the table is the dimensions of C. Since Cx^3 is a force, C has dimensions of force over length cubed, namely $ML^{-2}T^{-2}$. These four quantities made out of three dimensions produce one independent dimensionless group. Its simplest form is $Cx_0^2T^2/m$. Because there is only one dimensionless group, it must be a constant. In other words, $T \propto 1/x_0$. The only matching curve is curve E.

Comments on page 2

It was really cool revisiting this problem. I remember on the diagnostic I was totally lost, but looking at it again, dimensional analysis gave me the correct answer. It feels good to apply what we've learned to what we've seen before!

Interesting spring, do these exist by any chance?

I guess in reality, spring constants don't apply since springs aren't actually perfectly constant in tension acorss the entire spring.

Didn't we get this problem for the diagnostics? I thought it was cool to come back.

Yes, I think it was on the diagnostic. I agree, it was really nice to see this one come back and be able to solve it much more quickly.

Just curious, why do you specify the potential energy when the force equation is enough to find the dimensionless group? It threw me off a bit because I thought I should be doing something more complicated.

This way of doing it makes so much sense! I had no idea how to do it and made it much more complicated than it actually is and still didn't get the right answer!!

Yea, I had no idea how to do this problem on the pretest but it is quite simple using dimensional analysis!

Agreed!...although I wasn't too confident that I was doing it right since it seemed way too easy. Guess I am learning something in this class.

Oh... it completely slipped through my mind that I was supposed to use dimensional analysis.

I confused myself into knots on this one, and now feel very foolish for not figuring it out on my own.

So I did use dimensional analysis this time, but on the pre-test I found it easiest to use a comparison to a linear spring, which we know gives a constant T.

At large x0, this non-linear spring has a larger restoring force, so it should have a smaller period. At small x0, the restoring force is less, so T should be bigger. Curve 'C' represents the linear model, and the curve that satisfies this comparative relationship is E!

At first I tried to compare it to a linear spring, but found it too confusing to map between the two equations. For this problem, I definitely found that dimensional analysis was the easiest way to go

When I saw this problem I quickly jumped to physical intuition first using exactly the same reasoning as you. I like using dimensional analysis when using physical reasoning doesn't work.

I think this problem should be in the reading as an example - it was the first problem we did where I needed to use DA and it fits so well.

I might even use it in two sections. The 'compare with a linear spring' method, mentioned in these comments, is a nice example of easy cases reasoning. So, this example could be done twice, once in dimensions and once in easy cases.

Just to be a bastard, maybe I should make a new choice that is monotonic decreasing, like curves D and E, but that starts above curve C (e.g. twice curve D everywhere). Then one has to reason quite carefully about the low-amplitude extreme case to decide between this new choice and E.

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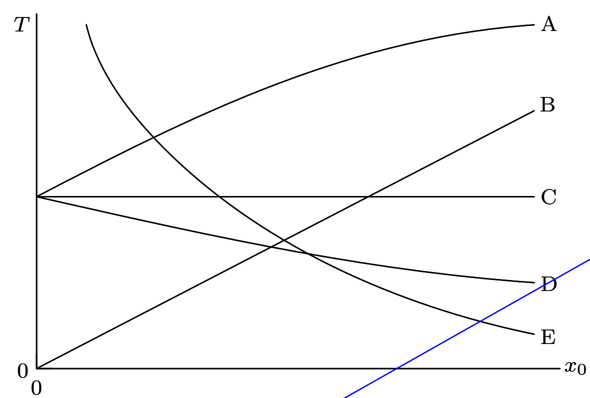

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I realized that here we basically have 2 equations for force, so I set them equal to each other giving L^3=MLT^-2. When I solved I got T proportional to T^-1. I'm trying to decide if I just got lucky.

I liked how this problem was easy to check by trying out the case I knew the answer to - namely the case when F = Cx.

I just thought that as the amplitude increases the period should decrease, and somehow my "gut" told me that it should be an exponential decrease rather than linear

This is a great approach! I was going to try and use simple cases to see if I could then apply that but this is easier.

I had a hard time trying to apply dimensional analysis for this problem, so I ended up trying to think about it in a different way and got the answer completely wrong!

> I would have too, but looked at the review of dimensional analysis for the standard Hooke's Law that was in the notes (I forget where).

> I thought my work using dimensional analysis was a bit too simple but it turned out to be correct. This was a very good example of how the process is used.

I liked this problem - you had to think about the proper units for C and that was quite helpful.

> I never thought to try and find the dimensions of C, for some reason I assumed that C must be a constant. This problem makes a lot more sense now

> Yeah, I had a complete brain fart on this quesiton–I totally messed up on the dimensions of C. this makes everything so much clearer.

>> Same here...which is funny, because (in hindsight) I remember working through units of C on the diagnostic.

I had just assumed it would be constant like a normal spring

this isn't that tricky because we were given the relation, so we automatically know the dimensions.

this helps a lot- I was so confused about how to approach this- is there any way to help us decide what technique to use to solve the problem?

How do you know there is only one dimensionless group? I feel like it's easy to combine dimensions of the variables until they cancel out, but how can you be sure you aren't missing another grouping?

> The way to figure out how many dimensionless groups there are is explained in either r20 or r21

Conceptually why does the curve E near infinity when the displacement is almost 0. This is what tore me away from E

> the period is very very fast (nearly undetectable) when there is basically no displacement of the "spring"

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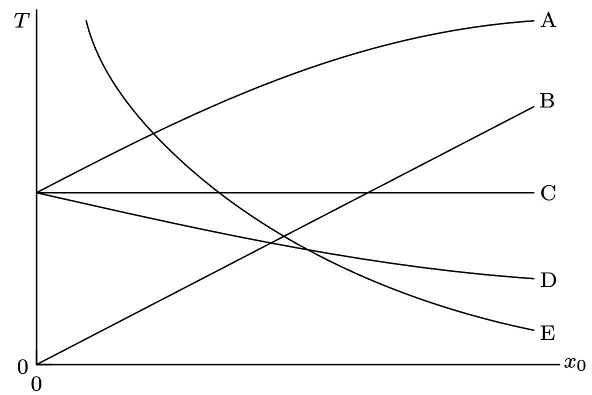

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The trickiest entry in the table is the dimensions of C. Since Cx^3 is a force, C has dimensions of force over length cubed, namely $ML^{-2}T^{-2}$. These four quantities made out of three dimensions produce one independent dimensionless group. Its simplest form is $Cx_0^2T^2/m$. Because there is only one dimensionless group, it must be a constant. In other words, $T \propto 1/x_0$. The only matching curve is curve E.

I found this problem very difficult to think through conceptually. This method really helped me figure it out.

Wow..I too had pages of calculations. I didn't realize dimensional analysis would make it so much simpler!

yeah, I tried just comparing dimensions used in the regular hooke's law to the dimensions used in the new equation. I knew the formula for the period was T (m/k)ˆ.5 ; i just looked at the dimensions and found out what C had to be. then found out how to make the period equation work. It ended up being dependent on 1/x

Yeah i didn't understand the "physics" of it either, but you don't need to if you use this method. That's a good thing to know.

I think that's the whole beauty of dimensional analysis. You don't need to know anything about the physics so long as you know the important parameters and their dimensions.

I found it difficult too, but after talking to a friend in the class it became easier - I feel like a lot of these are easier to approach when you bounce ideas off others/the problem set's nb page.

I used easy cases to figure out that this was the right curve once I got T 1/Xo. I used "If Xo goes to zero... T should go to infinity" And the only curve that fits that is E.

That makes sense.

what about at Xo = 0, shouldn't T = 0 too?

No, as stated above, as you approach xo = 0, T approaches infinity. The period at xo =0 is sort of a tricky case, since there's no motion. You could think of having to wait an infinite amount of time for an oscillation to happen since it's not moving.

Is that considered using easy cases? Its just thinking about the limits of the function...

That is indeed easy cases. As you noticed, in this problem the limiting cases are easy cases to think about.

I arrived at the same answer here but got lost with my dimensions working on my spring constant C. I'm not sure if I got lucky with my reasoning but it's good to see it completely done out this way

I did pretty much the same thing but got the wrong answer! That's unfortunate. Even in estimation, it makes sense to check work!

How is the period inversely proportional to the distance. This implies the period would be really long for a short change in Xo. This seems really counter intuitive....

Agreed, I got the right answer, but it was totally not what I expected.

It is a bit surprising. Think of it this way: The spring is very weak at small displacements (the xˆ3 in the force law makes the force go to zero very fast), so it can hardly make the mass move.

Problem 3 Power radiated by an accelerating charge

If the velocity and acceleration of a (nonrelativistic) electric charge are doubled, how does the power radiated by the charge change?

- ☐ The power increases by a factor of 16.
- ☐ The power increases by a factor of 8.
- ☐ The power increases by a factor of 4.
- ☐ The power increases by a factor of 2.
- ☐ The power increases by a factor of $\sqrt{2}$.

The first question is: On what does the radiated power depend? First, c (the speed of light), because that is the speed at which the power travels; second, the charge's acceleration or velocity (or both); and third, the charge itself q. We probably also need ϵ_0, the horrible constant when using SI units for electromagnetism. But ϵ_0 and q will show up only together as $q^2/4\pi\epsilon_0$, so let's combine those two quantities accordingly.

The remaining question is what to include from among acceleration or velocity. If the power radiated depended on the velocity, then we could use relativity to make a perpetual motion machine: Generate more energy simply by using a different reference frame, one moving with just the right velocity. No way! So, the power depends on the acceleration but not the velocity.

The list of variables, including the radiated power, is:

P	radiated power	ML^2T^{-3}
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c	speed of light	LT^{-1}
a	acceleration	MLT^{-2}

These four variables, again with three dimensions, result in one independent dimensionless group–for example,

$$\Pi_1 \equiv \frac{P}{q^2/4\pi\epsilon_0}\frac{c^3}{a^2}. \quad (3)$$

With only one group, it must be a constant, so

$$P \sim q^2/4\pi\epsilon_0 \frac{a^2}{c^3}. \quad (4)$$

Except for needing a factor of 2/3, this result is correct (the full result is called the Larmor formula).

To answer the particular problem, doubling the velocity and acceleration quadruples the power radiated.

Problem 4 Local black hole

What is roughly the largest radius the earth could have, with its current mass, and be a black hole (i.e. light cannot escape from its surface)?

$10^{\square} \pm \square$ m *or* $10^{\square} \ldots \square$ m

Comments on page 3

Was this from the pretest?

I do think something similar appeared on the pretest.

What does this mean?

I think that he means the net velocity and acceleration, as opposed to individual charge velocity and acceleration. I think.

I just ignored it. However I got the wrong answer so maybe I shouldn't have...

He means that the charge is not moving at speeds close to the speed of light.

Then why did he use the speed of light in his dimensional analysis?

since it's dimensional analysis, it's not the number part (which would give you how close to the speed of light you actually are) that matters, it's the units (which are the same no matter what the number is)

I was confused because I don't know what this even is talking about. Basically, why is the charge radiating power? Is this quantum physics or is it light or something else?

In classical electromagnetism, an electron orbits a proton and is constantly acceleration towards the proton. As the electron is orbiting and falling closer to the proton, it loses energy, which radiated.

I was really lost here because I'm not familiar with power radiation, so I made a complete guess based on the dimensions of power. Of course, we've been in situations before where we have no intuition but dimensional analysis helps us reason about the problem. How do you determine when you have enough knowledge about the problem to make a reasonable approximation?

How do we know whether we need it or not?

I think this just has to come with experience with physics- no guessing for this one!

That's a more tricky jump to make. I don't think I would ever know to do that.

I agree. I wouldn't know to do this. I suppose it helps to make things simpler, but I wouldn't know if it would cause me to skew the answer.

I had trouble jumping to this. I reasoned we needed q, but could not think of anything to cancel the current dimension.

I left the q and the 4*pi out. Honestly, I didn't even know I was supposed to put them in. I don't know enough about EM to do so. But the 4*pi is just a constant and the q doesn't contribute any dimensions to the dimensional analysis, so why is it necessary to use anything but the e_0?

I'm not good at physics and I don't use it in my field, was it ok for me to use the internet to find out that epsilon is part of this problem? I get frustrated that I have no idea what extra variables I'm expected to know about.

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This whole simplification requires a pretty intimate familiarity with electromagnetism. Though I guess next time I see an E&M problem, I'll know to put epsilon_0 in.

Agreed, I felt like I didn't know enough about EM to be able to properly understand this problem. I know this class is supposed to teach us approximation in a variety of fields that we may not be familiar with, but I felt like the solution to this problem hinged on your knowledge of a particular subject.

I didn;t have a good enough understanding to assume this

I feel like this explanation would be more clear if you explicitly said power radiated by the charge should be the same regardless of our frame of reference. Therefore, velocity cannot influence power radiated.

damn, got that wrong

I suppose you'd have to know this equation to get the right answer.

Yes, I don't particularly like this problem as much as other dimensional analysis problems because this expression requires some previous knowledge.

I did not use dimensional analysis and instead just basic physics, which i think in this case makes more sense!

I did this too, and I think it was probably much easier and faster.

yeah I just broke it down to power=Force*velocity=Mass*acceleration*velocity. If you double acceleration and velocity then you quadruple the power.

I am still having trouble coming up with the group elements in these problems...

yeah...my main problem with this question was coming up with dimensions for radiated power...

I used v instead of c (so I had the same dimensions) but my answer was totally wrong. I should have noticed that I was missing c...

I used dimensional analysis and groups to get P mva. or some form of mv^2t ma^2/t and just figured that regardless, it was going to be 4x. My approach fails miserably if it was only the velocity though.

I followed the same reasoning, but we would also have been wrong if the other crazy parameter had taken care of one of the L/T or L/T^2 factors, so that P was proportional just a^1.

This should be LT^-2... no M.

You're right. At least that mistake didn't propagate (the subsequent formulas are okay).

I totally confused radiated power for electric power.

Yeah, I'm not sure if that's a completely wrong assumption to make, but I used P = IV and got the correct answer...

I pulled up too many equations too determine the variables involved in radiating power before i got down to looking for independent groups. Ended up getting more variables than i needed.

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instead could we not use: P = Force * velocity, force = mas* accelerations; therefore doubleing both gets; P = 4*mass*a*v

I got this problem wrong because I doubled velocity and the speed of light. I knew something was fishy about it...

This is really cool. I already knew the Larmour formula before doing this problem, so it wasn't maybe the most enlightening problem, but it's comforting to know that if I ever need to re-derive it I can get pretty close just using dimensional analysis.

I again used physical reasoning that I gained from my E&M classes, so i guess its unfair. But I intuited that doubling the velocity/acceleration doubles the number of electrons flowing or doubling the amount of current. And power is proportional to current squared, thus quadrupling the power.

I did this in a much simpler way and got the same answer- I modified the power equation P=W/t= F*d/t= (ma)(vt)/t= mav.

Same here, only I just remembered P=F*v=mav. So although this gets the same answer, you get 4 because of a^2 but no dependence on v, whereas this answer depends on a and v. The two equations would seem to be mutually exclusive (P cannot be both proportional to a*v and a^2), so was this just pure luck that it worked out?

I did this is a simpler way and got the same answer

I got to this answer by totally negleting the q^2/(4*pi*epsilonnot). speed*acceleration*mass make power.

I still don't see how doubling velocity affects the power, or does it?

It doesn't have any effect. Only doubling the acceleration affects the power.

I'm confused - I said P=F*v and got the right answer. How does velocity not affect power?

I think I got this with a lot less work. I 'spose it was just a lucky guess then?

I think I took too simplified of an approach to solve this problem.

That sounds dangerous. ;)

What is the mass of a black hole generally? Is it comparable to earth? I know so little about this.

Don't black holes have almost no mass?

this was a really fun question to think about!

In gravity problems, the quantity GM/Rc^2 is dimensionless. For most objects, it is very tiny. A likely candidate criterion for a black hole is when this quantity is around 1. Therefore, the radius of the black-hole earth would be

$$R \sim \frac{GM}{c^2}. \qquad (5)$$

Putting in numbers,

$$R \sim \frac{7 \cdot 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2} \times 6 \cdot 10^{24}\,\text{kg}}{10^{17}\,\text{m}^2\,\text{s}^{-2}} \sim 4\,\text{mm}. \qquad (6)$$

(The true black-hole radius, based on general-relativity calculations, is twice this value based on dimensional analysis and a bit of guessing.)

Problem 5 Wire

Roughly what is the number density of free (conduction) electrons in a copper wire?

$10^{\square} \pm \square\ \text{m}^{-3}$ *or* $10^{\square} \ldots \square\ \text{m}^{-3}$

Copper has, as a guess, one conduction electron per atom. Each atom occupies roughly a 3-Angstrom cube – i.e., a volume of roughly $3 \cdot 10^{-29}\,\text{m}^3$. The number density is the reciprocal of the atomic volume (since there is only one conduction electron per atom). Thus $n \sim 3 \cdot 10^{28}\,\text{m}^{-3}$. (The true value is roughly 2.5 times greater, due to the atomic volume being slightly smaller than I estimated here using the 3-Angstrom rule of thumb.)

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Use dimensional analysis to work out the relation between radius R, time t, blast energy E, and air density ρ. Then use the data in the table to estimate the blast energy E:

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Four quantities composed of three independent dimensions make one independent dimensionless group. The only quantities whose dimensions contain mass are E and ρ. So E and ρ appear in the group as E/ρ, whose dimensions are L^5T^{-2}. Therefore the following choice is dimensionless:

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With only one dimensionless group, the most general statement connecting those four quantities is

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$:

Comments on page 4

is this the speed of light? how do you know that we have to use this constant

I plugged in c for v because a black hole doesn't let light escape.

And yes, c is the speed of light.

I still a little confused by this problem. What is the equivalent of GM/Rc^2. The number 4 mm seems a bit small to me. Am I thinking of this in a wrong manner?

I didn't know how to go about this problem in the homework....seeing the solution clarifies it. I had a feeling that escape velocity was a good method, but I have no knowledge of black holes and assumed my hunch would be completely wrong

Why is that quantity dimensionless? I couldn't solve this problem. I used F = GMm/R^2 but don't know how we arrived at this.

I figured this out after reading through reading 25 and remembering that this is a dimensionless group equal to the angle of the bended light.

I ended up going a different way around to dimensionless groups because I didn't see what/why I should set the quantity to.

Why is this true?

Dunno. I assumed that theta (bending of light at the surface) 1, instead. it put me sooo far off.

1 radian shouldn't have put you too far off. I used pi/2 and wasn't that far off. But I agree, where is this logic coming from about GM/Rc^2 being about 1?

Is that from the light bending when the constant was 1, 2, or 4 depending on the model of gravity? And we assume 1 because its the most simple case?

Is that from the light bending when the constant was 1, 2, or 4 depending on the model of gravity? And we assume 1 because its the most simple case?

I used pi/2 also which I suppose is close enough to 1?

Yay I did it just like this!

Me too!!...although I only did it because I didn't know what else to do and the only problem I could even slightly compare to it was the light bending problem. I don't know if that's useful or not...

I did this problem using the light bending, and the given 30 degree = light cant escape, and I got a very similar answer.

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

Sweet.

This is what I did:

The gravitational force equation is F = -mMG/r^2 In order to escape a black hole from radius R, a mass m must be moving with KE = 1/2 m v^2 that is equivalent to the work done by gravity as the mass moves from R to infinity. (The mass has just enough kinetic energy to come to decelerate to zero velocity at infinity)

Integrating, we set GM/R = 1/2 m v^2

Thus, R = 2 GM/v^2

In this case, v = c for light so

R = 8.873 x 10^-3 m = 10^-2 /pm 0.5 m

This seemed fishy to me, because the theory used a mass that had kinetic energy. But light doesn't have a mass... so substituting in lightspeed at the end made me uncertain.

I had no idea what the dimensionless group was until I saw the solution, and then I also saw this in reading 25 which jogged my memory that this quantity is angle of the bended light, which is why it's dimensionless.

I actually also used a different approach and got the same answer. I used the approach that we used in class when calculating how light bends around the sun. I just extended the analogy for the case when light bends 180 degrees and "can't escape"

yippie! I got one!

I used the same method but I got about 1.5 mm...

It's still within an order of magnitude. Besides, there was probably some variation in estimating division. At least your process was correct.

Used the wrong "G"

It might help to point out this is what would give the enormous density required for a black hole.

I used divide and conquer and got an answer in the same ballpark as this. Awesome

I find this result very interesting.

Having very little experience with black holes outside of a few SciFi movies (which often portray black holes as large, vast entities), this problem helped me gain a little bit more tangible intuition for similar problems.

Yeah, I was expecting some kilometers or something. The answer here really surprised me.

I used a similar approach, but came up with a different answer (probably because of my estimations). I got around 1 m and thought that was way too small–guess not!

I came up with this as well and figured it was completely wrong, but put it down anyway. Upon second glance, I trust my reasoning! This is a very cool problem.

I got it too, but did not trust my gut and completely changed my answer....,

Did you get 1 meter by initially assuming acceleration due to gravity needed to equal the speed of light? I did that at first before really thinking about it and realizing that light would still escape.

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I think that is really cool to think about. it having to be so small.

I came up with this same answer using the same method, but I was so surprised by how small it was I thought I had made a mistake.

I'm having a hard time identifying when dimensional analysis would be useful. It didn't occur to me in this problem, or the spring-mass problem.

me too and as a result i didn't end up using dimensional analysis for several of these problems

Which is called the Schwarzschild radius.

That's what I used.

That's what I used.

i've never heard of this...

I'm so proud of myself that my answer was really close to that.

I thought this problem was a bit tricky, and kept searching for a way to figure it out using the methods we have recently learned. I guess dimensions are always a good way at looking at a problem.

I like how this problem tied in with the stuff we have been doing in class, such as the work with hydrogen.

I was having a hard time understanding what is really meant physically

Is it valid to just use the density of copper and the atomic mass instead of the Angstrom stuff?

Is this really a guess? Don't we know that it has "one free electron" in the outermost ring?

On the diagnostic I just assumed this, but then I thought about it more and thought, that's why it's such a good conductor, so I had more justification for one conduction e- per atom. Similarly, gold also has one e- in the outermost shell.

I thought it was two.

In fact, Wikipedia says copper 2+ is the most stable state, despite having 1+, 3+ and even 4+ states.

In any case, that will only make us be off by a factor of 2, which in this class is hardly worth worrying about.

I think those are oxidation states in reacting with other elements, not the number of conduction or valence electrons/atom (1) in solid copper.

Confused about this as well. I just took what wikipedia said and ran with it.

Instead of using angstroms, I just used the density of copper and divided by its molar mass to get a volume/mol. Using this I still got about 10^28 electrons per unit volume

I did the same thing. I like the size of the atom better as it doesn't require looking up numbers that I didn't already know.

yes. i'm very surprised that avagadro's number didn't come up in your equations. i thought it'd be the easiest way to get myself in roughly the correct order of magnitude.

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Putting in numbers,

$$R \sim \frac{7 \cdot 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \times 6 \cdot 10^{24}\,\mathrm{kg}}{10^{17}\,\mathrm{m^2\,s^{-2}}} \sim 4\,\mathrm{mm}. \quad (6)$$

(The true black-hole radius, based on general-relativity calculations, is twice this value based on dimensional analysis and a bit of guessing.)

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After talking with a friend I realized this but I was definitely a bit rusty on my chemistry going into this problem

I thought copper has 29 electrons per atom? Then I used that fact to say that the number of conductive electrons is between 0 and 29, and used that for my estimation (10ˆ1 +- .5).

I may have cheated and used wikipedia... but anyway you just need to know how many electrons are in the outer shell.

Oops I didn't distinguish between electrons and conduction electrons...

For some reason, I thought copper had 2, since I usually see it as "Cu 2+"

Yeah, why is it just one?

I tried using simple geometric shapes, but I never got anything close to 3A. How exactly do you get that number?

isnt a copper ion normally +2?

I thought I had this one nailed. Gotta work on my error budget calculatioins.

wow, this solution is so much simpler than I thought. I was doing a bunch of crazy calculations. If i had directly gone to using atomic volume then it wouldve made a lot more sense

can we go over this in class... I'm unfamiliar with much of this.

I got this answer just by guessing essentially. Either I'm lucky or my estimating sense is getting better.

Blame your gut. I mean, intuition.

This problem is much easier than I made it out to be, and for some reason, it just goes over my head.

I got something close to the answer, but still don't really get it...

I ended up with 10ˆ30 is that close enough?

I took the same approach of estimating how much space the atoms take up but where did you get the volume from? how did you know how thick a wire to choose?

I did something similar, but had to look up the numbers- most times I can't even estimate the numbers, because I don't know enough background info

I agree, and sometimes if i'm going to look up the value i could just look up the answer.

It's a lot more accurate if you use the known density of copper as well as the molar mass. Though I suppose looking up these values is not the preferred method for the class.

This can already be explained by copper having 2+ as its most common oxidation state. In which case, there would be 2 free electrons per atom.

Wow, this solution looks so simple compared to the numerous calculations I did. My method was completely wong

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I used a pretty different method. I calculated the weight of one copper atom based on its number of protons and neutrons (29 each), then used copper's density to estimate the number of copper atoms per meter cubed of copper. My mistake was not understanding what "free" electrons were so I multiplied by 29 to find the number of electrons per meter cubed.

I also used this method. I looked up the atomic mass and density of copper and with Avogadro's number, calculated the number of copper atoms in a cubic meter. Then I multiplied that by 2 since a copper atom has 2 valence electrons.

I liked this problem as well - very helpful for understanding how to apply dimensional analysis.

I liked this question because its basic dimensional analysis but we have actual data to plug in to the equation we get, so its not really estimation.

I agree, I thought this and the first were the best problems of the set. And they were somewhat related.

This problem was one of the most difficult for me to approach that we have seen in this class. It really just left me confused, seeing the solution is a great thing and I appreciate the option and will try to remember it for the future.

Funny!

I'm really glad i got the dimensions to this problem.

not sure how you chose this...is this a common equation?

You use the dimensional analysis techniques discussed - the solutions just don't list the table (but the same information is represented in the paragraph of text above)

I used this since it was the only thing that worked, but i thought it had to be wrong because R^5 is so large an exponent!

yeah I had the same problem too just that we barely see such a high power in the problems presented in class

I worked on this problems for almost an hour trying to find a solution that didn't involve R^5.

This is where I have a complaint.

How are we supposed to know to pick 1 as the arbitrary constant? Depending on what we pick, the energy will be arbitrarily large or small.

most of the constants we deal with are on the magnitude of 1, which seems like the safest place to start anyways. If you find out later (this constant is def. possible to look up), you can easily alter your answer, which could be confusing if you picked something like 1289

I did not see how to take the jump to equating the dimensionless group to a constant, but I do see how it would be hard to pick anything but 1 since, 1 is such a nice number.

This otherwise horrible problem was so straight forward with dimensional analysis. Absolutely delightful!

I was fairly surprised by (and skeptical of) the R^5 term when I was working this out.

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Wouldn't it be more accurate to use (R2-R1)/(t2-t1) for velocity as opposed to R/t?

this is a good point since we do have the data points – but i'm sure it doesn't affect it much.

This result is very exciting. You couldn't suggest it right off the bat, but dimensional analysis spits it right out.

Yay!!! I did this the same way!!!

t (ms)	R (m)	E (10^{13} J)
3.26	59.0	6.7
4.61	67.3	6.5
15.0	106.5	6.1
62.0	185.0	5.6

The data are not perfectly consistent about the predicted blast energy E, but they hover pretty closely around $6 \cdot 10^{13}$ J.

This blast energy, expressed in more common units for such devices, is roughly 15 kilotons of TNT – in close agreement with the then-classified value of 20 kilotons. Dimensional analysis triumphs again!

I missed that this was ms instead of s. Threw me off by 10ˆ6.

I just realized I made the same mistake.

I caught myself when I thought about what this measurement is saying. Although 10ˆ7 J passes the "makes sense" test for me, an explosion taking 3 seconds to expand 59 m does not. (Think about it counting to 3 Mississippi.)

I wasn't really sure if at this point I should average them or what. Why are they different? Where is the energy going?

At this point, they are all the same order of magnitude and maybe the difference is just experimental error.

I originally thought it meant that as time progressed, the radius got larger, and possible energy is being dissipated.

I was confused about this too. Perhaps clarification should be offered in the problem.

I agree. I wasn't entirely sure what to do with the table either.

I took that chart as how the "shock" from the blast traveled with respect to time. At t=0, the bomb goes off. As t increases, the blast radius would increase as the "shock" traveled outward. From there, we could estimate E of the "shock" over time and distance. I took E decreasing as energy dissipated.

Similar to what they said a little lower. The order of magnitude stayed the same. I took the decrease to be roughly due to a dissipation of energy.

I really liked having 4 data points here. Seeing that there was clearly a similarity in the values of E I got for each point really helped me sense that I was on the right track.

I really liked doing this problem...it made sense, but still needed thinking.

So close...forgot to change ms to s and got 10ˆ7 instead.

I did the same thing! I was so proud of my finding it was Rˆ5 and I didn't even see the time scale. I wonder how many people did that.

Yeah I made that mistake too. So many units to keep track of in this problem..

Oh, ha! That explains why my answer is so off!

Nice, me too!

Just curious, what exactly is the ratio of kilotons of TNT to energy. Does TNT always have the same energy density?

Whoa! This just sounds scary, altough an atomic blast always has on it's own too.

That is pretty cool.

This is really cool how close it got.... and a lot of TNT...

Triumphs over the US government?