6.055J/2.038J (Spring 2010)

Solution set 7

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Problem 1 No need to remember lots of constants

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$$\frac{e^2/4\pi\epsilon_0}{\hbar c} \equiv \alpha \approx \frac{1}{137} \quad \text{(fine-structure constant).}$$

Use those values to evaluate the energy of a visible-light photon. Note: The photon energy is $E = hf$, where h is Planck's constant and f is the frequency of the radiation; equivalently, $E = \hbar\omega$, where $\hbar = h/2\pi$ and ω is the angular frequency of the radiation.

$10^{\square} \pm \square$ eV ***or*** $10^{\square} \ldots \square$ eV

$$E = hf = 2\pi\hbar f = 2\pi\hbar\frac{c}{\lambda},$$

where f is its frequency and λ is its wavelength. For green light, $\lambda \sim 600\,\text{nm}$, so

$$E \sim \frac{\overbrace{6}^{2\pi} \times \overbrace{200\,\text{eV nm}}^{\hbar c}}{\underbrace{600\,\text{nm}}_{\lambda}} \sim 2\,\text{eV}.$$

Global comments

the the kinematic viscosity of air one of the numbers we should know?

It was on the table of useful constants linked from the course website.

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COMMENTS ON PAGE 1

Here is the solution set – memo due on Sunday at 10pm.

I realize I am inconsistent with my approximations especially with this pset. Sometimes, I do specific calculations and it seems to be throwing my answers wayy off. should we totally stray away from anything specific even if we are able to do the complex calculations in our head?

Personally, I thought this pset was a lot tougher than previous ones. There were several problems that I just had no idea how to address. Also, I feel like the pset problems have changed. In the beginning of the class, there were problems ranging across many topics that we solved by breaking them down into information we knew or could find out. But now all the problems are heavily focused on physics and are solved by digging through the notes to find the appropriate equation. I'm not getting anything out of this, and it took me significantly longer to do this pset than usual.

The differences in the psets most likely have something to do with the topics we are learning. When we were learning divide and conquer it was considerably easier to be able to break down a problem. On the other hand, I agree with you about not really getting anything out of this and taking much longer on this pset. Somewhere down the line, everything did seem to change into equations.

Wholeheartedly agree. It's cool to learn about some of the things using physics as examples, but also harder if you're not too familiar with all of it...and ends up with me losing interest or getting confused.

I too agree this pset was a lot tougher and does seem to be testing our knowledge of the equations we derived and not necessarily our ability to derive them through approximation. I feel like I can't "talk to my gut" for these problems since I lack a lot of intuition for the physical systems we're discussion. I understand physics is probably the most common use of approximation, but I'm really starting to get bogged down in the examples.

Totally missed these units!

Yeah me too. My answer was ridiculously high because of it.

I found it confusing that we were given all these values here, when only the first one was actually relevant to the first problem. I spent a while trying to figure out a way to use all of them in the first problem before doing the problem the way I knew how to do it and ignoring all of the values.

I agree. I approached it the same way that he did but I couldn't figure out how to resolve the frequency term. I thought that you were supposed to only use some combination of the given constants.

These are very good numbers to remember, though I'm not sure I would store the fine-structure onstant in my brain. Seems just too arbitrary.

This problems make me dig through my 5.111 to remember the equations!

But you still need to memorize these values... so what's the point?

The point is that these numbers are easier to remember than the exact values.

Some of these numbers are exact, they are just easier to remember in combination. For instance, I always remember c but can't necessarily remember the wavelength of something like a radio wave. But I can think of the frequency and then do the math, like we had to do here with hc.

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what exactly does that mean?

I didn't think we needed a 2 pi term if we used E=hf?

It's not hf though, it's h-bar times f. h-bar is h/2pi.

I liked this problem, but what was the approximation methods we had to apply? It seemed mostly like algebra and plugging in numbers.

I too am a little confused where approximation is used here (other than reducing the number things we have to memorize). I didn't really like this problem, since I was able to arrive at a reasonable answer by just blindly re-arranging the equations, but don't really have a good grasp of all the relationships used in the problem.

wow i totally missed this part, with the different h's. i cant believe it–this makes so much more sense now

Same. The writing is hard to read though; I could only see the difference at 151%.

I feel stupid now. That was something I easily could have done in high school.

This was reminiscent of something in high school for me - what I'm confused about is what this has to do with the later material we've seen. In other words, what tools did we not have at the beginning of the term that prevented us from doing this problem?

As I understand it, the lesson here was lumping of the constants into easy to remember values. At the beginning of the semester, we might have been tempted to solve for each constant individually and plug it into the equation.

I eventually arrived at this formula, but I think I did a bit more arithmetic first. Perhaps it would be useful to show all the steps.

These problems seemed a lot more daunting than they actually turned out to be. I thought we didnt need 2pi if using E=hf and c=f*wavelength

that's not Plank's constant. That's "h bar" which equals h/2pi.

Idon't even remember equations like this one, but as soon as someone mentions it I can figure out what to do with it. Its frustrating that I can't just recall these simple equations.

Why did we assume and use the wavelength of green light? I was confused about the value so I just used the first one I saw while looking online.

Visible light varies from 380 nm (purple) to 750 nm (red). He picked a number that was within the range and that made multiplication easier.

It was really helpful for me that I remembered the range of visible light.. otherwise I think I would have been totally lost at this point!

This would have been nice to include in the original problem I think - although it does test some estimation skills. I was way off...

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this is the piece of information i was missing! i was trying to do the problem by rearranging terms and doing dimensional analysis and wasn't getting too far.

I tried that method too, then realized that I must be missing something b/c it was taking so long!

Agreed. I knew the range of visible light, but I assumed that we weren't expected to use it and ended up wasting a lot of time trying to combine the given terms. I think the way this problem was presented made it seem like those three pieces of data were all necessary for the problem.

i mean.. he did say "visible light", and this class has sort of taught us to just go ahead and pick an easy-to-multiply value.

I agree that the "visible light" was the real clue. He doesn't often (if ever) give us information we don't need to know. So when you see things like this, you can assume it serves some purpose. You should also realize that different frequencies of light must carry different energies, so the frequency must come in to play somehow.

When you said, "Don't look anything up", what if we didn't know this value?

Well, I think you should know the wavelength of SOMETHING in the visual spectrum. As long as you pick a wavelength that's visible, you're okay, since the question asked for just a visible photon. Remembering the visual range isn't that hard.

everyone learns this in 5.11x or 3.091!

which was 3 years ago for some of us!

i took 3.091 and never had to memorize it!

A good fact to know is that visible light is in the range of400-700 nm. I think it's a good takeaway from any class involving optics, no matter your major.

Why did you choose to use green light?

I would imagine because green light is about in the middle of the visible spectrum.

That's why I chose green.

I didn't really remember which wavelength corresponded to which color, but I remembered from high school that visible light goes from 400nm to 700nm. For lambda, I decided to choose the middle value of 550.

I forgot f = c/lamda silly me

same here. i also wouldn't have known the wavelength for visible light off the top of my head. this was a strange question.

Thats exactly how I did it! :)

This would explain why Sanjoy told us not to spend too much time on these questions. These approximations shouldn't be that overly complicated. I can't believe I forgot that equation.

Yeah, I also missed that. It completely threw off my answer. Additionally, I was unhappy that there was too much information; I couldn't remember that throwing out information was ok. Eventually I just said screw it and used just some of it, thinking I had to use all.

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Wow, I've spent the last hour trying to figure out why my calculations were so astronomically wrong when I realized I was in joules. Fail.

This explanation really helps make it clear. I guess it makes sense to use light in the middle of the visible range

This problem was pretty self explanatory

This makes perfect sense; I have no idea why I didn't do it this way.

I was way way off on this problem...I confused h and h_bar, silly.

The difference is a factor of 2*pi, so you should be off by less than an order of magnitude.

Yeah wow, I must have seriously messed up something - I used the same equation (forgot about the 2pi) and got like 10ˆ9......oops.

I did it correctly, but it seems so odd to have such a large number from a single photon! also, how would we estimate the range, given that we don't know the exactly values the above constant clusters are off by?

For some reason I had a lot of trouble with this problem. I think I was just making it more complicated than I needed to.

Problem 2 Boundary-layer thickness

How thick is the boundary layer on a golf ball traveling at, say, $v \sim 40\,\mathrm{m\,s^{-1}}$?

$10^{\square} \pm \square$ m *or* $10^{\square} \ldots \square$ m

The thickness δ is roughly $\sqrt{\nu t}$, where ν is the kinematic viscosity of air, and t is the time for air to travel a distance comparable to r, the radius of the golf ball. So

$$\delta \sim \sqrt{\frac{\nu r}{v}}, \qquad (1)$$

A golf ball has a diameter of about 5 cm so $r \sim 2$ cm. The kinematic viscosity of air is $\nu \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$. So

$$\delta \sim \sqrt{\frac{10^{-5}\,\mathrm{m^2\,s^{-1}} \times 2 \cdot 10^{-2}\,\mathrm{m}}{4 \cdot 10^{1}\,\mathrm{m\,s^{-1}}}} \sim 10^{-4}\,\mathrm{m} \qquad (2)$$

(after neglecting a factor of 0.7). Compared to this thickness, the dimples are plenty thick enough to trip the boundary layer into turbulence and thereby reduce the drag coefficient.

Comments on page 2

Is this actually how fast a golf ball goes??

You could estimate it pretty easily. I drive about 200 meters, and the ball is usually in the air about 5 seconds. That's 40m/s right there.

I liked that this problem was in the pset, right after we did it in lecture

Sad that I missed lecture :(

Was it meant to matter if the golf ball was dimpled to begin with? I calculated it ignoring the dimpling, but now I see that that comes up in the solution notes. If the BL is disturbed by dimples, is it still the same size as the BL calculated for a smooth ball?

maybe it's just cause i'm a golfer- But I love doing calculations about golf and learning more about the physics of it. maybe you can put more sports examples in because those I always find the most interesting

how do you know it's comparable to r?

He discusses this point on R27 (last paragraph of second page). It's how he deals with the time component, by making t (v/r) where v is velocity of the ball and r is it's radius.

i was thinking a distance like the distance the golf ball was traveling. bad error...

I thought I had to incorporate Reynolds number into this..

I wonder if there was a way to set this problem up such that we could see the difference between dimples and no dimples

There's no "dimples" parameter in this equation, so we're not concerning ourselves with dimples or not. Where do you see a dependence on dimples/no dimples?

oh I know that dimples are not involved in this question. I'm saying that since we learned about why there are dimples on golf balls, it would be cool to see a problem in which we solve how much it changes.

Didn't we see this in the reading?

I re-read the readings a few times but I still don't really understand what the significance of the dimples either. How would that change this problem?

They make the flow go a bit turbulent earlier so the boundary layer stays attached till further back on the ball. That being said, I have no idea how that would change this answer! I think the layer thickness might stay the same or close to the same, it just holds on longer?

Damn, I think I switched the V's by accident (velocity and viscosity)

I wasn't sure which v was which, so I used dimensional analysis to figure it out.

I forgot the square root. Damn.

I think I did too. 10^-9?

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(after neglecting a factor of 0.7). Compared to this thickness, the dimples are plenty thick enough to trip the boundary layer into turbulence and thereby reduce the drag coefficient.

Oh I ended up using the wrong equation.. I think I calculated delta/r...

I did that too–i also didn't know the kinematic viscosity of air. so i directly made a ballpark estimate of reynolds number, saying 1000. then the reciprocal of the square root of that was ratio of boundary layer to radius. boundary layer to radius = 1/100, so using 2 cm, the boundary layer is around 1*10^-4 m

I ended up using this equation and then double checking with a reynolds number guess

Whoa...I totally used the wrong equation...I used delta = Re*kin. viscosity / velocity, and got a bogus answer of .35 m...I thought I was doing something wrong!

I just realized I used the wrong Re, I used the actual Re, not the boundary layer Re. If I used Re (bl) = sqrt(Re), then I got the correct answer! yay!

Darn units!!!

Me too. I used a very wrong value for this.

Ahh... I used this value as my radius... What an overestimation of the size of a golf ball!!

i think i screwed up and used the diameter for r. silly mistakes :(

I got this problem right but did it using the inverse of the Reynolds# (I got it from the readings somewhere) and the radius ...guess it worked out

Yeah that's the method I used to solve this problem. It also worked out for me.

Since the boundary layer readings were more confusing, my work on this was more guessing that knowledge, but I still think I got the right order of magnitude. Sweet!

So I also looked up this equation in wiki because at first I couldn't find it on the lecture notes and it has a value of 5 multipled to the equation. that confused me...

Woah, I got this number even though I was using dramatically different assumptions. Pretty cool how that all works out.

hmm... off by a factor of 5. I used a similar method and similar numbers. bad math probably.

how'd you go about it?

i forgot to take the square root! i hate it when i get problems wrong for stupid mistakes like this. only 2.005 test, i missed a question because i said that 2+.5+2=2.5

Is that the limiting constraint when designing boundary layer obstructions, that they have to be bigger than the boundary layer thickness?

I believe so. Intuitively, if something is smaller than the thing it's trying to block, it won't block it.

Makes perfect sense to me.

So are there golf regulations for how deep/how many dimples the golf ball can have?

Also, would super deep dimples actually make that much difference? Like once it's reached a certain point, is there any difference having a deeper dimple?

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I'm glad you added this sentence. It seems like a good sanity/order of magnitude check

Good to see this works out again!

Yep, I totally had thunk (in the beginning of hte year) that the dimples were there just for show or tradition!

yippie! I got one :)

Feels good to get one right!

Got the right answer, I think. Pretty fascinating stuff, really. It's good to put the answer back in perspective, relating the number back to the size of the dimples - I forgot to think about that after finishing the problem.

Problem 3 Viscous versus form drag

The form drag (drag due to moving fluid aside) is

$$F_d \sim \rho v^2 A. \quad (3)$$

The viscous (skin-friction) drag is

$$F_\nu \sim \rho\nu \times \text{surface area} \times \text{velocity gradient}, \quad (4)$$

where $\rho\nu$ is the dynamic viscosity η. The velocity gradient is v/δ, where v is the flow speed, and δ is the boundary-layer thickness.

The ratio F_d/F_ν is dimensionless, and must therefore be a function of the only dimensionless measure of the flow, namely the Reynolds number Re. In fact, the function is a power law:

$$\frac{F_d}{F_\nu} \sim \mathrm{Re}^n, \quad (5)$$

where n is the scaling exponent. What is n?

☐ ± ☐ *or* ☐ … ☐

Using v/δ as the velocity gradient and A as the surface area, the skin-friction drag becomes

$$F_\nu \sim \rho\nu A \frac{v}{\delta}. \quad (6)$$

Therefore, the ratio of drag forces is

$$\frac{F_d}{F_\nu} \sim \frac{\rho v^2 A_{cs}}{\rho\nu A v/\delta'} \quad (7)$$

where A_{cs} is the cross-sectional area. For objects that are not too elongated (e.g. not a long train), the surface and cross-sectional areas are comparable. Then the areas in the numerator and denominator cancel out. Additionally, the factors of ρ and one factor of v also cancel. What's left is

$$\frac{F_d}{F_\nu} \sim \frac{v\delta}{\nu}. \quad (8)$$

From the reading (`r27-lumping-boundary-layers`), $\delta \sim r/\sqrt{\mathrm{Re}}$, so

$$\frac{F_d}{F_\nu} \sim \frac{vr}{\nu} \times \mathrm{Re}^{-1/2}. \quad (9)$$

The fraction vr/ν is the Reynolds number, so

$$\frac{F_d}{F_\nu} \sim \mathrm{Re}^{1/2}. \quad (10)$$

Thus, $n = 1/2$.

For most everyday flows, $\mathrm{Re} \gg 1$. Thus, most of the drag is form drag rather than skin-friction drag. The exception to this rule is very long objects (freight trains), where the surface area is much greater than the cross-sectional area.

Comments on page 3

can you explain more what viscous drag is? I had a lot of problems with this problem just because I didn't really understand what this was.

I like these scaling problems. I think it's a cool way to look at things.

wow i forgot the "gradient" part

Wow I did this totally wrong...I think I just couldnt keep track of v versus nu, my paper was a mess of things that kinda looked like "v"

definitely the different v's made this problem take longer for me than it would have otherwise

I agree, even knowing how the equations should look I was still a little confused by the two

Also agree, I thought I had it figured out when I went back into the readings, but I kept on subst. the wrong v for one another in my rearranging...

Yeah it definitely poses a problem. But the best thing to do in this case would be to zoom into the page pretty significantly to make sure that they're different variables.

i guessed this just to make things easier, and im glad it worked out :)

I didn't realize you could make this approximation..this would have made things a lot easier!

I was stuck on this for a while as well, and made the approximation for simplicity. How much error does this introduce?

I agree. I think this assumption is crucial to solving this problem.

Yeah, this fact definitely is necessary to simplify the problem.

It took me a while to figure this out. I finally decided it was okay, after trying to determine a good ratio (such as 1/2 r) was too complicated to put back into Re.

I got down to this part with no problems, but couldn't get it to relate to the Reynolds number. This way makes sense though.

I actually canceled too many terms because I couldn't tell between the v's and the nu's. It's a messy and careless mistake, but one that could have been avoided with better typesetting.

Whoops, I think I thought this value *was* the Reynold's number, from another point in the reading. Now I'm confused why it isn't...

yeah, I did the same thing. I think its because that would give us the reynold number of the flow inside the boundary layer as opposed to for the object overall...

I think my problem was that I accidentally focused only the del and neglected velocity and viscosity. Thus my entire estimation was overly simplified to F_d/F_v del. That would explain why my answer was n=-0.5

Likewise. Is it wrong to assume this is a Reynold's number of sorts? Doin it this way does make the next question a lot easier...

even after the note to look at boundary layers (and re-reading that part), I really didn't get this one...however, this explanation was _very_ helpful. Thank you

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The viscous (skin-friction) drag is

$$F_v \sim \rho\nu \times \text{surface area} \times \text{velocity gradient}, \quad (4)$$

where $\rho\nu$ is the dynamic viscosity η. The velocity gradient is v/δ, where v is the flow speed, and δ is the boundary-layer thickness.

The ratio F_d/F_v is dimensionless, and must therefore be a function of the only dimensionless measure of the flow, namely the Reynolds number Re. In fact, the function is a power law:

$$\frac{F_d}{F_v} \sim \text{Re}^n, \quad (5)$$

where n is the scaling exponent. What is n?

☐ ± ☐ *or* ☐ . . . ☐

Using v/δ as the velocity gradient and A as the surface area, the skin-friction drag becomes

$$F_v \sim \rho\nu A\frac{v}{\delta}. \quad (6)$$

Therefore, the ratio of drag forces is

$$\frac{F_d}{F_v} \sim \frac{\rho v^2 A_{cs}}{\rho\nu A v/\delta}, \quad (7)$$

where A_{cs} is the cross-sectional area. For objects that are not too elongated (e.g. not a long train), the surface and cross-sectional areas are comparable. Then the areas in the numerator and denominator cancel out. Additionally, the factors of ρ and one factor of v also cancel. What's left is

$$\frac{F_d}{F_v} \sim \frac{v\delta}{\nu}. \quad (8)$$

From the reading (`r27-lumping-boundary-layers`), $\delta \sim r/\sqrt{\text{Re}}$, so

$$\frac{F_d}{F_v} \sim \frac{vr}{\nu} \times \text{Re}^{-1/2}. \quad (9)$$

The fraction vr/ν is the Reynolds number, so

$$\frac{F_d}{F_v} \sim \text{Re}^{1/2}. \quad (10)$$

Thus, $n = 1/2$.

For most everyday flows, Re $\gg$ 1. Thus, most of the drag is form drag rather than skin-friction drag. The exception to this rule is very long objects (freight trains), where the surface area is much greater than the cross-sectional area.

ahh, I used this for the previous problem but didn't even think to use it here!

I know. I got a lot of the canceling out through A A and the v, but I completely forgot to use the definition of d for this. It would've made this a cinch.

tricky

I remember seeing this in the reading!

I left it as Re^ -1/2. I forgot about the additional portion.

I'm just a little upset by the variables involved: nu and v, they both look exactly alike, I think at one point or another when squinting at the section notes or here, i mixed up the two and ended up with a bad equation. I wish we could've used more distinctive letters for these.

Yea, I confused these and it messed me up. That is a rather unnecessary thing to get bogged down on.

Shouldn't this be = Re^-.5 instead of *Re^-.5?

No, the delta is replaced by the r/Re^.5 so instead of dividing by Re^.5, you multiply by Re^(-.5), which is mathematically equivalent.

I understand this answer, but I would argue n = 1 is also correct because the problem did not specify which Reynolds number. Re^(1/2) is correct for Reynolds number based on the radius. However, Re^1 is correct for Reynolds number based on delta, which is also a commonly used parameter.

Yeah I got n=1 also.

Ohhh oops, I totally missed this part. Tricky to put the Reynolds number in there.

I agree, I saw this in the reading but wasn't sure when you could use the Re estimate. Why wasn't it used in the gold ball example?

That's why I still had a factor of r. I went back and used the equation I used in problem 2 which complicated things more for me.

Nice! This one made me happy to get right.

I got -1/2 for this problem.. my gut told me it was +1/2.. Like we learned in class, intuition seems to be the best bet!

I was also debating between whether it should be negative or not, but ended up leaving it at negative. Should have trusted my intuition also...

Same here, I went with intuition but am glad to see where it was right.

I got this one wrong, I somehow got n = 1 after manipulating the variables, i didn't realize delta r/sqrt(Re)

Wasn't this very similar to something we did out in one of the memos?

Man, I missed the square root part of the problem.

I really don't know what technique/principal this problem is trying to illustrate

Problem 3 Viscous versus form drag

The form drag (drag due to moving fluid aside) is

$$F_d \sim \rho v^2 A. \quad (3)$$

The viscous (skin-friction) drag is

$$F_\nu \sim \rho\nu \times \text{surface area} \times \text{velocity gradient}, \quad (4)$$

where $\rho\nu$ is the dynamic viscosity η. The velocity gradient is v/δ, where v is the flow speed, and δ is the boundary-layer thickness.

The ratio F_d/F_ν is dimensionless, and must therefore be a function of the only dimensionless measure of the flow, namely the Reynolds number Re. In fact, the function is a power law:

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where n is the scaling exponent. What is n?

☐ ± ☐ *or* ☐ ... ☐

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$$F_\nu \sim \rho\nu A\frac{v}{\delta}. \quad (6)$$

Therefore, the ratio of drag forces is

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The fraction vr/ν is the Reynolds number, so

$$\frac{F_d}{F_\nu} \sim \text{Re}^{1/2}. \quad (10)$$

Thus, $n = 1/2$.

For most everyday flows, Re ≫ 1. Thus, most of the drag is form drag rather than skin-friction drag. The exception to this rule is very long objects (freight trains), where the surface area is much greater than the cross-sectional area.

Got something right finally

I used this idea to set the exponent, to something greater than one but I guess i didn't consider that Re is really big.

It's good to actually see this painted out for us, as it confirms what we probably already knew intuitively.

I like how these problems were inter-related. It helped me feel assured of my method as I went through the problems. However, The problem with this is that if you miss the first problem, it is difficult to move forward with the next dependent problem

Problem 4 Viscous versus form drag while walking
Use the result of **Problem 3** to estimate the ratio

$$\frac{\text{form drag}}{\text{viscous drag}} = \frac{F_d}{F_v} \quad (11)$$

for a person walking.

$10^{\square} \pm \square$ *or* $10^{\square} \ldots \square$

The ratio is roughly the square root of the Reynolds number, where

$$\text{Re} \sim \frac{\text{size} \times \text{speed}}{\text{kinematic viscosity}}. \quad (12)$$

For a person, the size is roughly $r \sim 1\,\text{m}$ (using the geometric mean of 2 m for the height and 0.5 m for the width). For walking, $v \sim 1.5\,\text{m}\,\text{s}^{-1}$. The viscosity of air is, conveniently, $1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}$, so the Reynolds number is roughly

$$\text{Re} \sim \frac{1\,\text{m} \times 1.5\,\text{m}\,\text{s}^{-1}}{1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}} \sim 10^5. \quad (13)$$

The square root is $10^{2.5}$ or 300. Form drag, which is mostly independent of viscosity, is the big source of drag.

Comments on page 4

So we could have done it either way, right? That's a good way to check how accurate our approximations are.

I like this a lot - most of the approximations I feel more comfortable when there's another way to check it.

i didnt realize this problem was really just asking us to calculate the re #

Oh, I didn't even think of this. I kind of just picked a number in between, and used 1 for convenience.

Oops. That's not the way I calculated size.

oops. I was thinking in feet and knew that the "radius" of a human was approx three feet, but then I wrote it down as 3m, though this did not really affect my answer!

I got this partwrong. Ruined my entire equation.

Same here...I always forget to use kinematic viscosity, and I think I accidently used the wrong viscosity.

I've always wondered if it's kosher that so many numbers were calibrated on every day objects, like the density of water being a convenient number like 1,000, for instance. I know it all balances out, but it seems fishy, you know?

What is this based off of?

I think most people can walk 3-4 miles per hour, which is roughly 1.5 m/s.

I didn't even think of using the results from the last problem. I actually used the equations for the forces to solve it. Obviously I didn't read the entire question or just skimmed it.It clearly says to use the result from the last problem...

I did this problem correctly but my n exponent was wrong from Problem 3.

Same with me. Although, I believe my answer was just a factor of 10 off, which isn't SOO bad.

I was also a factor of 10 off. Now i see my mistake...

Awesome to see we got the same answer here, and how much easier this problem is with the information from problem 3!

I agree, these two problems work very well together explaining how you can combine these formulas to answer questions.

Dang, since I missed calculated problem 3, my answer for this question was way off. I guess this is my issue with relying on possibly wrong estimates, it carries over elsewhere.

yeah, this problem set was more frustrating for me than others because a bunch of the problems relied on correct answers from other problems, which I didn't always get.

From my mistakes in Problem 2, I accident got the inverse of the correct answer.

Yea I agree with that. Maybe there is a way in the future to have us rely on concepts from other questions but not necessarily the exact numbers.

Problem 4 Viscous versus form drag while walking

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The square root is $10^{2.5}$ or 300. Form drag, which is mostly independent of viscosity, is the big source of drag.

It's important to note that you're assuming all the constants of proportionality (ratio of areas, drag coefficients, etc.) to cancel out to about 1.

I got the right concept, that form drag is the major source of drag (over viscous drag), but I messed up on the first part, and didn't get the factor of 1/2, so my value here was 10^5

Again, I understand this solution, but I don't know why the way I did it doesn't work. From before, Fd/Fv = v*delta/nu. Then I used delta = (v*r/nu)^1/2.

Problem 5 Rolling down the plane
Four objects, made of identical steel, roll down an 30-degree inclined plane without slipping. The large objects have three times the radius of the small objects. Which object has the greatest acceleration?

- ☐ a large spherical shell
- ☐ a large disc
- ☐ a small solid sphere
- ☐ a small ring

The goal is to find the acceleration a along the plane. It depends on g, θ (which is 30° here), the object's moment of inertia I, its mass m, and the rolling radius r.

$$a = f(g, \theta, I, m, r).$$

Use dimensional analysis to simplify this function of five variables. The six variables in total make three independent dimensionless groups:

$$\frac{a}{g} \quad \theta \quad \frac{I}{mr^2}.$$

Therefore,

$$\frac{a}{g} = f\left(\theta, \frac{I}{mr^2}\right). \tag{14}$$

Probably

$$\frac{a}{g} = f\left(\frac{I}{mr^2}\right)\sin\theta.$$

$$a = f\left(\frac{I}{mr^2}\right) g \sin\theta.$$

The ratio I/mr^2 is a dimensionless measure of where the mass of an object lies. The farther toward the edge and away from the rolling axis, the greater the ratio. Most importantly, I/mr^2 is independent of an object's radius; for example, a big and a small ring have the same ratio.

The bigger the ratio, the bigger the fraction of energy consumed by rolling motion compared to translational motion. Therefore, the object with the smallest I/mr^2 will have the greatest acceleration. The solid sphere (choice C) has the most mass near the rolling axis, so it will be the fastest.

Comments on page 5

I thought this problem was pretty intuitive. The most dense and aerodynamic object should have the greatest acceleration.

If they all weighed the same, would the ring be the fastest?

The size didn't end up making a difference?

I find that when I see problems I've solved before I jump right into the math. keep the fresh problems coming.

Aha...there's the variable I missed. My attempt at dimensionless analysis totally failed without it.

yeah thats the problem with dimensional analysis.

I thought we were supposed to consider drag in this problem... oops!

same! intuitively i would think that the large disc would be fastest because it would have the largest ratio of mass:drag force.

I didn't even think to use dimensional analysis here, I just went straight for Inertia and neglected g, m, theta etc. Got the right answer too

Same for me. I went from torque=I*alpha, and after making simplifications using proportionality, got down to acceleration was inversely proportional to radius ˆ4....I found that hard to believe, but it does give you the right answer (you also get that acc is proportional to mass, so figuring same radius amongst the small object, you know that the solid sphere must have more mass than the ring)

I did the same, a quick flip back to 2.003 gave me the right answer in a couple seconds, although it's good to see how we can use dimensional analysis here to arrive at the same conclusion.

I did the same thing and found it effective.

I also didn't think of using dimensional analysis for this problem. However, since I'm not course 2, I didn't have any notes or information from previous classes to help me, and I just had to go with my "gut".

I managed to get the correct answer without using a, g, or theta. I used dimensional analysis to find a relationship between I, m, and r and from there used reasoning. Though I didn't get the "same answer", it's nice to know my approach was right.

I thought this one was actually simpler than this. I forgot about angular acceleration...

I thought this was a reynolds number problem instead of a general dimensional analysis problem. Still I got the right answer from intuition.

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I didn't even use these equations, I just did it by calculating which mass was greatest- which I thought would have the greatest acceleration

Same although I considered the SA for air friction as well.

I agree... I was looking for some sort of relation to something we have done recently and attempted to analyze it via Re forces.

I had no idea how to approach this problem via dimensional analysis (which I see now), but when I did it I used tools learned in high school physics and in 8.01.

Same here. I basically used the equations F=ma and I=mr^2.

Agreed. Moment of inertia and Rotational Kinematics could also get you to the answer of this problem.

I didn't even go into all this. I just looked at what had the most concentrated mass

so you don't need to know what f is? how you know whether or not a/g is positively correlated to 1/m^2? meaning as 1/m^2increases, a/g increases? can't it be the other way around

I actually used a different set of variables here – I used the actual radius of the object and the average radius of its mass distribution, which relates to its moment of inertia.

I thought we were supposed to use cross-sectional area and look at drag resistance. That seemed in the subject of what the pset was about

That's what I used, and I assumed that mass was a little more important than air resistance, so I got the right answer.

This is true. The focus on drag in the beginning of the homework might have thrown several people on the wrong track.

So is I proportional to r^2? I don't see how the ratio is independent of r if there's an r^2 in the denominator.

I think I=mr^2, so maybe they cancel out?

I=c*mr^2 for most simple objects (more specifically, solids of revolution about the axis of rotation), where c is a constant depending on geometry.

See here: http://en.wikipedia.org/wiki/List_of_moments_of_inertia

I didn't think to consider the ratio. I just said that since the acceleration will depend on how far the mass is from the rolling axis the object with the most mass the furthest away will roll faster.

I also said that the object with the most mass the furthest away will roll faster....however, it seems to me that's not how it worked? or does the small solid sphere really have more mass farther from the center than any of the others?

what is?

I've lost track of how many times I've seen experiments proving this fact, and it still seems counter-intuitive to me.

Agreed, but because of that I tend to remember it

I agree. I pulled the same mistake again too! I definitely don't know why it's so counter-intuitive.

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I for some reason did not even think to apply energy here...

I think the energy balance is more clear. Assuming you know enough about the moment of inertia, you can see why one would roll faster.

I looked up moments of inertia for different objects... I is always proportional to mr^2, but there are different constants for different shapes (ring, disc, solid sphere).

I like this explanation more than the 2.003 one. it puts it more in a physical context

I identified this but somehow convinced myself the ring was better...should've thought about it a little harder.

This is basically what I did. The explanation is clear.

I notice there is no consideration for drag here. Is that simply being lumped in with acceleration?

Interesting; this makes sense when I imagine it visually, but it's pretty hard to reason intuitively unless you find all the variables for dimensionless analysis.

I actually reasoned it intuitively only and didnn't do the math. The way I thought about it is: 1) you want a lot of mass so you get more force downward 2) at the same time you don't want it to have a large radius that will cause it to roll slower ie. have more radial inertia.

I did the same thing, but then I got stuck on how the ring seemedl ike it would have the least drag, but the sphere had the lowest moment of inertia.

@sat1:03 – Awesome way to think about it! Thanks

I answered this based on intuition instead of doing out the math.

I had the whole concept flipped. This explanation makes a lot of sense.

sweet, again my intuition worked.

haha, my intuition was totally wrong–i approached this from a drag perspective and said the solid disc...but this makes sense–with more mass near the center/rolling axis, more energy goes into translational motion

Glad I remembered 2.004 well enough to do this problem successfully.

I'm sure there are a lot of comments similar to mine...but I totally did this wrong...didn't even think of using dimensional analysis. I need to remember to use all the techniques!

Problem 6 Hydrogen binding energy
In lecture and readings we analyzed hydrogen (`r26-lumping-hydrogen.pdf` on NB), which is one electron bound to one proton. Using those results, one can show that the binding energy is

$$E \sim \frac{1}{2} m_e (\alpha c)^2, \tag{15}$$

where α is the fine-structure constant, c is the speed of light, and m_e is the mass of the electron.

Use the methods of **Problem 1** to calculate the binding energy in electron–volts.

$10^{\square} \pm \square$ eV ***or*** $10^{\square} \ldots \square$ eV

Rearranging the powers of c,

$$E \sim \frac{1}{2} \times m_e c^2 \times \alpha^2. \tag{16}$$

Because $\alpha \approx 1/137$, which is roughly $1/141$,

$$\alpha^2 \sim \frac{1}{1.41 \cdot 10^2} \sim \frac{1}{2} \cdot 10^{-4}. \tag{17}$$

Since $m_e c^2 \sim 0.5 \cdot 10^6$ eV, the binding energy is

$$E \sim \frac{1}{2} \times \frac{1}{2} \cdot 10^6 \text{ eV} \times \frac{1}{2} \cdot 10^{-4} \sim \frac{1}{8} \cdot 10^2 \text{ eV}. \tag{18}$$

The result is 13 eV.

Comments on page 6

This problem was nice and easy once you realized that it was a simple manipulation of variables!

Haha I forgot to square alpha, no wonder my answer was high....silly mistake

Ah crap. Same. =(

forgot the 1/2 damn

same here. crap.

I'm not really sure I understand how this helps?

Oh nevermind, because when you square it you get 2. It's not immediately obvious though, so you might want to make a note.

Ah, that is clever. It took me a little bit to get this, but it definitely makes the math easier.

Another option is to make this just 10^2, and compensate for that by calling .25 * 10^6 = "few" * 10^5. You end up getting a very similar answer, and definitely within when given the bounds.

yeah, i also converted it to a few * 10^4.

Definitely a clever trick, I did not think of this.

me neither, but good thing the problem didn't hinge on it.

Yea, I just arbitrarily rounded while doing these calculations and got a close answer.

It's the fine-structure constant again!

I didn't really think to simplify the numbers this early in the stage.

I forgot to square the alpha term, which threw off my answer by a factor of 10^2. I did the problem correctly but messed up the easiest step!

I used the method explained here, but i got a major overestimate. Its probably due to me just arbitrarily picking values instead of thinking through them and figure out stuff like the 1/141.

I don't really see how this is a useful exercise. Sure, we're seeing how those 3 values from problem 1 can be used rather than the individual constants, but otherwise this is just a plug and chug problem.

I agree, I was surprised that this problem was so simple, I feel most of the problems on these psets the hardest thing is finding the right equations to use

I think it's an exercise to show you how much easier the problem becomes when you know a few groups of constants.

Also, it's an exercise on re-arranging variables in a way that makes it convenient and simpler.

Maybe he was just being nice? He said the pset had more problems but they were simple

It also builds up to the next couple problems.

it was also good practice for quick, approximating-type math

i ignored all those 1/2s but they clearly add up - oops!

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The result is 13 eV.

i ignored all those 1/2s but they clearly add up - oops!

what is this problem trying to show us?

Problem 7 Heavy nuclei

In this problem you study the innermost electron in an atom with many protons (i.e. with a heavy nucleus). So, imagine a nucleus with Z protons around which orbits just one electron. Let $E(Z)$ be the binding energy. The case $Z = 1$ (**Problem 6**) is hydrogen.

Find how $E(Z)$ depends on Z. Namely, what is the scaling exponent n in

$$E(Z) \propto Z^n \tag{19}$$

or, equivalently, in

$$\frac{E(Z)}{E(1)} = Z^n\,? \tag{20}$$

$\square \pm \square$ *or* $\square \ldots \square$

With Z protons pulling on one electron, the electrostatic energy contains the factor $Ze^2/4\pi\epsilon_0$. So instead of using $e^2/4\pi\epsilon_0$ as one quantity in the dimensional analysis, we should use $Ze^2/4\pi\epsilon_0$. The other quantities – a_Z, m_e, and $\hbar$ – are unchanged except for a_Z replacing a_0. The Z propagates along with the e^2 through the calculation of the radius a_Z and the energy $E(Z)$.

Since the radius a_0 has one factor of e^2 in the denominator, the a_Z picks up a factor of Z in the denominator relative to a_0. Therefore,

$$a_Z = \frac{a_0}{Z}.$$

The electrostatic binding energy is inversely proportional to the radius a_Z:

$$E(Z) \sim \frac{Ze^2}{4\pi\epsilon_0}\frac{1}{a_Z}. \tag{21}$$

One factor of Z is directly visible, and the second factor is part of $1/a_Z$. The energy $E(Z)$ thus has a factor of Z^2:

$$E(Z) = E(1) \times Z^2.$$

Therefore, $n = 2$.

Comments on page 7

I got help to do this problem but I don't think I would have been able to do it alone..

I forgot Z was originally in the equation even after looking for it.

Where exactly was this in the readings? I totally guessed at this for the purposes of the problem.

I couldn't find it either...and so I couldn't figure out how to incorporate Z.

Missed how this was proportional to the number of protons. Does doubling protons double the potential energy?

I thought the explanation in this paragraph was enough to explain the answer, all the stuff after this just confuses me.

what is az?

I tried to relate Z and energy to the size of the nucleus and got this completely wrong.

I'm a little confused - when did we ever find an expression for a0?

It's an intermediary step. You can find an expression for a0 based on what was given in Prob. #1.

I am still not sure how this was calculated either...

I'm also confused how this was calculated... actually I was pretty confused about this problem and the next one in general. I didn't really have any idea how to solve them. I just don't know enough about chemistry or physics to understand the problems.

I also agree about not really understanding this step from the first problem or on this one and would like to see it explained in class.

I diddnt know this radius changed, w=or would be calculatable in these circumstances

I feel like if you didn't think this equation, there's no way to get the answer. I tried thinking of how Z changed the radius using the equation in the notes (the one similar to the Bohr radius), but couldn't conclude any changes.

I also tried to think of this equation, but couldn't...I did, however, remember that if Z increases, there's more charge pulling the electron in, therefore the radius is smaller...i just couldnt come up with something that made sense...this seemed a little too simple for electron math

I don't think I would have guessed this step

It makes more sense if you rearrange the equation as a_0 = Z*a_z and think about how atoms pack and electric attraction scales.

I really had no idea on what to do. A friend explained it to me, and I was still clueless on how to get the exponent.

Interesting conclusion, I ended up getting close by abstracting this further by energy for each shell. I had no idea to use e^2/4pie0 for this.

Problem 7 Heavy nuclei
In this problem you study the innermost electron in an atom with many protons (i.e. with a heavy nucleus). So, imagine a nucleus with Z protons around which orbits just one electron. Let $E(Z)$ be the binding energy. The case $Z = 1$ (**Problem 6**) is hydrogen.

Find how $E(Z)$ depends on Z. Namely, what is the scaling exponent n in

$$E(Z) \propto Z^n \tag{19}$$

or, equivalently, in

$$\frac{E(Z)}{E(1)} = Z^n\,? \tag{20}$$

☐ ± ☐ *or* ☐ . . . ☐

With Z protons pulling on one electron, the electrostatic energy contains the factor $Ze^2/4\pi\epsilon_0$. So instead of using $e^2/4\pi\epsilon_0$ as one quantity in the dimensional analysis, we should use $Ze^2/4\pi\epsilon_0$. The other quantities – a_Z, m_e, and $\hbar$ – are unchanged except for a_Z replacing a_0. The Z propagates along with the e^2 through the calculation of the radius a_Z and the energy $E(Z)$.

Since the radius a_0 has one factor of e^2 in the denominator, the a_Z picks up a factor of Z in the denominator relative to a_0. Therefore,

$$a_Z = \frac{a_0}{Z}.$$

The electrostatic binding energy is inversely proportional to the radius a_Z:

$$E(Z) \sim \frac{Ze^2}{4\pi\epsilon_0}\frac{1}{a_Z}. \tag{21}$$

One factor of Z is directly visible, and the second factor is part of $1/a_Z$. The energy $E(Z)$ thus has a factor of Z^2:

$$E(Z) = E(1) \times Z^2.$$

Therefore, $n = 2$.

I completely missed the second factor 1/az and came up with n=1, which threw off my answer to 8 as well.

Isn't this just saying that if you sub in the actual equation for a_z, you will get the answer?

I couldn't figure out the 2nd factor of Z, but after solving the next problem, I knew it had to be Z^2, so I redid this problem.

I made a mistake with the charges and thought it was proportional to one.

this was my initial guess - then I changed it, over thought it I guess

I'm not sure the way I did this was correct, but somehow I got the right answer. Perhaps I didn't interpret this solution correctly.

Problem 8 Heaviest nuclei

Consider again the system of **Problem 7**: a nucleus with Z protons surrounded by one electron.

When the binding energy $E(Z)$ is comparable to $m_e c^2$ – the rest energy of the electron – then the electron has enough kinetic energy to produce, out of nowhere, a positron (an anti-electron). As a result of this process, which is known as pair creation, the positron leaves the nucleus, turning one proton into a neutron. That makes the atomic number Z drop by one. The consequence is that, for large-enough Z, the nucleus is unstable! Relativity sets an upper limit for Z.

Use the results of **Problem 7** to estimate this maximum Z set by relativity (feel free to ignore factors of 1/2 in $E(1)$).

☐ ± ☐ *or* ☐ . . . ☐

To include in the explanation box: Compare your estimate with the Z for the heaviest stable nucleus (uranium).

Since the binding energy $E(Z)$ is $E_0 \times Z^2$ and $E_0 \sim m(\alpha c)^2$, the binding energy is

$$E(Z) \sim mc^2 (Z\alpha)^2. \tag{22}$$

Whe $Z\alpha \sim 1$, this energy is comparable to the electron's rest energy. That is when the electron becomes significantly relativistic, which permits pair creation to destabilize the nucleus. So the maximum Z is roughly α^{-1} or about 140. The heaviest stable nucleus is uranium with $Z = 92$, so the explanation for the stability of the elements looks pretty good.

Comments on page 8

I was pretty lost about this problem in general. How are people who don't have much knowledge about physics and chemistry expected to do these problems? I was searching for information for hours, and still didn't know how to do this problem.

Yeah this one got me too...

I like having related problems on these psets.

I completely misread this problem. I don't particular like it, nor did I find it easy to understand.

i didnt really understand what was going on here. i guess i need to brush up on chemistry, or physics, or something.

Same with me, i had to look up some of this stuff to get a better understanding of the problem.

This class makes me remember so much knowledge I've long-since deemed useless.

I knew this was related to alpha but for some reason I couldn't understand the exact relation. Now it all makes sense...

I couldn't figure out how to do this problem using problem 7 but my chemistry knowledge was enough for me to reason what the answer should be..

This formula (obviously) would've been very useful...I'm failing to recall, was it in one the readings somewhere or did this require a little knowledge of relativity?

it was derived from the previous two problems. at least, that's how i figured it out.

Yeah this comes directly from 6 and 7

I didn't even think of this approach... very clever.

I managed to re-write the equation into a similar form. But since my reasoning was wrong in Problem 7, my entire approach to this question was off from the start.

Don't quite understand this. Can someone please explain in more depth.

typo

I got 130. I'm surprised as to how close I was to the correct number.

would the factor of 1/2 help or hurt us here?

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I was unsure of whether or not a value of 140 was good. If you think about it, the numbers are close but this answer would actually predict that all of the elements that we know of would have a stable nucleus.

Me too! I was conflicted. In the context of the estimations we've been doing in this class, 140 is good enough compared to an actual answer of 92, but in the context of the periodic table, it is pretty useless in predicting where the instability starts since there are less than 140 elements overall.

If we hadn't ignored that factor of 1/2 wouldn't it have been even larger? I also didn't consider this a very close approximation.

yeah I agree, the numbers seem pretty far apart...at what value would Z have been a bad approximation?

Not that I was close since I did a horribly simple arithmetic mistake in #6, but when he asks us to disregard factors so that our answers come out closer to the actual value it makes me even more skeptical/suspicious of a technique or solution.

That's a great point – can we go over this in class. What threw off the accuracy?

I think there are additional factors, besides relativistic ones. For example, space limitations, as the nucleus gets bigger.

I felt more comfortable with my guess here than i did on the previous question (my gut told me 102 +/- 10)...I ended up getting this problem ready to solve, after further contemplation on #7, but ended working backwards and solving 7 using my gut guess on 8

I like how this problem set worked off of itself