6.055J/2.038J (Spring 2010)

Solution set 8

*Submit your answers and explanations online by **10pm on Wednesday, 05 May 2010**.*

Open universe: *Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Comments on page 1

Here is solution set 8 (this time in the right NB site – sorry about that). The memo will be due Friday at 9am.

Problem 1 Should you be worried?

Assume that 1 in 10^4 bridges in the United States are in danger of collapse unless repaired soon, and that a new test for bridge integrity has been devised. This test is 90% accurate: It always detects an unsafe bridge (no false negatives); and 10% of the time it says that a safe bridge is unsafe (10% false positives).

You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

$$10^{\boxed{} \pm \boxed{}} \quad \textit{or} \quad 10^{\boxed{} \ldots \boxed{}}$$

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

Let's do it by the natural-frequencies approach. Imagine a population of 10^4 US bridges. Given the base rate of 1 in 10^4, assume that one bridge among them is actually in danger of collapse.

Now imagine applying the bridge-integrity test to all 10^4 bridges. It will spot the one unsafe bridge. But from among the nearly 10^4 safe bridges, it will also mark 10% or 10^3 bridges as unsafe. The bridge you use is among the roughly 10^3 bridges with a positive test. But only one of those bridges is actually unsafe, so $p_{\text{unsafe}} \approx 10^{-3}$. Therefore, the odds are 10^3 to 1 that the bridge is safe (or simply 10^3).

Now let's use Bayes theorem to get the same result. The odds form of Bayes theorem is

$$O(H|E) = O(H)\,\frac{P(E|H)}{P(E|\overline{H})}, \tag{1}$$

where H is the hypothesis that the bridge is unsafe, $O(H)$ is the odds in favor of that hypothesis being true, E is the evidence that the bridge failed the integrity test, $P(E|H)$ is the probability of a failed integrity test given that the bridge is unsafe (the false-negative rate), and $P(E|\overline{H})$ is the probability of a failed test given that the bridge is safe (the false-positive rate).

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$$\frac{P(E|H)}{P(E|\overline{H})} = \frac{1}{0.1} = 10. \tag{2}$$

Therefore, the new odds are 10^{-3} in favor of the bridge being unsafe (or 10^3 in favor of it being safe, as computed above).

For an excellent article in the newspaper (of all places) on how to do Bayes theorem using natural frequencies, see Steven Strogatz's recent column in the *New York Times* (thanks to Sean Clarke for pointing me to it), available at **http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/**

Comments on page 2

Somehow I totally missed this when I read the problem. Woops.

That actually seems rather low

i feel like that's "okay" because there are no false negatives.

i took this to mean that for every 9 unsafe bridges, it incorrectly said 1 was unsafe. i guess it depends on what you mean by "10% of the time"

Pretty crappy test if you ask me. That's a lot of false positives. So the US would fix 1000 times more bridges than it needs. Go budget deficit.

It's probably reasonable given that you have to ensure no false negatives (seen as unsafe when it's safe), as those are unacceptable given the conditions here

As another post said, you don't want to miss anything (no false negatives). And few false negatives usually means lots of false positives. Even then, you don't have to fix everything that fails this test. There might be other, more detailed tests (e.g. taking samples of the material, doing X-rays, who knows what) that are more reliable although more expensive (hence not worth doing on all bridges).

Lots of medical tests work that way. If you are positive on the cheap but not-super-reliable test, then you go in for "further screening". As an example, if you present at the hospital with chest pains and shortness of breath, you have just tested positive on a very crude test for a heart attack – so then you are sent for further tests (which someone with more medical knowledge than I have will have to describe!).

despite what our math just told us, can you imagine the media outrage if this were ever to actually happen? sigh...

Ah, I never quite understood the meaning of odds.

or you could write this as odds=(1-punsafe)/punsafe

I feel foolish. I just could not figure out O(H). I was trying to figure out the unsafe bridges on a country-wide scale, using an approximation of 600,000 bridges. It didn't even occur to me to use a sample of 10^4 bridges.

I did it this way; in fact, I think I wrote this exact statement (almost exact) in my explanation.

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I really like this approach, I tried to use Bayes theorem and got lost in the probability aspect of the problem rather than trying to use logic to get through it

What were you confused about? Bayes Theorem is not that hard, but you do have to think about what you're plugging in.

I ended up setting the problem up like an asymmetric binary channel and solving it using Bayes' theorem.

I used Bayes' theorem but this is actually a lot more intuitive.

I also used Bayes' but this so clear. It really makes a lot of sense to me.

for me, bayes' theorem is not necessarily hard, but my intuition is obscured by the math.

Yeah, this intuitive reasoning actually makes much more sense to me than the Bayes rule calculations I used!

I do too. I highly recommend it for making the Bayes theorem result intuitive. Gigerenzer's studies showed a huge improvement in accuracy when the 'natural frequencies' method was explained to doctors (see the NYT article and references therein).

My analysis doesn't make any sense now, oops.

Me too. I completely overlooked this. In retrospect I definitely should not have assumed it was so simple.

wow, I must have been really tired...I did not take 10% of 10^4, I just converted it to a decimal!

I thought this method is faster and easier than the bayes theorem

Me too. I was about to try Bayes theorem then I realized how simple this way was so I did the problem this way.

this is hard to follow

where did you get only one of them is unsafe? I thought is 1 out of 10^4 not 1 out of 10^3

I wasn't sure how to put this into my calculations...

This method is so much more simpler than using Bayes theorem. Why couldn't I remember this?

maybe you should make it clear there that punsafe=1-psafe. There seems to be confusion about how to use punsafe in the formula for odds that you gave above.

It would be helpful to see bayes theorem form probability first, then change it to odds

Yea, the Bayes theorem is definitely more commonly known in probability form and that would also be consistent with the way you presented it in the reading. But it doesn't really matter, its all Bayes theorem anyway.

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I didn't think about using the Bayes theorem...

That's unfortunate. It's useful!

The reasoning with words is much easier to understand. I went straight to Bayes theorem because that is what we were shown in class, but failed to get the right answer.

That's what I used for this - took me a few tries to define the probabilities correctly

Same. I really wonder where I went wrong in it, I guess the number I used for P(unsafe read)

this probability stuff isn't the most intuitive to me either. after seeing these examples, I think Bayes theorem anytime a test is used to validate something.

I think my problem with this was I was trying the traditional method.

This is exactly what I did! And I got the correct answer! Thank you 18.440...

Totally forgot about using this form...I just used the normal Bayes rule!

Yeah using this form definitely saves a little bit of time at the end. It's important to change the odds back to the odds that the bridge is safe, not unsafe.

I tried Bayes' formula as well, without the odds figure. The odds figure somewhat confuses me.

for some reason the wording in these problems always throws me off, the probability that the bridge is save given it failed and the probability that the bridge failed given that it is save seems like the same thing to me

That's what I got as my answer

Yeah, I guess I stopped at the initial odds, and didn't continue to calculate the likelihood ratio.

As a visual comment, when I'm looking at the solutions, I see this equation and immediately think that its saying the answer is 10. I t would be nice if you could clearly display what the answer is.

Yeah, I agree. I often find it annoying to wade through all this description when I just want to see what the answer is to compare to mine. It would be much better to display the answer more clearly.

Good point. For the next pset, I'll make the answer the final displayed equation in the solution (or maybe with a gray background, in case there's discussion after the answer, so that the answer still stands out).

where is the "likilihood ratio" in the notes? I didn't see it.

Reading 28, page 153

This was my answer but it took me a while because I kept thinking there was more to it.

These are just the initial odds though

I still don't understand odds very well. Probability makes a lot more sense to use for an answer. Why did you ask for the odds?

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This should actually be 1/999.9 exactly, right?

Hooray! I did this correctly! I still feel like the odds fraction is a slightly less intuitive way of using Bayes' formula, especially for those who already have probability.

I agree with you, but odds just seems to be the ratio between probabilities of the bridge being safe over bridge being unsafe

Maybe not less intuitive, but the Bayes thm just seems most systematic and like it is covering all the bases.

Agreed, knowing how to do Bayes gave me a lot of confidence in the answer, not just the ability to solve the problem.

I answered in terms of the odds of the bridge being safe (10ˆ3) instead of being unsafe... Same thing.. wrong reading of the question.. oops

This pretty cool hope this works - I got that answer, but and still impressed at how unlikely it is that the bridge is bad!

haha. Sounds like an article we should all read.

can this be a hyperlink?

This is supposed to be a book, so probably not.

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Yeah, and its not too much trouble to just copy and paste the link.

I made it a hyperlink in the PDF file itself (so if you download the file and view it on your computer). But when NB turns that file into images, I think the hyperlink gets lost. But it's something that I think Sacha mentioned as on his 'to do' list (if not, I'll suggest it).

Problem 2 Reusing plausible-range combinations

In lecture, we saw that if the width of an object has a plausible range $w = 1\ldots 10\,\mathrm{m}$ and the length has a plausible range $l = 1\ldots 10\,\mathrm{m}$, then the area $A = lw$ has the range $2\ldots 50\,\mathrm{m}^2$.

If instead the plausible ranges are $w = 2\ldots 20\,\mathrm{m}$ and $l = 5\ldots 50\,\mathrm{m}$, what is the plausible range for the area?

$\boxed{} \pm \boxed{}\ \mathrm{m}^2$ *or* $\boxed{} \ldots \boxed{}\ \mathrm{m}^2$

Multiplying everything (the top and bottom) by a constant changes the midpoint of the range and the lower and upper endpoints, but does not change the width itself (the ratio of the upper to lower endpoints). For example, a factor of 25 uncertainty is still a factor of 25, just around a new midpoint. In fancy words, the width of a range is invariant to changes of scale.

From lecture, we are given that $1\ldots 10 \times 1\ldots 10 \approx 2\ldots 50$. Now multiply the lower and upper endpoints of the first range by 2; and multiply the lower and upper endpoints of the second range by 5. Those changes multiply the lower and upper endpoints of the product by 2×5. So,

$$2\ldots 20 \times 5\ldots 50 \approx 20\ldots 500. \tag{3}$$

Problem 3 Singing a logarithm

Estimate 1.5^{40} using the singing-logarithms method from lecture (a copy of the handout is on the course website).

$10^{\boxed{} \pm \boxed{}}$ *or* $10^{\boxed{} \ldots \boxed{}}$

1.5 is 3/2, which is 7 semitones (a perfect fifth). Each semitone is a factor of $2^{1/12}$ which is also $10^{1/40}$ (40 semitones make a factor of 10). Therefore, $1.5 \approx 10^{7/40}$ and

$$1.5^{40} \approx \left(10^{7/40}\right)^{40} = 10^7. \tag{4}$$

The true value is just above $1.1 \cdot 10^7$.

Comments on page 3

This description of how to calculate the plausible range makes a lot more sense than the formula i used from the notes

I missed the "factor of" part some how from the readings and was confused for a while about how 2...20 had a range of 25. Then I realized it was a factor of 25. doh.

Yeah, I didn't get this at first, (where the 25 came from) but I still got the right answer–now its really clear to me, though after reading this solution

i didn't think of it this way. i just used the formula in the readings, although i noticed the trend between the two examples.

Same here, I used the formula in the readings.

I mainly used a Symmetry argument for this problem. If the initial range was 2-50 for a value between 1-100, then I figured 20-500 made sense for 10-1000.

It really helps to think in log space here, or in factors like 2x or 10x.

I didn't even think of log space, but it makes a ton of sense when you mention it.

i just tried to think about factoring out the right numbers so that we are left w/ quantities we know about.

I feel like the key to this problem was realized the ranges were identical so you already knew what 'r' was and could go from there easily

I got it correct! :)

how did you choose 2 and 5? not arbitrarily right

I used the method in the notes to find it. I dont really understand this method...

i agree. I also used the method from the notes. Calculating the plausible range using this is not very intuitive to me. I don't really get how the width of the range is invariant to changes of scale.

Yeah I agree. Although I used the method and arrived at the correct answer, it is not very intuitive for me either.

Also agreed, the lecture notes, were a great reference and thoroughly explained, step-by-step, so I relied on the readings.

I ended up using the longer explanation used in the lecture notes. I don't remember where we used this method.

So did I. I used the formulas in the notes that give values for midpoint and range.

I did this by noticing the pattern. I guess it's an estimation technique of sorts, but I'm a little hazy on this. I'll probably ask you about this after class tomorrow.

I did the entire analysis out. I guess I wasn't completely confident that you could carry factors through like this. I guess this will definitely just make my life easier in the future. It's nice to double check though.

I also did the entire analysis, but I used the factors to see how it carried through the problem. I think it was more useful to see how it worked, than to be told that something works.

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wow, never thought about doing this problem with patterns from previous questions....what if you changed the endpoints by a lot – meaning a change in the width itself? would you have to recalculate, or could you add and subtract?

> If you double all the widths (on a logarithmic scale), then you'll double the output's width. That's all on a log scale, so that means squaring the actual widths. For example, if 1..10 x 1..10 makes 2..50 (a width of 25), then 1..100 x 1..100 (doubling each logarithmic width, from 1 to 2 factors of 10) will make 4..2500 (a width of 25^2).

i understand the solution for this part but at the same time, it could probably be written out a bit more to be explicit (separate lines, use brackets to show what to multiply/why)

It is important to remember to multiply correctly in this problem, multiplying lower bounds together and upper bounds together.

So where does doing the logarithum procedure come into play? can you explain this solution in further detail?

For some reason I entered this in completely wrong but I got the same answer using the method from the reading I didn't notice the invariant but that would have made this problem a little quicker.

ooh i think i got that! or something like it?

I did this the hard way and got a different result. This way is much easier!

This solution makes sense.

yes! finally got one right

Same here- sometimes I have the right idea, but do the calculation wrong, but definitely got this right!

I did something way more complicated although this was my initial inclination

I used a different method to solve this problem, not the one shown in the lecture notes. Instead of finding the geometric means and then the midpoint, and then the range, I used a pattern in the example, multiplying the first term by 2 and the end term by 1/2, and got the same answer.

At least for me, this concept was pretty tough to understand so I resorted to using simple logarithms (not singing logarithms)

even with my notes and the handout i struggled with this problem.

Although, I don't think I'm ever going to use it outside this class, I think this is a really neat trick.

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i didn't understand this in the handout at all. it just seems like a really complicated process that you need a handout for. why not just use a calculator?

Sometimes you just don't have a calculator handy. Personally i thought this was a really cool method that I hadn't seen before, and it was cool to connect music to math.

I agree even though it might be a little round about, the method is definitely cool and something I had never thought about before

I still also don't know how to do this without memorizing a few of the intervals, which I don't know, since I haven't studied music much.

I understand we might not always have a calculator at hand, but I'm definitely more likely to have a calculator than I am to have a copy of that handout, and this method seems useless without the handout. Why are we doing this?

It does seem complicated to me, but someone explained it as A^B = B*P1 = C*40+D*P2 then A^B = 10^C * E^D where P1 is the semitones for A and P2 is the semitones for E. That made it easier.

i think the handout would be easier to understand if the steps for each situation were listed out more in an outline form instead of just written in paragraph form. if there's anything latex's taught me, it's the power of white-space.

This explanation definitely helped clear a bit of it up - honestly the handout was the only way I would've gotten through this problem at all.

Yeah, not having studied music at all; I still don't understand this method...I couldn't get it from the handout and couldn't find any other information on it online...

I don't think you need music theory to solve it. Since I never learned anything about music, I just ignored the music terminology and focused on the patterns themselves.

I try and try to understand the singing logarithms and I still cannot get them. the worst part is I play the violin. any suggestions on understanding them?

This makes more sense, i approximate 1.5 as 1 and tried to do it but got very confused

I really enjoyed this problem. It's an interesting way to estimate exponents.

I agree–it's really cool how we can estimate exponents like this. I got the right answer, but obviously i feel like in an everyday situation i'd have no possible way of applying this method because I probably wouldn't have the chart with me

I liked getting practice with this singing logarithms method - it's quite useful. Will we be allowed to use the logarithms sheet during the final?

Yeah i really liked this problem. It was simply but useful for understanding the basics of using the algorithm.

Well I'm sure we aren't expected to memorize the whole table!

I was shown this method after doing it out with 3^2 = 10 and 2^10=10^3. Need more practice here and with singing logs in general.

can we have a white paper that explains how to use the semitones handout?

Problem 2 Reusing plausible-range combinations

In lecture, we saw that if the width of an object has a plausible range $w = 1 \ldots 10\,\mathrm{m}$ and the length has a plausible range $l = 1 \ldots 10\,\mathrm{m}$, then the area $A = lw$ has the range $2 \ldots 50\,\mathrm{m}^2$.

If instead the plausible ranges are $w = 2 \ldots 20\,\mathrm{m}$ and $l = 5 \ldots 50\,\mathrm{m}$, what is the plausible range for the area?

$\boxed{} \pm \boxed{}\ \mathrm{m}^2$ *or* $\boxed{} \ldots \boxed{}\ \mathrm{m}^2$

Multiplying everything (the top and bottom) by a constant changes the midpoint of the range and the lower and upper endpoints, but does not change the width itself (the ratio of the upper to lower endpoints). For example, a factor of 25 uncertainty is still a factor of 25, just around a new midpoint. In fancy words, the width of a range is invariant to changes of scale.

From lecture, we are given that $1 \ldots 10 \times 1 \ldots 10 \approx 2 \ldots 50$. Now multiply the lower and upper endpoints of the first range by 2; and multiply the lower and upper endpoints of the second range by 5. Those changes multiply the lower and upper endpoints of the product by 2×5. So,

$$2 \ldots 20 \times 5 \ldots 50 \approx 20 \ldots 500. \tag{3}$$

Problem 3 Singing a logarithm

Estimate 1.5^{40} using the singing-logarithms method from lecture (a copy of the handout is on the course website).

$10^{\boxed{} \pm \boxed{}}$ *or* $10^{\boxed{} \ldots \boxed{}}$

1.5 is 3/2, which is 7 semitones (a perfect fifth). Each semitone is a factor of $2^{1/12}$ which is also $10^{1/40}$ (40 semitones make a factor of 10). Therefore, $1.5 \approx 10^{7/40}$ and

$$1.5^{40} \approx \left(10^{7/40}\right)^{40} = 10^7. \tag{4}$$

The true value is just above $1.1 \cdot 10^7$.

I was very confused by the singing a logarithm handout. This explanation helped a lot though.

The handout could use a lot more explaining. I'm actually sit a bit confused on how to do this w/out looking at the sheet.

Yes I agree, this explanation makes a lot more sense than the handout.

yeah I agree...I'm still really confused about the singing logarithms even though I got the right answer by following the examples.

I really liked this problem, the "singing logarithms sheet" was extremely helpful. It's just gonna be hard to memorize for the final exam

Will we have the table of singing logarithms for the final? I feel like that's a LOT of information to memorize if we don't.

Yeah agreed. If we are given the handout then singing logarithms won't be that bad. I can't imagine we wont get it if it's on the final.

I got something similar to this.

I was off by a factor of 10 for some reason. I'll have to go back and check what I did.

This was just hard until I looked into semitones. Then the conversions weren't difficult.

I missed this class but was able to find a friend to explain it - not that bad.

I had a little trouble understanding the theory of semitones at first but was able to get the answer using the chart in the document on the website.

this is such a cool trick! and as a plus, I was able to wow my roommate for a bit, as she thought I just had a .pdf open from another class.

I did this one right!

I did this one wrong, but my answer wasn't really off by all that much.

I thought this one was really simple but I liked it a lot! It made me feel like i learned something useful from a simple knowledge of music

Problem 4 Estimating a mass

You are trying to estimate the mass of an object. Suppose that your plausible range for its density is $1\ldots5\,\mathrm{g\,cm^{-3}}$ and for its volume is $1\ldots5\,\mathrm{cm^3}$. What is (roughly) your plausible range for its mass?

$\boxed{} \pm \boxed{}$ g *or* $\boxed{} \ldots \boxed{}$ g

Each range is a factor of 5 in width. In semitones,

$$5 = \underbrace{\frac{5}{4}}_{4\ \text{semitones}} \times \underbrace{2}_{12\ \text{semitones}} \times \underbrace{2}_{12\ \text{semitones}} = 28\ \text{semitones}. \tag{5}$$

Because 40 semitones make a factor of 10, the 28 semitones means a base-10 logarithm of 0.7. Add the squares of (logarithmic) widths to get the new (logarithmic) width squared:

$$0.7^2 + 0.7^2 \approx 1. \tag{6}$$

So, the new plausible range has a width of 1 decade (a factor of 10). The range is centered at 5 g:

$$m = \rho V \sim \underbrace{\sqrt{5}}_{1\ldots5}\,\mathrm{g\,cm^{-3}} \times \underbrace{\sqrt{5}}_{1\ldots5}\,\mathrm{cm^3} = 5\,\mathrm{g}. \tag{7}$$

So the plausible range is $1.7\ldots15$ g. (A full calculation, without using the semitones approximation, gives $1.6\ldots15.6$ g.)

Problem 5 Which is the wider range?

Suppose that your knowledge of the quantities a, b, and c is given by these plausible ranges:

$$\begin{aligned} a &= 1\ldots10 \\ b &= 1\ldots10 \\ c &= 1\ldots10. \end{aligned} \tag{8}$$

Which quantity – abc or a^2b – has the wider plausible range? (The 'width' is the ratio of the upper to lower endpoints; so a, b, and c are each a factor of 10 wide.)

☐ abc

☐ a^2b

☐ Both quantities have the same width.

Both choices have b in them, so ignore it and instead compare ac versus a^2. When computing ac there is a chance that an overestimate in a will compensate an underestimate in c (and vice versa). However, when computing a^2, any error in estimating a is magnified – a factor of 2 error in a becomes a factor of 4 error in a^2. So, a^2 has a wider plausible range than ac. Numerically,

$$\begin{aligned} ac &= 2\ldots50. \\ a^2 &= 1\ldots100. \end{aligned} \tag{9}$$

Comments on page 4

it does still seem weird to try and do this in semitones

agreed, that never occurred to me. is that necessary?

There was no indication to use this method in the problem and I didn't think it was related to the semitones thing. Seems weird to use it here for no reason. I used the same formula for plausible range from the other reading.

Same, I wish this problem would have mentioned that method.

I'm also a bit surprised that this problem used semitones. I tried to reverse calculate a range in a fashion similar to the paper area method.

You don't have to do it that way and can use the formulas from the reading. It was just more fun for me (one of the benefits of teaching after all) to not use a calculator and thereby keep my singing-log muscles in training.

I think I guesstimated a lot to do this problem in stead of doing it step by step like the solution.

Nice, I did this one somewhat differently but had the same estimated answer. I still used singing logs, but estimated range = 5^sqrt(2) =5^1.41 = 10^1.41 log(5) = 10^1.41 * 28/40 = 10^1.

whats the point of doing it this way?

This was the step I missed. Coo'

i actually didn't spot this as a semitones problem at all.

I agree. This problem was kinda tricky about it. But it makes you wonder if you should be using the semitone estimation more often.

Where do the 2s come from?

The twos come from our (arbitrary) breakdown of 5 into 5/4*4 = 5/4*2*2, which then happens to be easy to calculate in terms of semitones.

I did not even see this as a way to approach this problem. I ended up just using the range equations.

I think I will have to go through this problem and try it again.

where did 40 come from?

You can find this information on the approximating logarithms chart...

So if it weren't 1, we'd have to take the square root to get the width, right? (which might be hard)

i don't follow this part of the solution

my computer must have really bad resolution, because that looks like a minus sign, but that would make no sense.

Try zooming in NB, it's a double tilde for approximately.

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Each range is a factor of 5 in width. In semitones,

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$$a = 1\ldots10$$
$$b = 1\ldots10 \quad (8)$$
$$c = 1\ldots10.$$

Which quantity – abc or a^2b – has the wider plausible range? (The 'width' is the ratio of the upper to lower endpoints; so a, b, and c are each a factor of 10 wide.)

☐ abc

☐ a^2b

☐ Both quantities have the same width.

Both choices have b in them, so ignore it and instead compare ac versus a^2. When computing ac there is a chance that an overestimate in a will compensate an underestimate in c (and vice versa). However, when computing a^2, any error in estimating a is magnified – a factor of 2 error in a becomes a factor of 4 error in a^2. So, a^2 has a wider plausible range than ac. Numerically,

$$ac = 2\ldots50.$$
$$a^2 = 1\ldots100. \quad (9)$$

I found the same new logarithmic width, but got values for the endpoints closer to those of the full calculation.

Based on hidden step of 5*1/sqrt(10) and 5*sqrt(10), or is there an easier way just knowing the width of the range and the midpoint?

> I should have written out that step. For me, sqrt(10)=3 is so automatic, as is 5/3=1.7, that I didn't even think of it as a step – but that's a reason to explain it so that it'll become automatic for everyone else.

i used the same method here as i did in question 1. it seems more intuitive to me.

I also did the same method as i did in question 1. It might be a few more steps, but I still think I got the answer faster than if I had tried the log approach.

You were still taking logs though, just not using the semitones method, right?

I also used the method from problem 1. It makes a lot more intuitive sense to me and seems less contrived than singing logarithms (though I still think the singing logarithms are an incredibly useful and interesting tool).

I did also! dont know where these semitones came into the picture here

What is the full calculation- the method from the notes?

yes - that is what I did and I got the 1.6 to 15.6 g, I found the method from the notes much easier to use than the semitones!

agreed...the application of semitones here took a couple of minutes to understand how it broke down properly. I like seeing these things applied elsewhere though!

if you had wanted us to use the semitones method, perhaps you should have said so. I naturally just assumed this was another range-calculation and i didn't even have a feeling that it was supposed to be done another way

> I don't mind what method you use. My solutions are just one approach. One of the purposes of making the solution sets a reading memo is that I hope others will suggest alternative solutions, and everyone can benefit.
>
> I did it by singing logarithms (which are just logarithms to the base 10ˆ1/40) because it was more fun than using a calculator to do the logarithms.

I think I got this; and I used the non-semi-tone-method with the mus.

I solved it without using semitones, but I ended up with a much smaller plausible range. However the semitones method seems easier to use.

I got here without using semitones. They seem a lot more complicated here than following the method given in class to find these.

I arrived to this answer intuitively based on our discussion in class about the inherent difficulty in eyeballing area versus the greater ease in estimating two lengths.

I can't believe I also thought to do this, I feel so smart! :)

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$$\begin{aligned} ac &= 2 \ldots 50. \\ a^2 &= 1 \ldots 100. \end{aligned} \quad (9)$$

Oh this is extremely clever. I can see how this would be a problem if you thought you could eliminate a factor of a from both equations too.

Invariant! Which I forgot...

oo, i get it now, i thought the opposite. since in ac, there are 2 variables, and you will get a wider range, since is more "varied"

See, this is what i felt when i saw a "squared". I thought "oh, that must have a larger error because it's getting squared". But then i reasoned out that since a and c have the same uncertainty, and the uncertainty is the only thing taken into finding plausible ranges, that they must be equal. How do you account for this using the formulas you gave?

Exactly my question–could we see how this plays out in the math? I don't know how to distinguish the square of one term from the product of two terms in the range calculation.

This problem brings out an assumption that I did not make explicit enough in the reading. Namely, the formula for combining the plausible ranges assumes that the errors are uncorrelated. Said differently, information about the accuracy of one factor does not tell you anything about the accuracy of the other factor's estimate.

That assumption is fine when you are multiplying say number of people in the US * cars per person. But that assumption is not true when you are talking about a*a. There, any error in a becomes twice (on a log scale) that error in a^2. So, if the range for a is x..y, then the range for a^2 is x^2..y^2.

Exactly what I thought! I don't see how this problem used an approximation method

Hahahaha I got exactly the opposite! That was silly, I should have thought that through more. I just assumed that not having another variable w/error would reduce the width – but of course, they balance each other out!

I actually didnt think about the compensating for underestimates, or errors...I just thought that if a is either really small or really large, the range becomes huge–either 1^2 or 10^2, the two ends of the spectrum. And if we include c, c doesnt have to be the same as a (if a is 1 or 10, c can be something in between), which narrows the range

Wow...I didn't even think of canceling out the b. I just tried to include it in the formulas. Probably why I didn't get the right answer...

I like this explanation. I never really thought of this problem until this hw assignment.

I also like this explanation but I find it difficult to resolve the fact that a and c also have more diversity than a^2. This seems to suggest that they should have a wider potential range.

wow this never occurred to me, i just took the fact that they have the same ranges at face value!

This is so much easier to say than what I said. I'm actually kind of confused by my answer now that I go back, even though I got this right. Glad I have a simpler explanation now.

I chose "both" for this problem, because all values are equal. Thinking about in terms of how an error will be magnified makes a lot more sense though!

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Ah, this is quite insightful. For some reason my intuition said they would be the same width because each of the values for a and c were equally probable. That doesn't make much sense now. Cool problem!

Yeah, I spent a while reasoning about this in different ways too... this was one of my favorite (or at least most insightful) problems on this homework

I kept trying to reason my way into thinking that a^2 would have the wider range, but was never quite able to get to it.

Now I realize that my thinking was quite close to the solution here.

I did not think of that. a^2 will magnify an error in an approximation while (ac) can have its factors canel out its approximation errors.

Yeah, I definitely thought both quantities would have the same width, since you're multiplying 3 values that all have the same range. I didn't realize we were supposed to think about any potential error in the ranges.

I had the right answer and the right intuition, but I think putting my explanation into writing was definitely not as concise or understandable as it is here. Well put.

Oh I get it now, I also failed to think in terms of probability for this problem. Tricky!

Ah this makes so much sense. Great explanation. Its actually is really intuitive.

That does make a lot of sense. I used a dummy variable to represent the range 1...10, but I didn't realize that in doing so I neglected to include information about the errors present in the measurements.

I put the same and then explained that ac was probably better for some reason. Good explanation.

I thought this intuitively, but figured the math wouldn't work out that way, considering they were all the same. This analysis definitely makes sense though.

this is what i guessed too. the fewer variables you have the greater chance that no errors will cancel out.

I disagree with this answer. Sure, there' sa chance error in c will cancel with error in a, but isn't there also a chance that the error in c is not only in the same direction, but actually of greater magnitude (both overestimate, but c does so by 10% more) in which case ac would have a wider range than a^2...

I liked this problem - it made sense to me.

this makes sense now but when I was doing it I was totally convinced it was the other way around

If we said this without any math behind it, is that ok?

I hope so! I definitely had the same reasoning, just a much poorer explanation.

I understand why the plausible range should be larger with a^2, but I don't know how he was able to get those ranges.

i mean, the way this class works, it doesn't matter whether we got a certain problem perfectly right or not, right? you understood what was happening and by getting to see the solutions, you get to further understand it. that's it

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$$\begin{aligned} ac &= 2 \ldots 50. \\ a^2 &= 1 \ldots 100. \end{aligned} \tag{9}$$

I thought that a^2 was smaller, since we only picked 1 random number rather than 2. I realize now why that's wrong.

yeah exactly! that's the same thing I thought too!!

I didn't understand why the ac factor didn't affect the range the same way as aa did. But now the explanation makes sense because the c gives a chance to cover up for mistakes in estimating the a

I knew there was probably a reason for the a^2b term having a greater range, but I didn't know what. This makes so much more sense.

Agreed, it seemed to easy to be the same but I still couldn't understand exactly what the difference would be.

I thought it seemed to easy to blame the square term for making a larger range and somehow reasoned out (incorrectly) that they were the same. funny how that works out

I was originally thinking of this, but then got confused by the reading. I should have trusted my intuition...

I agree. I tried to use the range method from the reading to no success.

Thing that got me was that a^2 could be smaller than ac. But once I realized this would increase the range, it made sense.

but c could also be overestimated? this is assuming someone is estimating - i assumed numbers were chosen randomly from the range

i see what you're saying here, but i came up with a reason that made perfect sense to me, but gave the wrong answer. i said that -abc depends on 3 terms, while a^2*b depends on 2. (or 2 terms and 1 term, if you cancel out the b), so there's a bigger chance that something will go wrong with the -abc. why is this logic incorrect?

It's because more (independent) terms means more chance for errors to cancel. There's no guarantee that they will cancel, but it's at least possible. Whereas if everything depends on one quantity a, then in figuring out a*a, the errors have no chance of cancelling.

Problem 6 Golf-ball dimples
Why do golf balls have dimples?

- [ ] The dimples make the main airflow around the ball become turbulent.
- [ ] The dimples stabilize the flight.
- [ ] The dimples are there by tradition but have no physical justification.
- [ ] The dimples make the airflow turbulent in the thin boundary layer adjacent to the ball.

Let's first calculate the Reynolds number (a good first instinct in understanding a fluid flow). Let's say that the golf ball is hit at $30\,\mathrm{m\,s^{-1}}$ (70 mph!). It's diameter is a few centimeters, say 3 cm. Using $\nu \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$ for the viscosity of air, the Reynolds number is

$$\mathrm{Re} \sim \frac{30\,\mathrm{m\,s^{-1}} \times 3\cdot 10^{-2}\,\mathrm{m}}{10^{-5}\,\mathrm{m^2\,s^{-1}}} \sim 10^5. \tag{10}$$

That means the Reynolds number in the boundary layer is roughly $\sqrt{\mathrm{Re}} \sim 10^{2.5} \sim 300$. This is not high enough for turbulence, so the boundary layer is laminar.

A laminar boundary layer separates easily on the back of the golf ball, creating a large turbulent, low-pressure region behind the ball – that means lots of drag. If only the boundary layer could be made turbulent! Then the boundary layer would stick to the golf ball farther along the back side, and the drag would be lower. That's just what the dimples do: They trip the boundary layer into turbulence at a lower Reynolds number than is otherwise required (Choice D).

Comments on page 5

This problem didn't really test my knowledge since it was taken directly from the reading. Although, I don't think I would have been able to re-derive this on my own.

I liked that it was here- we saw it on the diagnostic test when we had no knowledge of the subject and now we can actually reason it!

I explained this using theory but I guess working with numbers creates a better explanation.

Why don't airplaines and rockets, and bullets, and such have dimples on them? Is the boundary layer airflow already somehow turbulent? Or is the shape too complex, or some other reason.

I didn't even bother calculating the Reynolds number- didn't we go over this in a previous pset?

yeah that's what I thought, so I just put D as my answer.

I think this question was more about understanding physical concepts, instead of the usual "find the approximation".

Yea I feel like we went over this before. Maybe it was on the 1st exam?

yeah it was on the diagnostic... a lot of the pset problems have been

same here. i also hate thermo so taht doesn't help either.

I understand this and know how to get the solution to this problem but I still don't understand intuitively why a turbulent boundary layer wants to stick to the surface of the golf ball.

I'm not really sure, but I think maybe it has to do with the vorticies that turbulent flow produces in comparison to laminar flow. When it's laminar, the flow is less likely to change direction, so going over a shape would not "bend" the flow downwards - it would want to continue more straight. However, if turbulent - the flow is much more likely to be pulled closer to the surface as there is no sense to the flow. this is just my guess though, I don't know for sure.

I still dont understand how a turbulent boundary layer sticks better and helps

This make perfect sense especially after the pset last week.

it was straight from lecture!

I just remembered the fact from lecture, not so much math as shown here.

It's interesting to see applications of the boundary layer. I didn't understand it very well when we first learned about it.

I think choice A is pretty similar to D. D is the better answer, but I don't see how A is wrong

The main flow is already turbulent, and that's true with or without the dimples (because the main-flow Reynolds number is so large for any actual golf-ball stroke). So, the dimples don't change anything about the main flow (directly). But they do make the boundary layer turbulent (which indirectly changes the main flow because the bdy layer stays attached farther back).

Problem 6 Golf-ball dimples
Why do golf balls have dimples?

- ☐ The dimples make the main airflow around the ball become turbulent.
- ☐ The dimples stabilize the flight.
- ☐ The dimples are there by tradition but have no physical justification.
- ☐ The dimples make the airflow turbulent in the thin boundary layer adjacent to the ball.

Let's first calculate the Reynolds number (a good first instinct in understanding a fluid flow). Let's say that the golf ball is hit at $30\,\mathrm{m\,s^{-1}}$ (70 mph!). It's diameter is a few centimeters, say 3 cm. Using $\nu \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$ for the viscosity of air, the Reynolds number is

$$\mathrm{Re} \sim \frac{30\,\mathrm{m\,s^{-1}} \times 3\cdot 10^{-2}\,\mathrm{m}}{10^{-5}\,\mathrm{m^2\,s^{-1}}} \sim 10^5. \tag{10}$$

That means the Reynolds number in the boundary layer is roughly $\sqrt{\mathrm{Re}} \sim 10^{2.5} \sim 300$. This is not high enough for turbulence, so the boundary layer is laminar.

A laminar boundary layer separates easily on the back of the golf ball, creating a large turbulent, low-pressure region behind the ball – that means lots of drag. If only the boundary layer could be made turbulent! Then the boundary layer would stick to the golf ball farther along the back side, and the drag would be lower. That's just what the dimples do: They trip the boundary layer into turbulence at a lower Reynolds number than is otherwise required (Choice D).

this was easy if you remembered the explanation from the reading. no calculations required! also mythbusters told me this...

I believe it's also stated in the reading that by creating turbulence in the boundary layer, there would be less drag..etc

I agree - my statement was something along the lines of "Cause you told me so"

got it right! However, I didn't do any calculations. I just reasoned that tradition wouldn't sell, the flight path isn't stabilized, from my experience, and then guessed btwn a and d.

got it right! However, I didn't do any calculations. I just reasoned that tradition wouldn't sell, the flight path isn't stabilized, from my experience, and then guessed btwn a and d.

Problem 7 Singing logarithms to combine plausible ranges
You are trying to esimate the plausible range for the volume of an object. You have assigned the length, width, and height the plausible ranges

$$\begin{aligned} l &= 1\ldots 10\,\mathrm{m} \\ w &= 1\ldots 10\,\mathrm{m} \\ h &= 1\ldots 10\,\mathrm{m}. \end{aligned} \tag{11}$$

In other words, each range is a factor of 10 wide (the 'width' is the ratio of the upper to lower endpoints). Convince yourself that the plausible range for the volume $V = lwh$ is a factor of $10^{\sqrt{3}}$ wide and is centered on $10^{1.5}\,\mathrm{m}^3$.

Each factor in lwh is centered at $10^{0.5}\,\mathrm{m}$ (the geometric mean of the lower and upper endpoints). Therefore lwh is centered on

$$\left(10^{0.5}\,\mathrm{m}\right)^3 = 10^{1.5}\,\mathrm{m}^3. \tag{12}$$

To compute the width of the range for lwh, note that each factor in lwh is 1 factor of 10 in width. For plausible ranges, add the squares of the (logarithmic) widths to get the square of the final (logarithmic) width:

$$1^2 + 1^2 + 1^2 = 3. \tag{13}$$

So the plausible range for V is $\sqrt{3}$ wide (in its base-10 logarithm); in other words, the range has width $10^{\sqrt{3}}$.

For the answer box, use $\sqrt{3} \approx 1.7$ or $\sqrt{3} \approx 1.73$ and the singing-logarithm method from lecture (a copy of the handout is on the course website) to estimate $10^{\sqrt{3}}$.

$\boxed{} \pm \boxed{}$ *or* $\boxed{} \ldots \boxed{}$

I'll first use $\sqrt{3} = 1.7$. Then

$$10^{\sqrt{3}} = 10^{1.7} = 10 \times 10^{0.7}. \tag{14}$$

But $10^{0.7}$ is 28 semitones (40 semitones is a factor of 10).

$$28 \text{ semitones} = \underbrace{\frac{5}{4}}_{4\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} = 5. \tag{15}$$

So, $10^{1.7} \approx 50$.

For fun, let's correct that estimation slightly by using $\sqrt{3} = 1.73$. The extra factor is $10^{0.03}$. Since 0.03 is roughly 1/40, $10^{0.03}$ is roughly 1 semitone. Based on the observation that 1.25 is 4 semitones, 1 semitone is given by

$$1.25^{1/4} = (1 + 0.25)^{1/4} \approx 1 + \frac{0.25}{4} \approx 1.06. \tag{16}$$

So, we should raise the earlier estimate of $10^{1.7}$ by 6%, which gives 53. [An exact calculation gives $10^{\sqrt{3}} \approx 53.96$.]

Comments on page 6

I made this problem harder than it could have been. I essentially did two problems: estimating the plausible range of l*w and then estimating the plausible range of (l*w)*h.

me Too!!

Yeah, for some reason I didn't think to add.

I definitely did not see this trick for this problem, pretty cool

I don't quite get where this came from.

This is pretty cool, I tried to do it in a much more complicated way.

This one was kinda fun to do

I did so much more work than what was shown here.

I'm not sure where my confusion is on this, because I follow my notes and I think I'm doing it right and then I get it wrong.

I thought this was the same kind of question. After doing the others, I knew I needed to estimate using semitones again.

this is the trick i was missing :(

I broke this up differently. I noticed that 1.7 was about 70/40

confused about this. seems that ur examples make sense if i stare at them, but singing logs didn't click for me like the other stuff. i guess practice makes perfect...

Perfect practice makes perfect...

This method was really fun to work through! I don't really understand the practicality of it yet (access to a singing logarithms table implies that you would also have access to a calculator, maybe on the Internet) but it was surprisingly accurate and very interesting to learn.

Yeah i agree. Its interesting to see the application and a different way of thinking about calculating, but I don't really see its applicability in everyday life.

Yeah agreed. I had a lot of fun working through the problem and even got the right answer, but I don't know when else I'll use this method.

With a bit of practice, the singing-log table will become part of your mental toolbox. I never have the table in front of me. For example, I wrote all the solutions that used singing logarithms without the table. You need just a few ideas to derive the whole table, or any item in it as needed. First, 2ˆ1/12 = 10ˆ1/40 [which is from 2ˆ10 = 10ˆ3]. Then you need two more numbers: 7 semitones = factor of 3/2 (perfect fifth); 4 semitones = factor of 5/4 (major third). Everything else can be figured out from those.

For the book itself, I'm going to make the singing logarithm handout into an actual section with explanations, and we'll build up the table step by step using the preceding information (like what wedid in lecture).

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So, we should raise the earlier estimate of $10^{1.7}$ by 6%, which gives 53. [An exact calculation gives $10^{\sqrt{3}} \approx 53.96$.]

This is pretty incredible accuracy for such a goofy method.

I can't believe I got this one right. I thought I had it wrong for sure

surprisingly close approximation though

Problem 8 Perfume

If the diffusion constant (in air) for small perfume molecules is $10^{-6}\,\mathrm{m^2\,s^{-1}}$, estimate the time for perfume molecules to diffuse across the lecture room.

$$10^{\boxed{}\pm\boxed{}}\ \mathrm{s} \quad \textit{or} \quad 10^{\boxed{}\ldots\boxed{}}\ \mathrm{s}$$

To include in the explanation box: Now try the experiment, at least mentally. How long does it actually take to smell the perfume from across the room? Explain the large discrepancy between the theoretical estimate and the experimental value.

The dimensions of a diffusion constant D are $\mathrm{L^2T^{-1}}$, so the diffusion time is given by $\tau \sim x^2/D$, where x is a length. The lecture room is perhaps 10 m deep (and maybe 15 m wide). It doesn't matter exactly which length I use, so I'll use the one that is simpler to square: $x \sim 10\,\mathrm{m}$. Then

$$\tau \sim \frac{10\,\mathrm{m}^2}{10^{-6}\,\mathrm{m^2\,s^{-1}}} \sim 10^8\,\mathrm{s} \sim 3\ \mathrm{years}. \tag{17}$$

That time does not agree with experiment! In reality, it takes perhaps a minute to notice that someone has opened a bottle of aromatic stuff. The discrepancy is that the molecules must travel not just by diffusion; in fact, the unavoidable air currents in the room transport the molecules much farther and faster than diffusion can.

Comments on page 7

I actually found this problem somewhat out of place on this pset - dimensionless groups were some time ago.

yeah, although we talked about diffusion when we talked about random walks more recently.

Yea, the formula I found for this was from the probabilistic random walks reading

Did this mean given a few molecules released at an instant or is the bottle left open? What's the role of concentration or flux here?

the final answer was sooooooooooo many orders of magnitude off that i don't think those factors really matter much here.

I am glad this was included in the problem, or I would have been really confused, b/c the approximating was not that difficult, nor was the math!

This was so interesting, especially because of the huge discrepancy between the estimated value and real life.

Forgot to square the length, otherwise would have gotten the same answer...

This problem was really simple, but I didn't see it. I didn't trust my gut and confused myself by trying to find the velocity of air (which doesn't make sense).

I took this into account by looking at the surface area of a sphere created by the perfume. This has the same factor, x^2, as the way solved here.

I totally missed this. i thought the room was a lot larger!

i did too. i said the room was 100m long... i did math backwards and said that 10m was 3ft instead of 30 ft. oooops.

Should this be 100m^2 since x 10m?

this is exactly what I got, at least now I know why the numbers didn't make sense. It just shows that problems in real life are more complicated than you think they are... except for the easy cases.

I agree–i got this answer too, and at first i thought to myself, doesnt this just show us that our estimations can be completely off? then I thought–well in a good estimation we're not just using one method–here, if we just apply a little common sense and relate it back to our daily experiences, we add a lot of insight to our estimation

I just thought my answer was off because I missed a step, when it was actually correct. The opposite explanation makes sense

same... i got a few years and thought it couldnt be right since it was so far from the convection result

I remember you talking about this in class, so I was vaguely confident about my answer, but it definitely seems off intuitively.

Yeah so I got this number and wrote it as my answer. I completely forgot to convert units to check if the answer was realistic. I don't know how I would have felt if it was multiple years, but I guess it would have worked out.

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That time does not agree with experiment! In reality, it takes perhaps a minute to notice that someone has opened a bottle of aromatic stuff. The discrepancy is that the molecules must travel not just by diffusion; in fact, the unavoidable air currents in the room transport the molecules much farther and faster than diffusion can.

This was an interesting problem, I was a little worried when I got 3 years, but the fact that you need air currents, etc... helps reassure that our estimation was off

its pretty amazing how large the discrepancy is, epically when I feel like the air int he class room isnt' really moving

This was a really surprising fact. I didn't realize that small air currents can move particles that much faster compared to if they weren't present.

Lots of air currents. Don't know what we'd do without them.

Hmmm. Makes sense that the air currents would be making the difference. Now I kind of wish people didn't use this perfume example to explain diffusion all the time.

Are there any accessable environments that do not have significant air currents that this sort of test could be performed?

I got a crazy long answer too! So, if there wasn't air currents in the room, it would really take THAT long for perfume odor to disperse? And wouldn't the odor go away in that time?

is there a way to account for that?

My mass transfer education is minimal, but maybe you could find an effective diffusion constant given convective mass transfer.

Air flow due to pressure and temperature gradients might also be considered in analyzing the air flow.

it's sort of like the difference between conduction and forced convection.

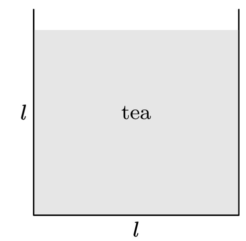
Yeah i was really surprised at how small of a role diffusion plays in this scenario

if it were a sealed room, would it be true?

Very likely, because any temperature differences between the top and bottom of the room would produce convection (air currents). It's really hard to get rid of convection.

Problem 9 Teacup spindown

You stir your afternoon tea to mix the milk (and sugar if you have a sweet tooth). Once you remove the stirring spoon, the rotation starts to slow. In this problem you'll estimate the spindown time τ: the time for the angular velocity of the tea to drop by a significant fraction.



To estimate τ, consider first a physicist's idea of a teacup: a cylinder with height l and diameter l, filled with a water-like liquid. Tea near the edge of the teacup – and near the base, but for simplicity we'll neglect the effect of the base – is slowed by the presence of the edge.

Because of the no-slip boundary condition, the edge creates a velocity gradient. Because of the tea's viscosity, the velocity gradient produces a force along the edge. This force tries to accelerate each piece of the edge along the direction of the tea's motion. The piece in return exerts an equal and opposite force on the tea. That is how the edge slows the rotation. Now analyze this model quantitatively using the following steps. Keep the results in symbolic form until the final step (**Step e**) when you get a numerical value for τ.

a. Convince yourself that the spindown time τ is given by

$$\tau \sim \frac{\rho l^5 \omega}{\sigma l^3} = \frac{\rho l^2 \omega}{\sigma}, \qquad (18)$$

where ρ is the density of tea, σ is the viscous stress (the viscous tangential force per unit area), and ω is the initial angular velocity. *Hint:* Consider the torque on and the angular momentum of the rotating blob of tea. In addition, drop all dimensionless constants like π and 2 by invoking the Estimation Theorem 1 = 2.

If the tea is spinning at angular velocity ω, then it has angular momentum $L = I\omega$, where I is the moment of inertia. The moment of inertia is given by mass times a squared distance from the origin:

$$I \sim \underbrace{\rho l^3}_{m} \times l^2 = \rho l^5. \qquad (19)$$

Not all of the mass is at a distance l from the center, but the twiddle accounts for the omitted dimensionless constant. With that I, the angular momentum is

$$L \sim \rho l^5 \omega. \qquad (20)$$

The viscous stress produces a torque that reduces this angular momentum. The viscous torque is

$$\text{viscous torque} \sim \underbrace{\text{viscous stress}}_{\sigma} \times \underbrace{\text{area}}_{l^2} \times \underbrace{\text{lever arm}}_{l} \sim \sigma l^3. \qquad (21)$$

Because torque is dL/dt, it has dimensions of L/t. So a time is given by L/torque:

$$\tau \sim \frac{\rho l^5 \omega}{\sigma l^3}. \qquad (22)$$

b. Now estimate the viscous stress σ by using the idea that

$$\text{viscous stress} \sim \rho\nu \times \text{velocity gradient}. \qquad (23)$$

The velocity gradient is determined by the thickness of the region over which the the edge significantly affects the flow; this region is the boundary layer. Let δ be its thickness (you'll find δ

Comments on page 8

I like this problem very much. There is no sure procedure to solve the problem that jumps to mind immediately. But just playing with dimensions and intuition gets you a decent solution.

I got this problem wrong... solution is still hard to follow.. Could we maybe go over this in class?

I also got it wrong. I can follow the solution, but the class might benefit from an in-person explanation.

Agreed, I thought this problem was pretty difficult, and this solution doesn't make much sense either. I think this problem was a little too heavy on physics, particularly for those of us who don't use 8.01 knowledge everyday.

I would similarly appreciate a comment or two in class in relation to this problem.

I will discuss it, hopefully with a related demonstration that just occurred to me (details to be worked out in time, hopefully!).

where were these in the first place?

For example, in the moment of inertia of a spinning cylinder of water.

How did I forget this??

this is were I messed up!

Yeah, I never attempted this one, wouldn't have thunk of this!

well i definitely didn't get this on my own, but i believe you.

Yeah I agree, I definitely did not know how to even start this question, let alone work through it and finish.

in **Step d**). In terms of δ, estimate the velocity gradient near the edge. Then estimate the viscous stress σ.

The velocity gradient is

$$\frac{\Delta v}{\Delta x} \sim \frac{\omega l}{\delta}. \tag{24}$$

Therefore the viscous stress is

$$\sigma \sim \rho\nu \frac{\omega l}{\delta}. \tag{25}$$

c. Insert your expression for the viscous stress σ into the earlier estimate for the spindown time τ. Your new expression for τ should contain only the boundary-layer thickness δ, the cup's size l, and the viscosity ν.

After substituting,

$$\tau \sim \frac{\rho l^5 \omega}{l^3 \times \rho\nu\omega l/\delta} \sim \frac{l\delta}{\nu}. \tag{26}$$

[The information in the problem statement is sufficient to arrive at this result, because $l\delta/\nu$ is the only way to make a time from l, δ, and ν.]

d. Now estimate the boundary-layer thickness δ using your knowledge of random walks. The boundary layer is a result of momentum diffusion – and ν is the momentum-diffusion coefficient. In a given time t, how far can momentum diffuse? This distance is δ. Estimate a reasonable t for the rotating blob of tea. [*Hint:* After rotating by 1 radian, the fluid is moving in a significantly different direction than before, so the momentum fluxes no longer add.] Use that time to estimate δ.

A reasonable time is the time to rotate 1 radian, namely $t \sim 1/\omega$. In that time, the diffusion distance δ is

$$\delta \sim \sqrt{\nu t} \sim \sqrt{\nu/\omega}. \tag{27}$$

e. Now put it all together. For a typical teacup stirred with a typical stirring motion, what is the predicted spindown time τ? [Tea is roughly water, and $\nu_{\text{water}} \sim 10^{-6}\,\text{m}^2\,\text{s}^{-1}$.]

$10^{\square} \pm \square$ s ***or*** $10^{\square} \ldots \square$ s

Substituting for δ in the expression for the spindown time τ gives

$$\tau \sim \frac{l\delta}{\nu} \sim \frac{l}{\sqrt{\nu\omega}}. \tag{28}$$

Now put in numbers. My nearby teacup is a few inches across, so $l \sim 10\,\text{cm}$. When I stir the tea, it rotates at a frequency $f \sim$ few Hz, so $\omega = 2\pi f \sim 20\,\text{s}^{-1}$. The result is

$$\tau \sim \frac{0.1\,\text{m}}{\sqrt{10^{-6}\,\text{m}^2\,\text{s}^{-1} \times 20\,\text{s}^{-1}}} \sim 20\,\text{s}. \tag{29}$$

Comments on page 9

The velocity gradient formula is what I lacked to get this problem right

I actually did this correctly! I was just trying something although I didn't know if it was reasonable to make the velocity gradient into omega*L.

i didn't get this step. i guess i didn't understand what it was asking for.

Ooops, I guessed it as omega*delta instead. That threw my final answer of significantly, but I realized this in my submission online

This was the furthest I could get on the problem.

For some reason, I always forget to consider dimensional analysis in approximating. If I had remembered this, I could have finished the problem.

I still think random walks was a pointless unit where I learned nothing.

I found this part really confusing. In particular, I was unsure how to apply random walks to analyze the momentum diffusion. I tried taking slices, etc. but eventually gave up and set t 1s.

Definitely the hardest question on the test. Even reading the question took a few goes. But this explanation does walk you through it well. As long as you don't get discouraged by the fact it's sooo long.

THis question really confused me. The hint not helping much at all

this is the piece that I was missing to completely solve this problem. Now that makes sense.

I only thought to do this because the problem statement said to drop all constants like pi, and i immediately thought of reducing w.

Oh interesting, this never crossed my mind. I went straight to t=1sec.

Ya, this estimate was elusive to me. I ended up going with smaller t, and still wound up with a huge huge number for my time constant answer (thousands of seconds)...

huh?

It took me a while to convince myself that it made sense to have omega in the denominator, here. Basically it means that the faster you spin it, the faster it slows to a fixed percentage of its initial momentum – not to a momentum constant for all cases!

ahhh i messed plugging stuff in here!

this seems like a really long time to me. i guessed it would only be 1 or 2 seconds.

I got 100,000 years

I actually got this right, but only because I got a lot of help on this one. It was really hard.

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i think the most frustrating part of this problem for me was that this is about the answer i would get if i just guessed off the top of my head. i don't think allllllllllllllllll the work that was just done made the answer tooooooo much better.

And now you also know why (physically) it takes about that much time.

wow i think i was way off!

To include in the explanation box: Estimate τ_{exp} experimentally by stirring tea. Compare the experimental time with the predicted time.

I just tried it, and $\tau_{experimental}$ (the time for the rotation to slow significantly) was around 10 s. Not bad!

Comments on page 10

This was the problem I didnt manage to do from the problem set, the step by step was fantastic.

I wasn't able to do this problem either. But you're right, the solution step-by-step does a great job of explaining how to go about the problem!

I had assumed it was until the water pretty much stood still...

I made this assumption as well - I wasn't sure exactly what a "significant fraction" meant. My experimental result was closer to 1-2min.

I think "slow significantly" roughly means until its speed is reduced by a factor of 1/e, (i.e. about a factor of 1/3).

Hmm, I got significantly lower for my tested result, maybe because I used water in a glass and it appeared to stop earlier than it actually did?

I also got lower.

Yeah I tested it and got around 5 seconds. The answer I got using this problem was 10. I guess it comes down to how you define when the rotation significantly slows down.

This pset felt like it had just the right amount of computation. As a result, I enjoyed it the most out of all the psets we had this semester.