6.055J/2.038J (Spring 2010)

Solution set 1

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That criterion explains why the range narrows after you estimate using divide and conquer. At first, you have little idea about the true value, so you would not be surprised were it to fall outside a fairly large range; after the estimate, you know more, your confidence in the estimate increases, and your plausible range shrinks.

Warmups

1. **One or few**

   Use the 1 or few method of multiplication (and division) to estimate

   $$161 \times 294 \times 280 \times 438$$

   (a random multiplication problem generated by a short Python program).

   $$10^{\boxed{} \pm \boxed{}} \quad \textit{or} \quad 10^{\boxed{} ... \boxed{}}$$

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   > The first step is to convert each factor in the product to the nearest power of ten, perhaps also including a factor of a few. For example, 161 contains two factors of 10 and a factor of 1.61; and 1.61 is closer, on a log scale, to 1 than it is to few ($\sqrt{10}$). So 161 becomes simply 100 or 10^2. Here are the conversions for all four factors:
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These answers were very well explained and helped me to see step by step where I could have improved my answer- thanks!

man, it's really satisfying to have all your answers come so close to the solution set. I guess divide&conquer works pretty well!!

Looking through this solution set, I realize that using divide and conquer not only is a quick and clever way to simplify problems, but it also makes error analysis much easier. when you look at the final answer, and you're off, by say a factor of 10^2 (which was the case in my estimation of mass of CO2, you look back at your tree. the only value with a large enough order of magnitude that I could have estimated off by 10^2 was the world oil consumption; all the other values are small, and I most likely wouldnt under/over estimate them by 2 orders of magnitude. Indeed, looking through the solution and explanation, I was off in my original estimate of world oil consumption by 10^2

One thing that I struggled with was how small/large my error was... It seems here we have fairly strong confidence in our answers so would the error be +/-0.5?

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> Here is solution set 1. Submit your memo on it by 10pm on Thursday. Maybe you'll find mistakes (I didn't put it any on purpose) or suggest new solutions or spot confusing places.

should this be a+b? [later edited:] sorry i just zoomed in more. i couldn't see it until i got to 181%...

This was definitely a good starting problem. Took maybe 30 seconds. I think we could have maybe 3-4 of them , very short, very easy to turn into 1 and few, and get us into good shape for doing it. Maybe 1-2 per PSET from now on just to get really quick.

I agree, these are kinda fun.

I also think these problems are good to warm up on and aren't too strenuous. Maybe you could also add problems that help us get better at listening to our gut or that help us learn nice numbers that are important to know (numbers like number of seconds in a year)

I like the idea of having just simple estimation problems for things we don't often think about. How many meters in a roll of toilet paper? How many trees on campus? Just something to give us a feel for very rough estimation

I like having these warm-up type questions. Basic questions that test our speed with fundamental ideas.

I ended up doing the multiplication by splitting the numbers up, similar to the abstraction 3 memo, instead of using the 1 or few method to find the exact answer. Going back now, I see that I read the problem wrong...

It feels almost like cheating because we are estimating so much, but it works out! I feel like these are good for forcing me to be ok with the lack of accuracy.

I'm still a little confused as to when we "borrow" an overestimate from another number. Since 438 is a little over a "few" and we rounded down, I thought we should round the 161 up to a "few", but this put my answer off by a larger factor. How close must the estimation differences be before we're allowed to "borrow"?

You actually borrowed or compensated well. Think of it in multiplicative terms. Rounding 161 up to 300 is almost a factor of +2X, and rounding 438 down to 300 is -1.3X. These actually offset each other nicely – instead of rounding BOTH down by factors of -1.6X and -1.3X – so I think your answer (10^10?) is actually closer to the correct answer in terms of the ratio of correct/estimate. (You OVERestimate, but by LESS than a factor of 2, but the solution underestimates and is off by a factor of 2.)

i did the same thing, and when you compare it to the actual value, it becomes apparent that it's a good decision

I did something pretty similar to this rather than using the "few" idea and feel like both are pretty good estimates with one being over by a little less than the solutions' is under.

I also just saw I would be overestimating by calling 2.9 and 2.8 a few, so it seemed all right to call 1.61 a 1.

I estimated 1.61 as a few because of the overestimation of the 4.38. How does this work out on a log scale?

This balancing error in either direction is easy to see when we know what numbers we are dealing with from the start, but it bothers me more when we do it in problems such as divide in conquer when you don't know what you might have to over or under estimate later along in the problem.

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I did exactly this but I was wondering what the limit to a few would be...would 500 and 600 still be considered a few. I tried to round up for some and round down for others to compensate

When I solved this I took few to equal 3 instead of 10^.5 but it still ended up the same. :)

> i think the point is that 3 10^.5, since 3^2=9, which is close to 10.

the deviation of 161 and 438 roughly cancel each other out

Woohoo - this is exactly the same way I did this problem, with the same answer. Good to know that we're learning...

> Me too! The class time dedicated to "few" really helped me with the concept- which I even used in other problems besides this one! :)

> I agree. This concept of few is really handy and useful in lots of situations. Also, using this few vs. 10 method, it's easy to see where we are overestimating and underestimating, so we can relatively easily have a sense of which direction and around how much we might be off.

>> I agree as well - that was very obvious after having gone through the class (before I would have done a lot of tedious math to estimate it)

>>> Yeah, I thought the method of "few" has been the strongest methodology taught. I actually use in day to day approximations.

>>>> I agree, I also think it was important to understand when you are approximating a little bit lower then the value and a little bit higher in order to make up for this compensation later in the problem.

Is this the preferred format for this number or is it equally acceptable to express it only in powers of few?

> Yes, the preferred format is a power of 10 (may be 0) times nothing or few.

This was the one I wasn't sure about – whether 4.38 can be approximated as a few, or closer to 10^.7 (like 6 is). When should I consider a number big enough to no longer approximate as a few? (i.e., what about 5?)

Now the product is easy to do mentally. There are eight factors of 10 and three factors of a few. Since $(\text{few})^2 = 10$, three factors of a few becomes $10 \times \text{few}$. So

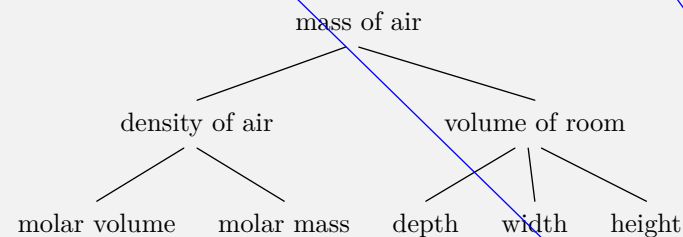$$161 \times 294 \times 280 \times 438 \approx 10^8 \times 10 \times \text{few} \approx 3 \cdot 10^9.$$

In the form 10^x, the estimate is $10^{9.5}$ because 3 (or few) is one-half of a power of 10. The estimate is only a factor of 2 smaller than the actual value of 5805041760 or roughly $6 \cdot 10^9$.

2. Air mass

Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

$$10\,\boxed{} \pm \boxed{}\ \text{kg} \quad \textit{or} \quad 10\,\boxed{} \cdots \boxed{}\ \text{kg}$$

One way to estimate the mass is to subdivide into the volume of the room and the density of air. The volume of the room subdivides into its length, height, and width. I remember that the density of air is roughly $1\,\mathrm{g\,\ell^{-1}}$ (or $1\,\mathrm{kg\,m^{-3}}$) because I have used the value often in estimation problems. Alternatively, you can use a useful fact from chemistry, that one mole of an ideal gas at standard temperature and pressure occupies 22 liters, and combine that fact with the molar mass of air. Using that method, the tree is

```
                   mass of air
                  /           \
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          /        \        /    |     \
  molar volume  molar mass  depth width height
```
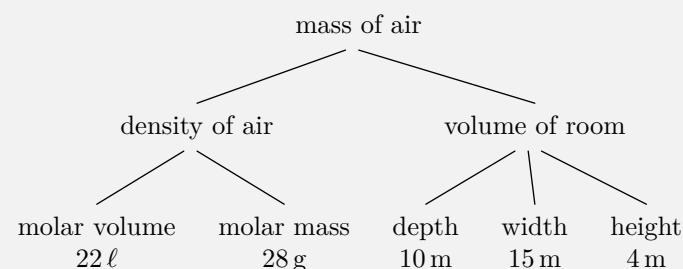
Now put values at the leaves.

For the room dimensions, the MIT schedules office webpage gives the room area, but let's estimate the dimensions by eye. Most rooms are 8 or 9 feet high but our classroom (4-163) has high ceilings, so let's say 12 feet high or $4\,\mathrm{m}$. The room has about 10 rows, spaced around $1\,\mathrm{m}$ apart. So the depth is about $10\,\mathrm{m}$. The room is perhaps 1.5 times as wide as it is deep, so the width is roughly $15\,\mathrm{m}$. As a check, these estimates mean the area is $150\,\mathrm{m^2}$ or about $1600\,\mathrm{ft^2}$; the MIT classroom-inventory page (linked from the course website) says that the area is 1303 square feet, so our estimate of the area is accurate to 25%.

The molar volume for air (like any ideal gas) is 22 liters. The molar mass is, roughly, the molar mass of nitrogen, which is $14\,\mathrm{g}$. But nitrogen is diatomic, so the actual molar mass is $28\,\mathrm{g}$.

The tree with values is:

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                   mass of air
                  /           \
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  molar volume  molar mass  depth width height
     22 ℓ          28 g      10 m  15 m   4 m
```

i wrote my answer as fewE9, should i instead have written 3E9?

> Either is fine.

> > Something that also helps is knowing that approximately: 10^0.5 = 3 10^0.7 = 5 10^0.9 = 8

> > > Yah, he covered this in class more recently, and it actually seems rather cool. At least for our +/- correction factors

is there a reason we have to enter all our data in the form of 10^x and not as __ *10^x. because I always just drop the first number and I don't think that provides the best estimate

How would you have calculated your error for this?

> i don't think that it's so much a "calculating" error as thinking about where error might have come from and estimating how much that would be off...for this one, you know that each of your "estimates" are not off by much, so the final value should be quite close.

> > regardless, i'd also like to know what a reasonable error would have been to understand how far off an estimation like this could/should end up being

For a problem of this scale, you could even have a sense of the direction and magnitude of the error. The middle two are really close to 300, and the other two are rounded down, so we'll be underestimating, and we know it'll be by a factor of about 1.6 x 1.33 = 2. This is one way to start refining our estimates, since people have been concerned about how accurate to be.

> I agree. I'd be kind of embarrassed to get a problem like this wrong to a factor of 2... Especially since the answer can be determined quite precisely. I can really only see this accuracy being useful for sense-checking another result.

So when our answer is fewx10^(5) we should actually input our answer as 10^(5.5)? I had previously only been taking the exponent and not factoring the few into it.

> sanjoy replied to someone else who asked this question above and said either is fine

> > So long as its within the confidence interval, i think its ok

I would have expected the actual number to be less because 438 is the only number above 3 in the mantissa, while the rest our below 3.

When we do the range of answers? Do we use the actual answer to figure out the range or just do a random estimate? I have just been adding 0.5 because I figure that's a reasonable factor.

I really liked this problem, it was very clear how to use divide and conquer to solve it, and I had a lot of fun doing it!

I used a similar divide and conquer method on the homework, however I had trouble conveying it because I was limited to writing in text. Perhaps in the future you may want to add an option to attach a jpeg

> On a similar note, I wish the text box for inputting explanations on the homework were larger so I could see my whole answer without scrolling. Or maybe I write to much for the explanations...

What does that variable represent?

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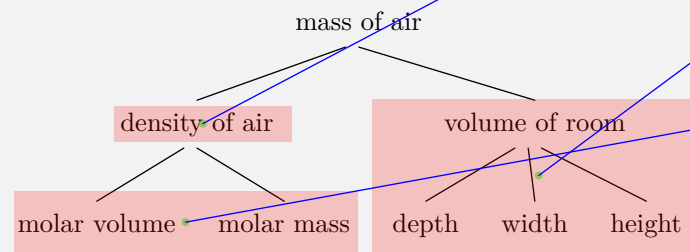$$161 \times 294 \times 280 \times 438 \approx 10^8 \times 10 \times \text{few} \approx 3 \cdot 10^9.$$

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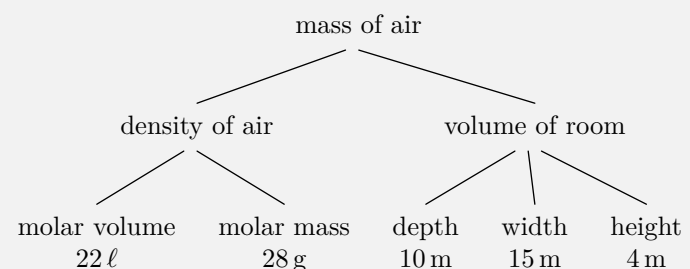
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This variable represents gram per liter. The liter is raised to the negative one because it is in the denominator.

I think that the most tricky part of these problems are the units.

there is no "molar mass of air". you'd make the assumption that it's mostly nitrogen, i suppose

Thankfully, this is also on your sheet of constants :)

I took into account the space taken up by the stadium seating and assumed it reduced the volume by about 1/5 or so. But it probably was a small enough compensation were error somewhere else probably overshadowed it.

I did the same, but I also thought I'd overestimated the height, so I kept the adjustment.

Yah, same here with the keeping the final amount because the ceiling is really high

I didn't think of accounting for the furniture in the room, but that is a good way to approach it. I do agree though since we are doing a lot of assumption making, it doesn't matter in the end.

I don't really see how this could help get the density any more accurately than just using intuition, since the numbers involved are so big/small and could have large errors.

I think the purpose of the tree is also to help organize our thoughts better.

MIT floorplans are also available on the facilities page. I used these to decide which dorm rooms I wanted during in-house lotteries.

also, just fyi, these are usually only available with certificates, which shouldn't be a problem for any of us.

This was hard for me to recall, since I rarely look up.

You could probably figure it out by looking up at the ceiling of the room you are in when working on this problem, then estimate from there.

I broke this down by estimating the size of my room, thinking about how many people could fit in my room, and multiplying until I had enough room for all my classmates.

wouldn't this be basically the same as saying 10 ft high or few meters high, since we're just doing an approximation anyway?

That I did not realize, i think calculating depth was the hardest part for me

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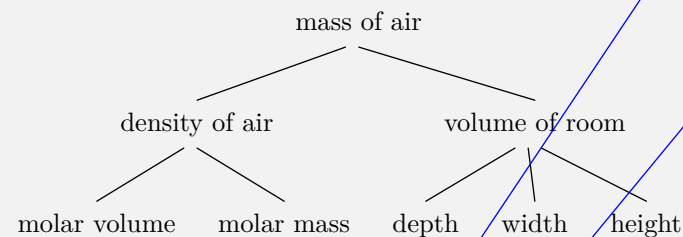$$161 \times 294 \times 280 \times 438 \approx 10^8 \times 10 \times \text{few} \approx 3 \cdot 10^9.$$

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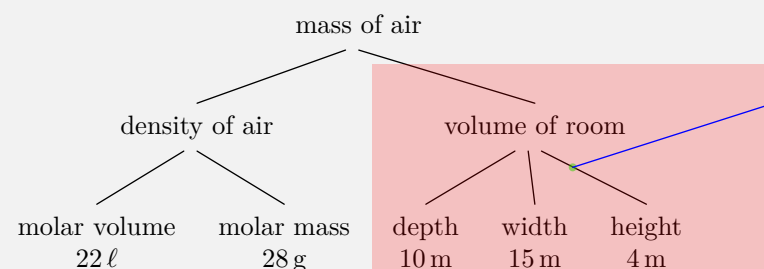

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The tree with values is:

i overestimated here thinking that it was twice as wide, but upon further inspection, this makes more sense.

> Me too... but I had the height at "few" meters high... I ended up with 10^3 kg in the end so it all worked out.
>
> I overestimated even more, the room seemed wider to me.
>
> > I guessed 20m, personally

I had no idea that we had a classroom-inventory page.

Out of curiosity, are we allowed to consult answers to subproblems before we move onto solving the main problem? As in, can we use other resources to make sure that our values at the leaves are correct before computing the root of the tree?

> That would seem to defeat the purpose of the process. It might make you feel better, but trust yourself all the way through and just keep your confidence limits in mind. On a real problem, you might not be able to check partway through, although things like building early prototypes or making quick toy models might be ways to check your prediction before you go on.
>
> > Ideally, or at least randomly, your errors might cancel out, and you can end up in the same place anyway
>
> I agree. Checking your answers would be nice, but the point here is to estimate this final value, not to check a bunch of answers to subquestions that will eventually give you the exact value. Plus, in this case, even if you were terribly off–if you guessed that two of the dimensions of the room were twice as big (huge overestimate), your final answer would only be a factor of 4 off, and you'd probably have a sense that you overestimated.

My biggest problem on this homework was figuring out when to try and think of a way to estimate a quantity and when to just give up and look for the answer somewhere else. For instance, is there a good way to estimate the density of air if you don't know the molar volume and molar mass off the top of your head?

> I think it's better to estimate from experience then wasting time looking for answers if you don't know where to look. Also it depends on how actuarate you're trying to be.

I had used the table on constants on the p-set from the pre-test. should I have estimated the values a different way?

I didn't know this piece of information. So i likened air to water, which i know the density of, and then pulled the oxygen out of the water and fudged for the new spacing.

> That's an interesting approach. The volume of 1 mole of air is pretty easy to estimate if you remember $PV=nRT$ and that $R=.0821\,\mathrm{Latm/molK}$, which I think is pretty easy to remember since we use it so often in a lot of different classes.
>
> > Wow, those are both good methods. I actually went to wikipedia for this... I had no idea how to estimate molar volume.

I simply assumed that there are 100 people in the class and that each person takes up 1 m^3 and that the classroom is 3 m high. That gave me 10^2.5 kg, which was on the lower range

> My volume was much smaller than displayed, yet my answer (and range of confidence) was nearly the same as this one. That's interesting.

Now the product is easy to do mentally. There are eight factors of 10 and three factors of a few. Since $(\text{few})^2 = 10$, three factors of a few becomes $10 \times \text{few}$. So

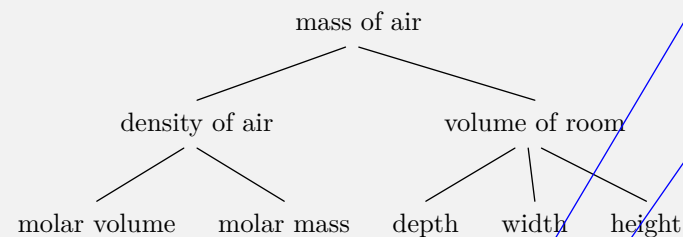$$161 \times 294 \times 280 \times 438 \approx 10^8 \times 10 \times \text{few} \approx 3 \cdot 10^9.$$

In the form 10^x, the estimate is $10^{9.5}$ because 3 (or few) is one-half of a power of 10. The estimate is only a factor of 2 smaller than the actual value of 5805041760 or roughly $6 \cdot 10^9$.

2. Air mass

Use divide-and-conquer to estimate the mass of air in the 6.055J/2.038J classroom and explain your estimate with a tree. If you have not yet seen the classroom, try harder to attend lecture!

$$10\,\boxed{} \pm \boxed{} \text{ kg} \quad \textit{or} \quad 10\,\boxed{} \cdots \boxed{} \text{ kg}$$

One way to estimate the mass is to subdivide into the volume of the room and the density of air. The volume of the room subdivides into its length, height, and width. I remember that the density of air is roughly $1\,\mathrm{g\,\ell^{-1}}$ (or $1\,\mathrm{kg\,m^{-3}}$) because I have used the value often in estimation problems. Alternatively, you can use a useful fact from chemistry, that one mole of an ideal gas at standard temperature and pressure occupies 22 liters, and combine that fact with the molar mass of air. Using that method, the tree is

```
                    mass of air
                   /            \
          density of air      volume of room
           /        \         /     |      \
   molar volume  molar mass  depth width height
```
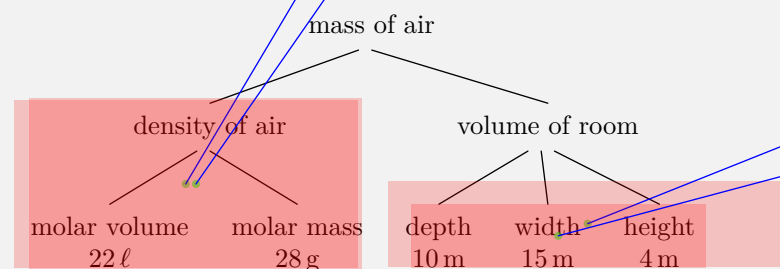
Now put values at the leaves.

For the room dimensions, the MIT schedules office webpage gives the room area, but let's estimate the dimensions by eye. Most rooms are 8 or 9 feet high but our classroom (4-163) has high ceilings, so let's say 12 feet high or $4\,\mathrm{m}$. The room has about 10 rows, spaced around $1\,\mathrm{m}$ apart. So the depth is about $10\,\mathrm{m}$. The room is perhaps 1.5 times as wide as it is deep, so the width is roughly $15\,\mathrm{m}$. As a check, these estimates mean the area is $150\,\mathrm{m}^2$ or about $1600\,\mathrm{ft}^2$; the MIT classroom-inventory page (linked from the course website) says that the area is 1303 square feet, so our estimate of the area is accurate to 25%.

The molar volume for air (like any ideal gas) is 22 liters. The molar mass is, roughly, the molar mass of nitrogen, which is $14\,\mathrm{g}$. But nitrogen is diatomic, so the actual molar mass is $28\,\mathrm{g}$.

The tree with values is:

```
                    mass of air
                   /            \
          density of air      volume of room
           /        \         /     |      \
   molar volume  molar mass  depth width height
      22 ℓ         28 g       10 m  15 m   4 m
```

oh i just used the value from the table on the diagnostic. was i not supposed to?

> There's no real 'supposed to' in estimation. For a first estimate, feel free to use the table values. Then, to learn more about estimation, see if you can figure out ways to estimate the values that you used. Myself I try to estimate everything totally closed book, just to stay in training.

Since the density of air was given on our "useful equations" sheet for the pre-test, is it alright if we used that value instead of breaking it down into volume and mass (although I understand that this way is more fit to the tree approach)?

> I have the same question. In my opinion, the point of breaking things down is to get to a point where you have numbers that you can be reasonably sure are accurate. If you have the value already, why not use it??

> > Agreed, if you know it, you should use it. Don't pretend not to just to fulfill some sense of duty to break down the problem to an arbitrary level. There is always some deeper branching possible, but it may not add any value to your estimate.

> > > I would amend that slightly to say: "If you know it, use it in your first estimate. Then if you have time and interest, and it seems fun, see how you might derive any of the numbers that you took as a given."

> > > I used the 1 kg/m^3 value instead of bothering with the molar volumes but I still think it is a useful exercise in case one forgets these values.

> I guess I just did some multiplication wrong, because I over estimated the size of the room but underestimated molar mass, but forgot that N is diatomic.

> I also took this value from the equations sheet from the pretest. In the future, is it acceptable for us to use these numbers, or should we stick with only numbers we know off the top of our heads?

> > I think "about 1" is such an easy number (or abstraction?) that there wouldn't be any reason to just plug it in.

> > One reason is to stay in training. The practice in estimating quantities you already know (and can therefore easily check whether your method was sensible) helps you when you get to quantities that you do not know.

> > I haven't played baseball for eons, but I remember as a child practicing swinging a bit with a heavy ring at the end. One could say, what's the point, since in real life you won't swing such a bat. But it helped develop our arm strength and control for the harder problem that was about to come (with a pitcher throwing fast baseballs right near you).

> > Another reason to break it down is that it helps you explain the estimate to someone who doesn't have the same set of known values in his/her head.

my estimates were really similar, which, though a minor accomplishment, is exciting.

you choose very even round numbers, how do you decided between trying to estimate the best and fudging the numbers to make your calculations easier

Now the product is easy to do mentally. There are eight factors of 10 and three factors of a few. Since $(\text{few})^2 = 10$, three factors of a few becomes $10 \times \text{few}$. So

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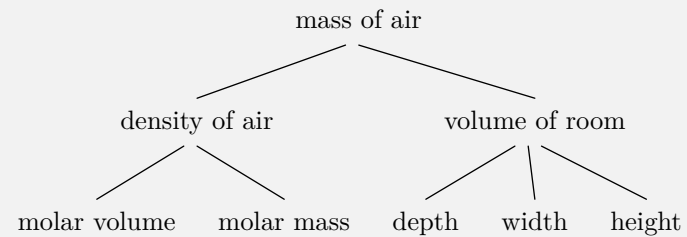
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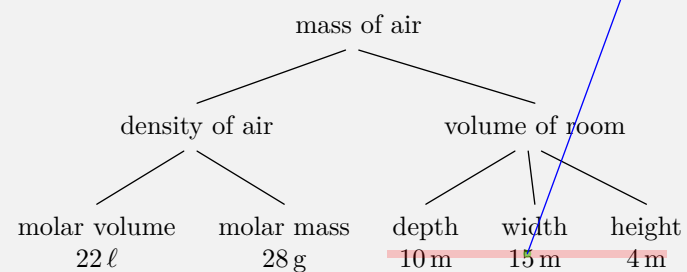
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```

I totally overestimated all of these! I was off most in the height...i thought the room was about as tall as it was deep...I need to get better at estimating distances!

Now propagate values upward. The volume of the room is $600\,\mathrm{m}^3$. The density of air is roughly $28/22\,\mathrm{g}\,\ell^{-1}$, or roughly $1\,\mathrm{kg}\,\mathrm{m}^{-3}$. Therefore, the mass of air in the room is roughly $600\,\mathrm{kg}$. In the form $10^x\,\mathrm{kg}$, it is halfway between (on a log scale) few $\times\,10^2\,\mathrm{kg}$ and $10^3\,\mathrm{kg}$. Because few is one-half of a power of 10, the mass is in the middle of the range $10^{2.5}..10^3\,\mathrm{kg}$. So let's call it $10^{2.75}\,\mathrm{kg}$. Either $10^{2.5}\,\mathrm{kg}$ or $10^3\,\mathrm{kg}$ would also be a reasonable estimate if you are rounding the exponent to the nearest 0.5.

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where r is the radius of the earth, and ρ is the density of the earth. Note that even in this first step we have already approximated by assuming that the earth is spherical and that it has a uniform density.

To estimate r, I remember that California is about 3000 mi away from Boston (a typical flight at 500 mph takes about 6 hr) and it is also 3 time zones away. So each time zone is about 1000 mi, meaning that the circumference of the earth (24 time zones) is $C \sim 2.4 \cdot 10^4$ mi giving a radius of $r = C/2\pi \sim 4000$ mi. In metric units, that is $6.4 \cdot 10^6$ m.

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I used the normal sized ceilings and a little but of an undersized room so I was off by a little but still within about 100kg

I got this value. I used a 10*5*4, and a 1kg/m density.

ended up missing a decimal in my estimation of volume, I was really confused how I ended up so far off.

why not just use the cube method, as we did with the CD and a square? Radius 6000m, so cube approx w/ side length 10,000m?

why not? we know the formula and since pi = 3, you even cancel out the fraction. then it's just a few times the rest.

That's a good idea. Let's see how it works out: 10^7 m (you meant 10,000 km) gives 10^21 m^3. Then with the density you get 5*10^24 kg.

So it works exactly the same

I didn't incorporate density into my approximation... I just did F=ma=GMm/r^2. The m cancels out and we know acceleration due to gravity, G and the radius of the earth from the numbers sheet.

I did the same thing and got an answer of 10^24.7. I'm positive I would not have been anywhere as close using the density/volume approximation (which actually seemed like the most inituitive way to do it but I was at a total loss for approximating the density of rock and iron...)

yeah I ended up doing the same thing because earth's density not very constant, and least dense on the surface.

It is really easy to sometimes forget all these other approximations that we make, I completely forgot that the earth is probably not completely spherical.

i also used the 3000 number, but estimated how many US's it would take to go around the earth.

To get r, I used a value from the useful numbers sheet. Are we allowed to use this sheet as a reference when doing these problem sets or should we work from scratch?

I used the back of the envelope numbers from the diagnostic test to find the radius of the earth. Is that unacceptable, should I have approximated the radius using some method like you did here?

I also used the number from the sheet.. This is impressive logic but I don't know how I would have thought of this.

I just use the constant from the sheet that you provided us

That's useful to know!

I really like this approach to finding the radius, For some reason I have the number 24000 miles stuck in my head as the earth's circumference, so I could figure it out, but now when I think about it, if I had not known that I would have been at a loss as to how to find the earth's radius.

I agree, I never would have thought of estimating the radius, i just looked it up online

Now propagate values upward. The volume of the room is $600\,\text{m}^3$. The density of air is roughly $28/22\,\text{g}\,\ell^{-1}$, or roughly $1\,\text{kg}\,\text{m}^{-3}$. Therefore, the mass of air in the room is roughly $600\,\text{kg}$. In the form $10^x\,\text{kg}$, it is halfway between (on a log scale) few $\times\,10^2\,\text{kg}$ and $10^3\,\text{kg}$. Because few is one-half of a power of 10, the mass is in the middle of the range $10^{2.5}..10^3\,\text{kg}$. So let's call it $10^{2.75}\,\text{kg}$. Either $10^{2.5}\,\text{kg}$ or $10^3\,\text{kg}$ would also be a reasonable estimate if you are rounding the exponent to the nearest 0.5.

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Then the mass is, using $\pi \sim 3$,

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I wouldn't have thought of using time zones.

again, for these values i used the table from the diagnostic.

I found this method of using the timezones to gauge the earth's circumference to be rather clever. A good trick to remember.

> Yeah, I would never have thought to use the time zones. I guess I was just lucky to know trip distance to China.

> I feel like random trivia is a big part of being able to approximate on the fly. Will we be learning lots of just that in class? The stuff about the pianos and cochlea was pretty neat.

> I agree. This is a really clever and quick way to get a value for the radius of the earth. I used the value given in the constants, but Idefinitely would not have come up with this, and whatever method I used probably would have taken much longer.

> > Agreed, this is very clever. I feel like this is a value similar to the 300 million people in the US, something we should have memorized.

> > > This was very clever. I had no idea where to start so I had to look up the radius.

> > > > I agree, the radius of the earth comes up in a lot of physics problems in the gravitational unit

This is actually a pretty good way of measuring distances. Its easier to remember about how long a flight was as opposed to the distance traveled and planes always fly around 500mph.

Are we supposed to estimate everything for the homework? I just used the values from the list of useful numbers you provided on the course website.

Iron and magnesium... peridotite mainly. But that information is useless if you aren't studying Geo.

in my hw, I just used the density of water, because I thought the earth was composed of 70% of water. I think it is why my answer was off by a factor

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I approached this problem very differently. I equated the familiar F=mg with F=GmM/r^2 and solved for M. This worked out extremely well.

The density for rock, as listed in the constants page, is 5 * 10^3. I always figured that the materials below the surface of the earth grow more and more dense as they get closer to the core, making the density much higher.

> It cannot get too much higher: Solids, even under very high pressure, have almost the same density as at standard pressure: the atoms don't have any room between them. So the core, which is iron at very high pressure, has the same density as iron at the surface (about 7 g/cm^3).

I too did this and when I checked my final answer I was much closer than when I attempted the density*volume method.

I also used that method, of a = Gm/r^2 and solved for m since I knew 9.8 m/s^2 (aka 10)

using the force of gravity is definitely an interesting approach. I wish I had thought of that.

That's what I did too, I thought it was much easier than trying to approximate the average density of the earth.

Yeah I started out doing this problem as a density volume problem but got lost in the numbers and then realized how easy it would be using GmM/r^2

I remember when I did this problem I did it both ways and got the same answer (within some error, obviously)

Are we supposed to be able to justify our numbers like this? I just looked them up..

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I also found this factor of 5 very unintuitive.

I figured if it was on our constants sheet it was fair game and did not require justification.

I was really confused on how to estimate the density of Earth. I instead used the F=mg type equations.

I still don't understand your use of a factor of 5.

Are we supposed to be able to justify our numbers like this? I just looked them up..

Should we not use the numbers table on homeworks? I didn't on the pretest but everyone else said they had, I felt at a disadvantage.

I never would have thought of this. Very good.

I estimated this first to 3x10^6. It's probably why I got 23.5 instead of 24.5.

> There's a general principle in that point, which we'll see often: When quantities are raised to a high exponent (e.g. 3), a moderate inaccuracy in the quantity turns into a large inaccuracy in the result. For example, a factor of 2 in the radius turns into a factor of 8 (almost an order of magnitude) in the volume and mass.

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I rounded 4 down to a few an 6^3 as few*10*10 (round 2 of the 6s up and 1 down). This still gives me the same answer.

missed superscript (10^24), but clear from context

Thanks. A frequent bug in my use of TeX. I have a short TeX program that typesets scientific notation. So 5 edot5 turns into 5*10^5. And 5 edot24 turns into 5*10^24. But, because of how TeX parses procedure arguments, 5 edot24 turns into 5*10^2 followed by the 4.

I had caught one mistake just like that (the 5*10^24 earlier on the same line originally read 5*10^2 followed by the 4).

Having made it a second time on the same line, it's time to make an abstraction. Here is a grep invocation that will catch the same kind of error:

grep ' edot[0-9][0-9]' hw01.tex

Before I fix the mistake that you caught, it finds this line:

is $6 edot24 kg$.)

Which I will now change to

is $6 edot24 kg$.)

Thanks for giving me the chance to illustrate another use of UNIX. (As a further abstraction, I've now filed that grep line as one line in my "check TeX files for common errors" script.)

I definitely missed the answer by a lot (a factor of 10^12!!) because I used the density of rock as the density for the entire volume, using the number provided on the constants page. I guess instead of assuming it was equal to that number, I should have thought more about the actual composition of the Earth's mass.

I really like this problem as its the only one in the class so far that is directly related to my area of expertise. I never thought about file systems as an abstraction before this term.

This was a good problem. Unfortunately I got it wrong. Never learned how to code

I understand how the development of unix was a method of abstraction, but I don't understand how us simply using it really helps us learn anything about estimating. It just teaches us how to use Unix. I don't find this very helpful or instructive for this class.

The class is partly about estimating, but only partly. The broader theme is "solving hard problems", and to that end divide and conquer is useful, as is making good abstractions (divide and conquer can be considered as using good abstractions).

I totally switched this around! arg. i was thinking ls -t(head(tac(head -1))) in stead of one after the other...i should have know it wouldn't be a "trick" questions!

unfortunately tac isnt in mac os's darwin. Rather straight forward though. I enjoy learning these.

Now propagate values upward. The volume of the room is $600\,\text{m}^3$. The density of air is roughly $28/22\,\text{g}\,\ell^{-1}$, or roughly $1\,\text{kg}\,\text{m}^{-3}$. Therefore, the mass of air in the room is roughly $600\,\text{kg}$. In the form $10^x\,\text{kg}$, it is halfway between (on a log scale) few $\times\,10^2\,\text{kg}$ and $10^3\,\text{kg}$. Because few is one-half of a power of 10, the mass is in the middle of the range $10^{2.5}..10^3\,\text{kg}$. So let's call it $10^{2.75}\,\text{kg}$. Either $10^{2.5}\,\text{kg}$ or $10^3\,\text{kg}$ would also be a reasonable estimate if you are rounding the exponent to the nearest 0.5.

Problems

3. **Mass of the earth**

   Estimate the mass of the earth.

   $$10^{\boxed{}\ \pm\ \boxed{}}\ \text{kg}\qquad \textit{or}\qquad 10^{\boxed{}\ \cdots\ \boxed{}}\ \text{kg}$$

   After choosing your range (in either form), check it against the measured value.

   The mass breaks into density times volume:

   $$m \sim \frac{4}{3}\pi r^3 \rho, \tag{1}$$

   where r is the radius of the earth, and ρ is the density of the earth. Note that even in this first step we have already approximated by assuming that the earth is spherical and that it has a uniform density.

   To estimate r, I remember that California is about $3000\,\text{mi}$ away from Boston (a typical flight at $500\,\text{mph}$ takes about $6\,\text{hr}$) and it is also 3 time zones away. So each time zone is about $1000\,\text{mi}$, meaning that the circumference of the earth (24 time zones) is $C \sim 2.4 \cdot 10^4\,\text{mi}$ giving a radius of $r = C/2\pi \sim 4000\,\text{mi}$. In metric units, that is $6.4 \cdot 10^6\,\text{m}$.

   To estimate ρ, I start the density of water: $10^3\,\text{kg}\,\text{m}^{-3}$. The earth is made up mostly of iron and dense rock, both much denser than water – maybe by a factor of 5. Why a factor of 5? A factor of 3 would be too low, since that is the density of typical surface rocks, and they are the material that floated to the top when the earth was cooling, so they are less dense than the rest of the earth. A factor of 10, on the other hand, sounds way too dense. So I'll choose a factor of 5, making $\rho \sim 5 \cdot 10^3\,\text{kg}\,\text{m}^{-3}$.

   Then the mass is, using $\pi \sim 3$,

   $$m \sim 4 \times (6.4 \cdot 10^6\,\text{m})^3 \times 5 \cdot 10^3\,\text{kg}\,\text{m}^{-3}. \tag{2}$$

   Do the arithmetic by divide and conquer. The powers of 10 total to 21: 18 from the cubed radius and 3 from the density. Then there's the factor of 4, a factor of 6.4^3, and a factor of 5. If the 6.4^3 were 6^3, it would be 216, so let's pretend that 6.4^3 is 250. Then the factors are $4 \times 250 \times 5 = 5 \cdot 10^3$. The result is a mass of $5 \cdot 10^{24}\,\text{kg}$ or $10^{24.7}\,\text{kg}$. (The true value is $6 \cdot 10^{24}\,\text{kg}$.)

4. **Explain a UNIX pipeline**

   What does this UNIX pipeline do?

   ```
   ls -t | head | tac | head -1
   ```

   If you are not familiar with the individual UNIX commands, use the man command on Athena or on any other handy UNIX or GNU/Linux system.

Does having UNIX syntax pose relevance to abstraction? I'm all for learning UNIX, I wowuld just like to know if we should start making a point of getting to know it

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5. **Atmospheric carbon dioxide**

What is the mass of CO_2 generated by the world annual oil consumption?

$10\boxed{} \pm \boxed{}$ kg/year   *or* $10\boxed{} \ldots \boxed{}$ kg/year

Here is the (unbalanced!) combustion of a generic hydrocarbon (including oil, gasoline, and kerosene):

$$CH_2 + O_2 \longrightarrow CO_2 + H_2O.$$

I'll start with the US oil consumption, roughly $3 \cdot 10^9$ barrels/yr. Then increase it by a factor of 4 to get the world oil consumption: I often remember reading that although the United States has 5% of the world's population, it uses 25% of the energy. I don't remember whether the 25% was talking about energy overall or just oil, but maybe it doesn't matter. I'll then convert barrels to liters using $160\,\ell$ per barrel and then to mass using $1\,kg\,\ell^{-1}$ (assuming oil and water have comparable density).

Finally, I'll convert this mass of oil into a mass of carbon dioxide. According to the (unbalanced) chemical reaction, one mole of hydrocarbon (CH_2) becomes one mole of carbon dioxide (CO_2). I might just either ignore the effect of balancing the equation; on the other hand, it is not hard to determine: No other products or reactants involve carbon, so the coefficients in front of CH_2 and CO_2 must be identical. In other words, balancing may give strange coefficients for the other products and reactants, but it leaves the $1:1$ mole ratio between CH_2 and CO_2. A mole of CH_2 weighs $14\,g$ whereas a mole of CO_2 weights $44\,g$, almost 3 times as much as the mole of CH_2. So, to convert mass of oil into mass of carbon dioxide, I'll multiply by 3 (or few).

The overall calculation is then:

$$3 \cdot 10^9 \text{ barrels/yr} \times 4 \times \frac{1.6 \cdot 10^2\,\ell}{1\,\text{barrel}} \times \frac{1\,\text{kg oil}}{1\,\ell} \times \frac{3\,\text{kg } CO_2}{3\,\text{kg oil}}. \tag{3}$$

Now do the numbers. There are 11 powers of 10 and then the following factors:

$$3 \times 4 \times 1.6 \times 3 \sim 60. \tag{4}$$

So the estimate is $6 \cdot 10^{12}$ kg per year.

Out of curiosity, I wanted to compare this number to the actual world production of carbon dioxide. It's hard to find the carbon-dioxide production due just to oil. But oil might be one-third of the world energy consumption (there's also natural gas, hydroelectric, coal, etc.). Multiplying the above estimate by 3 gives an estimate for the world production of carbon dioxide: $18 \cdot 10^{12}$ kg per year or roughly $2 \cdot 10^{13}$ kg per year. The actual total in 2006 was $3 \cdot 10^{13}$ kg.

Comments on page 4

were we really supposed to know this, and is it really that important to know?

> You weren't supposed to know it off hand, but I wanted people to look it up using the "man" command, because it is important to learn how to use the "man" command. "Teach a (wo)man a UNIX command, and you show him/her how to solve one problem. Teach a (wo)man the UNIX 'man' command, and you help him/her solve any problem."

i didn't have easy access to a place where i could use the "man" command, but wikipedia actually had articles on each of the commands and that's what i ended up using to work this problem :)
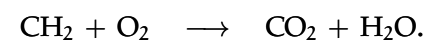
You could argue that most things in this class aren't "really important" to know, but they are all pretty interesting and quite useful to know, and the same thing here since Unix is the basis of file systems and computers have such an important role in everyday transactions nowadays.

i didn't know it automatically took the first 10 lines when you didn't specify a number

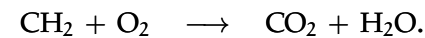**I still don't really understand what it does after reading this.**

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So I totally botched this example - I wonder if other people had trouble with the UNIX pipelines as well? I thought that the rest of the problems were quite fun, but this one really did confuse me.

I liked this. I didn't understand at first, so I looked up the commands. Got it; even the summary at the end.

It took me about 2 minutes to look up the commands on Wikipedia and another 2 minutes to figure out what was happening, so I didn't find it that difficult (and I don't have much programming experience). That being said, I did feel that this problem was not very connected to the other estimation problems.

I messed this up because I didn't realize each line was one of the files. I thought it would be the content of each. But now that I think more about it, this makes way more sense.

It does involve breaking a non-obvious, if arbitrary, problem (display the 10th most recent file) into discrete parts, like d&c.

Yeah I was kind of confused for a little bit because I actually typed in the code into my athena. I only got the name of some random file, so I was pretty confused.

Actually Athena was a great tool for answering this question. Knowing that each | denotes a new command, if you apply each command separately, you will find that the commands act upon a list of file names, thus the result of MacData.

someone mentioned that this problem was kind of random but isn't the point seeing a bunch of different problems to get a better idea of the flexibility of this concept?

I agree, Athena was an excellent resource for this question since it was easy to man each command and checking the final result was as simple as running a few commands in succession.

With no programming experience I was happy to be able to figure this out. After reading these comments I no longer feel bad about looking up some commands.

I was able to figure out all the meanings but for some reason I couldn't tie than together to derive a meaning.

I liked this question - taught me something new. A friend showed me.

I kind of got this problem. I had to look up all the commands and go through everything in my head but I still arrived at the wrong answer. I think for those of us who have no UNIX experience these problems tend to be much more difficult. With some practical experience I'm sure these would become cake.

Whoops. For some reason my computer has no idea was tac is so I looked it up and misunderstood it to mean that it reversed file names letter by letter...

That would be a kinda cool program, but I thought we went over that one in class to find the ending letters?

I understood the combine part as them combining the files together, not just the files name. So I thought i printed the last line of the 10th most recent file.

In the man page, it uses the "file" to mean input - when you pipe in the results of the previous commands, those become the input "file"

i just don't know why it took the top 10 lines – aren't you supposed to specify? i didn't see it define the number of lines. for the first instance of "head"

The `ls -t` lists all the filenames in the directory ordered by recency with the most recent first. The next step, `head`, takes the first 10 lines. Therefore so far we have a list of the 10 newest files. The `tac` reverses this list so that we still have a list of the 10 newest files but it is ordered from 10th newest at the top to newest at the bottom. The `head -1` takes the first line from this list, giving us the 10th-newest file.

do you have a directory in mind when you give UNIX examples? I think you should have given an overview of UNIX in general before jumping in to specific pipeline examples. I asked some of my course 6 friends about this and they were confused as well, especially as to why we were learning pipelines in this class...

Isn't it technically the tenth most recently modified file/folder?

> You are right, that is a more accurate way to describe it.

5. Atmospheric carbon dioxide

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$$10^{\boxed{}} \pm \boxed{} \text{ kg/year} \quad \textit{or} \quad 10^{\boxed{} \ldots \boxed{}} \text{ kg/year}$$

oops. i didn't read world. i guess i was just thinking US again.

Here is the (unbalanced!) combustion of a generic hydrocarbon (including oil, gasoline, and kerosene):

$$CH_2 + O_2 \longrightarrow CO_2 + H_2O.$$

When I initially read the problem, I had trouble figuring out how I would use the equation. Once I started thinking about it more and one I had figured out the mass of oil consumed in kilograms, I figured out that I was supposed to look at the ratio between CH2 and CO2.

> I was really surprised that this showed up with this examples. Had you not included this reaction, I would never have thought to approach this problem using chemistry.

> > Yea, I really enjoyed incorporating chemistry into the solution. It really is breaking it down into the smallest bits we can.

I'll start with the US oil consumption, roughly $3 \cdot 10^9$ barrels/yr. Then increase it by a factor of 4 to get the world oil consumption: I often remember reading that although the United States has 5% of the world's population, it uses 25% of the energy. I don't remember whether the 25% was talking about energy overall or just oil, but maybe it doesn't matter. I'll then convert barrels to liters using $160\,\ell$ per barrel and then to mass using $1\,\text{kg}\,\ell^{-1}$ (assuming oil and water have comparable density).

We derived this in class, right? Otherwise I would have had no idea how to do this problem.

When I was doing this problem I took the fact that barrels are crude oil into consideration which had a big impact on my answer, how can we just ignore this?

> I think the barrels should refer to the amount of crude oil. Are you trying to distinguish between crude oil and refined oil? Because if that's the case, I have read that a barrel of crude oil actually translates to about a barrel of refined products. Therefore, your answer should not be largely impacted.

Finally, I'll convert this mass of oil into a mass of carbon dioxide. According to the (unbalanced) chemical reaction, one mole of hydrocarbon (CH_2) becomes one mole of carbon dioxide (CO_2). I might just either ignore the effect of balancing the equation; on the other hand, it is not hard to determine: No other products or reactants involve carbon, so the coefficients in front of CH_2 and CO_2 must be identical. In other words, balancing may give strange coefficients for the other products and reactants, but it leaves the $1:1$ mole ratio between CH_2 and CO_2. A mole of CH_2 weighs $14\,\text{g}$ whereas a mole of CO_2 weights $44\,\text{g}$, almost 3 times as much as the mole of CH_2. So, to convert mass of oil into mass of carbon dioxide, I'll multiply by 3 (or few).

How did you approximate this number?

> I believe we went over this in class. But one way to do it is divide and conquer. We estimate how many cars there are and how much gas each one uses, etc.

Is this a leaf from one of the examples that we did in class? (I know we had a similar problem, but don't recall our exact answer)

> I didn't remember that number off the top of my head from class either, so maybe it would be beneficial to have the calculation for that also posted along with the rest of the solution.

The overall calculation is then:

$$3 \cdot 10^9 \text{ barrels/yr} \times 4 \times \frac{1.6 \cdot 10^2\,\ell}{1\,\text{barrel}} \times \frac{1\,\text{kg oil}}{1\,\ell} \times \frac{3\,\text{kg } CO_2}{3\,\text{kg oil}}. \tag{3}$$

> It is a leaf from the oil imports example we used - however, this was the number we found for number of barrels per year that the US imports, not the amount of barrels that the US consumes. That number would be roughly $6*10^9$, but since that's only a factor of 2, it probably won't change the answer too much.

Now do the numbers. There are 11 powers of 10 and then the following factors:

$$3 \times 4 \times 1.6 \times 3 \sim 60. \tag{4}$$

So the estimate is $6 \cdot 10^{12}$ kg per year.

Out of curiosity, I wanted to compare this number to the actual world production of carbon dioxide. It's hard to find the carbon-dioxide production due just to oil. But oil might be one-third of the world energy consumption (there's also natural gas, hydroelectric, coal, etc.). Multiplying the above estimate by 3 gives an estimate for the world production of carbon dioxide: $18 \cdot 10^{12}$ kg per year or roughly $2 \cdot 10^{13}$ kg per year. The actual total in 2006 was $3 \cdot 10^{13}$ kg.

> Ah, that factor of 2 is probably where a lot of the underestimate in my total world CO2 production/year comes from.

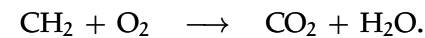**I messed up this factor, I thought it was larger, I said 7**

> The `ls -t` lists all the filenames in the directory ordered by recency with the most recent first. The next step, `head`, takes the first 10 lines. Therefore so far we have a list of the 10 newest files. The `tac` reverses this list so that we still have a list of the 10 newest files but it is ordered from 10th newest at the top to newest at the bottom. The `head -1` takes the first line from this list, giving us the 10th-newest file.

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For those who don't know about this, wouldn't it make more sense to say that US probably uses a significant portion of the world's oil (I don't think many people would dispute that it's > 10%) so you can multiply this number by "few" instead of 4?

> Both ways seem fine. I also remembered the 25% figure, but your approach is smart and logical.

>> yeah I multiplied it by few also.

>>> I took a combined approach, remembering that it was about 25% but also realizing that using the "few" estimation would make the end product easier to manage. They both make sense and "few" is very close to 4

For this problem, I could not remember the number of barrels of oil the US (or world) consumed (though I guess I should start) - so I used a maximum of the earth's mass, and then assumed some fraction of it that could be oil (I think I guessed billionth), and assume 200 years of use...which I think gave me the right ball park numbers.

> That's an awesome way to look at it! Thanks for sharing, it's really interesting.

I was able to estimate the number of barrels correctly but I ended up botching the rest of the problem. I got 2 moles of CO2 and never broke down the barrels into masses themselves

I totally messed up the chemistry stuff..

It's been so long since intro chem so I've forgotten about the moles, etc and I think there would be others in the same position (I used mass directly instead of converting to moles). I guess the factor of a "few" isn't too significant in the final answer though.

> I definitely screwed this part up.

>> I also completely forgot how to do this portion of the problem. Wikipedia really helped though!

>>> Wow, I actually did this right! It was nice to have this equation though, all the elements had masses that I actually remembered from chemistry.

>>> Is Wikipedia fair game? Doing all of these problems I thought we couldn't look anything up or even use a calculator. (Except for comparing my estimation to the actual for #1.)

I did not think about this ratio at all. Oops.

Are we supposed to divide by 3 here? I thought we were going to multiply by 3 because the mass of CO2 is 3 times that of oil?

> yeah, isn't the ratio 1 kg of oil gives 3 kg of CO2?

>> If you look at the next line, it says 3 x 4 x 1.6 x 3. I think the 3kg oil is just a typo, since in the next line it's treated as 1kg oil.

>>> Yes, that's my mistake.

This seems like a bit of a large number. I had about 10^8 barrels per day
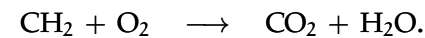
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$$3 \cdot 10^9 \text{ barrels/yr} \times 4 \times \frac{1.6 \cdot 10^2\,\ell}{1\,\text{barrel}} \times \frac{1\,\text{kg oil}}{1\,\ell} \times \frac{3\,\text{kg } CO_2}{3\,\text{kg oil}}. \tag{3}$$

Now do the numbers. There are 11 powers of 10 and then the following factors:

$$3 \times 4 \times 1.6 \times 3 \sim 60. \tag{4}$$

So the estimate is $6 \cdot 10^{12}$ kg per year.

Out of curiosity, I wanted to compare this number to the actual world production of carbon dioxide. It's hard to find the carbon-dioxide production due just to oil. But oil might be one-third of the world energy consumption (there's also natural gas, hydroelectric, coal, etc.). Multiplying the above estimate by 3 gives an estimate for the world production of carbon dioxide: $18 \cdot 10^{12}$ kg per year or roughly $2 \cdot 10^{13}$ kg per year. The actual total in 2006 was $3 \cdot 10^{13}$ kg.

So I feel a little guilty because my estimate was pretty close but I couldn't think up a way to calculate this. After reading though it makes sense.

So when I entered my answer, I estimated 10^12.5, which was a little off, but my question is when we enter our answers, since we can't put 6*10^12, should we put 10^12.8 instead?

Wow, I have to say the biggest advantage of these homework problems (for myself) is finding out how close my estimates are to reality, because it gives me this confidence that I'm doing something right; and it definitely helps with the forcing myself to "listen to my gut" because I'm realizing I'm close to right (this was the question I was most unsure about).

I agree, I'm still having a bit of trouble understanding how close to the correct answer I should be trying to get.

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$$10^{\boxed{}} \pm \boxed{} \qquad or \qquad 10^{\boxed{}\cdots\boxed{}}$$

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3. How often a piano needs to be tuned. Judging by our own piano, it needs to be tuned every year, but we somehow don't arrange it that often; maybe once every 2 years is more realistic.

4. How long it takes to tune a piano. Piano tuning looks like an intricate task investigating all the strings, etc.; maybe it takes half a day. I'll estimate 3 hours for it.

5. How many hours of work a piano tuner needs to stay afloat. A regular work week of 40 hr times 50 weeks gives 2000 hr in the year. Perhaps piano tuning involves lots of traveling; plus it's hard work. So maybe a fulltime piano tuner spends 1500 hours per year tuning pianos.

Now I use convenient forms of unity to find the number of piano tuners:

$$10^7 \text{ people} \times \frac{1\,\text{family}}{4\,\text{people}} \times \frac{1\,\text{piano}}{10\,\text{families}} \times \frac{1\,\text{tuning/piano}}{2\,\text{yr}} \times \frac{3\,\text{hr}}{1\,\text{tuning}} \times \frac{1\,\text{yr of work}}{1500\,\text{hr tuning}}. \qquad (5)$$

There are a total of 3 powers of 10: 7 from the 10^7 and 4 in the denominators (10 families and 1500 hours of work). What's left is

$$\frac{1}{4} \times \frac{1}{2} \times 3 \times \frac{1}{1.5}. \qquad (6)$$

The 3 and the 2×1.5 cancel leaving 1/4. The number of tuners is therefore $10^3/4$ or 300. In the form 10^x, it is roughly $10^{2.5}$.

7. Your turn to create

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I really liked how this problem utilized d/c. I think it would be a great example to walk through in the readings for d/c.

> This was the hardest problem in the pset for me.because I had no knowledge of any of the information need to get the answer. after looking at the solution, even if I had had the numbers, I would not know what percentage I would have used of each to calculate the numer of piano tuners in New York

To be honest, I'm not too familiar with the maintenance of a piano, so I had to pause for a moment to consider if this was a person or some sort of device.

Would Fermi come up with random questions for fun? Also, what sort of techniques did he use? Did he ever write an essay about solving these problems systematically (like George Polya did for solving problems)?

I foolishly forgot to split people into families.

I didn't know that number but I figured that US population is 300mill and NY must have at least 1/6 of that and so NYC must have like 1/10 of USpop. I got 3e7 that way

i estimated that nyc was 1% of the us's pop and got 3e6 and am pleasantly surprised to see how close that was.

i didn't use a fraction of families, but rather just a fraction of the population

> Me too... and I put it as 1:1000

>> I also used this method, but I said 1/50 people

> Me too, because I figured there were pianos in public places, not just homes.

I am curious what the ratio of single people to families is; I thought that there were many more single people in NYC than families and used 1 piano per 100 people.

> OP: although I still ended up with the exact same answer you did, sweet!

Ha, cool to use personal experience like that to generalize to the number of families with pianos in all of NY.

But I feel like the story is different in NYC. That city is not really family oriented. I would think young working people.

I feel like 1/10 is too large of a fraction to estimate how many people have pianos in NYC. In fact, the apartments in New York are really small, so I feel like the fraction is more close to 1 in 30.

> I thought this too, but I made the same estimation as well and somehow my numbers worked out to give me a pretty accurate answer

Given the income level of most of New York, the size of the housing stock, and building height, I think this is where this estimate goes most wrong. The Westchester suburbs might give you a higher fraction like 1/10.

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I guess that seems fairly high to me, given how densely packed NYC is (I assumed most people in apartments do not have pianos)

I agree - 1/10 seems high, as growing up, I was the only family with a piano I could think of. Then when you add all the homeless people in and others who could not own one, I estimated 1/100.

I grossly over estimated this number, though, I agree 1/10 seems too high still.

but I think the fraction should be higher because you need to account for the pianos in schools, especially universities and music schools, so I would increase the fraction by a bit to account for these factors?

These two numbers were the hardest for me to estimate since I have no idea what goes into maintaining a piano and how many people actually have one. ended up having to ask others to get a reasonable estimate on this one.

I used 1/yr. I'm sure some pianos are turned way more; like many of the pianos at music schools which NY has a bunch of. I'd say 1 each 2 years is too low, but that's just me.

Is there another way to reason this out if we don't have any experience with owning/maintaining a piano?

Perhaps (and this method also supports the 1/yr estimate): Pianos need tuning because of temperature and humidity changes (those change the tension in the strings, the responsiveness of the sounding board, and maybe a lot else). So after a whole year of all the seasonal fluctuations in both temperature and humidity, the tuning probably gets quite a bit out of whack.

Also in support of the 1/yr estimate: Our piano needs tuning once a year (it's now quite a bit out of tune). But maybe people don't always do what they need to, so 0.5 tunings/yr is closer to what really happens (basically, until you really cannot stand the mistuning any more).

And overall, it's only a factor of 2. So don't worry. There are probably other factor of 2 errors, e.g. in the number of families with pianos. I'm not at all confident about the 1/10 number.

If you don't know anything about pianos (like me) you might read this question as simply how many pianos are there... I didn't know a piano tuner meant a person who tunes them.

I don't think I used a time component when using divide-and-conquer. Having never owned (let alone played) a piano, there's no way I could have gotten a reasonable answer with this reasoning.

I didn't consider this aspect, just the number of pianos a tuner needs to tune per year based on the cost of living in NYC.

I thought another easy way to tackle this problem would be to relate it to something you actually know. For instance, I live in a close suburb of Boston and know my towns population and stores pretty well. I can guess as to the number of tuners in my town, and adjusted the population to match that of NYC. The estimate came out almost exactly correct (a few * 10^2)

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For this I just estimated a ratio of tuners to pianos. The solution seems much more detailed though and makes more sense.

> I did the same thing. Also I'm not sure how fair it is to assume that most piano tuners work only one job. It seems like it might be hard to stay afloat just by tuning pianos.

>> I also did tuners to pianos, it seems too difficult to know how long tuners spend at their job. It seems like it would be the second job of a lot of people. I try to move toward information I'm more certain about when doing these problems.

>>> I thought it was an interesting way to look at the problem. Estimating how often people need a tuner and how much they need to work to make a living.

Something kind of funny: I sort of miss read this problem. I thought tuners, meant little hand-held devices. But my estimate was still close since I assumed only professional who would tune would own them. But perhaps you could reword.

I took a piano tuner as something like a tuner for a guitar... not an occupation. This put me off by one order of magnitude

I just guessed that piano tuners did NOT spend all of their time tuning, because that made no sense to me. I ended up way off as a result!

I made a serious over estimate...I wish I knew more about tuners. Is this something you also estimate or is it fair to look some of this up?

i thought this part was interesting- I wouldn't have thought to do that this way

I fee like this information is unnecessary. What I did was simply estimate about how many piano tuners there are for a given population or given area then multiplied for the area or population of NYC.

It's really satisfying to see the same answer come up using a different way. I got a really close answer through totally different methods.

I feel kinda silly because I guessed 1000. Maybe that's just because I think being a piano tuner is an alright backup job??

> I did something similar - I figured that not all piano tuners are full time piano tuners, and as such the number would be higher since some people would be part time teachers or something

I totally thought that 'a few hundred' just made sense...however, I think that I totally goofed and wrote down 10^3.5 instead of 10^2.5 ... I really should have paid more attention when i moved my stuff from the paper i did all my work on to the online submission thing...

I got the same answer. Awesome

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I got the same exact value. It's really interesting because while I used the same general strategy of figuring out how many people are in NYC and how often pianos are tuned, etc., I think the estimates I made at each step were a little off from all the estimates made here, and yet my answer ended up exactly the same.

This is probably because all your overestimates and underestimates basically cancelled out in the end. I used a slightly different approach as well–I didn't divide it into families, I just directly said that about 1 in 20 people might own a piano, since not everyone who plays piano owns one. I also didn't divide a year of work for a piano tuner into hours–I just said 200 days, and about 3 pianos a day.

The method I used which produced the same answer was organizing NYC into a grid of avenues and streets. There are 12 avenues and 150 streets, so 1800 blocks. Then I assumed that there is a piano tuner on 1/5 blocks, so 1800/5 = 360 = 3.6 * 10^2 = 10^2.5. Thoughts?

That's a very interesting and different way to look at it - I guess I'm wondering, how did you reason that there was a tuner every 1/5 blocks? My personal guess would have been 1/10 or 1/20 since I personally think piano tuners aren't particularly common.

I guess if you assume an equal distribution of people with pianos and a fairly dense area (such as NYC) where the number of pianos would be high enough, the 1/5 number works. I still think an approach based on the population is slightly better.

I origianlly had 10^3, thinking there'd be about 1 tuner per 100 pianos (I went by a method figuring # of pianos in the population of New York), but felt that this was a bit too low for a population near 10 million. Is it actually that few?,

NYC is more than just Manhattan (10X factor of population) and 1 every 5 blocks seems like a big overestimate. There isn't even a Starbucks every 5 blocks, even if it might seem that way.

As to the actual answer here, I think it's considerably lower, maybe even <100. I couldn't quickly find a reliable count, but this Yellow Pages listing gives a pretty good indication. http://bit.ly/be1PGE Piano tuners aren't big operations, as you can tell by the fact that most of the listings are just men's names.

I got about a factor of 10 less when I did it. It came from my assuming 1 piano/ 100 families, instead of 10. Do that many people in NY actually drag pianos all the way up their apartment buildings?

So I simplified even more. I said one in ten thousand people is a piano tuner, since one in thousand seems far too many and one in one hundred thousand seems too few. My estimate turned out to be 10^3 and I only made one assumption, which seems very reasonable.

How far off is "too" far off to be reasonable? My answer was 10^1.5, a factor of 10 off from yours. Is this totally unreasonable? It was based on 10^7 people in New York, 1/1000 have pianos (10^4 pianos), 250 work days/year*2pianos/day to 500 pianos per year. This leads to 20 piano tuners... I guess it seems low...

I could not even begin to estimate the key parameters such as pianos/families and the times associated with tuning. I merely guessed that there needed to be at least 10 but not more than 1000.

I honestly have no comments on the solutions. They all seem clear to me. The only question I don't think I quite got was the Piano tuners one, and that's because I used more questionable values. All yours make more sense to me.

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---

i didn't really understand the point of this question. didn't help me learn anything.

> I think this question was a good one to include. While it doesn't serve any real educational purpose, it really shows you that divide and conquer can take you a long way. I was thinking of a bunch of estimation quesitons, and when you really break a question down, some seemingly complex questions start to look a lot easier.

>> i didn't think it mattered whether we learned from it. it gives sanjoy an idea of what we want to know and might give him good ideas for examples later on.

>> It was pretty useful for me to have to sit down and try to think of a "cool or interesting" problem that could be solved using divide and conquer - I wanted to find a problem that wasnt just an interview question I'd heard of but something hopefully original

>>> It would be nice to see our problems come up in problem sets and lecture.

Are we going to go over any of these questions?

I liked doing this

> I agree. This problem works like reverse engineering. It presented a new way of looking at divide-and-conquer.

Where are all the responses to this question? I would be interested in knowing.

How many questions have you gotten from the class/from this problem?

You should post all the questions!

> I agree, it would be kinda fun to see what my classmates came up with at the end of the semester!

> Maybe they'll make it into our final.

>> I would definitely like to see some of these questions in one way or another, I am sure there are some great ones.

>>> Has anything happened here? I'd be really excited to see some of the old questions... i had to go back to the homework to remember what I had written.

6.055J/2.038J (Spring 2010)

Solution set 2

Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 24 Feb 2010**.

Open universe: *Collaboration, notes, and other sources of information are* **encouraged.** *However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

On `linux.mit.edu` (the Athena GNU/Linux machine), an (American) English dictionary lives in the `/usr/share/dict/words` file.

For the (optional) question that references `decline.txt`: It is the plain-text file on the course website that contains volume 1 of Gibbon's *Decline and Fall*. It is also available – as is any other file on the course website – on any Athena machine as `/mit/6.055/data/decline.txt`

Warmups

1. **Direct practice with one or few**

   Here is another 'one or few' problem generated by my Python script:

   $$985 \times 385 \times 721 \times 319 = ?$$

   (1)

   $$10\boxed{} \pm \boxed{} \quad \textit{or} \quad 10\boxed{} \cdots \boxed{}$$

   Here are the approximations for each number:

   $$985 \to 10^3,$$
   $$385 \to \text{few} \cdot 10^2,$$
   $$721 \to 10^3,$$
   $$319 \to \text{few} \cdot 10^2.$$

   (2)

   The approximate product has 10 powers of 10 and two factors of a few, giving 10^{11}. The exact value is $87,221,370,775$ or roughly $0.9 \cdot 10^{11}$.

2. **Land area per capita**

   Here is another problem on which to practice the 'one or few' method of multiplication and division: Estimate how much land area each person would have if people were evenly distributed on the (land) surface of the earth.

   $$10\boxed{} \pm \boxed{} \ \text{m}^2 \text{ per person} \quad \textit{or} \quad 10\boxed{} \cdots \boxed{} \ \text{m}^2 \text{ per person}$$

Comments on page 1

Read through Solution Set 2 and do your memo by Thursday at 10pm.

Are we supposed to comment on both sol. set 2 and the reading by Thurs or is commenting on the sol. set optional?

Comment on both by Thursday. I try to keep the Thursday reading short so that you are not overloaded.

I always forget to comment on the solution set. Not sure what I can do to remind myself...

I actually didn't know until today that comments on the solution set were even necessary!

I've often been breaking coefficients down further and temporarily leaving in numbers such as 5 or 2, since they often cancel out later down the line (or I get rid of them later if they don't). Is that generally an extraneous step?

nice. got it

I thought this was in between few and rounding to 10, but I guess not. In the future I will round up.

For me personally, I rounded this up because I thought that 7^2 = 49, which is half of 10^2, but 5 times 3^2. Since we work with squaring stuff a lot, I figured it was closer to the upper bound. Also, we round down both 385 and 319, so it doesn't really hurt to round the 721 up.

I also rounded this up but because of that I rounded 319 down to 10^2. Don't you need to compensate for the large round up somewhere?

How do we know at what point to round up and when we should round down? What if it had been 621 or 521? I rounded down but gave a large error to account for it.

My intuition was that it was closer to a few than 10.... in retrospect i can see how this is 10 by thinking about how we deal with powers here (from the comments above).

Like someone said previously, I figured since I rounded down on 385 and 319, I could round up on 721 as it was half of 10^2.

yay!! i did this right!

These problems really help me crunch large number together in the real world much faster.

Solution set 2

Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 24 Feb 2010**.

Open universe: *Collaboration, notes, and other sources of information are* **encouraged.** *However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

On `linux.mit.edu` (the Athena GNU/Linux machine), an (American) English dictionary lives in the `/usr/share/dict/words` file.

For the (optional) question that references `decline.txt`: It is the plain-text file on the course website that contains volume 1 of Gibbon's *Decline and Fall*. It is also available – as is any other file on the course website – on any Athena machine as `/mit/6.055/data/decline.txt`

Warmups

1. **Direct practice with one or few**
   Here is another 'one or few' problem generated by my Python script:

   $$985 \times 385 \times 721 \times 319 = ? \tag{1}$$

   $$10^{\boxed{} \pm \boxed{}} \quad \textbf{\textit{or}} \quad 10^{\boxed{} \cdots \boxed{}}$$

   > Here are the approximations for each number:
   >
   > $$985 \rightarrow 10^3,$$
   > $$385 \rightarrow \text{few} \cdot 10^2,$$
   > $$721 \rightarrow 10^3,$$
   > $$319 \rightarrow \text{few} \cdot 10^2. \tag{2}$$
   >
   > The approximate product has 10 powers of 10 and two factors of a few, giving 10^{11}. The exact value is $87,221,370,775$ or roughly $0.9 \cdot 10^{11}$.

2. **Land area per capita**
   Here is another problem on which to practice the 'one or few' method of multiplication and division: Estimate how much land area each person would have if people were evenly distributed on the (land) surface of the earth.

   $$10^{\boxed{} \pm \boxed{}} \text{ m}^2 \text{ per person} \quad \textbf{\textit{or}} \quad 10^{\boxed{} \cdots \boxed{}} \text{ m}^2 \text{ per person}$$

you know that you overestimated, but how can you tell by how much you overestimated (aka what to put in the other box)?

> What you put in the box depends on how closely you look over the calculation.
>
> For example, here is a quick and crude estimate. The 985 going to 1000 is almost exact, within a couple percent. The 3.85 going to "few" is maybe a 20% error. The 721 to 10^3 is maybe 30%. And the 3.19 to "few" is nearly exact. So at worst, the percentage errors conspire, meaning that they add, giving about 50% or a factor of 1.5. That's plus/minus 0.2 in the exponent because log_10 (1.5) = 0.2.
>
> Upon a closer inspection, the errors almost cancel: The 3.85 to "few" is an underestimate, and the 7.21 to 10 is an overestimate. So the total error is maybe 10%, namely a factor of 1.1 (or plus/minus 0.05 in the exponent).
>
> Does that help? Perhaps all of the above belongs in the solution set.

That is a really good explanation. I thought it was clear that the 7 should go to 10, but some people didn't. When introduced in this way it makes perfect sense, and even explains how this rule should be used in the future

I forgot to distinguish land from sea.

Oh man, I totally forgot to take out the water too.

Me too–I totally forgot about that

I estimated that 60% was water...guess I was a little off.

I remembered the land/water distinction, but I was wondering: how does Antarctica, Greenland, et al figure (mostly ice)?

Haha that's a good point...I just assumed people could live on anything 'solid', including ice

The surface area of the earth is $4\pi r^2$, where r is the radius of the earth. The land area is some fraction f of the total surface area. I half remember that f is about 0.25, which seems plausible: There is a lot of land, but the oceans are giant. The radius of the earth (from Homework 1) is $r \sim 6.4 \cdot 10^6$ m. The world population is about $5 \cdot 10^9$.

So, the per-capita area A is

$$A \sim \frac{4\pi \times (6.4 \cdot 10^6 \text{ m})^2 \times 0.25}{5 \cdot 10^9}. \qquad (3)$$

There are three powers of 10. In the remaining factors, the 4 and the 0.25 cancel each other. The $\pi \times 6.4^2/5$ is (using $6.4^2 \sim$ few $\times 10$) few $\times$ few $\times 10/$few or few $\times 10$. The final per-capita area is few $\cdot 10^4$ m^2.

3. **Nested square roots**

Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \qquad (4)$$

The computation is recursive in that it contains a copy of itself. To see that, define

$$P \equiv \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}}. \qquad (5)$$

Notice that P is repeated inside the square root:

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The solution to this equation is $P = 2$.

4. **Searching for . . . gry words**

What English words, other than `angry`, ends in `gry`?

Humans are much worse than computers at this question, because we store words not by their endings but more by their beginnings and meanings. For a computer, it's all bit strings, and computers don't care whether the bit string happens at the beginning or end of the word (and there's no meaning).

The regular expression that matches words ending in `gry` is `gry$`. In the following pipeline, the first `grep` finds all those words, and the second `grep` excludes `angry` from the list:

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Comments on page 2

Instead of working with pi and earth being round, I thought of the Earth as a cube. Turns out I shouldn't have, so how would I know when to change circular shapes into rectangular?

did you underestimate the radius when you did this? my guess is that is the only way that it actually works, ever

> Yeah, you'd want to underestimate the radius to make it work. The surface area of a sphere is pi*d^2 where d is the diameter. For a cube it is 6*d^2, which is about a factor of 2 larger than the sphere's area. So, underestimate the radius by 30% and you'll be home free (0.7^2 = 0.5). Here, try something like 5000 km.

yeah, I also approximated this as a cube since last time we approximated Earth as a cube.

For this i looked at the people on earth and estimated their weight...as well as adding a couple of factors of 10 for all the large animals. Everything else I kept negliblie.

I too also remembered this, but I remembered it as water being 75%. It's not a very rare piece of information.

I think I estimated something slightly different: that land takes up 1/3 of the earth's surface.

this is the key. because "the oceans are giant" isn't a good reason.

> While I agree that this is key, a vague mental recollection of a map would surely lead one to pick a number between .1 and .5 - which would give fairly reasonable answers.

> I thought it was 2/3 water? Not a huge difference, but this is what I used and have always heard.

>> I have also always heard 2/3rds water.

Also on the constants sheet. Does looking at the "back of the envelopes" sheet count as looking stuff up or is that acceptable to look at?... I've been using it all semester.

I just realized I should have mentioned something about how I am assuming that the earth is a perfect sphere...

> I didn't realize there was that much of a difference, its almost a sphere right?

I wasn't sure if we were allowed to use previously established numbers- that requires reference. I guess we could just calculate it again though...

I thought there were 6 billion?

> Me as well, in fact I believe it's halfway to 7 by now. Perhaps this figure should be updated?

>> All things considered, this isn't actually important for solving the problem since 5 6.

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The result is just one line: 'hungry'.

I didn't know this value off the top of my head, and I ended up having to look it up. Is there an easy way that we could have derived an estimate for this value if we didn't know it and we couldn't look it up (in an interview, etc.)?

The simplest method would probably be to just memorize the number (world population is somewhere in the 5-7 billion range).

The only methods I can think of that would allow you to derive it is remembering vague numbers for certain countries (US 400m, China 1bill, etc), which seem like a lot more work.

Hopefully this number will continue to come up so that it will be easier to recollect by the end of the term.

I remember this being quite a bit bigger (not enough to change the estimation of course..), but closer to 7 billion. Maybe this example is slightly outdated? Also, wait a few more years and 'f' might drop a lot, too :)

The world population isn't changing that fast and a billion is a lot. I think a good way to remember this is to think in terms of upper limits. Human population is somewhere in the low billions and you will at least be correct to an order of magnitude.

I just guessed this as 10^10.. I guess that's a big error.

Same here, I approximated upwards to 10^10 since I figured population was around 7*10^9. I guess 5 makes a little more sense though.

Should we attempt to put these numbers in terms of just 10 to a power or few x 10 to a power?

Funny, I actually used 10^8 here and still got 10^4 as my answer.

Yeah I rounded up too, since I knew it was 7*10^9...I thought we were supposed to only use 1, few or 10 so I decided to round up to 10...should I have kept it at 7?

I'm pretty sure the number is closer to 7 billion, but given that we just end up calling it a few billion anyway, it's not much of an issue. But for the sake of correctness in the solutions maybe this should be changed.

Wow, you are right! I think I must be remembering the number from back when I first learnt it. I have a clear memory of learning it in 6th grade. Then I must have slightly increased it to get 5 billion.

This is exacty what I did, except that I assumed f=1/3 instead of 1/4

I used 0.33 here to make the math simpler (the pi in the numerator cancels with the 3 in the denominator). Not much of a difference, but slightly easier to compute.

i used the same method. nice

That is a lot of land per capita!

Very suprising!

this is a really confusing sentence to read. you could have used numbers.

i got 10^2, but I approached it in the exact same way

The surface area of the earth is $4\pi r^2$, where r is the radius of the earth. The land area is some fraction f of the total surface area. I half remember that f is about 0.25, which seems plausible: There is a lot of land, but the oceans are giant. The radius of the earth (from Homework 1) is $r \sim 6.4 \cdot 10^6$ m. The world population is about $5 \cdot 10^9$.

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You would also get the same answer (10^4.5) if you assumed that land is a 1/3 of the total surface area and that the world population is 6 x 10^9.

I tried both methods and got this answer - I think its nice to be able to confirm your answer with a different estimation

Hoo boy, I got that one completely wrong... unfortunately, I can't find the paper where I wrote by calculations, but my method seemed the same, so hopefully I just messed up the math.

Did anyone else just enjoy this answer a little bit? I found it incredibly surprisingly and gave me a new perception on overpopulation.

Yah, its a bit scary to think about. It was one of those problems where I sat there going "that can't be right..."

So I got that and actually almost changed my answer because I thought it was so absurdly high. I guess my gut just totally lied to me.

I had a very similar response - I immediately thought of all the places like china that are overcrowded. I wonder how much this would be if we took out uninhabitable places...

I think I solved this problem without recursion, although looking back, that is definitively the best way to solve it. I just started working it out, and quickly realized that it could never rise above square root of 4.

It took me a little bit to get how this one worked. But once I realized it was a recursion problem, this as well as the other geometric series problems became a lot clearer.

After looking at this for a long enough time, I realized it was just the multiplicative equivalent of 1 + 1/2 + 1/4.....and it all made sense!

That's how I first approached the problem.

This problem is a famous problem in mathematical analysis

That was soooo much easier than I thought. Damn, I feel stupid.

Agreed, i took forever trying to figure out a way of estimating this until I realized it was recursive. Recursion is definitely my favorite method, it's so simple and neat!

I always have trouble figuring out how to start problems...

I would have never thought or known about this- can we do a few more similar examples

I'm pretty sure the sum of the series 1/2^n is 2 so the answer should be 2^2 which is 4....?

I think that's totally right except the sum of $1/(2^n)$ is one, not two. Even if you just try the first 4 terms or so it appears to be converging at one (.9375).

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I did this a little differently. I changed the radicals to powers, so it was 2^1/2 * 2^1/4 * 2^1/8 ... or 2^(1/2+1/4+1/8...) and I noticed that the power was a geometric series, which I used abstraction to get the sum of.

I accidentally said P = sqrt(2xsqrt(P)) and thus got a different answer. Hopefully no one else made the same mistake.

Wow, this now seems like a very obvious abstraction. I used the radical into power method described above, but this simple abstraction is so much more straight forward.

I understood that this is what the problem was asking but writing it with the use of P helps make the solutiona lot more clear.

I kept on telling myself that it was just root2 times the whole thing over again, but never actually made the step in my head to write out p=sqrt(2*p). I wrote out the terms and summed the geometric series.

Using a variable definitely helps a lot. I had the same problem with trying to think of it as "oh, it's just the original thing all over again."

Really cool way to use abstraction..and it helped a lot to have done this problem in order to solve the resistor problem after.

I used the power method mentioned above. I feel like these abstractions are very useful but at times very difficult to make or see, especially when dealing with mathematics and such. They may be easier in another arena however, i still feel that I would not be able to make these extremely simplifying abstractions like those above, in math problems.

You hit the nail on the head when you say, "Using a variable definitely helps a lot." Indeed, a variable is an abstraction, so it's another example of the usefulness of abstraction.

looking back, it would have been helpful to know invariants here...!

I didn't realize the series; I just tried to figure it out as is. It seems a lot easier when figured out with powers.

Nice. I tried rewriting it with exponents but wasn't really sure of how it would look.

I definitely was not great at seeing abstractions before but I'm starting to know how to look at these problems in a more useful way

This only works for things that repeat infinitely, right?

oh that makes a lot of sense and now i feel like a ditz

Don't feel bad, thats the point of a problem set! So now you learn to recognize these patterns in the future and apply the things you learned

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The result is just one line: 'hungry'.

For some reason, it took me awhile to realize this, but I felt a certain level of satisfaction that something so complex at first reduces to something so simple.

I too had to work a little at this, but it really helped put me in the recursive mindset for the later problems. Like you said, it's satisfying seeing the simple answer in the end.

Agreed - this question made me feel really good about myself, and was very helpful for the other recursive problems.

yeah I was able to do the other ones after finishing this problem.

For me, this was one of those that I stared at for hours and could never even get started. Now the answer makes perfect sense once I see it, it's just kind of frustrating.

so i knew it converged to 2 but i didn't know what i would put as the error...since it should be 2. is it then 2 +/- 0?

I just put 0 for the error since I was able to solve it. In this case, I didn't do any approximation per se so there isn't any error margin.

Is there a reason for all of the UNIX stuff in this class? I find it mostly annoying because i'm not interested in learning more about it.

There is. For one, UNIX is an incredibly useful tool, and someday you might be able to use it to solve a problem. See for example one of the comments on Problem 8, about running a website and needing to convert mailing-list data from one format to another.

Second, UNIX illustrates the generality of divide and conquer and of choosing good abstractions.

So, try not to be too annoyed.

I think this question was poorly worded, I had no idea how you wanted us to go about this- actually list out all the words? But then I remembered the previous problem using UNIX

Apparently there are many riddles surrounding this.

This is also a really famous riddle, often starting with "there are 3 words ending in -gry".

Yah, I realized after the due date that he wanted a UNIX answer, not just an answer to the riddle!

Yeah I did not realize that we were supposed to come up with a unix pipeline for this problem until just now, but I guess that makes more sense

I didn't really understand the reason for this question. Just looking something up?

more practice using UNIX commands?

I used google... was this a bad idea?

This is something you can't just look up yourself the way you usually would though. It requires figuring out how to tell the computer what to do in general terms (not necessarily UNIX), which is a useful problem-solving skill.

It was intended to be a UNIX example we could try out ourselves. Turned out to be pretty fun.

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The subject-verb mismatch of "words" and "ends" made me suspicious that hungry was the only one when I thought about it before running grep.

but if he had only said 'word', and you thought of 'hungry', then you wouldn't have bothered doing the problem, right?

Well, I would have still done it out of curiosity since I don't have a complete mentai dictionary. I just think the problem would be better stated vaguely as, What English word(s) end in gry?

I automatically started thinking of proper nouns, like Pingry. But I laos thought of hungry immediately. I'm not sure I felt this was the best problem; but it did force the use of linux shells.

Good catch!

My brain handled it OK – though I didn't have the certainty that I hadn't missed anything. Maybe if it were harder I would see the necessity of this problem.

good to know, this will be helpful for later, although I ended up using a different solution to get the same result

I used a different solution as well: ".gry" The solution gry$ works here because the file we were searching was organized so every word was on a new line, but if we were searching any text document I think the .gry pattern is more robust at finding what we want.

I never would have thought of this method for answering this question.

why does 'gry$' work? i thought that the $ allowed all endings, but gry is our ending here.

$ in a regular expression means "end of line", so it forces the word to end in "gry".

I don't know UNIX and I totally forgot about this $ operator. I inverted the dictionary and then found words starting in yrg. Is this valid?

I did the same thing. If it works then I don't think you can say it's not valid. It's just not the easiest way to do it.

Ha yeah I'm glad other people did it that way too. The embarassing part is I am familiar with unix and I still di it this way...

I looked this up: '-v' stands for 'invert' in the sense of matching. so find everything except for 'angry'

I was actually thinking about reversing the words, until I just looked for grep 'gry' and got only hungry and angry as results.

I did reverse the words. I got the answer but took up more computing power I guess. Is that bad?

the method in the reading seemed a lot longer

I didn't see that in the reading..

were we supposed to use code to do this, I just brainstormed for a long time- any other ideas to help that do not require a computer

The surface area of the earth is $4\pi r^2$, where r is the radius of the earth. The land area is some fraction f of the total surface area. I half remember that f is about 0.25, which seems plausible: There is a lot of land, but the oceans are giant. The radius of the earth (from Homework 1) is $r \sim 6.4 \cdot 10^6$ m. The world population is about $5 \cdot 10^9$.

So, the per-capita area A is

$$A \sim \frac{4\pi \times (6.4 \cdot 10^6 \text{ m})^2 \times 0.25}{5 \cdot 10^9}. \tag{3}$$

There are three powers of 10. In the remaining factors, the 4 and the 0.25 cancel each other. The $\pi \times 6.4^2/5$ is (using $6.4^2 \sim \text{few} \times 10$) few $\times$ few $\times 10$/few or few $\times 10$. The final per-capita area is few $\cdot 10^4 \text{ m}^2$.

3. Nested square roots
Evaluate

$$\sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}} \tag{4}$$

The computation is recursive in that it contains a copy of itself. To see that, define

$$P \equiv \sqrt{2 \times \sqrt{2 \times \sqrt{2 \times \sqrt{\cdots}}}}. \tag{5}$$

Notice that P is repeated inside the square root:

$$P = \sqrt{2 \times P}. \tag{6}$$

The solution to this equation is $P = 2$.

4. Searching for . . . gry words
What English words, other than `angry`, ends in `gry`?

Humans are much worse than computers at this question, because we store words not by their endings but more by their beginnings and meanings. For a computer, it's all bit strings, and computers don't care whether the bit string happens at the beginning or end of the word (and there's no meaning).

The regular expression that matches words ending in `gry` is `gry$`. In the following pipeline, the first `grep` finds all those words, and the second `grep` excludes `angry` from the list:

```
grep 'gry$' /usr/share/dict/words | grep -v '^angry$'
```

The result is just one line: 'hungry'.

Not true for my dictionary. I got a whole lot of weird words I've never heard of before.

> I think wikipedia has a list of those words but they are considered to be "archaic" that no one uses them anymore. So they dont include them in modern dictionoaries

>> That sounds fun. What were the words?

I wrote a pipeline to find all words ending in -gry, and since it only spit out 2 words, angry and hungry, I just stopped there and said the other word was hungry. Is that acceptable? (I figured it was, since the question doesn't specify writing pipelines that give a certain result.)

> I did this as well. But I suppose the problem did specifically ask for solutions other than angry, so we should not let angry show up at all in our output.

>> I also used grep to return all words ending in -gry. I think the second grep would have been a more obvious command if the output had been a lot more than two; it would have been the time saver of deleting the word out of the list.

> For the purposes of practicing using grep this might be true. But I don't see any other reason why one shoudn't just find all the words ending in "gry" and then subtract one from that number. Especially since it's probably faster to just subtract one in your head than type out the additional command to remove angry from your output.

For this, I just downloaded a text file of a dictionary and searched for the key 'gry'; does this qualify as a valid intelligently redundant method?

> I think it does. The given solution is a mini-program, and it's always a good idea to check whether a program is giving nonsense – which your method accomplishes.

Problems

5. Geometric series

Use abstraction to find the sum of the infinite series

$$1 + r + r^2 + r^3 + \cdots. \tag{7}$$

> Similar to **Problem 3**, look for a repeated motif or abstraction. Here, define S as the sum
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> So, $S = 1 + rS$, whose solution is
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> $$S = \frac{1}{1 - r}. \tag{10}$$

6. Pool temperature

A large outdoor swimming pool in the Arizona desert has a time constant of 4 days for exchanging heat with the air. Roughly how large are the peak-to-peak fluctuations of the water temperature caused by 30 °F (peak-to-peak) night–day fluctuations in the air temperature?

$$10 \boxed{} \pm \boxed{} \,°\text{F} \quad \textbf{\textit{or}} \quad 10 \boxed{} \cdots \boxed{} \,°\text{F}$$

> The daily temperature oscillation has an angular frequency
>
> $$\omega = 2\pi f = \frac{2\pi}{1\,\text{day}}. \tag{11}$$
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> With a thermal time constant of $\tau = 4$ days, the dimensionless paramater $\omega\tau$ is $2\pi \times 4$ or about 25. Since $\omega\tau \gg 1$, we are in the limit of fast oscillations. In this limit, the low-pass filter – the combined system of thermal resistance and reservoir (the swimming pool) – attenuates the inputs oscillations by a factor of $|\omega\tau|$. So the 30 °F fluctuations in air temperature become a 1 °F fluctuation in pool temperature.
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> The practical consequence for swimming is as follows. These fluctuations happen around the average daily temperature (the DC or zero-frequency input signal). In the Arizona winter, the daytime temperature is often 70 °F, but the nighttime temperature can be only 40 °F (the 30 °F variation). Therefore, the pool will sit mostly at 55 °F. It is far too cold for swimming. The small fluctuation of 1 °F around 55 °F does not make the pool comfortable even at its peak temperature.

7. Resistive network

In the following infinite network of $1\,\Omega$ resistors, what is the resistance between points A and B? This measurement is indicated by the ohmmeter connected between these points. (If you want to read about series and parallel resistances, a useful reference is the Wikipedia article 'Series and parallel circuits'.)

---

i felt like this problem here was what made me finally understand recursion. it might be nice to have this problem in the text/readings to illustrate another way of using abstraction

> Yeah, I sort of agree. I wasn't entirely getting recursion, and got help with one of the recursive problems in this assignment, and suddenly the other two made sense. The coin example wasn't quite as useful, possibly because the symbolic layout of these homework problems really helped it hit home.

this was brought up in the comments on the HW, but should there be a requirement that 0<r<1?

> Depends on how you want to define convergence.
>
> > Could you be more specific?
> >
> > > I think there is still some confusion...doesn't r have to be 0<r<1?

I didn't know the formula for this so I thought the result would be infinity- this explanation of the solution clarifies how I should have been thinking

> I think this is a very intuitive guess...it does seem like it would keep increasing; it is an infinite sum after all, and the numbers keep getting bigger!

I think I finally got recursion. Neat.

> pretty cool–just finding a pattern that is inside itself...
>
> I agree. This is the most helpful example of recursion.

Thanks to whoever posted this method in the reading memo :)

I couldn't get the equation I learned in high school out of my head when I was doing this.

this is kind of hard to see because it requires factoring r into 1*r. you can also multiply both sides by r in order to realize that it's recursive.

> This is exactly what i did.
>
> I did the problem this way also. I subtracted S*r from S and solved for S.

looks like recursion

I don't really understand why the "formula" for the geometric sequence is always taught to people in high school, but it seems very few people learn how simple it is to derive it.

> I agree. I never remember this even though I've taken plenty of classes that cover the material. I also don't get why its called "geometric" although I'm sure there's a good reason for it.

Problems

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This makes sense. For some reason I thought he wanted a concrete number though. I guess because everything else is approximating?

> I think the difference here is that there is an actual variable (r) in the problem without given a value for it.
>
> > Yeah I'm sure there's no way to get an exact number without a defined value for r.
> >
> > > I didn't expect that we would be able to solve these problems with an answer that puts no weight on the size of the series. This is a lot more helpful then I thought.

Ooh, this makes sense. It reminds me of the formula for the sum of geometric series that I learned in high school but have since forgotten.

> This is the geometric sum from high school. I thought against using this at first, thinking that the answer couldn't be a simple formula, but I couldn't think of anything else.
>
> > I still think that unless a person has seen this before it would be very difficult to make the jump to factoring out the r. This seems like something that just comes to you in a stroke of genius and that there is no real systematic way to approach it. Basically if it doesn't just come to you then you are out of luck.
> >
> > > It's true, that I saw this and remembered doing it before, so it wasn't much of a challenge, however this concept is still a good abstraction, as recognizing this sort of pattern (recursion) is very useful, and can help solve many otherwise challenging problems.
> > >
> > > > For some reason I found this equation harder to come by than the other recursion problems.

I feel like this type of thing is related to discount rates (finance theory). There could probably be a nice application to be shown regarding bonds and coupons.

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The solution should be dependent on the value of r. Everybody seems to forget that.

> Right, but the only way us non-mathy people could remember that is through remembering the reading or what he did explicitly in class. Otherwise, it seems more logical to look at what was done above with the recursive squares and try and find an actual "answer," as in a number.

This made more sense after lecture Wednesday

So what would have been the best inputted answer in the form asked for.

> I think the 1 F variation is reasonably reliable, perhaps even more accurate than a factor of 3 (which would be plus/minus 0.5 in the exponent). So maybe
>
> 10^(0 plus/minus 0.2) F

how do you know it relates to the angular frequency? this is kinda abstract to me

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I didn't understand the real answer until now, but my estimation was right.

i got confused by the 2pi*f*tau equation in the notes. there's no tau in this particular equation.

tau is 4days. w*tau=2pi*f*tau is the dimensionless parameter that appears in the transfer function.

This is just the equation for converting our period of one day into an angular frequency for our input before we consider the time constant (tau) of the system.

I forgot the 2*pi and thus I ended up with 6 degree F.

parameter

This is a nice example of the modeling of physical system using circuit model/transfer functions.

What does this mean?

i'd like to know too. i'm not sure what "limit" refers to.

> Sorry for the jargon. (By the end of the book and course, it'll be very familiar.) It means just that the oscillations are very fast, as if omega*tau were tending to infinity (hence the use of "limit").

I'm not sure about the cutoff for >> here. Is it, in general, when the lesser part is < 1% of the greater part?

This problem makes sense after seeing the solution, but I sure I no idea how to do this. My answer was somewhat close, but then again it's obvious that the temp doesn't change by much.

Yeah my answer was higher than this and I assumed instant changes in temperature which might have thrown my numbers off by more than i expected.

This kind of problems always eluded me a little bit. We did them in some classes before and I never really quite got it... seeing it done here is a lot nicer .

Agreed, I had to look at the solutions in order to solve this problem, and even then it took me quite some time

this is kinda confusing- not how I approached it

This seems so simple now after the fact but before I was just not able to make any connections with what I knew to this problem. I started writing a book of physics equation and that and your notes have been really helping me with the psets.

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Wow, I can't believe I got this exact answer. I was pretty confused when I read this section of the heat flow notes, and then an answer of 1 degree seemed a bit low to me. It's an interesting result.

Is this just 30/abs(25) = 1?

yes

yeah, I got a pretty low number also (3). however, this make sense because if you have ever been in a swimming pool in a hot climate, the water never really seems to change temperature that much.

That's right. What prompted the question is the swimming pool in my parents backyard. They live in Tempe, Arizona. When I visit, I often think how nice it would be to go swimming. But my visits are often during winter break (summer break is baking hot in Arizona). The water is always cold even when it is 70 degrees during the day. And that puzzled me, until I thought about it as an RC circuit.

I forgot about this part—I got omega*tau=about 25, but completely forgot about this—that's why my answer was so high...

This answer makes sense after reading it, but I don't think I would have been able to come up with it my own.

Wow, I did not do this.... My answer is totally wrong... :(But it helps to look at the solution so we know how to do it next time we face a problem like this

So if the pool was somehow artificially heated up to 70 deg one day, would it take the full time constant (4 days) to settle at its 55 deg oscillation?

You'd see an exponential decay toward 55 degrees (and the decay would have a 4-day time constant). The 1-deg oscillations would be superimposed on that exponential decay.

Linearity means that you can work out the responses to each input separately, then add them to get the response to the total input. Another example of divide and conquer...

wet suit in a swimming pool?

I got an answer very close to this but didn't understand the "correct" logic until after reading this

This is 1 degree peak to peak, right? So it should be about 0.5 degrees around 55 degrees?

I was a bit confused about how to enter this with such a small variation

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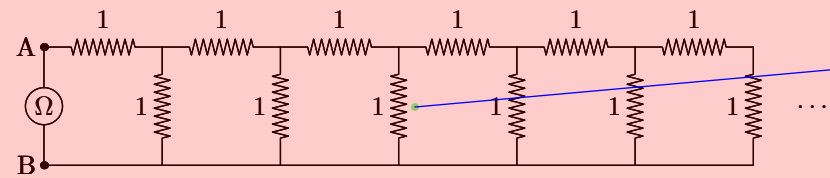
This seems like a good problem to solve using symmetry (and that's actually what I tried to do for my own solution). It might be a worthwhile exercise to do.

Would you mind explaining your symmetry argument? I'm having a little trouble understanding the process and it might help to see it on a problem we've already thought through.

The answer shown uses symmetry. The network is size-invariant. If you increase the size by an additional "ladder rung" the whole thing looks the same and has the same resistance.
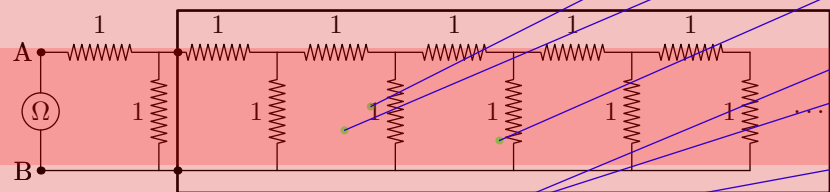
I think you're confusing recursion (or self-similarity) with symmetry.

Why be stingy with methods? It uses both. For finite systems, the symmetry is broken, so you're using just recursion. For infinite systems like this one the symmetry is not broken, so you have recursion and symmetry.
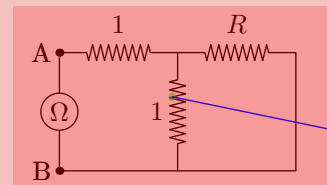
This resistive network contains a copy of itself (enclosed in the box):

Call R the resistance of the network inside the box, measured between the two dots as the terminals. Then the original network, which also has resistance R, is

It is a $1\,\Omega$ resistance in series with the parallel combination of $1\,\Omega$ and R. So

$$R = 1 + \underbrace{\frac{R}{1+R}}_{1\,\Omega \| R}, \tag{12}$$

or $R^2 - R - 1 = 0$. The positive solution is

$$R = \frac{1 + \sqrt{5}}{2} \approx 1.618, \tag{13}$$

which is the Golden Ratio.

An alternative, direct method is the following continued fraction that accounts for the infinite cascase of series and parallel resistors:

$$R = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}. \tag{14}$$

This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)

Now that I understand recursion, I feel I could get this problem.

This is so classic 8.022...

This problem took up a whole recitation in 8.022 back when I took it. It's nice to have the same argument we use to solve infinite series also solve resistors. They seem so different.

Does it matter what we define as the box?

I think you need to be more explicit here about why the the resistance of the original network is R also.

self-symmetry. Its the same principle used in several of the other problems on this pset.

How is the resistance inside the box equal to the total resistance? I don't understand this abstraction.

its not the total resistance, it's just "R" a variable for right now.

well, it is in fact equal to the total resistance. The idea is that the pattern inside the box is identical to the pattern of all of the resistors, since they stretch on to infinity. So the resistance between the vertex near the top-left of the box and point B is equal to the resistance between A and B. This is ok because of the way resistances add in series and in parallel.

Imagine the diagram is like the series in problems 3 and 5. These infinite series contain themselves.

It's because you're still measure from point A to b. Look at the box he's drawn. He's drawn new dots that are analogous to the dots at A and B. So the resistance of the box is the same as the resistance of the whole circuit.

The trouble I had with this question was going from the original diagram to this simplified one. This was the key. However, once I saw this diagram, I was able to do the problem easily and got the same answer as the rest of the solution.

The same was true for me - the reduced diagram does make the problem much easier, and it was great that the previous problems put us into the recursive mindset.

I thought the resulting box would be in parallel and not in series with the left side. Then suprisingly I got 1/2 of the compliment of the golden ratio
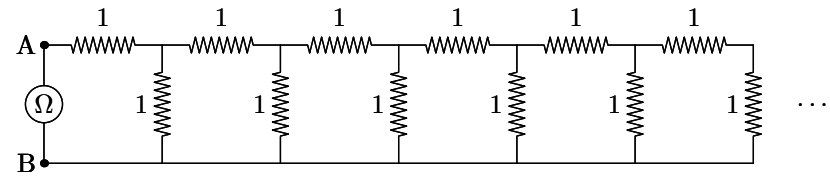
I think I did this completely wrong...oh well

I understood it much better now! always look for symmetry! this is indeed similar to the square root of 2 problem above, but just in a different format/setting

I understood the recursion in this problem, and the idea in this simplified diagram. But I didn't get the next step--noticing that it's one ohm + the parallel resistance of 1 and R...(I wrote down the formula for parallel resistance, but totally blanked out on the concept)
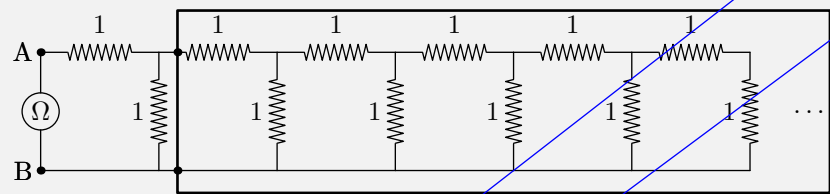
can we please go over this one in class

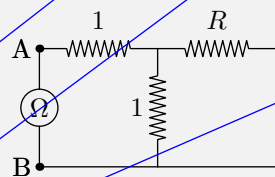I need a little help with the resistive networks I think

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This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)

Here it seems you are finding the total resistance. However, if so you denote the total resistance as R, the same as the resistance in the box. How does this work?

What was confusing for me in this problem was the equivalence of R...its first in series, then in parallel, etc...and I didn't get how to actually add those up correctly. The last operation is the in parallel resistors, but how does this account for all the other resistors, as the R on the right hand side is a different R than the left hand side?

> One way to look at it is to take the network and add another repitition of the series parallel 1 ohm resistors. Does the overall resistance change? How does the the addition of these elements affect the overall resistance if you write it out in symbolic form?

> > You didn't do a great job of explaining this one in class, but I think I get it now.

> They aren't different R, they are the same, just like S for the sum in the earlier problem is the same S on both sides of the equation.

I don't really understand how this got simplified down so quickly.

I'm a little confused on this problem. Why wouldn't the current just take the path of least resistance? and shouldn't the answer be like 1.4 (from the first resistor and then the 1 and R)?

this is close to what i got, but by totally different logic.

I didn't really know how to do this mathematically, but I did guess the range correctly!

This is pretty cool. I didn't notice when I did the problem myself.

> Me neither, but it seems to make sense. The ability to look at problems and break down a series into a definite answer is so useful and something I never really looked for.

> Yeah, I agree–this is very interesting that an infinite series can converge to something like this, and that we can solve it out using this quite simple method (if you see the pattern)

> > Yeah wow. I'm kind of mad at myself for not seeing this before.

> > > I remembered that it was the Golden Ratio, which I thought was really cool also

> Yep, I too liked this a lot.

> I did not realize this, definitely cool

i wouldn't have seen this. the previous example was what i thought though, because it's similar to the other recursion problems.

> this is what i did! yay!

> After starting from here I had a fun time solving it...forgot how algebra actually gets tricky at times.

> > Seeing the previous examples helped me get to this infinite sum

This is what I ended up doing but had a hard time spotting the exact recursion. ended up adding an extra 1 in somewhere from the beginning of the recursion

This resistive network contains a copy of itself (enclosed in the box):

Call R the resistance of the network inside the box, measured between the two dots as the terminals. Then the original network, which also has resistance R, is

It is a $1\,\Omega$ resistance in series with the parallel combination of $1\,\Omega$ and R. So

$$R = 1 + \underbrace{\frac{R}{1+R}}_{1\,\Omega\|R},\qquad(12)$$

or $R^2 - R - 1 = 0$. The positive solution is
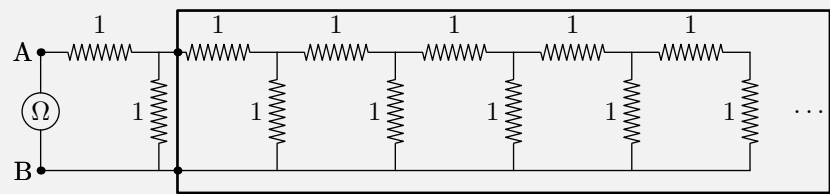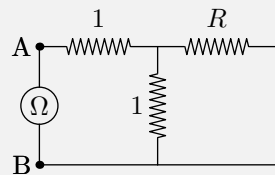
$$R = \frac{1+\sqrt{5}}{2} \approx 1.618,\qquad(13)$$

which is the Golden Ratio.

An alternative, direct method is the following continued fraction that accounts for the infinite cascase of series and parallel resistors:

$$R = 1 + \frac{1}{1 + \frac{1}{1+\cdots}}.\qquad(14)$$

This famous continued fraction converges (slowly) to the Golden Ratio. (One special feature of the Golden Ratio is that it has the the slowest-converging continued fraction of any real number.)

Why is this called the Golden Ratio?

This number suprisingly appears in a large amount of places, including Da vinci paitings and flowers and nature itself. There is a book on it and its quite good called suprisingly "The Golden Ratio"

Thanks for the note. I'll look for the book later.

The golden ratio is incredibly fascinating, It would be great to hear more about it in class and how it applies to this sort of thing.

It pretty much all seems clear and concise.

Optional

These problems are optional in case you want more practice or want to try a (possibly large) project.

8. **Email indexer**

   Design a set of shell scripts for doing quick keyword searches of a large database of emails. Assume that each email is stored in its own plain-text file. Perhaps one shell script generates an index, and a second script searches the index.

9. **Running time**

   Ordinary long multiplication requires $O(n^2)$ digit-by-digit multiplications. Show that the Karatsuba multiplication method explained in lecture requires $O(n^{\log_2 3}) \approx O(n^{1.58})$ digit-by-digit multiplications.

10. **Counting empires**

    How often does the word `Empire` (uppercase E, then all lowercase) occur in `decline.txt`? [*Hint:* Look up the `tr` command.]

> Divide and conquer! First turn all non-letters into newlines (squeezing out repeated newlines); second, look for lines that exactly match 'Empire'; and third, count the lines. Those three stages are the three stages of the following pipeline:
>
> ```
> tr -cs 'a-zA-Z' '\012' < ./data/decline.txt | grep '^Empire$' | wc -l
> ```
>
> It produces '37'.

This is an interesting problem. I manage the website for my lab, and I have just been given the task of converting an Excel file with 75+ contacts into a list that can be printed in HTML. I thought about brute-forcing it, but was too lazy and am now writing a script for this.

> In such situations, I often use gnumeric (nonproprietary software) to read the excel-format file, then ask it to export the file in tsv (tab separated value) format. Now I finally have a text file!
>
> Then I might use awk (the -F option is useful here) and sed to generate an HTML table. Or if it gets too messy I'd use Python.

Another idea is to simply grep for all the lines with Empire. This is almost right except it doesn't account for lines with multiple 'Empire's. You can use sed to replace "Empire" with " nEmpire" (newline + Empire). grep for "Empire" again and count the lines.

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   $$10 \boxed{} \pm \boxed{} \text{ passenger–mpg} \quad \textit{or} \quad 10 \boxed{} \ldots \boxed{} \text{ passenger–mpg}$$

   A roundtrip economy ticket from New York to San Francisco costs roughly $400. The journey is about 2500 miles each way, so a 5000-mile journey costs about $500 (rounding up the $400 to make the math easier). That's about 10 cents/mile. Perhaps one-half of that cost is fuel. [Although the service – in the air, on the phone, and at the counter – is so lousy due to understaffing that perhaps two-thirds of the cost being fuel would be a better estimate!] At 5 cents/mile for fuel, and at $3/gallon for fuel, the fuel efficiency is 60 passenger–miles per gallon.

2. **High winds**

   At roughly what wind speed is the force on your body from the wind approximately equal to your weight?

   $$\boxed{} \pm \boxed{} \text{ m s}^{-1} \quad \textit{or} \quad \boxed{} \ldots \boxed{} \text{ m s}^{-1}$$

   A typical person is maybe $m \sim 65\,\text{kg}$, so a weight of $mg \sim 700\,\text{N}$. The drag force is $F \sim \rho v^2 A$. Therefore,

   $$v \sim \left(\frac{mg}{\rho A}\right)^{1/2}. \tag{1}$$

   The density ρ is roughly $1\,\text{kg}\,\text{m}^{-3}$. For a person, the cross-sectional area is roughly $2\,\text{m} \times 0.5\,\text{m}$ (height times width) or $1\,\text{m}^2$. So

   $$v \sim \sqrt{\frac{700\,\text{N}}{1\,\text{kg}\,\text{m}^{-3} \times 1\,\text{m}^2}} \sim 25\,\text{m}\,\text{s}^{-1}. \tag{2}$$

   That's roughly 55 mph.

Here is the solution set. Apologies that i posted it late. Do the memo by Friday at 9am.

Could you list these on the main list of "things we need to do"? I keep forgetting that these are due.

Is there someway that the system could be set up so we can see our answers that we submitted, sometimes I forget what I had come up with.

Wow. I completely misinterpreted this "passenger-miles per gallon."

Isn't the point just to have a baseline energy comparison between different modes of transportation? Like if I drive my car I get 25 miles per gallon but if I fly I get 50 miles mpg. So if I'm traveling alone, flying is about 2x as efficient.

Oh.. I guess I was very off from guessing the price of the ticket. I used a one way ticket price because I think round trip tickets are very discounted.

I live in California, and I've never been able to find a roundtrip ticket home for this cheap!

most of the time I pay 300 - 360 to get to sacramento.

For a while I was paying 600 for a roundtrip ticket to california, but now it's a lot closer to 250-300, I was also really confused with my answer because I remember hearing somewhere that flying is much less efficient than cars, but I guess there's more people in a plane too

For estimating this price, is it best to use the maximum, minimum, or average value these tickets generally go for?

Wow, I did not think to use the price of a plane ticket!

I did, the problem stated to. But I felt that using the price of a plane ticket lacked precision because of so many more approximations that it would open up, like the fraction of cost to fuel, and there seems to be so many other factors. It seems to me that fuel efficiency more intrinsically a physical estimation, with size of airplane?

I guess I had overlooked that detail.

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   That's roughly 55 mph.

I definitely forgot to count the backwards trip, but I still had a similar number. What is the "actual" answer to this question?

Yep, an actual answer would be nice. I ended up with 30 passenger-miles per gallon, which is roughly the same fuel efficiency as a car, and I got confused...

You need to multiply the fuel efficiency for a car by the number of passengers it can carry.

Oh gosh, I forgot to account for the return trip as well! It's always in the assumptions at the very beginning of the problem that I trip up

I think I'm confused – I thought the only point of considering the round trip was just to make the math easier?

I also forgot the return trip in the cost, so maybe I overestimated. I also didn't bother thinking about costs other than fuel, which in hindsight probably wasn't logical.

I didn't account for the return trip either but I got about the same answer just using a one way trip. I think as long as your cost and mileage accounted for just one way it works out to be about the same.

Me too... I also underestimated the price. I guess most people aren't poor college students going for the cheapest flight.

Oops... I forgot the each way part! But my ticket price was lower and journey distance higher, so everything evens out in the end.

I liked this question because I had real world experience with all the estimations I had to make.

That is a very cool way to solve this problem. I tried to estimate the drag on a plane and on a car and used that ratio. Unfortunately, I was way off.

i dont see why you would use this considering it said use the cost of a plane ticket.

I did it the same way as the solution but I had trouble deciding how much of your plane ticket went to fuel. How did you come up with this number?

i didn't think that not all the ticket went to fuel, and my answer was close

i used a much lower fraction considering there are a lot of other business expenses to account for.

Yeah I'd have to disagree with that fraction. It seems a bit high. There are many other costs incorporated into plane ticket prices: inflation, business, profit, etc.

And this was hard to estimate because there's not much in my life that has given me experience to how much it costs to run an airline. I did this by estimating how much the fuel costs, and based my estimate on my knowledge of gas prices for the kind of gas for my car. Don't know if this was more or less accurate.

maybe leave it in dollars?

oops, I completely forgot about this part, but ended up with the same answer...

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That's roughly 55 mph.

I started this the same but halfway through did it differently by estimating the cost of jetfuel. I got a very different answer, which makes sense because I remember fudging the units in mine once.

Yeah me too. My strategy was pretty much the same but I just estimated the fuel cost differently.

I think I ended up close; same process. It took me a bit to connect the ticket price to the fuel cost.

doing a problem like this made me realize how little real world knowledge i have

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Yeah me too. But with a little help from Google you get to learn a few stuff. I also like this class because you learn real world stuff like that

This is a great question, It would be interesting to compare it to trains or boats or something as well.

yeah! this is similar to my thought process!

I just cut out all the services and went with pure fuel cost - but my plane ticket estimate was on 300 dollars

Yeah I didn't even think of the money going anywhere but fuel...

I didn't either, but in retrospect of course other things are being paid for with our money than just gas.

Is fuel actually two-thirds of the price? I was wondering this when solving the problem and figured it couldn't be that much bc then they would hardly make money.

While this definitely works for order of mag estimates, I have heard that jet fuel is more expensive. perhaps even up to 3 times more expensive.

This can be true, as it fluctuates with the oil market (similar to automobile fuel fluctuation). \$3-\$5 per gallon is a pretty good estimate (like you said, it wouldn't get much more than \$9)

I said they get fuel closer to \$2 a gallon. I know that airlines often will buy options for their fuel consumption but I'd also assume they can get it at a somewhat discounted price as compared to car drivers because of the large quantity they buy.

while they do get "breaks" on the cost for buying large quantities, the gas itself is much more expensive because it has to be processed more. [at least that's what I've heard]

i was surprised to find that jet A (gas for aircraft is not that much more expensive than regular unleaded and \$3/gallon is actually a fairly accurate assessment.

crap I missed a decimal in the answer box, at least my estimate was only off by a factor of two

what exactly is a passenger-mile per gallon. I think a got a similar number for gas mileage however, I assumed that you must multiply this by the number of passengers flying. You didn't really talk about passengers in the solution so I am just wondering what these units actually represent. Nevertheless, I made almost all the same assumption so I think I at least learned the lesson here.

I had this same question as to how the passenger part comes into play. Based on this I think the final solution I came up with was actually a mpg rating for the aircraft...

Oh I misunderstood and ended up divided by the number of people on the flight at some point.

6.055J/2.038J (Spring 2010)

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   > That's roughly $55\,\text{mph}$.

I feel like this was such an easy problem for most people, but coming up with the relevant parameters is still difficult for me.

Isn't m 9.8? So I would think we would round down to 600 here.

   That's true - that's what I would have done as well.

   I jsut completely forgot to figure in gravity for some odd reason

   You're not the only one... oops!

   Forgetting gravity! Your high school physics teacher should be ashamed...

   Maybe he rounded to 700 to account for the obesity epidemic (65 kg = 143 lbs)?

   Random fact: the FAA sets the average weight for American passengers on planes to be 170 pounds...

   Nah, it's not really significant and easier just to say it's 10 and thus 700.

i thought that if you used pounds, that the pounds themselves are units of force, so you wouldnt multiply by gravity.

oops by mistake I had a v^3 term in my answer since I took the Power equation from the notes on drag (in the cycling section) instead of the Force equation!

Are we just supposed to know this off the top of our head?

   The equation was derived in the reading for this week, but I agree that it isn't something I would have known off the top of my head otherwise.

   Isn't it also true that if the wind speed is equal to the weight of a person, it is the same as if the person were falling from the sky at terminal velocity.

   Yes, it is the same thing as terminal velocity. That's how I figured out this problem.

   hm–I was thinking about using terminal velocity, because I thought it made sense mathematically, but I just couldn't wrap my head around it conceptually...

   It's really annoying that they don't teach this in 8.01. ignoring air resistance is so silly, knowing and internalizing this equation would be so much more helpful for everyday questions!

   I would love if equations like this were provided, I for sure don't know many off the top of my head, and they come up in a ton of problems for this class.

yeah used this, called on memory of values from 2.005, got similar answer

The problem I had here was that this equation states that v is "proportional" to that expression. Shouldnt there be a constant somewhere?

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   > That's roughly 55 mph.

A list of useful equations with the useful numbers would be helpful.

> this isn't really a standard equation. This particular equation was discussed in the reading and basically is just a force balance. m*g=ro*(v^2)*A. then you solve for v.

> the equation that relates F proportionally to the pv^2A was in the reading, and his equation here is just that same equation, but with v isolated

I was going to use Newton's third law to calculate it but it was a lot harder! this is much better!

got to stand in a 45 mph wind in the wind tunnel sophomore year. was a lot of fun to lean into the wind and and being buoyed up by it...

I wasn't sure what to do with the cross sectional area here so I just dismissed it by using 1... Luckily that worked out great!

> an A of 1 seems like a pretty large person... I used smaller area and a mass that was a little larger and got a wind speed of 55 m/s... I kinda doubt that your terminal velocity in free fall would be this small....? ... haha I just searched it and it's -56 m/s

> I also said that the cross-sectional area of a person is about 1. it's an overestimate for the whole population, but if you think about it, take a (built, and slightly taller than) 6-foot man, cut him in half and put the two halves together, it's about a square meter right...

>> This seems like a very tall and skinny person. But I guess its all close enough. Its closer to the cross sectional area of any person than it is to the cross sectional area of something else, like an elephant.

Ha! I considered the density of a person for some reason. That was dumb. And also explains my incorrect answer.

> I sometimes forget which density to use in these problems, but what helps me is to look at the equation and think about what should happen to the force when you increase/decrease the density.

> i did the same thing (used the density of the person!)

sick. I got the exact same answer. First time.

> Unfortunately, there is no "exact" answer in an approximation class...

> But congratulations!

>> I got that answer too. Woot! This is just the same as terminal velocity right?

>>> Nice! My test person was a bit lighter (I used 600 so it would work better)

>>>> haha... i messed up the math and got the way too low answer of 0.6. so i made that the low range of my answer and used something that i just thought made sense for the high end. i guessed 30, which is just right for "a little higher than correct"

> eh same order of magnitude counts right?

>> Lol, I got something close to the speed of light.... I used the formula for impulse instead of using the one for terminal velocity.

> It's nice to come close; i also got a very close answer.

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Do the following warmups and problems. Submit your answers, including the short explanation, online by **10pm on Wednesday, 3 Mar 2010**.

Open universe: *Collaboration, notes, and other sources of information are* **encouraged.** *However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Warmups

1. **Fuel efficiency of a 747**

   Use the cost of a plane ticket to estimate the fuel efficiency of a 747, in passenger–miles per gallon (passenger–mpg).

   $$10 \boxed{} \pm \boxed{} \text{ passenger–mpg} \quad \textbf{\textit{or}} \quad 10 \boxed{} \dots \boxed{} \text{ passenger–mpg}$$

   > A roundtrip economy ticket from New York to San Francisco costs roughly \$400. The journey is about 2500 miles each way, so a 5000-mile journey costs about \$500 (rounding up the \$400 to make the math easier). That's about 10 cents/mile. Perhaps one-half of that cost is fuel. [Although the service – in the air, on the phone, and at the counter – is so lousy due to understaffing that perhaps two-thirds of the cost being fuel would be a better estimate!] At 5 cents/mile for fuel, and at \$3/gallon for fuel, the fuel efficiency is 60 passenger–miles per gallon.

2. **High winds**

   At roughly what wind speed is the force on your body from the wind approximately equal to your weight?

   $$\boxed{} \pm \boxed{} \text{ m s}^{-1} \quad \textbf{\textit{or}} \quad \boxed{} \dots \boxed{} \text{ m s}^{-1}$$

   > A typical person is maybe $m \sim 65\,\text{kg}$, so a weight of $mg \sim 700\,\text{N}$. The drag force is $F \sim \rho v^2 A$. Therefore,
   >
   > $$v \sim \left(\frac{mg}{\rho A}\right)^{1/2}. \tag{1}$$
   >
   > The density ρ is roughly $1\,\text{kg m}^{-3}$. For a person, the cross-sectional area is roughly $2\,\text{m} \times 0.5\,\text{m}$ (height times width) or $1\,\text{m}^2$. So
   >
   > $$v \sim \sqrt{\frac{700\,\text{N}}{1\,\text{kg m}^{-3} \times 1\,\text{m}^2}} \sim 25\,\text{m s}^{-1}. \tag{2}$$
   >
   > That's roughly 55 mph.

So for this problem we are assuming that the wind is picking the body up such that feet don't create friction with the ground?

I had a very similar approach but for some reason came to a very different answer. This answer makes more sense from experience though.

I did this differently. The terminal velocity of a human being is about 120 mph. At the terminal velocity, the force of wind balances the force of gravity, so I estimated about 54 m/s.

> this makes _so_ much more sense to me! Thanks.
>
> > This is also how I did it. Although this does require that you know the answer, fortunately the answers from both methods are very similar.

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3. Daunting integral

Evaluate

$$\int_{-\infty}^{\infty} \frac{x^3}{1 + 7x^2 + 18x^8} \, dx. \tag{3}$$

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$

The integrand, $x^3/(1+7x^2+18x^8)$, is antisymmetric: When x becomes $-x$, the integrand changes sign. So, for every sliver of rectangle in the negative-x region, there's a corresponding sliver with the opposite sign in the positive-x region. The net sum is therefore zero.

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4. Solitaire

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Speaking of clues, is it a happy coincidence that $0.8^2 + 0.6^2 = 1$? That convenient sum suggests looking at sums of squares, and how those are changed by making a move. Replacing a and b by $a' = 0.8a - 0.6b$ and $b' = 0.6a + 0.8b$ makes the sum of squares $a^2 + b^2$ into $a'^2 + b'^2$. Expand that expression:

$$a'^2 + b'^2 = (0.8a - 0.6b)^2 + (0.6a + 0.8b)^2$$
$$= 0.64a^2 - 0.96ab + 0.36b^2 + 0.36a^2 + 0.96ab + 0.64b^2$$
$$= a^2 + b^2.$$

How fast were the winds that day?

Weather underground shows max wind gusts last month at 57 mph at the top of the Green building, and 36 mph in Central Square. The canyon effects on campus could definitely produce wind forces equal to your weight.

Haha yes, I remember this. I couldn't close the door in front of my dorm....

The winds here are terrible, and on top of that, half of MIT is a wind tunnel... especially dorm row!

This problem was so cool! How would the equation change if you were running rather than walking?

You'd add your increased speed to the relative velocity between you and the air, but you'd be providing forward force with your legs.

I think the force with your legs would be irrelevant to the problem if you're just trying to understand the force of the wind.
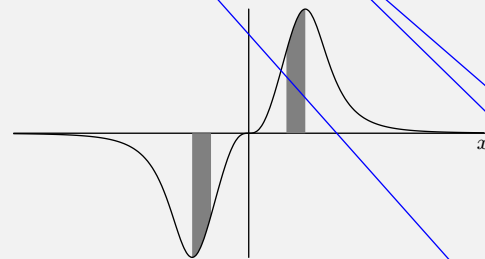
I kind of guessed it would be same amount negative as positive - my calculus is all fuzzy these days, but how could we identify that from just looking at the function?

yeah me too...I just kind of guessed that it had to be antisymmetric so that it would all cancel out, otherwise there wouldn't be a way to solve it without calculating it out..what's the correct way to get this type of intuition though?

The intuition is that the integrand is symmetric about the origin. This means that it is a mirror image using the origin as the axis. If we are then integrating over the same distance in both negative and positive x (in this case, negative infinity to positive infinity), then the integration before x = 0 will cancel out the integration after x = 0.

aw shoot, odd of course! I didnt notice that and just thought that is was symetric about the y axies, and then used a triangle to approximate.

I definitely stared at this for 10 to 15 minutes before even thinking about even or odd functions...I agree that it brings up a really nice symmetry that makes you feel silly afterward

I pretty much figured that this would be symmetric since there was no other way of solving this that we had learned, although before I realized it I did try to calculate it by estimating it as the integral of x^-4

i read your whole explanation as if this were "asymmetric." needless to say i was entirely confused (but understand now).

though while it makes sense, i don't know how i would have known it was symmetric without my ti-84

think about it at large values of x. at large values, the equation pretty much becomes 1/x^5, which is symmetric about the origin

you could look at the equation and realize that if you plugged in a number or that number times -1, the denominator did not change, but the numerator turned negative. this would mean that the curve is symmetric and positive to the right of x=0 and negative to left of x=0

how do you know this by looking at it?

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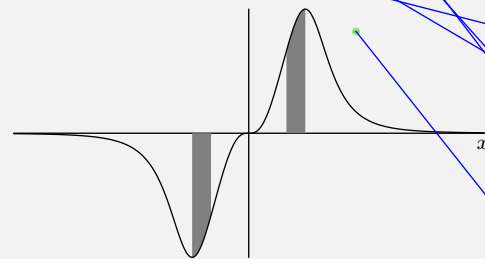
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$$

As soon as I graphed this, it made sense. My question is, would you like us to remember how to tell a symmetric integrand from an antisymmetric integrand without being able to graph it?

Look for odd or even functions.

Exactly. Like the professor, I typically start looking for symmetries whenever things don't fall into a typical 18.01 formula.

Does an odd function = antysymmetric function?

yes.

think of y=x [an odd function] – antisymmetric.

think of y=x^2 [an even function] – symmetric.

The way to determine if the integrand is to determine, as mentioned earlier, if the integrand is odd or even. Let f(x) = the integrand. If f(-x) = -f(x), then the integrand is odd and thus antisymmetric.

With multiple powers in this equation, sometimes determining this is a bit difficult. This one works out nicely though, seeing as the denominator has all even powers, it will never be negative, and in fact, never 0, with symmetry across the y axis. With this said, the numerator is clearly odd, making the function odd.

but this would have been hard if you don't have a calculator to graph it

I definitely stared at this for a solid 20 min without writing anything before realizing how simple the problem becomes.

It's simple, but I still don't understand, mathematically, why the integrand is antisymmetric (without the even odd function stuff).

I'd like to get more into mathy solutions in class as well as the approx.

This is the most useful thing I have learned about integrals since 18.01.

this seems like a _really_ complicated way of explaining odd functions...without actually doing any reasoning as to why is is an odd function.

does an odd function have to be perfectly antisymmetric to come out to 0 or does it go without saying that an odd function integrated from -inf to +inf is 0

You should check that the integral converges, though, before canceling sides (because -inf + inf !=0...). In this case, the denominator is bounded below by 1, and as x->+inf the function is bounded above by 1/(18*x^5), which converges (and it is similarly bounded as x-> -inf)

I was certain that there was an ever so small displacement from 0 for being anti-symmetric, although as an estimate it comes out to the same

I don't understand the graphs as much. Make later in the course or via office hours can you explain approximates in graphing in greater details? I just assume the answer would be zero based on what I know about integrals.

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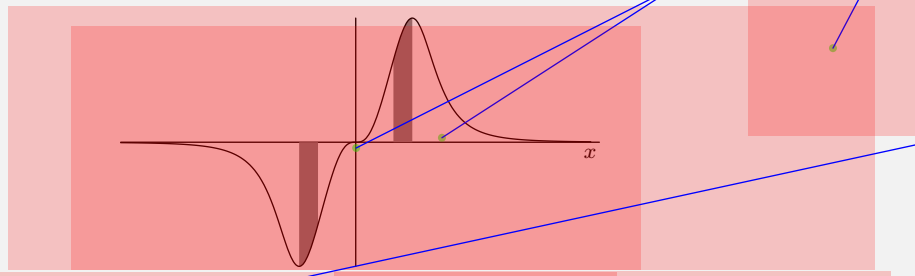
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I thought about this kind of the same way, but just used the rule of calculus that you can split the integral into two with different limits (-inf->0 and 0->inf) and saw the symmetry there

I'm not sure how valid this is, but I basically looked at the denominator, saw that it would get large much faster than the numerator (as the function went to infinity) and assumed that the function would go to zero.

I think the most sure way would be to just double check by plugging in numbers. right off the bat i plugged in 1 and -1 and realized that the magnitude of the function stayed the same but just changed signs

I wouldn't have realized the integral was antisymmetric just from looking at the equation.

Were we supposed to graph it?

I solved it without graphing by approximating the integrand to be x^-5 and evaluating.

I graphed the first few values. I really liked this problem, while it looks very complicated, the solution is quite simple and can be solved after a little bit.

I guess I will think about problems more visually from now on.

Graphing it was helpful, but I think the idea is to be able to recognize its zero from looking at it. Integrals with odd integrands and symmetric limits are always zero. this fits that description.

As a physics undergraduate, I saw this right away. Such symmetries are crucial to finishing p-sets in under 24 hours.

As an engineering major, I was pissed that I wasn't allowed to use my calculator.

hahaha nice

Ah yes, this did smack of many a physics pset.

This class is dragging up a lot of things I didn't think I would use again.

So when you solve this, do you need to take anything into consideration besides the x^8 in the denominator. Because I just saw that and figured you'd have 0-0 anyways.

This book should be required...yet I've never heard of it

i think i'm going to start using this phrase at 4am, when i'm still up doing math.

Even now, this is one of the things in the course that gives me the most trouble. Finding these little invariants in the game are always strange to me.

I really hate invariants. Is there anyway to get better at them? this whole seeing more of them thing is just making me hate them more and more.

I remember this in lecture...!

why are my guesses in this class never lucky?

I never would have thought about it like that

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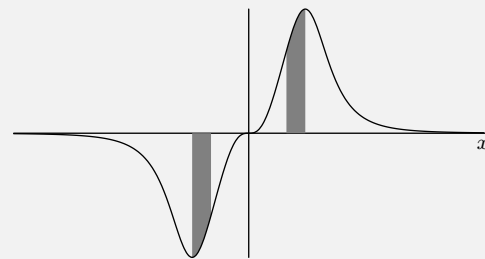
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\end{aligned}$$

interesting, Ii completely missed this

I did not see this coming at all.

i feel like this would have been easier to see if they'd been written as fractions, just because we've trained to spot 3-4-5 triangles, etc.

I like to think I'd notice these clues if I weren't so tired when I did my work, but I think that's wishful thinking.

d'oh. totally missed that equation.

I never thought to do this...I wish I had, now the rest of the problem isn't bad!

> Agreed. I couldn't figure out what the invariant was either...

>> Yeah..reading this makes it seem so much easier.

>>> Is there any way to derive this invariant, or is it just guess and check?

> I also never would have thought to do this, is there a good way for seeing these types of things other than practice?

wow, I used straight up sums–and didn't really feel too confident about it, but I didn't think of sums of squares...wow that makes this problem make a whole lot more sense

> I also used sums and agree that this was way more clever. I don't think using sums was a bad idea though, it still brought out a similar relationship between a and b that shows the final sum would be impossible to reach.

this was really hard, and how was i supposed to look for this- any hints?

Interesting. I guess it just takes a lot of practice in order to see these sorts of details readily.

> yeah i didn't get this, but tried a few variations to come to the proper conclusion instead.

It took me a while to find the invariant, but once I found this the answer to the problem came right along behind it.

> Agreed - these problems are all so simple once you solve them... but so tricky to get there!

>> Yeah Theres a lot of tricky hidden tricks that are hard to get even with some knowledge of symmetry.

>>> Definitely agree with this. Took forever to try to find an invariant...

>>>> Yea I never found it. What do you do then?

>>>>> I tried all sorts of sums and products and subtractions but never found the invariant :(I can't believe I didn't try sum of squares!

Great! Each move leaves the sum of squares unchanged. That sum started out with the invariant at $3^2 + 4^2 + 5^2 = 50$, so it remains 50. The goal state, however, requires that the invariant become $4^2 + 4^2 + 4^2 = 48$. It's not possible to reach the goal.

The invariant has a nice geometric interpretation (a picture). To see it, let $P = (a, b, c)$ be the coordinates of a point in three-dimensional space. Then each move leaves unchanged the distance to the origin, which is $\sqrt{a^2 + b^2 + c^2}$. So each move shifts P to another location equally distant from the origin, meaning that it moves P on the surface of a sphere. But it cannot escape the surface.

An interesting question to which I don't know the answer: Can you reach every point on the surface of the sphere? The distance invariant does not forbid it, but maybe other constraints do?

5. Maximizing a polynomial
Use symmetry to find the maximum value of $6x - x^2$.

$$\boxed{} \pm \boxed{} \qquad \textit{or} \qquad \boxed{} \ldots \boxed{}$$

The polynomial factors as $P = x(6 - x)$. As a symmetry operation, try replacing x with $6 - x$. That operation is a reflection through the vertical line $x = 3$. It turns P into $(6 - x)x$, which is again P just with the factors swapped. Let's call x_0 the value of x that maximizes P. Because changing x to $6 - x$ doesn't change the curve, it doesn't change the location of the minimum, which is at $(x_0, P(x_0))$. Thus x_0 turns into x_0 under the symmetry operation $x \mapsto 6 - x$. The only value of x that is unchanged by a reflection through the vertical line $x = 3$ is 3 itself, so $x_0 = 3$ and $P(x_0) = 9$.

6. Tiling a mouse-eaten chessboard
An 8×8 chessboard gets two diagonally opposite corners eaten away by a mouse. You have dominoes, each 2×1 in shape – i.e. each covers two adjacent squares. Can you tile the mouse-eaten chessboard with these dominoes? In other words, can you lay down the dominoes to cover every square exactly once (no empty squares and no overlaps)?

☐ yes

☐ no

Placing a domino on the board is one move in this solitaire game. For each move, you choose where to place the domino – which means you might have many choices for each move. Can you cover the whole board? The space of possible moves grows rapidly. Hence, look for an invariant: a quantity unchanged by any move of the game.

Because each domino covers one white square and one black square, the following quantity is invariant (unchanged):

$$I = \text{number of uncovered black squares} - \text{number of uncovered white squares.} \qquad (4)$$

With a regular chess board, the initial position would have $I = 0$, from 32 white squares and 32 black squares. With this modified board, two black squares have vanished, so I is $30 - 32 = -2$. However, in the winning position, all squares are covered; therefore $I = 0$. Because I is invariant,

I didn't catch this before.

I liked this problem a lot - it required a lot of thinking about invariants, and we had to find one that was nontrivial. You should use this problem (or some version of it) again!

I had trouble finding the invarient at the start, but once I found it the problem was clear and a lot of fun to prove

I agree, I think this was a good problem. Although I definitely spent the most time on this problem compared to the others because it took me a long time to find the invariant. Obviously there aren't any rules about how to find the invariant, but are there any helpful tips aside from "just look for what doesn't change"?

Yeah the invariant was a really tricky find...

I feel like this problem would perhaps have been enhanced by some easier warm up problems to practice finding invariants. I pretty much ended up solving this problem by just trying it out until I had decided it wasn't possible.

I think I'm a bit confused about where the cˆ2 comes from...?

Yeah, I think that it's not necessarily obvious why we can generalize from aˆ2+bˆ2 invariant to aˆ2+bˆ2+cˆ2 also invariant.

This is so much easier once you see the invariant! I spent so much time on this problem until it hit me!

This problem makes more sense now (I had no idea how to do it)...doing another problem similar to this might be useful to imprint the technique on solving it

I used a similar idea for my invariant, although it is not as concrete. I saw that the sum of 3,4,5 and 4,4,4 was 12 and that each move seemed to decrement the sum (at least move it away from 12) so the sum of 12 would be really hard to achieve

I completely missed this but once you see it along with the visualization everything becomes so much easier.

I believe we have the invariant that preserves "rational points", so I don't think we could reach any of the irrational ones on the sphere. Not sure if we could reach all the rational ones though. My guess is no.

I got curious and tried doing something like a change of variables or another invariant to see if it could be done but I couldn't find another one (I tried a couple of geometric things) and can't see any change that makes it more obvious...

Do you mean reach every point, using only that move option?

I realize now that I found the value of x that maximizes 6x-xˆ2, but forgot to plug that number back into the function to get the actual value.

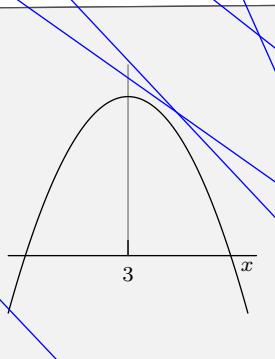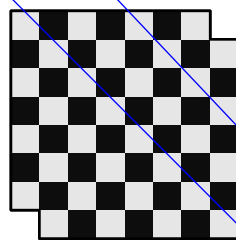**I actually didn't use symmetry for this, and just calculated it using the derivative...**

I did too. This seemed like too trivial of an example to really use symmetry.

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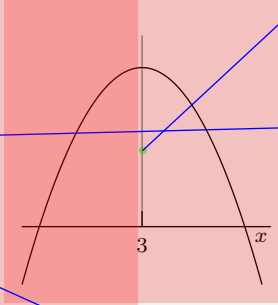
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   $\boxed{} \pm \boxed{}$ *or* $\boxed{} \dots \boxed{}$
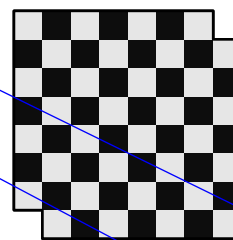
The polynomial factors as $P = x(6 - x)$. As a symmetry operation, try replacing x with $6 - x$. That operation is a reflection through the vertical line $x = 3$. It turns P into $(6 - x)x$, which is again P just with the factors swapped. Let's call x_0 the value of x that maximizes P. Because changing x to $6 - x$ doesn't change the curve, it doesn't change the location of the minimum, which is at $(x_0, P(x_0))$. Thus x_0 turns into x_0 under the symmetry operation $x \mapsto 6 - x$. The only value of x that is unchanged by a reflection through the vertical line $x = 3$ is 3 itself, so $x_0 = 3$ and $P(x_0) = 9$.

6. **Tiling a mouse-eaten chessboard**
   An 8×8 chessboard gets two diagonally opposite corners eaten away by a mouse. You have dominoes, each 2×1 in shape – i.e. each covers two adjacent squares. Can you tile the mouse-eaten chessboard with these dominoes? In other words, can you lay down the dominoes to cover every square exactly once (no empty squares and no overlaps)?

   $\square$ yes

   $\square$ no

Placing a domino on the board is one move in this solitaire game. For each move, you choose where to place the domino – which means you might have many choices for each move. Can you cover the whole board? The space of possible moves grows rapidly. Hence, look for an invariant: a quantity unchanged by any move of the game.

Because each domino covers one white square and one black square, the following quantity is invariant (unchanged):

$$I = \text{number of uncovered black squares} - \text{number of uncovered white squares}. \qquad (4)$$

With a regular chess board, the initial position would have $I = 0$, from 32 white squares and 32 black squares. With this modified board, two black squares have vanished, so I is $30 - 32 = -2$. However, in the winning position, all squares are covered; therefore $I = 0$. Because I is invariant,

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I found it much easier to use guess and check. Maybe the example problem for symmetry should be more difficult to solve unless you implement a symmetry strategy.

Yea, I just plugged in values 1-5 to the equation and it became obvious where the max was.

This is one thing that I just had a hard time with on this Pset – trying to some type of operation (or invariant point) that works! It always makes sense afterwords, but I'm not so good at coming up with them.

Yeah... this one took me a lot longer than the others. I would stare at a problem for a while and not see the answer. Some of them came to me later though while I was doing other things... funny how the brain works!

this one took me soooo long! I felt to stupid...the answer is just so simple

wow I didn't think about it like that; I looked for the zeros.

So when this problem said "use symmetry", I didn't think about finding an invariant that doesn't change, I thought about the graphically symmetric nature of parabolas (if it intercepts the x-axis at x=0 and x=6, then the max value must be at x=3). By "use symmetry", are you actually saying "find an invariant"?

I thought the same thing. I solved this and in my explanation tried to convince myself it was symmetry although I still don't completely understand.

I explained the symmetry by seeing that the curve was symmetric about the line x=3 and that meant that x=3 was the global max

I ended up using a different kind of symmetry for this - I know a parabola is symmetric so the max would be directly between the zeros. I was having issues finding an invariant.

I feel like the zeros method is a lot less confusing than the method here.

Agreed. Although the method used here may be more generally applicable. But I also just used previous knowledge of parabolas to solve this problem.

yeah, i used that same general symmetry of parabolas, but if you look at it, that's basically what the solution is doing too, but saying that the local extrema on a parabola remains unchanged with reflection through a vertical line through it, while everything else is symmetrical...basically the same thing–both sides of a max/min are symmetrical

I think I'd need this process if the polynomial were more complex; for this one I just pictured it.

i agree. this seemed like a really easy problem. something you couldn't graph in your mind would have proved a point a little better.

If I had thought to factor this i would have been able to see the symmetry much quicker.

I just solved it by using simple differentiation, I will admit that your approach applies more to the class, but with such an easy problem it is hard to not just do it the simple way, maybe next example could be made more difficult to solve based on math, and therefore would require the application of symmetry.

did you mean maximum here?

I think this was one of the trickiest problems. It's funny because the explanation seems so simple. However, I think that the problems without a lot of math often require the most thought.

Great! Each move leaves the sum of squares unchanged. That sum started out with the invariant at $3^2 + 4^2 + 5^2 = 50$, so it remains 50. The goal state, however, requires that the invariant become $4^2 + 4^2 + 4^2 = 48$. It's not possible to reach the goal.

The invariant has a nice geometric interpretation (a picture). To see it, let $P = (a, b, c)$ be the coordinates of a point in three-dimensional space. Then each move leaves unchanged the distance to the origin, which is $\sqrt{a^2 + b^2 + c^2}$. So each move shifts P to another location equally distant from the origin, meaning that it moves P on the surface of a sphere. But it cannot escape the surface.
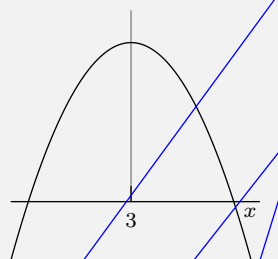
An interesting question to which I don't know the answer: Can you reach every point on the surface of the sphere? The distance invariant does not forbid it, but maybe other constraints do?

5. **Maximizing a polynomial**
Use symmetry to find the maximum value of $6x - x^2$.

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$
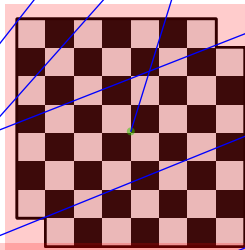
The polynomial factors as $P = x(6 - x)$. As a symmetry operation, try replacing x with $6 - x$. That operation is a reflection through the vertical line $x = 3$. It turns P into $(6 - x)x$, which is again P just with the factors swapped. Let's call x_0 the value of x that maximizes P. Because changing x to $6 - x$ doesn't change the curve, it doesn't change the location of the minimum, which is at $(x_0, P(x_0))$. Thus x_0 turns into x_0 under the symmetry operation $x \mapsto 6 - x$. The only value of x that is unchanged by a reflection through the vertical line $x = 3$ is 3 itself, so $x_0 = 3$ and $P(x_0) = 9$.

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☐ yes

☐ no

Placing a domino on the board is one move in this solitaire game. For each move, you choose where to place the domino – which means you might have many choices for each move. Can you cover the whole board? The space of possible moves grows rapidly. Hence, look for an invariant: a quantity unchanged by any move of the game.

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With a regular chess board, the initial position would have $I = 0$, from 32 white squares and 32 black squares. With this modified board, two black squares have vanished, so I is $30 - 32 = -2$. However, in the winning position, all squares are covered; therefore $I = 0$. Because I is invariant,

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This is one of those that if you have seen before you can solve it immediately, and will never forget it.

Yeah, I definitely have seen this problem before so I remembered how to prove it

I like this problem. It seems like a strange IQ test question, and indeed I think I remember it in a calendar of puzzles that I have, but it is fairly reasonable to work out.

I wonder if you were not given a picture of the chessboard if you would actually think in colors...I've tried it on my friends verbally and they could not get it

Well, the professor did say in class how people can understand things better with a visual of the problem.

I actually couldn't see or didn't think about looking at this problem as finding an invariant. But seeing it now, its a pretty nifty method of doing it.

If it wasn't for the theme of the homework set, I wouldn't have thought of it either... and it took me a while to realize how to think of it.

I am still a little shaky on the idea of invariants. I thought that the idea was more for the entire board not for the color of the squares.

It could be for the whole board but using squares greatly simplifies the complexity

ahhhh. that's it

very nice solution

Wow, I never would have found an invariant in this problem. I just realized that in order to fill this up with a 2x1 domino, you had to eventually make a piece whose area was 3, which isn't possible because we have 2x1 pieces...

Yea but I think thats only half the battle- you still need to prove it.

We've seen this problem before (well, a slight variation, but essentially the same). It would have been nice to see something a bit more different.

I wasn't able to find/use this invariant when solving the problem...instead i looked for visual "invariants;" that is, combos of white/black square in different shapes, trying to fit them into the board.

I went ahead and calculated all the pairs via counting and matching up. I did your method also but I assumed that counting would be more reliable. Perhaps I made a error in counting. Make approximating is more reliable.

instead of this i divided the board into sections and there eventually were sections that had an odd number of squares and couldn't be covered with a domino.

yea i did something different also, i made a smaller version of the board 3x3 and made it larger each time by scaling up

Great! Each move leaves the sum of squares unchanged. That sum started out with the invariant at $3^2 + 4^2 + 5^2 = 50$, so it remains 50. The goal state, however, requires that the invariant become $4^2 + 4^2 + 4^2 = 48$. It's not possible to reach the goal.

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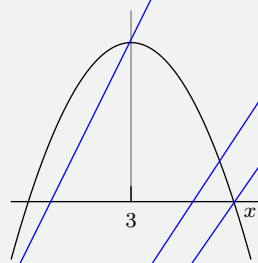
An interesting question to which I don't know the answer: Can you reach every point on the surface of the sphere? The distance invariant does not forbid it, but maybe other constraints do?

Oh this is very clever - I just kept trying to tile the board until I convinced myself it couldn't be done - I didn't even think about solving the problem this way.

I realized that it would have been possible if both missing corners had been on one side (one black and one white), but didn't attribute it to the color differences.

Same. I never even thought to look for an invarient, i just applied what I thought of as symmetry

Yeah, I had the same conclusion - I didn't even realize it was 2 black squares that were missing! If it was one white and one black, then it would have to work. huh, that's cool.

I'm curious to know whether or not all boards with the same number of white and black tiles could be tiled with dominoes. I feel like almost all the invariant questions we are dealing with are done to prove something is not true,

5. Maximizing a polynomial

Use symmetry to find the maximum value of $6x - x^2$.

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$
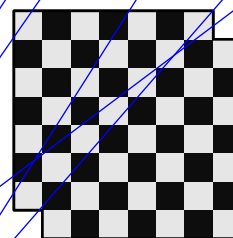
This explanation helped me a lot.

The polynomial factors as $P = x(6 - x)$. As a symmetry operation, try replacing x with $6 - x$. That operation is a reflection through the vertical line $x = 3$. It turns P into $(6 - x)x$, which is again P just with the factors swapped. Let's call x_0 the value of x that maximizes P. Because changing x to $6 - x$ doesn't change the curve, it doesn't change the location of the minimum, which is at $(x_0, P(x_0))$. Thus x_0 turns into x_0 under the symmetry operation $x \mapsto 6 - x$. The only value of x that is unchanged by a reflection through the vertical line $x = 3$ is 3 itself, so $x_0 = 3$ and $P(x_0) = 9$.

took me a while to see this. I approached this problem with recursion trying to break down the board into similar shapes, until I reached a base case.

I was pretty sure the answer to this was no, but I couldn't find a way to prove it...seeing that two 'black' spaces were removed makes it much clearer.

I also wanted to say no, but without being able to prove it and knowing the way this class can be, I expected to be shocked by some crazy simplification that solved the problem.

It really helps if you can visualize. I got this one much quicker then the integral.

this is really smart

6. Tiling a mouse-eaten chessboard

An 8×8 chessboard gets two diagonally opposite corners eaten away by a mouse. You have dominoes, each 2×1 in shape – i.e. each covers two adjacent squares. Can you tile the mouse-eaten chessboard with these dominoes? In other words, can you lay down the dominoes to cover every square exactly once (no empty squares and no overlaps)?

☐ yes

☐ no

So I don't understand why I not equaling 0 means it's impossible to cover the board. I had to cover this whole board. Also, I feel like a better way to solve this problem is to minimize it. Change it into a 2x2 board and you see how it's impossible.

Actually, when I=0 it means that it is possible to cover the board. The explanation says that with a regular chess board, the initial position would have I=0. This makes sense because the board is square with an even number of squares on each side. This explanation is saying that in this scenario, I=-2, and that's why it's impossible.

Placing a domino on the board is one move in this solitaire game. For each move, you choose where to place the domino – which means you might have many choices for each move. Can you cover the whole board? The space of possible moves grows rapidly. Hence, look for an invariant: a quantity unchanged by any move of the game.

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With a regular chess board, the initial position would have $I = 0$, from 32 white squares and 32 black squares. With this modified board, two black squares have vanished, so I is $30 - 32 = -2$. However, in the winning position, all squares are covered; therefore $I = 0$. Because I is invariant,

no sequence of ~~domino moves~~ can turn the initial uncovered board into the winning board (with all squares covered).

Optional!

7. Symmetry for second-order systems

This problem analyzes the frequency of maximum gain for an LRC circuit or, equivalently, for a damped spring–mass system. The gain of such a system is the ratio of the input amplitude to the output amplitude as a function of frequency.

If the output voltage is measured across the resistor, and you drive the circuit with a voltage oscillating at frequency ω, the gain is (in a suitable system of units):

$$G(\omega) = \frac{j\omega}{1 + j\omega/Q - \omega^2},$$

where $j = \sqrt{-1}$ and Q is quality factor, a dimensionless measure of the damping. Do not worry if you do not know where that gain formula comes from. The purpose of this problem is not its origin, but rather using symmetry to maximize its magnitude.

The magnitude of the gain is

$$|G(\omega)| = \frac{\omega}{\sqrt{\left(1 - \omega^2\right)^2 + \omega^2/Q^2}}.$$

Find a variable substitution (a symmetry operation) $\omega_{new} = f(\omega)$ that turns $|G(\omega)|$ into $|H(\omega_{new})|$ such that G and H are the same function (i.e. they have the same structure but with ω in G replaced by ω_{new} in H). Use the form of that symmetry operation to maximize $|G(\omega)|$ without using calculus.

When maximizing a parabolic function such as $y = x(6 - x)$, the symmetry is reflection about the line $x = 3$. In symbols, the transformation is $x_{new} = 6 - x$.

Let's transfer a few lessons from the parabola example to the problem of maximizing the gain. In the parabola example, the symmetry is a reflection about an interesting point (there, the point halfway between the two roots $x = 0$ and $x = 6$). Analogously, an interesting frequency is $\omega = 1$ because it makes the real part of the denominator in $G(\omega)$ go to zero, and making the real part go to zero helps minimize the denominator.

Therefore reflecting about $\omega = 1$ is worth trying, perhaps $\omega_{new} = 1 - \omega$. For frequencies, however, differences are not as important as ratios. For example, a musical octave is a factor of 2 in frequency, rather than a difference. So reflect in a multiplicative way: $\omega_{new} = w^{-1}$.

This transformation works either in $G(\omega)$ or in the magnitude $|G(\omega)|$. It's slightly easier in $G(\omega)$:

$$G(\omega) = \frac{j\omega}{1 + j\omega/Q - \omega^2} \mapsto H(\omega_{new}) = \frac{j/\omega_{new}}{1 + j/Q\omega_{new} - 1/\omega_{new}^2}.$$

Multiply numerator and denominator by ω_{new}^2:

$$H(\omega_{new}) = \frac{j\omega_{new}}{\omega_{new}^2 + j\omega_{new}/Q - 1},$$

which is the same function as $G(\omega)$, except for negating the real part in the denominator. Negating the real part in the denominator doesn't affect the magnitude of the denominator, so $|H(\omega_{new})|$ has the same form as $|G(\omega)|$.

This is very interesting... I wish I could think of it this way...

I thought about this in the same way but I couldn't find an argument strong enough to prove it. The proofs are always so subtle that I can't come up with them but they make perfect sense when I see it.

Are you ever going to give us a question where there is a solution?

> Haha that's true...most of the time now I begin with the assumption that the problem can't be solved.

>> Well he had us solve for f_1000 in the last class.

>>> This is a solution. there is one exact answer -"you can't do it". ha.

Oh this is interesting. I looked at it with a completely different method that now I think of it, is rather uncouth.

> And what different method was that?

> What I did was I thought about the 4X4 case and I reasoned that if it can't work for the 4X4 case, then it won't work for the 8X8 case

>> I did something similar - I realized that the board was symmetrical along the midline, and realized that in each half there were an odd number of squares so it wasn't possible.

>>> I did something similar as well - I looked at just the first two columns and identified that it wouldn't work when applied to the whole board.

> I did it your way at first and then the notion of the number of black and white squares hit me.

See I got this answer, but not through any of the means you've taught us. I'm worried that I'm not learning how to apply them well enough.

Since $\omega_{new} = 1/\omega$, the maximum value of ω_{new} will be ω_{max}^{-1}. That's one equation.

Since the two magnitudes $|G(\omega)|$ and $|H(\omega_{new})|$ are the same function, the maximum value of ω_{new} is also the maximum value of ω. That's the second equation.

Together they produce $\omega = \omega_{new} = 1$ (ignoring the negative-frequency solution $\omega = -1$). At that frequency, $|G(\omega)|$ is Q. For the electrical and mechanical engineers: The quality factor Q is also the gain at resonance.

8. Inertia tensor

[For those who know about inertia tensors.] Here is the inertia tensor (the generalization of moment of inertia) of a particular object, calculated in a lousy coordinate system:

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 4 & 5 \end{pmatrix}$$

Change coordinate systems to a set of principal axes. In other words, write the inertia tensor as

$$\begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix}$$

and give the values of I_{xx}, I_{yy}, and I_{zz}. *Hint:* What properties of a matrix are invariant when changing coordinate systems?

Whatever coordinate change I make, I will leave the x axis alone because the I_{xx} component is already separated from the y- and z submatrix. That submatrix is

$$\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$$

I have to figure out how changing the coordinate system changes this submatrix. Rather than find the coordinate change explicitly, I use invariants to avoid that computation.

One invariant of any matrix, not just of this 2×2 matrix, is its determinant. Another invariant is its trace (the sum of the diagonal elements). In the nasty coordinate system, the trace of the y- and z submatrix is $5 + 5 = 10$. So the trace is 10 in the nice coordinate system. The determinant is $5 \times 5 - 4 \times 4 = 9$, so it the determinant is 9 in the nice coordinate system.

Those facts are sufficient to deduce the submatrix in the nice coordinate system (without needing to figure out what the nice coordinate system is). In the nice coordinate system, the 2×2 submatrix looks like

$$\begin{pmatrix} I_{yy} & 0 \\ 0 & I_{zz} \end{pmatrix}$$

So I need to find I_{yy} and I_{zz} such that

$$I_{yy} + I_{zz} = 10 \quad \text{(from the trace invariant)}$$

and

$$I_{yy} I_{zz} = 9 \quad \text{(from the determinant invariant)}$$

The solution is $I_{yy} = 1$ and $I_{zz} = 9$ (or vice versa). So the inertia tensor becomes

$$\begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

I didn't do the optional questions, but I like reading their solutions. Thanks for taking the time to post them!

Agreed. They are fun to review.

I like the idea of applying abstraction to 2.001. This really honed in the concept for me.

I did this by eigenvalues, which are also invariant under basis set transformation. The eigenvalues of the 2x2 submatrix are 1 and 9, so just write out the diagonalized matrix with those values.

That's true, so the method outlined in the solution set gives you a way to compute eigenvalues. That's the method I use to compute eigenvalues in my head.

9. Resistive grid

In an infinite grid of 1-ohm resistors, what is the resistance measured across one resistor?

To measure resistance, an ohmmeter injects a current I at one terminal (for simplicity, say $I = 1$ Å), removes the same current from the other terminal, and measures the resulting voltage difference V between the terminals. The resistance is $R = V/I$.

*Hint:* Use symmetry. But it's still a hard problem!
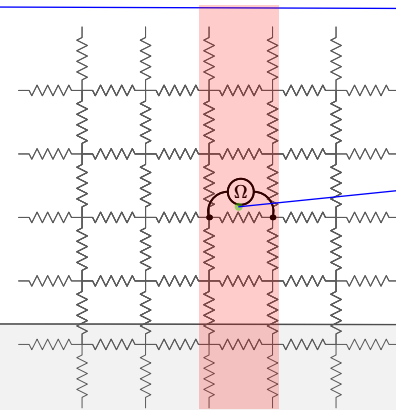
I'd like to find the current flowing through the resistor when 1 Å is sent into one terminal of the ohmmeter and removed from its other terminal. The solution has two steps, each subtle:
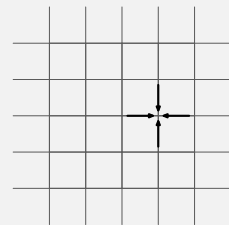
1. Break the resistance-measuring experiment into two parts, each having a lot of symmetry.

2. Analyze those parts using symmetry.

The current distribution that results from the full resistance-measuring experiment is not sufficiently symmetric because it has a preferred direction along the selected resistor. However, if I break the experiment into two parts – inserting current and removing current – then each part produces a symmetric current distribution.

By symmetry – because all four coordinate directions are equivalent – inserting 1 Å produces 1/4 Å flowing in each coordinate direction away from the terminal. Let's call this terminal the positive terminal. So inserting the 1 Å at the positive terminal produces 1/4 Å through the selected resistor, and this current flows away from the positive terminal.
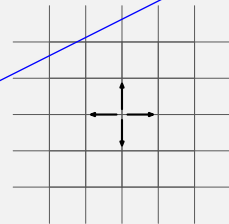
By symmetry, removing 1 Å produces 1/4 Å in each coordinate direction, flowing toward the terminal. Let's call this terminal the negative terminal. So removing 1 Å produces 1/4 Å through the selected resistor, flowing toward the negative terminal. Equivalently, it produces 1/4 Å flowing away from the positive terminal.

Now superimpose the two pictures to reproduce the experiment of measuring the resistance. The experiment produces 1/2 Å through the resistor, flowing from the positive to the negative terminal. The voltage across the resistor is the current times its resistance, so the voltage is 1/2 V. Since a 1 Å test current produces a 1/2 V drop, the effective resistance is 1/2 Ω.

If you want an even more difficult problem: Find the resistance measured across a diagonal!

COMMENTS ON PAGE 6

Do you think you could explain this problem in class?

This problem was awesome! i'd love to see it make its way into a lecture or even the textbook, provided its extremely well explained. I think there is so little E&M involved in the problem that it would be fair.

That's interesting. I did the resistive grid on just this part, i.e., infinite in one dimension, and got the same answer.

That result worries me. It means that the entire rest of the grid isn't participating in current flow – i.e. isn't providing alternative paths between the two ends of the ohmmeter.

Whereas I would expect the resistance of the outlined ladder to be slightly higher than 1/2. When I calculated the resistance of the outlined ladder and got a number slightly larger than 0.5.

I checked my answer again. I got (1+sqrt(3))/(3+sqrt(3)) approx 0.577

Sorry about the confusion.

In thinking about this, I was always worried that it didn't superimpose. That is, what if the fact that inserting current at one node and removing current at a second node affected the way the current flows because different directions have differing lengths between the nodes. That's pretty cool. I realize that I have to consciously fight against that worry in order to use superposition.

6.055J/2.038J (Spring 2010)

Solution set 4

*Do the following problems. Submit your answers and explanations online by **10pm on Wednesday, 10 Mar 2010**.*

Open universe: *Collaboration, notes, and other sources of information are **encouraged**. However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.*

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

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Solution set 4

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Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Here is the solution set, for the memo due Thursday at 10pm.

Is there a way to increase the time window that we can do this assignment? 24 hours just seems pretty short.

This hw set was very time consuming.

I agree. Although each problem took less time, the overall effect was a lot longer.

Well, this problem set had more problems on it. Unless there's an inverse relationship between time and number of problems, I guess it should take longer...

This problem set took me a lot longer than the other ones too.

It took longer but it fit in with the lectures really well which was useful in finding formulas and such.

Actually, for me, each problem seemed to take longer than usual. So combined with the fact that there were more problems, this pset took me a lot longer to do.

You're right, it did take a while, but it allowed us to use/test some of the more impressive skills we've learned recently. Who doesn't like a good challenge? I mean come on... we do go to MIT.

i don't think any of us are denying whether it was useful or good for us. the fact remains, it took a long time. i lost motivation by the end. usually i'm really excited by the findings i find but this time i just wanted to finish and didn't seem to care any more

That is useful feedback. I was thinking about having no homework set for this coming week (i.e. no homework due this coming Wednesday). We are at a good spot in the material for that pause.
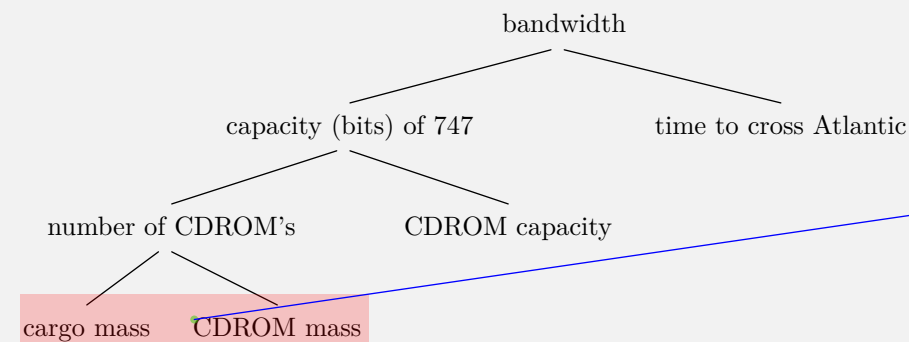
Given that this past set seemed to take longer than others, that confirms it. So, no new homework this Friday.

Problem 1   Bandwidth

To keep your divide-and-conquer muscles strong, here is an exercise from lecture: Estimate the bandwidth of a 747 crossing the Atlantic filled with CDROMs.

$$10^{\boxed{} \pm \boxed{}} \text{ bits/s} \quad \textit{or} \quad 10^{\boxed{} \cdots \boxed{}} \text{ bits/s}$$

Divide and conquer! Here's a tree on which to fill values:

bandwidth
  capacity (bits) of 747   time to cross Atlantic
    number of CDROM's   CDROM capacity
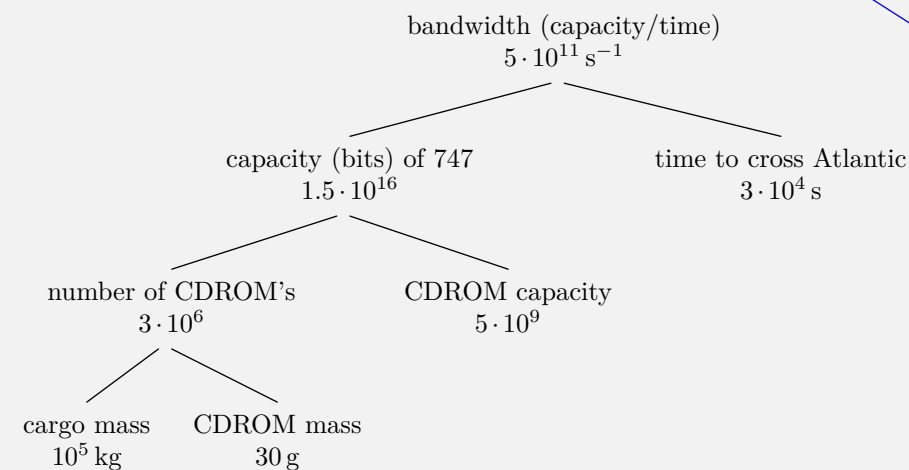   cargo mass   CDROM mass

First I estimate the cargo mass. A 747 can easily carry about 400 people, each person having a mass (with luggage) of, say 140 kg. The total mass is

$$m \sim 400 \times 140 \,\text{kg} \sim 6 \cdot 10^4 \,\text{kg}.$$

A special cargo plane, with no seats or other frills for passengers, probably can carry 10^5 kg.

Here are the other estimates. A CDROM's mass is perhaps one ounce or 30 g. So the number of CDROM's is $3 \cdot 10^6$. The capacity of a CDROM is 600 MB or about $5 \cdot 10^9$ bits. The time to cross the Atlantic is about 8 hours or $3 \cdot 10^4$ s.

Now propagate the values toward the root of the tree:

bandwidth (capacity/time)
$5 \cdot 10^{11} \,\text{s}^{-1}$
  capacity (bits) of 747   time to cross Atlantic
  $1.5 \cdot 10^{16}$ $3 \cdot 10^4 \,\text{s}$
    number of CDROM's   CDROM capacity
    $3 \cdot 10^6$ $5 \cdot 10^9$
      cargo mass   CDROM mass
      $10^5 \,\text{kg}$ 30 g

The bandwidth is 0.5 terabits per second or $10^{11.5}$ bits/second.

Despite the large bandwidth offered by a 747 carrying CDROM's (not to mention DVDROM's), trans-Atlantic Internet connections go instead via undersea fiber-optic cables. Low latency is important!

Did anyone else think the title of this was funny? bandwidth like a link has bandwidth but here we are just stuffing a bunch of CDs with bits of information into a plane and flying them to their destination...

I had to read over the lectures again to figure out bandwidth, not very familiar with that

It would be interesting for future versions to change the media to DVDs or even maybe bluray. It's the same thing and scales well, but the new media types are interesting.

I was a little frustrated with this problem because I felt like it was an exercise in remembering lecture. I'm unsure if getting a similar answer here is a sign that I'm learning the process or just storing the bits of data.

> While I agree that this was quite reminiscent of lecture, even remembering the processes from lecture is good practice. And, I think, helps solidify potential shortcuts for future problems.

i didn't use mass, but instead used volume as estimated by amount of luggage that can be carried. i feel like both are valid estimates. it's easier for me to estimate volume than weight.

> We discussed this in lecture, and determined that a plane probably couldn't take off if its volume were full. You need to verify what, exactly, is the limiting factor.

> > I went with volume just because we went with mass in lecture and I wanted to see the other side of it all. The numbers werent too incredibly different IIRC

> > > I also used volume and even though I knew that mass would be more accurate, volume was easier for me to work with. I figured that since we are doing first order approximations, it wouldn't be that bad.

> > > However I first tried to quantify how much thrust the engines could provide to determine lift as opposed to how many people a plane could carry. Using people would have made the mass method easier.

> > > > Anyone who knows anything about aviation will tell you go by mass. if you filled a passenger aircraft up to maximum volume with CD's it would not be able to leave the ground.

I had no idea a 747 could carry that many people

> Yeah, when I was trying to imagine how many people, I was thinking 50 rows with 6 people across was a big overestimate.

> > Also it might be harder to estimate because its not uniform, the people in first class are all spread out and everyone else is crammed.
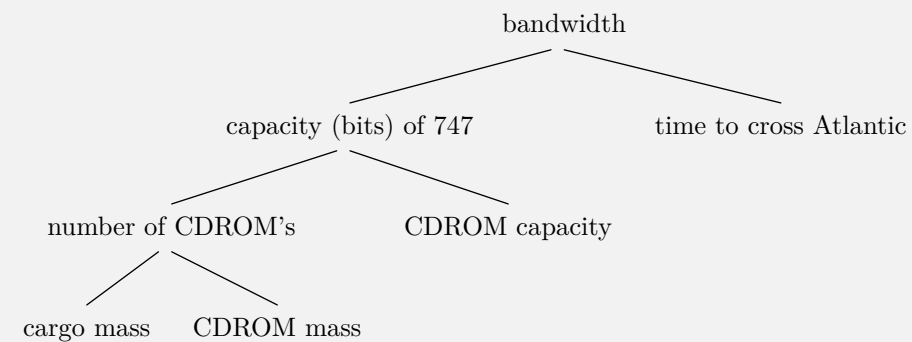
I think a lot of my estimates were inaccurate. I thought way fewer than 400 people could fit on a 747, and I would have never guessed that a special cargo plane could carry 10^5 kg. That number seems very high.

Problem 1 Bandwidth

To keep your divide-and-conquer muscles strong, here is an exercise from lecture: Estimate the bandwidth of a 747 crossing the Atlantic filled with CDROMs.

$$10\,\boxed{} \pm \boxed{} \text{ bits/s} \quad \textbf{\textit{or}} \quad 10\,\boxed{} \cdots \boxed{} \text{ bits/s}$$

Divide and conquer! Here's a tree on which to fill values:

bandwidth
├── capacity (bits) of 747
│   ├── number of CDROM's
│   │   ├── cargo mass
│   │   └── CDROM mass
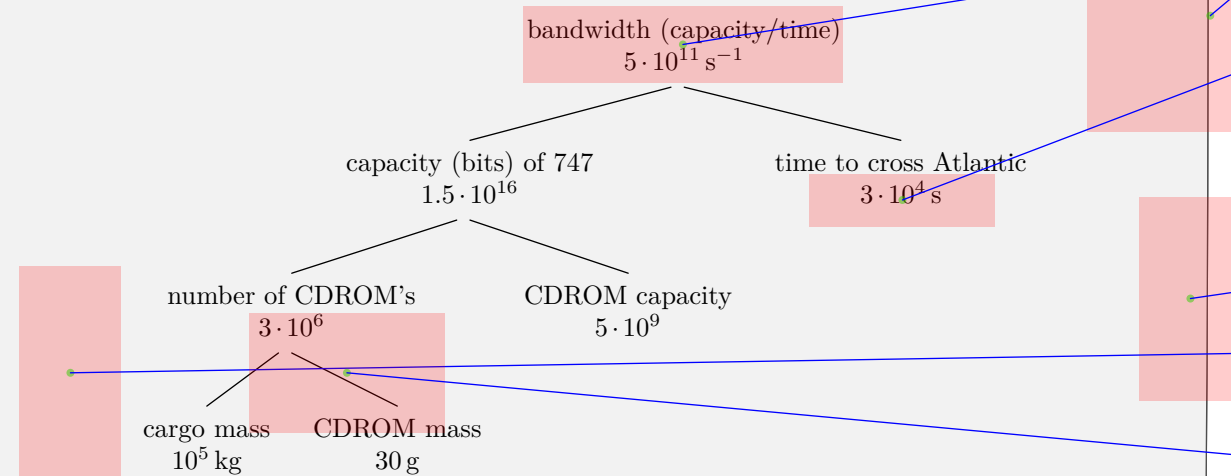│   └── CDROM capacity
└── time to cross Atlantic

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$$m \sim 400 \times 140\,\text{kg} \sim 6 \cdot 10^4\,\text{kg}.$$

A special cargo plane, with no seats or other frills for passengers, probably can carry 10^5 kg.

Here are the other estimates. A CDROM's mass is perhaps one ounce or 30 g. So the number of CDROM's is $3 \cdot 10^6$. The capacity of a CDROM is 600 MB or about $5 \cdot 10^9$ bits. The time to cross the Atlantic is about 8 hours or $3 \cdot 10^4$ s.

Now propagate the values toward the root of the tree:

bandwidth (capacity/time)
$5 \cdot 10^{11}\,\text{s}^{-1}$
├── capacity (bits) of 747
│ $1.5 \cdot 10^{16}$
│   ├── number of CDROM's
│   │ $3 \cdot 10^6$
│   │   ├── cargo mass
│   │   │ 10^5 kg
│   │   └── CDROM mass
│   │   30 g
│   └── CDROM capacity
│ $5 \cdot 10^9$
└── time to cross Atlantic
    $3 \cdot 10^4$ s

The bandwidth is 0.5 terabits per second or $10^{11.5}$ bits/second.

Despite the large bandwidth offered by a 747 carrying CDROM's (not to mention DVDROM's), trans-Atlantic Internet connections go instead via undersea fiber-optic cables. Low latency is important!

I made about the same estimate. I think this comes from "talking to your gut." Some parts in an estimation just can't be derived. I can now see how the "gut-talking" is being developed from exposure to all these problems.

> yeah, I did this too. After doing so many problems, these methods just come to you, which is awesoem
>
>> Same here - good to know we're learning :-).
>
> Yeah, I had no idea how much all the seats and luggage would weigh, so I just added an order of magnitude and it felt kind of right.
>
>> I went with volume and came out with 10^7 CDs, so off by a factor of 3, which turned into an order of magnitude in the final answer with other error in time. I feel like I did better on this problem earlier in the course. Since my standards have gotten more stringent, an order of magnitude seems like too much to be off in this case.

I thought we estimated this to be about 700MB in class. Though I understand how that difference is insignificant within our answer.

I had to lookup to remember that 1 byte = 8 bits...I guess us MIT students should remember this before we graduate

I remember in class having a debate about mass or volume... and I went with volume. I got a slightly different answer here but not hugely different in the end

> I went with mass and got a little different answer as well. My passengers and weights were different.
>
> i also went with volume. i said: a plane is like 6mx3mx50m ish and A cd is 1/9m"x1/9m"x1/100m". So # cds   1e7. not sure if my assumptions were right but it got me a similar #

I remembered the approach from the beginning, but had forgotten a lot of the numbers.

I didn't realize this was capacity/time...

I definitely did not estimate this time right. What distance did you consider for this time? It would vary by country of destination / origin.

> Since the 747 was traveling across the Atlantic, I just used the time from Boston to London, but the time didn't vary too much if I went from DC to London.
>
>> I feel like a general "time to cross Atlantic" or "max range/flight time of a 747" number would be useful, as we seem to come across it often.

I liked re-doing this problem to make sure that I still remembered how to do divide and conquer.

I forgot to account for there being more than one CD on board...but I also multiplied speed times distance to find the time instead of dividing, so my answer was closer than it could have been.

I was looking at the example done in class were we used distance travelled by 747 instead of time and arrived at the same solution. I am not sure if I am clear on the differences between the method. My notes weren't as coherent as I would have liked. Can you post that solution

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Divide and conquer! Here's a tree on which to fill values:

```
                    bandwidth
              /                  \
    capacity (bits) of 747      time to cross Atlantic
       /            \
  number of      CDROM capacity
  CDROM's
   /      \
cargo mass  CDROM mass
```

First I estimate the cargo mass. A 747 can easily carry about 400 people, each person having a mass (with luggage) of, say 140 kg. The total mass is

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Now propagate the values toward the root of the tree:

```
              bandwidth (capacity/time)
                   5 · 10^11 s^-1
              /                    \
    capacity (bits) of 747      time to cross Atlantic
        1.5 · 10^16                 3 · 10^4 s
       /            \
  number of      CDROM capacity
  CDROM's            5 · 10^9
   3 · 10^6
   /      \
cargo mass  CDROM mass
 10^5 kg      30 g
```

The bandwidth is 0.5 terabits per second or $10^{11.5}$ bits/second.

Despite the large bandwidth offered by a 747 carrying CDROM's (not to mention DVDROM's), trans-Atlantic Internet connections go instead via undersea fiber-optic cables. Low latency is important!

It looks like the conversion of 30g to 3*10^-2 kg was never made... if the conversion had been made, the number of CDroms should be 3*10^3, not 10^6.

I like the continuity from earlier in class, it was easy to double check my work by looking back at what we did, especially the "gut feeling" number estimates!

How do you estimate the error? I had the same process but got a different answer by a few orders of magnitude.

What would have been an acceptable range for this answer. I got 10^12.5 which I was happy with. Your thoughts?

What was your range? did you say 10^12.5 +/- 1 ? this answer would fall within your range at that point.

However, I agree, the solutions should also have the +/- errors for these estimations.

This just occurred to me as well. The part of the problem sets I often have the least idea about actually comes down to estimating a reasonable uncertainty in my answer. It would be instructive to see some estimate of the uncertainty in the solutions.

Hm, so after I did this problem I went and looked at the notes I took in class from when we did this problem. My result was closer to 10^14, but when we did it in class we got 10^12.5

I got an answer a lot bigger than this. In retrospect I see that you would divide the amount of data by the total time for the trip. I originally thought of it as how much data you could receive in one second if it were a continuous line of planes. This way you get about 5 planes worth of data in one second.

I often find these comments at the end more fascinating than the actual answer.

I had no idea that there are undersea fiber-optic cables

surely this is more because of cost than low latency? it seems like it would rather expensive to be burning disks and shooting planes back and forth.

while it would be relatively expensive to keep doing that, the cables to setup undersea connection were not cheap when built either. His point is time critical information would take much longer for it to be received, especially initially. It would be interesting to compare the throughput of the cables and the plane.

ooh. I thought it was a bit of a joke. I couldn't imagine anyone actually transmitting information in a plane when you could just use the internet!

I loved your analogy in lecture to this..."Imagine clicking a link on a website...and then a day later, receiving ALL the information, pictures, and videos on the site!"

Problem 2   Gravity versus radius

Assume that planets are uniform spheres. How does g, the gravitational acceleration at the surface, depend on the planet's radius R? In other words, what is the exponent n in

$$g \propto R^n?$$

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{} \tag{1}$$

The gravitational force (the weight) on an object of mass m is GMm/R^2, where G is Newton's constant, and M is the moon's mass. Thus the gravitational acceleration g is GM/R^2. But the mass M is proportional to R^3, so $g \propto R^1$. In other words, $n = 1$.

Problem 3   Gravity on the moon

The radius of the moon is one-fourth the radius of the earth. Use the result of **Problem 2** to predict the ratio g_{moon}/g_{earth}. In reality, g_{moon}/g_{earth} is roughly one-sixth. How might you explain any discrepancy between the predicted and actual ratio?

The ratio g_{moon}/g_{earth} should be proportional to the ratio of radii R_{moon}/R_{earth}, namely 1/4. The actual ratio is lower because of an effect neglected in the analysis of **Problem 2**: the differing density. When that effect is included, then the mass M is ρR^3 (except for a constant), so

$$g \sim \frac{G\rho R^3}{R^2} \propto \rho R. \tag{2}$$

If ρ_{moon}/ρ_{earth} is 2/3, that reduction in concert with the radii ratio would explain the factor of 6 difference in g.

Moon rock, which is less dense than the average earth rock, is comparable in density to rock in the earth's crust. This equivalence suggests that the moon was once a piece of the earth's crust that got scooped out probably by a large meteor impact.

I was unsure in this problem if we looked specifically at the radius 1/R^2 and ignored the mass's dependence on the radius or not.

> you just have to think about what is constant when you're looking only at proportionality. here, we're looking at the effect of different planet sizes on their gravity. so mass of the planet is changing. so you have to take that into account.

>> I thought the in-class discussion made it nice and intuitive. Thanks!

This is a good simple problem that really illustrates the point of proportionality very well.

While I think this problem is useful in science, it is not challenging for MIT students who have had this drilled into their head since high school.

The in-class explanation of this was incredibly helpful- Thanks!

> Yeah, I was totally lost on this question until we went over it in class.

This sort of problem seems weird to have a +/- for, when we determine the answer exactly (or solve for the answer, basically).

> I agree. Though I suppose if you were really stuck and thought your answer was way off you could add an arbitrary +/- to indicate you were unsure.

>> It's good to still be able to express your confidence. If +/- is zero, great, but if as above, people were wrong, then they were overly confident.

I can't believe I didn't think of this formula. I've used it a thousand times! I feel quite silly.

I definitely did not remember this formula, and thus did not get an answer for this question or the next....

I completely forgot about this equation. I think the problem for me with the proportional questions is that it requires some prior knowledge of some base equation beforehand.

> I approached this problem in a mathematical way, so I took the log of both sides to solve for the exponent, using Earth's radius and g. It didn't work too well–I am embarrassed that I forgot middle school physics.

>> Now I know to consult and index of equations before losing my mind trying to do some weird math manipulation.

> Yeah, I never know any equations to use.. It's really frustrating, I usually have no idea where to start.

>> I usually just fix this by looking at the equation sheet we got with our desert-island test

>>> I always feel like all the course 2 people in the class must know these equations by heart and I'm the only one who has to look them up... guess I'm not all alone though.

I thought this was pretty tricky. At first I completely forgot to consider how the mass was proportional to R. However, the answer seemed too trivial, and I finally figured it out. Having figured out this trick for this question, was helpful in finding the relationship between L and M in question 8.

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$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \dots \boxed{}$$

The gravitational force (the weight) on an object of mass m is GMm/R^2, where G is Newton's constant, and M is the moon's mass. Thus the gravitational acceleration g is GM/R^2. But the mass M is proportional to R^3, so $g \propto R^1$. In other words, $n = 1$.

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oops I forgot to take that into account

I did the same thing, thinking about it now I realize why it affects the formula.

Oh I did this too...this makes me sad.

It might be useful to include these formulas on the back of the envelope sheet. Just in case people forget the volume of a sphere, the gravitational force of an object in circular motion, etc.

Forgot. That's one of those things that I just need to REMEMBER. Figure everything into the equation!

Tricky little question.

i forgot it too, just remembering that gravitation force was based on r^2. however, the idea is to not look at equations and be able to do these things on the fly.

Chalk me up as one more that answered -2 because of forgetting about the M. I should have trusted my intuition that larger objects should have larger gravitational acceleration.

I made this mistake as well. The way the question was worded made me think that we could take the mass of the planet as a constant of the problem.

The reading this week really cemented the fact that M is proportional to l^3 so maybe we should do that reading before this question.

I think that this point needs to be stressed more. We should break up each variable in class more often to include all the things it depends on.

The idea is that for many planets (e.g. Mercury, Venus, Earth, Mars), they are similar in density but their radius varies a lot, and that variation accounts for most of the variation in gravitational field strength. Problem 3 is then a follow-up question on that idea. I do this kind of reasoning so often now that I forget how subtle it is.

I'll think how to reword the question to clarify that density is constant across the planets (but without giving away the whole idea in the rewording).

I definitely did not take this into account either. I sometimes have a hard time with proportional reasoning and isolation the one variable of interest and disecting equations to get the information we are looking for

I screwed up here too. This was tricky.

I noticed that when we have an equation I often plow head first trying to use it to solve the problem and forget about using proportions. I think adding a comment in the reading about when it is ok to use proportions and when it would be cumbersome. I myself am not completely sure when it is ok to use them (i am afraid of using them out of context).

I also forgot that equation and ended up screwing up a little. This was a lot easier than I made it out to be...

I feel as though my discrepancy can be explained by my getting it wrong, but this seems reasonable, too.

This problem was unusual since I didn't feel like it used any of the approximation methods that we learned

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> If ρ_{moon}/ρ_{earth} is $2/3$, that reduction in concert with the radii ratio would explain the factor of 6 difference in g.
>
> Moon rock, which is less dense than the average earth rock, is comparable in density to rock in the earth's crust. This equivalence suggests that the moon was once a piece of the earth's crust that got scooped out probably by a large meteor impact.

I'm pretty impressed that even though I had the wrong answer in 2, I explained the correct reason for why it was wrong in problem 3.

> I also did.

>> I found this somewhat intuitive though, as there aren't too many other things that could be the reason.

>>> also the fact that #2 asked you to assume that planets are uniform spheres kind of hinted to it for me – "uniform" kind of makes you not take density into account

>>>> I agree with the uniform comment- it made me feel like I didn't need to account for anything but G, M, or R. I guess that was the point though, otherwise I'm sure some people would have accounted for density and then this problem would have been trivial.

>>> I also agree. Not much else came to mind that could effect the gravity that we didn't account for besides the density.

> If you had the wrong answer in 2, wouldn't you have a crazy answer for 3, like the gravity of the moon should be 16x greater than on Earth? Even if you could guess that density was the reason for the discrepancy, would you think density would be 100x different? This is when reality checks are helpful.

I may have been off by an order of magnitude on problem 2, but I totally got this one! yippie

i guessed density initially and then postulated that G might have some sort of assumptions that is unique to earth. is this wrong? i have no idea what G actually entails, is it entirely independent of the body of mass with which it is being calculated on?

> G is a universal constant or fudge factor for the gravity equation. It applies to all objects and not just earth.

This is definitely a subtle detail that is way beyond the estimate, but I'm just curious.... wouldn't the density increase with R^3 just because gravity would make things more compact. I guess my question is do bigger things made of the same material have a higher density?

isn't the density of the rock on the moon the same? I thought that the moon is from the earth so it would be really close

isn't this also because there is less mass, which causes less pressure thus less dense rocks being formed. it seems that smaller planets/satellites should be less dense.

how big is the moon relative to the earth? can that mass really be scooped out?

> The moon is about 30% of the size of the earth, with a gravity that is about 1/8th.

> Seems like a rather large piece. But I suppose when comparing mass, the moon is 1% of the Earth. So I suppose the theory is plausible...

> it's big enough to affect the tides. Remember reading it somewhere. Some say it happened when the supposed asteroid that killed the dinosaurs hit the earth.

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Interesting fact!

Agreed - I was very curious as to why there was such a large discrepancy.

the reason why the total density of the moon differs is that the core is actually much smaller. It does have a core, but it's only 1-3% of the total mass, while Earth's core is like 33% of the total mass – meaning that the density is more like a silicate rock rather than iron (which is what the core is made of). Interestingly enough, dynamical models for the moon's origin show that the moon is more likely made of the impactor rather than from part of the Earth – which is quite problematic! (aka...the moon probably did not come from the core, even though it formed from an impact)

why is what you said problematic? also, it wouldn't have come from the core no matter what. it would have come from the crust. either way, it does make sense that because it is much smaller, it would have a smaller core, which is the massive part of the planet.

ah I should have been more clear, it's just a generally problematic because the isotopes of the Earth match those of the moon – which was originally explained by the Moon being made from the Earth. It just means that things just aren't like we though :) And the impact would have melted the entire Earth, meaning "scouping" doesn't really work (i think this is the right Canup model: http://www.flickr.com/photos/thane/3134316459/)

Hmm, I actually thought moon rock would be more dense due to its chemical composition. On the other hand, I guess due to graivty, atmosphere, etc it is more like a light powder?

I also thought that it might be the opposite based on my results from the first calculation which were off. This makes much more sense. The origin of the moon comment is interesting, does that mean that is is a piece of something that collided with the earth? Does that mean it bounced off somehow and was held in orbit?

does that mean the layers of the earth and their compositions haven't changed in the millions of years since the moon was formed?

I found this interesting as well.

Problem 4   Minimum power

In the readings we estimated the flight speed that minimizes energy consumption. Call that speed v_E. We could also have estimated v_P, the speed that minimizes power consumption. What is the ratio v_P/v_E?

$$\boxed{} \pm \boxed{} \quad \textbf{\textit{or}} \quad \boxed{} \dots \boxed{}$$

The zillions of constants (such as ρ) clutter the analysis without changing the result. So I'll simplify the problem by using a system of units where all the constants are 1. Then the energy is

$$E \sim v^2 + \frac{1}{v^2},$$

where the first term is from drag and the second term is from lift. The power is energy per time, and time is inversely proportional to v, so $P \propto Ev$ and

$$P \sim v^3 + \frac{1}{v}.$$

The first term is the steep v^3 dependence of drag power on velocity (which we used to estimate the world-record cycling and swimming speeds). The energy expression is unchanged when $v \to 1/v$, so it has a minimum at $v_E = 1$.

To minimize the power, use calculus (ask me if you are curious about calculus-free ways to minimize it):

$$\frac{dP}{dv} \sim 3v^2 - \frac{1}{v^2} = 0,$$

therefore $v_P = 3^{-1/4}$ (roughly 3/4), which is also the ratio v_P/v_E.

So the minimum-power speed is about 25% less than the minimum-energy speed. That result makes sense. Drag power grows very fast as v increases – much faster than lift power decreases – so it's worth reducing the speed a little to reduce the drag a lot.

If you don't believe the simplification that I used of setting all constants to 1 – and it is not immediately obvious that it should work – then try using this general form:

$$E \sim Av^2 + \frac{B}{v^2},$$

where A and B are constants. You'll find that v_E and v_P each contain the same function of A and B and that this function disappears from the ratio v_P/v_E.

I found this problem to be very confusing and difficult. Not that the work was difficult, but figuring out what I was supposed to do was very difficult. If you use this problem again in the future, I would suggest giving a few more hints for this, because I definitely wasted a few hours on this problem.

yeah i agree... i had no idea what the question was really asking for.

isn't the point of this class to be able to figure out how to solve a problem? (and if you're really stuck, asking a question on the homework page itself?) it basically consisted of doing a derivation almost identical to the one that was in the reading...

I'm not saying it wasn't a hard problem, but it certainly wasn't unreasonably hard. It's good to have to think about what tools to use. I think problems like this are the more interesting ones.

But this problem wasn't about trying to figure out the "trick" and it wasn't like we had to figure out whether to use divide and conquer, etc. The only point to this problem was, let's see if you can sift through pages of notes to find an obscure formula, with no mention of what formula we should be using. That's not useful or helping me learn any techniques, it's just a waste of my time.

But with this class you should be able to derive that formula on your own – that's the point. Either remembering or rederiving (which would have been faster than a few hours) the lift and drag proportionalities that have been some of the main points of this unit (F.D v^2 and F.L 1/v^2) and then combining them to get E v^2+1/v^2 isn't at all out of the realm of questions to ask MIT students.

I like that we aren't being handed answers in this class as much as in most classes. Knowing how to plug-and-chug isn't engineering.

it took me a while to find exactly in which reading we covered this...it would have been useful to reference it back

Why would you want to minimize power instead of energy?

We want to minimize both, and look at how they compare.

If you have plenty of energy, but can't deliver it fast enough. Maybe this is a poor example, but I can walk a lot farther than I can run.

Why couldn't we just find the speed, why do we need to bother making a ratio? This ratio must have some significance that you will cleverly explain later.

I did this problem completely different- I just looked at the readings where it states that power is proportional to l^-1 and energy also is proportional to l^-1 so I just assumed that since P/E=1, that v_p/v_e also would equal 1. Probably too much of a simplification but it came out with the same answer..

I actuallyy had no idea on how to do it

I did not get rid of the constants and I think that hindered my ability to solve this problem

I really like this method of analysis, it seems like in class you do this in your head a lot - look at only the variables that will change and examine their relation. It is something that i never thought of before this class and is a great trick to seeing trends when a problem looks too intimidating.

can we go over this in class. i still don't follow

Problem 4   Minimum power

In the readings we estimated the flight speed that minimizes energy consumption. Call that speed v_E. We could also have estimated v_P, the speed that minimizes power consumption. What is the ratio v_P/v_E?

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$

The zillions of constants (such as ρ) clutter the analysis without changing the result. So I'll simplify the problem by using a system of units where all the constants are 1. Then the energy is

$$E \sim v^2 + \frac{1}{v^2},$$

where the first term is from drag and the second term is from lift. The power is energy per time, and time is inversely proportional to v, so $P \propto Ev$ and

$$P \sim v^3 + \frac{1}{v}.$$

The first term is the steep v^3 dependence of drag power on velocity (which we used to estimate the world-record cycling and swimming speeds). The energy expression is unchanged when $v \to 1/v$, so it has a minimum at $v_E = 1$.

To minimize the power, use calculus (ask me if you are curious about calculus-free ways to minimize it):

$$\frac{dP}{dv} \sim 3v^2 - \frac{1}{v^2} = 0,$$

therefore $v_P = 3^{-1/4}$ (roughly 3/4), which is also the ratio v_P/v_E.

So the minimum-power speed is about 25% less than the minimum-energy speed. That result makes sense. Drag power grows very fast as v increases – much faster than lift power decreases – so it's worth reducing the speed a little to reduce the drag a lot.

If you don't believe the simplification that I used of setting all constants to 1 – and it is not immediately obvious that it should work – then try using this general form:

$$E \sim Av^2 + \frac{B}{v^2},$$

where A and B are constants. You'll find that v_E and v_P each contain the same function of A and B and that this function disappears from the ratio v_P/v_E.

Didn't we determine that this is simplification removed many different important aspects of the problem, and could be off significantly, If so, would this make further analysis even more likely to be off?

why Ev, how did you get that

Power = Energy/time = Energy*(velocity/distance). (because distance/velocity=time, so you flip this to get the reciprocal).

what isn't P proportional to Force * velocity? according to the explanation above, power = Energy * velocity / distance... where'd the distance go?

I'd like to see how you'd arrive at this a bit more rigorously. I'll grant you that P is proportional to E*v (as this allows you to ignore distance as a constant in this situation), but I don't see how, given this ignorance, you can justify jumping to a more rigid assertion that P v**3 + 1/v? This seems very hand wavy to me.

If you look at the Energy equation that was used to minimize energy for flight (it had a drag and a lift term) and divide it by time, (power is energy over time) then you will get an extra velocity term (s/t=v). These V's multiply/cancel with the other V's and you get the (1/v)+v^3.

This was the catch I couldn't figure out. Makes sense now

me too! I didn't think about this...i just took the derivative of the Ve, but ended up getting the same answer...which was totally wrong. with this simplification, it totally works out!

Yay! I actually got one. Took me a while of just messing with units on a sheet of paper

yeah i missed the setup. i wasn't using paper and pencil though, and would have felt better writing the equations out rather than just in the submit boxes.

I was using a sheet of paper and still botched it up. arg

Why can't the two parts simply be set equal as they were to minimize the energy?

I think the point of the problem is that the minimum velocity for energy consumption is not the minimum energy for power consumption. So we use find the proportions of energy consumption and power consumption in terms of velocity, respectively, and take the derivatives using calculus to find the minimum velocities, respectively. Setting them equal as you mentioned would just give us the velocity at which energy consumption and power consumption is the same, but that is not what the question asks for.

I was thinking about doing it that way also. Hmm

It's very helpful when you explain the answers along with describing where their derivations are in the notes.

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This explanation is great. It's very easy to follow. I like that you discussed what each component of the equation is.

> I agree. In many explanations, I am confused because I don't know where some variables come from.
>
>> Instead of taking the derivative I set it equal to the Power used to keep the plane afloat. Why is that wrong - it's the same thing we did for energy.
>>
>> This is probably one of the clearest explanations so far and I'm glad it goes with one of the problems that confused me the most.
>
> Yes, it was simpler than I expected it to be.
>
> This was definitely a very involved problem, and it was explained really well. Before this problem I didn't realize there wwere two different velocities that minimized energy vs power consumption

I don't quite understand how you got this V_e=1. everything else was cool though.

Please explain, I definitely did this part incorrectly. I thought one of the points of the pset was to steer away from needing to us things like derivations?

> I think it's harmless to differentiate a simple and nice function like what we have here. If it were a harder equation, then we might want to use symmetry, etc. Estimation to me is all about getting the best answer in the least amount of time, and just doing the derivative seems to be a good way of going about that.
>
>> I would be interested in seeing the non-calculus way. I got to this point and spent a while looking for the symmetry for power (since we had already seen the one for energy), but gave up after a while and used calculus as done here.

Ahh... This makes so much sense! I was completely lost, and because we've been ignoring a lot of more standard math procedures, I didn't even think of differentiating.

At first I didn't understand why this velocity also equaled the ratio of velocities, might be worth mentioning again that vE = 1 even though it's also listed above.

Hm, I did this wrong, but I'm not sure why because I used the same process, I thought? Maybe I miscalculated something, or related Power to Energy incorrectly. It's helpful to see these solutions though so I can walk through step by step.

I got this far into the problem, except for figuring out that Ve=1. Without that I couldn't get a good answer...

i think that most people realize this. i'm not sure it's worth mentioning here.

> I disagree, I thought it was nice to see that A and B could be left in there, since people are often worried about dropping away constants.

I remembered your graph from class where the two plots intersected at a given point, but didn't remember how to apply that correctly.

Problem 5   Highway vs city driving

Here is a measure of the importance of drag for a car moving at speed v for a distance d:

$$\frac{E_{drag}}{E_{kinetic}} \sim \frac{\rho v^2 A d}{m_{car} v^2}$$

This ratio is equivalent to the ratio

$$\frac{\text{mass of the air displaced}}{\text{mass of the car}}$$

and to the ratio

$$\frac{\rho_{air}}{\rho_{car}} \times \frac{d}{l_{car}},$$

where ρ_{car} is the density of the car (its mass divided by its volume) and l_{car} is the length of the car.

Make estimates for a typical car and find the distance d at which the ratio becomes significant (say, roughly 1).

$$10^{\boxed{} \pm \boxed{}}\,\text{m} \quad \textit{or} \quad 10^{\boxed{} \cdots \boxed{}}\,\text{m}$$

To include in the explanation box: How does the distance compare with the distance between exits on the highway and between stop signs or stoplights on city streets? What therefore are the main mechanisms of energy loss in city and in highway driving?

A typical car has mass $m_{car} \sim 10^3$ kg, cross-sectional area $A \sim 2\,\text{m} \times 1.5\,\text{m} = 3\,\text{m}^2$, and length $l_{car} \sim 4\,\text{m}$. So

$$\rho_{car} \sim \frac{m_{car}}{A l_{car}} \sim \frac{10^3\,\text{kg}}{3\,\text{m}^2 \times 4\,\text{m}} \sim 10^2\,\text{kg}\,\text{m}^{-3}.$$

Since $\rho_{car}/\rho_{air} \sim 100$, the ratio

$$\frac{\rho_{air}}{\rho_{car}}\,\frac{d}{l_{car}}$$

becomes 1 when $d/l_{car} \sim 100$, so $d \sim 400\,\text{m}$.

This distance d is significantly farther than the distance between stop signs or stoplights on city streets. In Manhattan, for example, 20 east–west blocks are one mile, giving a spacing of approximately 80 m. So air resistance is not a significant loss in city driving. Instead the loss comes from engine friction, rolling resistance, and (mostly) braking.

However, the distance d is comparable to the exit spacing on urban highways. So when you drive on the highway for even a few exit distances, air resistance is a significant loss.

Interestingly, highway fuel efficiencies are higher than city fuel efficiencies, even though drag gets worse at the higher, highway speeds, and presumably engine friction and rolling resistance also get worse at higher speeds. Only one loss mechanism, braking, is less prevalent in highway than in city driving. Therefore, braking must be a significant loss in city driving. Regenerative braking, used in some hybrid or electric cars, would therefore significantly improve fuel efficiency in city driving.

the title confused me because i couldn't figure out what it had to do with our hardcore calculations. it makes sense now but it took me way too long to figure it out.

Yeah, I was a little confused too, especially because all we were actually calculation was a distance and then answering the highway vs/ city driving qualitatively.

This was a lot less frustrating than some of the other problems because I didn't have to search for miles to figure out what equations I should use. Once I had the tools it was fun to do this problem!

I thought this question was asking for a qualitative answer, not a mathematical one. I'll try to apply equations to more of my explanations.

i don't think he's expecting the same level of detail in our answers. he just wants really detailed answers for us to read and learn from.

How are we able to assume the answer to this?

I thought it just kind of made sense, the problem led to something to do with the distance at which drag becomes significant. The difference in distances on the highway vs city is the distances between stops- exits on the highway and stop signs on the highway. I liked this application.

I used the 3m * 3m we used in class. Why change the estimation this time?

i'm pretty sure he used the same #s last time. i recall something about lying down in a car = 2m and when you stand it's almost your height but not really

I'd used 2m (wide) x 1.5 m (high) = 3 m^2 as the area. It is easy to confuse 3 square meters (i.e. 3 m^2) with 3 meters squared (i.e. (3 m)^2).

Mine was much higher than this, as I underestimated the volume.

I find it very useful to double check all of my estimations when working in metric. For me metric still doesn't come intuitively.

I agree with this - my answer was off by a lot because I messed up the car volume estimation.

How did you get this number? This is where my estimate was really off... and my final answer ended up off by about a factor of 10

He did the calculation in the above line. he shows how he got rho car = 100 kg/m^3. Air has density of   1 kg/m^3. their ratio is 100.

This is confusing to me because you say the car/air ratio is 100 but the formula asks for the air/car ratio...

That's true - I had to do a double-take, might be helpful to flip one of the ratios.

I think the reason he wrote it that way is because he asked us to set the ratio equal to one, and when you solve for d, you get the equation d/l = car/air.

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I think my answer was around there? I went about it similarly, but didn't use your density ratio. Retrospectively, I don't see why I didn't.

I really liked this problem, it was fairly straight forward and an interesting result that I think we can all relate to. I know that when I was driving to Rhode Island this past weekend every time I had to stop I thought about how much energy I was losing.

This problem was interesting to see how drag affects the car when stopping, a common occurrence (when you're not a student in the city)

i was just in nyc. i hate that place and the traffic is ridiculous. i don't know how anyone drives there at all.

I just grouped this all into kinetic energy. Is that okay?

as we saw in class as well

didn't realize there were so many components that went into stop-go driving. my answer only took into account the obvious braking component, though looking back, I guess the other components are pretty obvious too...

I thought this was really amazing!

This is kind of interesting, because as someone who grew up in the country I have a completely different sense of scale. For me an average distance between highway exits on average 5-7 miles... and much much larger in sparsely populated regions. I don't have a good sense of scale for a city block really.

What is the significance of this?

how does this create more air resistance again?

it's the point of the problem–at distances of about 400m, the air resistance becomes important, so on highways, this is a big factor. (not so much in cities). remember, one way to look at air resistance is the mass of air displaced.

So the idea is when we drive longer, we end up displacing more air, right?

Is it also related to the fact that car engines are more efficient at the rpms/gear ratios that are typically used when driving at 55 mph

yeah, I'm assuming this plays a role...when doing this problem I psyched myself out because I did say that on highways there is much more drag force, but I also knew that highway mpg is typically more than city...

I wish they had mpg rates for different speeds, and highway vs city driving

I was really confused at first with this – I didn't really pay attention to what the results were giving me, and I assumed that drag was more important for shorter distances (Because I remembered that city mpg was much worse than highway mpg), but I quickly realized I was wrong. It's amazing to realize how inefficient braking really is!

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What is this?

Quoting wikipedia: "A regenerative brake is an energy recovery mechanism that reduces vehicle speed by converting some of its kinetic energy and/or potential energy (due to elevation) into a useful form of energy instead of dissipating it as heat as with a conventional brake. The converted kinetic energy is stored for future use or fed back into a power system for use by other vehicles."

An MIT lab recently produced a prototype for a bike which uses this type of braking system to store energy in order to help propel the bike forward for hills or just to give the rider a break.

So electric cars would not help much if at all in highway driving? Hrm.

that's not entirely true...the fuel efficiency for highway driving can be greatly improved in an area with hills/mountains. The downside of it is that the cruse control on hybrids _sucks_...it doesn't take slopes into account at all.

Hybrids can still take the electricity stored up from braking and from using the gas engine and use just the electric motor to power the car at constant highway speeds. It's rapid acceleration that tends to require the gas engine.

no, most of them can't (from my experience). if on a flat road, it will use both to maintain a constant speed...rapid acceleration also uses both.

going down a gentle grade on the freeway will frequently run on just the electric motor, but otherwise it still requires gas.

*Point of Interest: Our Hybrid SUV [not a contradiction, we need the 4x4] has more get up & go than most V8 engines.

Problem 6   Mountains

Here are the heights of the tallest mountains on Mars and Earth.

Mars   27 km   (Mount Olympus)
Earth   9 km   (Mount Everest)

Predict the height of the tallest mountain on Venus.

$$10\boxed{} \pm \boxed{} \text{ km} \quad \textit{or} \quad 10\boxed{} \cdots \boxed{} \text{ km}$$

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

> One pattern is that the larger planet (earth) has the smaller mountain. Large planets presumably have stronger gravitational fields at their surface, which keeps the mountains closer to the ground. The derivation in lecture on mountain heights dropped the dependence on g because we looked only at mountains on earth where all mountains share the value of g.
>
> The same derivation can be repeated but retaining g. The weight of a mountain of size l is $W \propto g l^3$, so the pressure at the base is $p \propto g l^3 / l^2 \sim g l$. When the pressure p exceeds the maximum pressure that rock can support, the mountain can no longer grow upward. This criterion is equivalent to holding $g l$ constant. Therefore,
>
> $$l \propto g^{-1}.$$
>
> Here are the gravitational field strengths on the three planets:
>
> a. Mars: $3.7 \, \mathrm{m\,s^{-2}}$
> b. earth: $10 \, \mathrm{m\,s^{-2}}$
> c. Venus: $8.9 \, \mathrm{m\,s^{-2}}$
>
> The product $g l$ for each planet should be the same. That hypothesis works for Mars and earth:
>
> a. Mars: $10^5 \, \mathrm{m^2\,s^{-2}}$
> b. earth: $0.9 \cdot 10^5 \, \mathrm{m^2\,s^{-2}}$
>
> If Venus follows the predicted scaling, then $g l$ should be roughly $10^5 \, \mathrm{m^2\,s^{-2}}$ with $g \sim 8.9 \, \mathrm{m\,s^{-2}}$. Therefore l should be roughly $11 \, \mathrm{km}$. Indeed, the tallest mountain on Venus, which is Maxwell Montes, has just that height. Scaling triumphs!
>
> Here is a fun question: Why aren't mountains on the moon $60 \, \mathrm{km}$ tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in **Problem 3**)?

What about Mauna kea? That is around 40,000 ft high, right (if you count the distance from the base in the sea)?

> actually, it's 9.100 km above the ocean floor...but the baseline that people use is sea level because that's where the center of gravity for a mountain tends to be (yes, they go under ground about as far as they are above "ground")

> ...yeah but the forces that keep mountains smaller on earth discussed here occur from mass differences above sea level.

> > Wouldn't the water weight make it even harder on the mountains with undersea parts?

I don't like the fact that weather, is not taken into account here, but what I really don't like is the fact that the definition of mountain height used in the answers is the height from the ocean, while those mountains can sit upon giant plateaus that can change the height by a factor of 2 or more. I can see the value in the analysis, but I think that when comparing the actual answers with the analysis the fact that they are so close is simply luck and not a factor of the analysis working.

I got the proportionality right, but I didn't think about the cause of it. That's pretty interesting reasoning.

> What else did you think was holding the mountains down? Seems pretty obvious that gravity plays a big role...

> > I don't pick up on these things either..

> > > But this is only true assuming that the planet densities are all equal.

I completely ignored g... damn

When I did this problem I implicitly used gravitational fields or do the math, but I just reasoned that since the matter on Mars is less dense than that on earth, the mountains on Mars wouldn't be under as much pressure so they could get taller.

is it better to blindly estimate things or to look up a few background things, especially for the psets

I thought about it in terms of gravity and still managed to get it wrong. I feel like I made a lot of silly mistakes on this pset.

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The product gl for each planet should be the same. That hypothesis works for Mars and earth:

a. Mars: $10^5\,\mathrm{m^2\,s^{-2}}$

b. earth: $0.9 \cdot 10^5\,\mathrm{m^2\,s^{-2}}$

If Venus follows the predicted scaling, then gl should be roughly $10^5\,\mathrm{m^2\,s^{-2}}$ with $g \sim 8.9\,\mathrm{m\,s^{-2}}$. Therefore l should be roughly 11 km. Indeed, the tallest mountain on Venus, which is Maxwell Montes, has just that height. Scaling triumphs!

Here is a fun question: Why aren't mountains on the moon 60 km tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in **Problem 3**)?

Should we have looked these up? I didn't bother. I just assumed that since venus was slightly less than the size of earth, it would have a slightly smaller g (from the earlier problem) and picked .9g because this made the math easier.

I guessed, and then looked it up to make sure. It makes sense, but I guess is NOT one of the constants we are required to know without looking up.

Yeah, I had no idea, so i just assumed it was around the Earth's size and therefore has around the earth's gravity...

I assumed it was somewhere in between but the answer still wasn't off by much.

I also didn't have these numbers but based my answer more on size and location to the sun. I got a similar answer but I'm not sure I got it for a good reason.

I went with a few other people and guessed that the gravity was slightly less so that the height would be a little larger than earth.

Yeah I did the same thing... I figured Venus is closer in size to earth than to mars but is still smaller than earth and made my estimation based on that.

We did an earlier problem relating radius to gravitational force – so I just used that proportional scaling to guess at g (since I knew the ratio of the radii was somewhere less than one but greater than 3/4)

I had to look it up, and when I did I was reminded of something I vaguely recall from middle school science about how Venus is sorta similar to Earth based on gravities.

Yeah I just assumed that the gravity on Venus was similar to that of Earth's, just a little smaller. I actually got quite close to the right answer.

Yeah. I had no intuition about the relative size of planets, so I just used 1.

I think it would be good to show the math here i.e. the mountain heights

I agree. A sentence or two explaining the main computation would make these two values a bit quicker to understand.

Um, I think the more important concept is that they are very similar, so they will scale similarly. I don't know why you'd need more specific math than what's given.

so if i remember, venus and earth were always the same-ish size but there was a lot more pressure on venus than earth. why? wouldn't this affect mountain height?

I would imagine because there is little geological activity for mountains to form or that the types rocks cannot support such large mountains

Problem 6   Mountains

Here are the heights of the tallest mountains on Mars and Earth.

Mars   27 km   (Mount Olympus)
Earth   9 km   (Mount Everest)

Predict the height of the tallest mountain on Venus.

$$10^{\boxed{}} \pm \boxed{} \text{ km} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \cdots \boxed{} \text{ km}$$

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

One pattern is that the larger planet (earth) has the smaller mountain. Large planets presumably have stronger gravitational fields at their surface, which keeps the mountains closer to the ground. The derivation in lecture on mountain heights dropped the dependence on g because we looked only at mountains on earth where all mountains share the value of g.

The same derivation can be repeated but retaining g. The weight of a mountain of size l is $W \propto gl^3$, so the pressure at the base is $p \propto gl^3/l^2 \sim gl$. When the pressure p exceeds the maximum pressure that rock can support, the mountain can no longer grow upward. This criterion is equivalent to holding gl constant. Therefore,

$$l \propto g^{-1}.$$

Here are the gravitational field strengths on the three planets:

a. Mars: $3.7\,\mathrm{m\,s^{-2}}$

b. earth: $10\,\mathrm{m\,s^{-2}}$

c. Venus: $8.9\,\mathrm{m\,s^{-2}}$

The product gl for each planet should be the same. That hypothesis works for Mars and earth:

a. Mars: $10^5\,\mathrm{m^2\,s^{-2}}$

b. earth: $0.9 \cdot 10^5\,\mathrm{m^2\,s^{-2}}$

If Venus follows the predicted scaling, then gl should be roughly $10^5\,\mathrm{m^2\,s^{-2}}$ with $g \sim 8.9\,\mathrm{m\,s^{-2}}$. Therefore l should be roughly 11 km. Indeed, the tallest mountain on Venus, which is Maxwell Montes, has just that height. Scaling triumphs!

Here is a fun question: Why aren't mountains on the moon 60 km tall (the Moon's surface gravity is about one-sixth of earth's surface gravity, as analyzed in **Problem 3**)?

Because meteors hit any "mountains" that form, due to the moons lack of an atmosphere.

There is no real history on the moon to create mountains. Mountains form from tectonics....and on the young moon, well, tectonics are minimal. While the moon is quite interesting (and dynamic - a liquid thin outer core!) it is not too active, nor has had an active history (like Mars) - it doesn't undergo the same processes that make volcanoes on other planets. The moon cooled from a magma ocean and produced a rather interesting crust – decoupling it from the crazy stuff going on below the crust. Mare on the moon (topography) were produced from large impacts – otherwise, the topography is quite flat.

I'm sure spacial collisions matter, but I think the bigger culprit is the lack of a strong magnetic field or internal shifting techtonics. As such, there is a pretty stable surface and no real impetus to have formed these mountains in the primordial era.

Great response. I didn't even think about that, but it makes sense.

Problem 7   Raindrop speed

Use the drag-force results from the readings to estimate the terminal speed of a typical raindrop (diameter of about 0.5 cm).

10 ⬚ ± ⬚ m s^{-1} *or*   10 ⬚ ... ⬚ m s^{-1}

To include in the explanation box: How could you check this result?

The weight of the raindrop is the density times the volume times g:

$$W \sim \rho r^3 g,$$

where I neglect dimensionless factors such as $4\pi/3$.

At terminal velocity, the weight equals the drag. The drag is

$$F \sim \rho_{air} v^2 A \sim \rho_{air} v^2 r^2.$$

Equating the weight to the drag gives an equation for v and r:

$$\rho_{air} v^2 r^2 \sim \rho r^3 g,$$

so $v \propto r^{1/2}$.

Bigger raindrops fall faster but – because of the square root – not much faster.

With the g and the densities, the terminal velocity is

$$v \sim \sqrt{\frac{\rho}{\rho_{air}} g r}.$$

A typical raindrop has a diameter of maybe 5 or 6 mm, so $r \sim 3$ mm. Since the density ratio between water and air is roughly 1000,

$$v \sim \sqrt{1000 \times 10 \, m \, s^{-2} \times 3 \cdot 10^{-3} \, m} \sim 5 \, m \, s^{-1}.$$

First convert the speed into a more familiar value: 11 mph (miles per hour). If one drives at a speed v_{car}, then raindrops appear to move at an angle $\arctan(v_{car}/v)$. When $v_{car} = v$, the drops come at a 45° angle. So one way to measure the terminal speed is to drive in a rainstorm, slowly accelerating while the passenger (not the driver!) says when the drops hit at a 45° angle.

You could also run in a rainstorm and note the speed at which a small umbrella has to held at 45° to keep you perfectly dry.

i just put a weight but I gues a volume times density makes more sense

I left this factor in my calculation, and got about 14 for my answer

I kept it as well, i thought it conveniently came out to a few.

How do you decide when a dimensionless factor should be neglected?

we've been using a cubed diameter to approximate the volume of a sphere quite frequently.

Are you allowed to do that, without re-incorporating it again at a later point in the problem?

...oh

Hardest part of this question was estimating the raindrop dimensions.

I hoped to minimize that problem by putting the dimensions in the problem statement ("diameter of about 0.5 cm")!

I liked this problem but I don't see which approximation method we are using here.

i looked this up, and for a 5 or 6mm raindrop, it's actually about 10 m/s. I think the discrepancy is from neglecting the 4pi/3.

maybe the raindrop is not a perfect sphere. Maybe it forms into a more aerodynamic shape

like a droplet shape? but if we can estimate a sphere to a cube we can surely make the smaller jump from a sphere to a droplet.

Not sure where this angle number comes from

I thought the measurement would have something to do with an umbrella but had no idea what to do with it!

Do all raindrops reach this terminal velocity? Or might some clouds form too low for this to happen?

How would you ever come up with this?

It actually varies by size, but its a cool result!

To 1:03 am:

This method works because to fall at 45 degrees, your x and y velocity should be the same. Therefore, if you know your x velocity via a car, the moment you find that a raindrop hits at a 45 degree angle, you know it's y velocity was the same as your x velocity at that moment.

I notice this on days when I'm late for class and it's raining. I find holding my umbrella vertical doesn't keep me dry, so I have to adjust it

except for raindrops forming a couple feet off the ground, you can assume that all raindrops are falling at terminal velocity.

If anyone tries this, don't do it by the green building. pick somewhere with no wind.

Problem 7   Raindrop speed

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$10\boxed{} \pm \boxed{} \,\mathrm{m\,s^{-1}}$ ***or*** $10\boxed{} \ldots \boxed{} \,\mathrm{m\,s^{-1}}$

To include in the explanation box: How could you check this result?

> Phrasing is weird. I understand what you mean but this sentence is not very clear.

The weight of the raindrop is the density times the volume times g:

$$W \sim \rho r^3 g,$$

where I neglect dimensionless factors such as $4\pi/3$.

At terminal velocity, the weight equals the drag. The drag is

$$F \sim \rho_{\mathrm{air}} v^2 A \sim \rho_{\mathrm{air}} v^2 r^2.$$

Equating the weight to the drag gives an equation for v and r:

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$$v \sim \sqrt{1000 \times 10\,\mathrm{m\,s^{-2}} \times 3 \cdot 10^{-3}\,\mathrm{m}} \sim 5\,\mathrm{m\,s^{-1}}.$$

First convert the speed into a more familiar value: 11 mph (miles per hour). If one drives at a speed v_{car}, then raindrops appear to move at an angle $\arctan(v_{\mathrm{car}}/v)$. When $v_{\mathrm{car}} = v$, the drops come at a $45°$ angle. So one way to measure the terminal speed is to drive in a rainstorm, slowly accelerating while the passenger (not the driver!) says when the drops hit at a $45°$ angle.

You could also run in a rainstorm and note the speed at which a small umbrella has to held at $45°$ to keep you perfectly dry.

Problem 8   Cruising speed versus air density

For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on air density ρ? In other words, what is the exponent β in $v \propto \rho^{\beta}$?

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \dots \boxed{}$$

From the lecture notes,

$$Mg \sim C^{1/2}\rho v^2 L^2,$$

where C is the modified drag coefficient. So

$$v \sim \left(\frac{Mg}{C^{1/2}\rho L^2}\right)^{1/2}.$$

The only dependence on ρ is the ρ itself in the denominator, leaving

$$v \propto \rho^{-1/2}$$

and $\beta = 1/2$.

The inverse relationship between the speed and density explains why planes fly at a high altitude. The energy consumption at the minimum-energy speed is proportional to the drag force, which is proportional to ρv^2. Because $v \propto \rho^{-1/2}$, the powers of ρ cancel in the energy consumption; in other words, the energy consumption (at the minimum-energy speed for that ρ) is independent of ρ. By flying high, where ρ is low, planes can fly faster without increasing their energy consumption.

I totally did not get any of these problems...they were very frustrating to me.

Yeah, these last 3 were especially hard. They made the pset way too long.

This phrase was kind of hard to understand, maybe add a clarification to it?

This problem was pretty straightforward once you found the section in the lecture notes.

based on this part, the relation seems fairly straight forward, so i was wondering if i was missing something.

I didn't realize at first that this was balancing the force and drag again.

What force does this represent?

And what are those variables?

I never know where to find the equations for these types of problems, and this p-set was full of them.

I used the relationship between lift energy and drag. It was easier to see the relationship.

I did too but I got my proportion for mass all wrong. I'm a little confused why you can just use that equation, wouldn't you want to take the derivative of the energy equation?

should I commit stuff like this to memory, it seems important and like we have used it alot

For some reason, the specifics of your approaches in lecture aren't sticking. I get the general idea, but your specific examples don't always form a cohesive picture. Perhaps this is my own mental limitation, but I thought I'd mention it.

this really clears it up- thanks

Should beta=-1/2? Otherwise I do not see how these two lines follow

probably a typo...

I think it's a typo too - I definitely got -1/2 for my answer on the pset.

I wonder how often he reads these comments to correct!

I'm not sure, I'm just happy to see I did this right and got the same answer.

I'm pretty sure that's a typo

I wish this were explained in more detail. It seems like your substituting back into the same equation which will always cancel terms

I don't see the purpose in saying that energy consumption is independent of p, when you write in parentheses that this only happens when something that depends on p is satisfied.

This is like saying, "Given something that depends on p, then energy consumption becomes independent of p." It seems a little counter-intuitive.

Would a better way of phrasing it be, "Regardless of the density, p, there is always a speed v, that uses the same minimum energy consumption."? Or would that be saying something else entirely?

Problem 8   Cruising speed versus air density

For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed v depend on air density ρ? In other words, what is the exponent β in $v \propto \rho^{\beta}$?

[ ] ± [ ]   *or*   [ ] … [ ]

From the lecture notes,

$$Mg \sim C^{1/2}\rho v^2 L^2,$$

where C is the modified drag coefficient. So

$$v \sim \left(\frac{Mg}{C^{1/2}\rho L^2}\right)^{1/2}.$$

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The inverse relationship between the speed and density explains why planes fly at a high altitude. The energy consumption at the minimum-energy speed is proportional to the drag force, which is proportional to ρv^2. Because $v \propto \rho^{-1/2}$, the powers of ρ cancel in the energy consumption; in other words, the energy consumption (at the minimum-energy speed for that ρ) is independent of ρ. By flying high, where ρ is low, planes can fly faster without increasing their energy consumption.

I wonder what the tradeoff is since flying at a higher altitude increases the distance traveled to get to the destination.

This point is very evident if you take a flight from Boston to New York or even Philadelphia. The entire flight you're able to see the ground and don't get that high because the plane doesn't go too high whereas a flight across the Atlantic requires more speed so you must go higher.

I don't think the trade off is that significant. I think planes normally go up to about 35,000 feet (about 6.5 miles) which isn't really adding that much distance when you are traveling hundreds of miles

That's really cool

Problem 9   Cruising speed versus mass

For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed ν depend on mass M? In other words, what is the exponent β in $\nu \propto M^{\beta}$?

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$

Again from the lecture notes,

$$Mg \sim C^{1/2}\rho\nu^2 L^2,$$

where C is the modified drag coefficient. So

$$\nu \sim \left(\frac{Mg}{C^{1/2}\rho L^2}\right)^{1/2}.$$

For geometrically similar animals, g is independent of size (they all fight the same gravity) and C is also independent of size (because the drag coefficient depends only on shape). But M depends on L according to $M \propto L^3$ or $L \propto M^{1/3}$. Because L^2 is proportional to $M^{2/3}$, the denominator contains $M^{2/3}$. The numerator contains M^1, so the ratio of numerator to denominator is $M^{1/3}$. After taking the square root, we find the scaling

$$\nu \propto M^{1/6}.$$

In other words, $\beta = 1/6$.

Large birds (and planes) fly slightly faster than small birds and planes. The design of the 737 was affected by this fact. The 737 is for medium-range flights and carries fewer passengers than a 747. However, if the 737 were merely a geometrically scaled 747 – retaining the shape but reducing M by, say, a factor of 3 – then it would have a cruising speed roughly 20% lower than a 747 (because $3^{1/6} \approx 1.2$). That reduction would be fine if the 737 were the only plane traveling the skies. But planes are directed along fixed flight paths where it is dangerous to have planes overtaking one another. Therefore, the 737 was designed not to be geometrically similar to the 747 but instead to have the same cruising speed as the 747. Scaling matters!

Just like in problem 2 where M was proportional to R^3, I didn't do this and got a totally wrong exponent. Thus my answer to problem 10 was skewed too.

> Yeah, I forgot the hidden M's too.

>> Wow I'm glad others did...reading these solutions was a nice wake up call for me to be more attentive in the future.

>>> Aye, I missed them as well. I knew I had done something wrong based on my answer to #10, but was unsure where I had messed up.

>>>> i dont really get this statement. i guess i need to think through this a little more in order to be able to get it right the first time on the homeworks.

So I 'spose this is a rhetorical question, but: how do you know all these things? I know many of them from my great love of machines and mechanics, but Lex Luther is to Superman as biology is to me – I simply hate learning it. I guess I'm saying I envy your ability not to deeply resent other sciences in the unreasonable way I do.

> This really seems like physics of animals more than biology. Think of them as robots if you like. Although I'd say biology is pretty amazing too.

I really like these comments at the end of the solutions. They really give meaning and matter to the problems. Gives me new insight on the world.

> It's definitely nice to see these explanations. I feel like this is one of the only classes I've taken at MIT where I actually learn something practical and interesting each class that may be entirely unrelated to what I learned the last class.

>> Yes, and the airplane examples definitely back up the solutions by making intuitive sense!

Interesting, I wasn't sure where this question was going: if it was just comparing birds to themselves or perhaps airplanes

did not know that. I wonder how the new 787 will affect this. Isn't it supposed to fly slower but more efficiently?

Problem 10   Speed of a bar-tailed godwit

Use the results of **Problem 8 and Problem 9** to write the ratio v_{747}/v_{godwit} as a product of dimensionless factors, where v_{747} is the minimum-energy speed of a 747, and v_{godwit} is the minimum-energy speed of a bar-tailed godwit (i.e. its cruising speed). By estimating the dimensionless factors and their product, estimate the cruising speed of a bar-tailed godwit. [Useful information: $m_{godwit} \sim 0.4\,kg$; $v_{747} \sim 600\,mph$.]

$$10\boxed{} \pm \boxed{}\,m\,s^{-1} \quad \textbf{\textit{or}} \quad 10\boxed{} \ldots \boxed{}\,m\,s^{-1}$$

To include in the explanation box: Compare your result with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8.5 days.

Assuming that the animals and planes fly at the minimum-energy speed,

$$\frac{v_{747}}{v_{godwit}} = \left(\frac{\rho_{high}}{\rho_{sea\;level}}\right)^{-1/2} \times \left(\frac{m_{747}}{m_{godwit}}\right)^{1/6}.$$

A plane flies at around 10 km where the density is roughly one-third of the sea-level density. The mass of a 747 is roughly $4 \cdot 10^5$ kg, so the mass ratio between a 747 and a godwit is 10^6. Therefore, the speed ratio is roughly

$$\frac{v_{747}}{v_{godwit}} \sim (1/3)^{-1/2} \times (10^6)^{1/6} = \sqrt{3} \times 10 \sim 17.$$

A 747 flies at around 550 mph so the godwit should fly around 550/17 mph $\sim$ 32 mph. The actual speed of record-setting godwit is almost identical:

$$v_{actual} \sim \frac{11,570\,km}{8.5\,days} \times \frac{0.6\,mi}{1\,km} \times \frac{1\,day}{24\,hours} \sim 35\,mph.$$

---

is there a way to not use the results of previous problems that we may have gotten wrong, or didnt understand.

Why do we not need to include L^2in here?

I think L^2 was already taken into account when we calculated the exponent for the mass relationship (since we're assuming these flying things are geometrically similar).

I still think this is cool that a bird could fly so far without stopping...

Is there a way to format the psets to get around doing a question wrong or not understanding it? i.e. I knew my answers to the last two were wrong, so I kind of figured whatever I got here would be a lost cause, too.

That would seem to be part of the learning process. Questions that build on simpler ones are standard. Add more bounds to your confidence interval if you're not sure. Turn it back into an approximation more like we were doing earlier in the course.

What if the simple questions aren't simple to me? uh oh

I really liked this problem. I actually figure out these proportions!

Why do they fly a minimal height?

Yea, is the godwit flying at sea level? I thought migratory birds also took advantage of high-altitudes, where they can use strong tail-winds to minimize their energy expenditure.

Maybe because there isn't enough oxygen higher up in the atmosphere.

Where does this come from? I'm not following this at all...

See problems 8 and 9, in those problems we found how the cruising speed is proportional to the density of air and the mass of the flying object.

this comes from the previous parts–you just find the ratios of the velocities, and in the first two parts, we found how velocity depends on density and mass

My mistake in 9 killed me here when I used 1/2 instead of 1/6 as the exponent. I wound up with something under a meter per second, which I knew didn't make sense (although the godwit speed seems amazingly fast). I just couldn't figure out where my mistake was–it didn't occur to me that it might be in the proportionality.

I forgot to consider that the air density at different altitudes is different, so I just assumed the two density terms canceled out. Although I still got an answer in the same order of magnitude as the solution, it made a huge difference at the end when I estimated the speed of a godwit based on the ratio and the speed of the 747.

Air density differences at different altitudes is something I would just never have thought to consider if not for this class.

So I neglected the Lengths in problem 9, but incorporated Area here, as well... I was confused at first as to how my answer still came out right.

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$$v_{actual} \sim \frac{11,570\,km}{8.5\,days} \times \frac{0.6\,mi}{1\,km} \times \frac{1\,day}{24\,hours} \sim 35\,mph.$$

---

this last bit of math doesn't seem like "approximation" to me.

I think root three is commonly used enough that a practiced approximator should know it.

Why not? sqrt(3) is 1.67, which is approximately 1.7. Are you saying you don't see how we got from $(1/3)^{-1/2}$ to sqrt(3)?

I completely forgot this factor and still got a reasonable answer.

that factor comes out to 1.73, so it wouldn't make a huge difference

Just curious, why is it you used 550 mph here instead of the 600 mph mentioned in the problem?

I wanted to use the speed that I used in lecture, and had forgotten that I had given a different speed in the problem statement. But what's a 10% speed difference among friends?!

Why use 550 here when the intro to this problem says v_747= 600 mph?

He probably just took the numbers from two sources, or forgot that he had included a number in the problem.

It's great that we can get approximations that are so close to the real world answers!

I especially liked this question because we used our results from the two previous questions to then do a calculation which was very close to the actual answer.

this is amazingly close- cool

i didn't get this answer because i got the previous one wrong.

....but if you got the process right it doesn't really matter at this point.

6.055J/2.038J (Spring 2010)

Solution set 5

Submit your answers and explanations online by **10pm on Wednesday, 07 Apr 2010**.

Open universe: Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Problem 1   9V battery
Roughly how much energy is stored in a typical (disposable) 9V battery?

$$10^{\boxed{}} \pm {\boxed{}} \text{ J} \quad \textit{or} \quad 10^{\boxed{}\cdots\boxed{}} \text{ J}$$

> I'll estimate it by working out the energy in my laptop battery, and then adjusting the estimate to compensate for the smaller size of a 9V battery. The energy in my laptop battery is the ideal candidate for divide and conquer: the power drawn by the laptop times the battery life. I know the battery life well: about 4 or 5 hours (I have the slightly larger 9-cell Thinkpad battery).
>
> The power draw is harder to estimate. The screen, the CPU, and the disk drive probably use comparable amounts of power, since reducing the power consumption of each item seems to be comparably important in extending the battery's life. The methods include using a lower screen brightness, putting the CPU into idle (technically, C2 and C3 states), or spinning down the disk. The screen is an LCD screen, which is much more efficient than an incandescent (standard) light bulb. So, although it is bright like a (weak) light bulb, say a 30-watt bulb, it may draw only 5 or 10 W.
>
> Three such items – which includes the disk drive and the CPU – add up to perhaps 20 W. (As a check, the Powertop utility that comes with my Debian GNU/Linux installation says that the laptop is using 16.6 W.)
>
> The product of power and time is energy stored in the battery:
>
> $$E_{\text{laptop}} \sim 20\,\text{W} \times 4.5\,\text{hours} \times \frac{3600\,\text{s}}{1\,\text{hour}} \sim 3 \cdot 10^5\,\text{J}. \tag{1}$$
>
> Now let's shrink that energy to account for the smaller size of a 9V battery. As a simple method, I'll assume that all batteries have a comparable energy density (energy stored per mass). In mass, my laptop battery feels like about 15 or maybe 20 9V batteries. So I'll divide $3 \cdot 10^5$ J by 15 or 20:
>
> $$E_{\text{9V}} \sim \frac{E_{\text{laptop}}}{15\text{ or }20} \sim 15\,\text{kJ}. \tag{2}$$
>
> For a rough comparison with actual values, Wikipedia quotes 20 kJ as the energy stored in a 9V battery.

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Problem 1 9V battery

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I'll estimate it by working out the energy in my laptop battery, and then adjusting the estimate to compensate for the smaller size of a 9V battery. The energy in my laptop battery is the ideal candidate for divide and conquer: the power drawn by the laptop times the battery life. I know the battery life well: about 4 or 5 hours (I have the slightly larger 9-cell Thinkpad battery).

The power draw is harder to estimate. The screen, the CPU, and the disk drive probably use comparable amounts of power, since reducing the power consumption of each item seems to be comparably important in extending the battery's life. The methods include using a lower screen brightness, putting the CPU into idle (technically, C2 and C3 states), or spinning down the disk. The screen is an LCD screen, which is much more efficient than an incandescent (standard) light bulb. So, although it is bright like a (weak) light bulb, say a 30-watt bulb, it may draw only 5 or 10 W.

Three such items – which includes the disk drive and the CPU – add up to perhaps 20 W. (As a check, the Powertop utility that comes with my Debian GNU/Linux installation says that the laptop is using 16.6 W.)

The product of power and time is energy stored in the battery:

$$E_{\text{laptop}} \sim 20\,\text{W} \times 4.5\,\text{hours} \times \frac{3600\,\text{s}}{1\,\text{hour}} \sim 3 \cdot 10^5\,\text{J}. \tag{1}$$

Now let's shrink that energy to account for the smaller size of a 9V battery. As a simple method, I'll assume that all batteries have a comparable energy density (energy stored per mass). In mass, my laptop battery feels like about 15 or maybe 20 9V batteries. So I'll divide $3 \cdot 10^5$ J by 15 or 20:

$$E_{9V} \sim \frac{E_{\text{laptop}}}{15 \text{ or } 20} \sim 15\,\text{kJ}. \tag{2}$$

For a rough comparison with actual values, Wikipedia quotes 20 kJ as the energy stored in a 9V battery.

Here is solution set 5 (memo due Thu at 10pm).

The amount of time we have to do the annotations on these is irritatingly short. Sorry this one is a bit late.

I spent quite a while just sitting and trying to figure out a way to approach this question, are there any other ways that people can think of? The way that I chose failed miserably (Off by several orders of magnitude).

This is a much more intuitive, no sources needed, method of going about this problem.

> I agree! I didn't think of using my laptop battery...

What's the simplest object you can use for this calculation?

Quite clever. I was at a bit of a loss as to how to do this one, as I couldn't quite recall any similar scenarios that would have been of help.

Thus, I relied on looking up values for battery life to fill that missing parameter.

> I used my laptop as well, but mine only last a few hours...time to get a new one I gues...

>> Also, I have the larger of the two Thinkpad laptop batteries (the 9-cell rather than the 6-cell). The 6-cell lasted only about 3 hours.

> I didn't want to look anything up so I avoided using actual durations/power consumptions.

> So the only method I could think of was to estimate the number of atoms in the battery, assuming an Angstrom is about the order of dimension for an atom, and then use 1 electron per atom as the total charge stored. (1/2 Q * V) yielded an upper bound on the energy of 10^7. I knew this was high, so I intuitively centered my range around 10^4.

I used a flashlight instead to do the calculation. Sometimes it's easier to choose small appliances.

> Yeah...I based my calculations of my old gameboy color.

I assumed it was equal to 2 AA's. This gave me 10kJ, only 30% below this estimate's result. Fairly consistent.

> I tried a similar approach using an alarm clock. It had the number of watts it ran on (4) printed on the bottom. I overestimated how long it would last on the backup battery enough to throw my answer off by a factor of 100.

This is a really good idea, I don't know why I didn't think of using something we know the life of so well.

> Also, I know personally that my 4-year-old laptop only has a battery life of around an hour, although it used to be closer to 3 or 4. so i wouldn't know which value to use..1hr would feel more natural but i'm pretty sure it would give me the wrong answer

>> I feel like this is a serious issue with using the laptop battery time...rechargeable batteries ware out.

I did not think to use my laptop battery either.

6.055J/2.038J (Spring 2010)

Solution set 5

*Submit your answers and explanations online by **10pm on Wednesday, 07 Apr 2010**.*

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$$E_{9\text{V}} \sim \frac{E_{\text{laptop}}}{15 \text{ or } 20} \sim 15\,\text{kJ}. \tag{2}$$

For a rough comparison with actual values, Wikipedia quotes 20 kJ as the energy stored in a 9V battery.

I didn't think of comparing to another kind of battery, but this nicely brings scaling and divide and conquer into play.

agreed. i had pretty mich no idea how to solve this one

This is a good way to do this - I didn't realize that all batteries had similar energy density. I used and RC car and looked at how far and fast itgoes

I tried comparing it to a smaller device that used AA batteries, but I at least ended up within a order of magnitude of the answer.

We used a similar method except using something that actually used a 9v battery, a smoke detector. The box of a smoke detector tells how much power it uses and we know the batteries are changed once a year.

i thought i smoke detectors we're powered by some sort of radiation?

Perhaps this is just me, and while this is a perfectly reasonable way to address the problem, what does it have to do with dimensional analysis/easy cases (what we've been doing in class these past few weeks)?

I think he's just making sure we stay up on our divide and conquer skills.

I agree, I find myself using totally different approaches to solve these problems from what we've learned recently in class.

yeah, I think it might be helpful to indicate what approach is "easiest" to use when attacking the problems. If your goal is for us to remember all the approaches and pick which one ourselves, then maybe label a section as "all approaches" and then for those that are using the most recent material you can label it as such

I dunno, I found it most intuitive to use divide and conquer, but the comments on the homework almost mislead me to think that it was "wrong" and I should be using dimensional analysis. I think we all fall into the trap of assuming we're supposed to do the problem in a set way.

Wow this seems so intuitive! I was scrambling around my desk for batteries..

Something you can try on a non-Mac laptop is to pop out the battery and check the mAh rating on the battery.

I used a lightbulb, which i think is a lot easier to calculate with than a laptop- i definitely did not know a laptop uses about 20W

How would you know how long a 9V can power a light bulb?

I think estimating how long the light bulb would last is about as difficult as estimating how much power is used up by the laptop. Does anyone know of any devices that they have a good intuition for how long they last and also for how much power they output?

Since my laptop battery isn't very good anymore, I used my cell phone battery. Seemed like the most logical choice. Hopefully people didn't have trouble coming up with an example to get this problem rolling.

between choosing to work with a lightbulb or a laptop, it seems that you're trading ease in estimating time (but not power) for ease in estimating power (but not time).

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This was an interesting way to approach the problem. I don't know enough about the power consumption of my computer to have used it myself, but it was fun to see the problem worked out this way.

I agree. the way i solved this was much shakier than this solution. If i had known all these details about my computer i wouldve been so straightforward

do you need to go through all of this thought to figure it out? isn't there an easier method that requires less guessing?

I was also surprised by the amount of calculations that was done for this problem. I feel like the memos cover much easier problems (or they're in smaller chunks), but the homeworks are incredibly difficult to do.

So I know we've used really rough estimates like this before, but I feel like this one wasn't really supported by a really strong argument. The fact that it added up to about the right, power just seems like a coincidence.

I agree with this statement. I tried to estimate how much power a smoke detector uses, and I had no idea what was a good estimate. This resulted in a final answer that was way off.

Perhaps it would be helpful just to let us know what 1W can power...

I had to look up the fact that most small batteries draw about 10 W

My guess is that smoke detectors have a very low power consumption. They have no moving parts and almost no lights (the only one in our apartment's smoke detector is a small red LED), and probably just use a very low-power infrared beam as part of the detector. Also, you want them to be designed to last a long time on one battery, because otherwise people get annoyed and just turn them off completely, which is dangerous. What is the actual power consumption?

I have a really hard time having a feel for power consumption. even light bulbs have such a wide range.

How does the 9V manifest itself in this equation?

He compares the mass of his battery in his laptop to that of a 9V battery, assuming it is about 15-20 times the size.

I tried to use the time that I thought a 9V battery could power a 100 watt light bulb. Which from the numbers looks like maybe half an hour.

I forgot to do this.

I tried thinking of it as like a parallel plate capacitor with a separation the length of the battery, but when i divided V/L, I got a much smaller answer. Why is this so far off?

I don't think that's the right equation for the energy stored in a capacitor. Also, it's hard to know what the internal geometry might be. Similarly sized batteries can have different capacities.

A battery is sort of like an accordian of capacitors, I believe. Having an accordian gives all of the capacitors a very small distance between the plates, allowing a higher capacitance. This would be useful if you knew how many folded capacitors there were inside the battery.

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It's amazing how well this works out, definitely an interesting way to approach the problem

how can you just say - oh this feels like 20 9V batteries?

Yeah, this seemed like the most "gut-following" problem yet.

I'm confused why we can divide by volts when that unit wasn't in the original estimation. Or does 20W = 15*9V or something like that?

We're not dividing by volts, but by a number. I think he's assuming that voltage is proportional to energy, so the energy for 20 9V batteries divided by 20 will produce the energy for 1 9V battery.

Interesting idea! I used my cell phone's battery life as the comparative benchmark instead.

I ended up with 2x this by using the same ideas, I guess it highlights the idea of approximations to me :)

15kJ seems much more reasonable – I tried to do some estimates based on basic chemistry and ended up being off by about 10^4

I actually got this almost exactly by approximating 6 AAA batteries.

For this problem I got about the same answer but used what I know about physics equations to derive an equation that combined what I know- I ended up using that the change in energy= voltage*area

Cool, that's what I got

I found something that said 9V batteries used 560 mAh, and I just used that. Is there a way to really consistently estimate this one? I've done it before, but I always use a device an try to guess; I think it's hard ot guess that laptop battery uses a certain number of watts (I think mine uses 60 actually, but does much faster), and also hard to guess how many 9V in a laptop battery.

I also used mAh...however, I remembered (from some 9V work that i did earlier this semester) finding that a standard 9V has about 600mAh. then I messed up the math. whoops.

Funny that Wikipedia is mentioned here... I am always told Wikipedia is not a reliable source.

Never question the all powerful might of Wikipedia. It's second to Google in knowing random information.

while i'm personally okay that wikipedia is mentioned here, id on't know if it'd be a good idea if you're planning to publish this book. especially since wikipedia changes all the time.
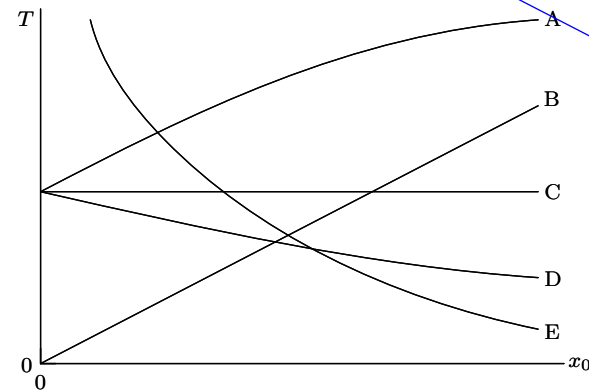
I was close to this, but I think I underestimated what it was capable of because the object I was considering was not one I was familiar with.

This problem wasn't really associated with what we were doing in lecture. Was this here to brush up our memory?

Right, it was a reminder of everyday estimations and divide-and-conquer reasoning.

Problem 2   Non-Hooke's law spring

Imagine a mass connected to a spring with force law $F = Cx^3$ (instead of the usual Hooke's law behavior $F = kx$) and therefore potential energy $V \sim Cx^4$ (where C is a constant). Which curve shows how the system's oscillation period T depends on the amplitude x_0?

☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

☐ Curve E

Dimensional analysis:

T	period	T
x_0	amplitude	L
C	spring constant	$ML^{-2}T^{-2}$
m	mass	M

The trickiest entry in the table is the dimensions of C. Since Cx^3 is a force, C has dimensions of force over length cubed, namely $ML^{-2}T^{-2}$. These four quantities made out of three dimensions produce one independent dimensionless group. Its simplest form is $Cx_0^2T^2/m$. Because there is only one dimensionless group, it must be a constant. In other words, $T \propto 1/x_0$. The only matching curve is curve E.

It was really cool revisiting this problem. I remember on the diagnostic I was totally lost, but looking at it again, dimensional analysis gave me the correct answer. It feels good to apply what we've learned to what we've seen before!

Interesting spring, do these exist by any chance?

I guess in reality, spring constants don't apply since springs aren't actually perfectly constant in tension acorss the entire spring.

Didn't we get this problem for the diagnostics? I thought it was cool to come back.

Yes, I think it was on the diagnostic. I agree, it was really nice to see this one come back and be able to solve it much more quickly.

Just curious, why do you specify the potential energy when the force equation is enough to find the dimensionless group? It threw me off a bit because I thought I should be doing something more complicated.

This way of doing it makes so much sense! I had no idea how to do it and made it much more complicated than it actually is and still didn't get the right answer!!

Yea, I had no idea how to do this problem on the pretest but it is quite simple using dimensional analysis!

Agreed!...although I wasn't too confident that I was doing it right since it seemed way too easy. Guess I am learning something in this class.

Oh... it completely slipped through my mind that I was supposed to use dimensional analysis.

I confused myself into knots on this one, and now feel very foolish for not figuring it out on my own.

So I did use dimensional analysis this time, but on the pre-test I found it easiest to use a comparison to a linear spring, which we know gives a constant T.

At large x0, this non-linear spring has a larger restoring force, so it should have a smaller period. At small x0, the restoring force is less, so T should be bigger. Curve 'C' represents the linear model, and the curve that satisfies this comparative relationship is E!

At first I tried to compare it to a linear spring, but found it too confusing to map between the two equations. For this problem, I definitely found that dimensional analysis was the easiest way to go

When I saw this problem I quickly jumped to physical intuition first using exactly the same reasoning as you. I like using dimensional analysis when using physical reasoning doesn't work.

I think this problem should be in the reading as an example - it was the first problem we did where I needed to use DA and it fits so well.

I might even use it in two sections. The 'compare with a linear spring' method, mentioned in these comments, is a nice example of easy cases reasoning. So, this example could be done twice, once in dimensions and once in easy cases.

Just to be a bastard, maybe I should make a new choice that is monotonic decreasing, like curves D and E, but that starts above curve C (e.g. twice curve D everywhere). Then one has to reason quite carefully about the low-amplitude extreme case to decide between this new choice and E.

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I realized that here we basically have 2 equations for force, so I set them equal to each other giving L^3=MLT^-2. When I solved I got T proportional to T^-1. I'm trying to decide if I just got lucky.

I liked how this problem was easy to check by trying out the case I knew the answer to - namely the case when F = Cx.

I just thought that as the amplitude increases the period should decrease, and somehow my "gut" told me that it should be an exponential decrease rather than linear

This is a great approach! I was going to try and use simple cases to see if I could then apply that but this is easier.

I had a hard time trying to apply dimensional analysis for this problem, so I ended up trying to think about it in a different way and got the answer completely wrong!

> I would have too, but looked at the review of dimensional analysis for the standard Hooke's Law that was in the notes (I forget where).

> I thought my work using dimensional analysis was a bit too simple but it turned out to be correct. This was a very good example of how the process is used.

I liked this problem - you had to think about the proper units for C and that was quite helpful.

> I never thought to try and find the dimensions of C, for some reason I assumed that C must be a constant. This problem makes a lot more sense now

> Yeah, I had a complete brain fart on this quesiton–I totally messed up on the dimensions of C. this makes everything so much clearer.

> > Same here...which is funny, because (in hindsight) I remember working through units of C on the diagnostic.

I had just assumed it would be constant like a normal spring

this isn't that tricky because we were given the relation, so we automatically know the dimensions.

this helps a lot- I was so confused about how to approach this- is there any way to help us decide what technique to use to solve the problem?

How do you know there is only one dimensionless group? I feel like it's easy to combine dimensions of the variables until they cancel out, but how can you be sure you aren't missing another grouping?
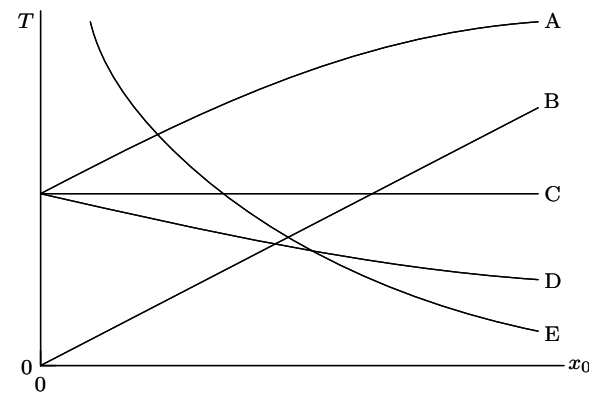
> The way to figure out how many dimensionless groups there are is explained in either r20 or r21

Conceptually why does the curve E near infinity when the displacement is almost 0. This is what tore me away from E

> the period is very very fast (nearly undetectable) when there is basically no displacement of the "spring"

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I found this problem very difficult to think through conceptually. This method really helped me figure it out.

Wow..I too had pages of calculations. I didn't realize dimensional analysis would make it so much simpler!

yeah, I tried just comparing dimensions used in the regular hooke's law to the dimensions used in the new equation. I knew the formula for the period was T (m/k)^.5 ; i just looked at the dimensions and found out what C had to be. then found out how to make the period equation work. It ended up being dependent on 1/x

Yeah i didn't understand the "physics" of it either, but you don't need to if you use this method. That's a good thing to know.

I think that's the whole beauty of dimensional analysis. You don't need to know anything about the physics so long as you know the important parameters and their dimensions.

I found it difficult too, but after talking to a friend in the class it became easier - I feel like a lot of these easier to approach when you bounce ideas off others/the problem set's nb page.

I used easy cases to figure out that this was the right curve once I got T 1/Xo. I used "If Xo goes to zero... T should go to infinity" And the only curve that fits that is E.

That makes sense.

what about at Xo = 0, shouldn't T = 0 too?

No, as stated above, as you approach xo = 0, T approaches infinity. The period at xo =0 is sort of a tricky case, since there's no motion. You could think of having to wait an infinite amount of time for an oscillation to happen since it's not moving.

Is that considered using easy cases? Its just thinking about the limits of the function...

That is indeed easy cases. As you noticed, in this problem the limiting cases are easy cases to think about.

I arrived at the same answer here but got lost with my dimensions working on my spring constant C. I'm not sure if I got lucky with my reasoning but it's good to see it completely done out this way

I did pretty much the same thing but got the wrong answer! That's unfortunate. Even in estimation, it makes sense to check work!

How is the period inversely proportional to the distance. This implies the period would be really long for a short change in Xo. This seems really counter intuitive....

Agreed, I got the right answer, but it was totally not what I expected.

It is a bit surprising. Think of it this way: The spring is very weak at small displacements (the x^3 in the force law makes the force go to zero very fast), so it can hardly make the mass move.

Problem 3   Power radiated by an accelerating charge

If the velocity and acceleration of a (nonrelativistic) electric charge are doubled, how does the power radiated by the charge change?

☐ The power increases by a factor of 16.

☐ The power increases by a factor of 8.

☐ The power increases by a factor of 4.

☐ The power increases by a factor of 2.

☐ The power increases by a factor of $\sqrt{2}$.

The first question is: On what does the radiated power depend? First, c (the speed of light), because that is the speed at which the power travels; second, the charge's acceleration or velocity (or both); and third, the charge itself q. We probably also need ϵ_0, the horrible constant when using SI units for electromagnetism. But ϵ_0 and q will show up only together as $q^2/4\pi\epsilon_0$, so let's combine those two quantities accordingly.

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These four variables, again with three dimensions, result in one independent dimensionless group– for example,

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With only one group, it must be a constant, so

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10 ☐ $\pm$ ☐ m   *or* 10 ☐ … ☐ m

Was this from the pretest?

I do think something similar appeared on the pretest.

What does this mean?

I think that he means the net velocity and acceleration, as opposed to individual charge velocity and acceleration. I think.

I just ignored it. However I got the wrong answer so maybe I shouldn't have...

He means that the charge is not moving at speeds close to the speed of light.

Then why did he use the speed of light in his dimensional analysis?

since it's dimensional analysis, it's not the number part (which would give you how close to the speed of light you actually are) that matters, it's the units (which are the same no matter what the number is)

I was confused because I don't know what this even is talking about. Basically, why is the charge radiating power? Is this quantum physics or is it light or something else?

In classical electromagnetism, an electron orbits a proton and is constantly acceleration towards the proton. As the electron is orbiting and falling closer to the proton, it loses energy, which radiated.

I was really lost here because I'm not familiar with power radiation, so I made a complete guess based on the dimensions of power. Of course, we've been in situations before where we have no intuition but dimensional analysis helps us reason about the problem. How do you determine when you have enough knowledge about the problem to make a reasonable approximation?

How do we know whether we need it or not?

I think this just has to come with experience with physics- no guessing for this one!

That's a more tricky jump to make. I don't think I would ever know to do that.

I agree. I wouldn't know to do this. I suppose it helps to make things simpler, but I wouldn't know if it would cause me to skew the answer.

I had trouble jumping to this. I reasoned we needed q, but could not think of anything to cancel the current dimension.

I left the q and the 4*pi out. Honestly, I didn't even know I was supposed to put them in. I don't know enough about EM to do so. But the 4*pi is just a constant and the q doesn't contribute any dimensions to the dimensional analysis, so why is it necessary to use anything but the e_0?

I'm not good at physics and I don't use it in my field, was it ok for me to use the internet to find out that epsilon is part of this problem? I get frustrated that I have no idea what extra variables I'm expected to know about.

Problem 3   Power radiated by an accelerating charge

If the velocity and acceleration of a (nonrelativistic) electric charge are doubled, how does the power radiated by the charge change?

☐ The power increases by a factor of 16.

☐ The power increases by a factor of 8.

☐ The power increases by a factor of 4.

☐ The power increases by a factor of 2.

☐ The power increases by a factor of $\sqrt{2}$.

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This whole simplification requires a pretty intimate familiarity with electromagnetism. Though I guess next time I see an E&M problem, I'll know to put epsilon_0 in.

Agreed, I felt like I didn't know enough about EM to be able to properly understand this problem. I know this class is supposed to teach us approximation in a variety of fields that we may not be familiar with, but I felt like the solution to this problem hinged on your knowledge of a particular subject.

I didn;t have a good enough understanding to assume this

I feel like this explanation would be more clear if you explicitly said power radiated by the charge should be the same regardless of our frame of reference. Therefore, velocity cannot influence power radiated.

damn, got that wrong

I suppose you'd have to know this equation to get the right answer.

Yes, I don't particularly like this problem as much as other dimensional analysis problems because this expression requires some previous knowledge.

I did not use dimensional analysis and instead just basic physics, which i think in this case makes more sense!

I did this too, and I think it was probably much easier and faster.

yeah I just broke it down to power=Force*velocity=Mass*acceleration*velocity. If you double acceleration and velocity then you quadruple the power.

I am still having trouble coming up with the group elements in these problems...

yeah...my main problem with this question was coming up with dimensions for radiated power...

I used v instead of c (so I had the same dimensions) but my answer was totally wrong. I should have noticed that I was missing c...

I used dimensional analysis and groups to get P mva. or some form of mv^2t ma^2/t and just figured that regardless, it was going to be 4x. My approach fails miserably if it was only the velocity though.

I followed the same reasoning, but we would also have been wrong if the other crazy parameter had taken care of one of the L/T or L/T^2 factors, so that P was proportional just a^1.

This should be LT^-2... no M.

You're right. At least that mistake didn't propagate (the subsequent formulas are okay).

I totally confused radiated power for electric power.

Yeah, I'm not sure if that's a completely wrong assumption to make, but I used P = IV and got the correct answer...

I pulled up too many equations too determine the variables involved in radiating power before i got down to looking for independent groups. Ended up getting more variables than i needed.

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---

instead could we not use: P = Force * velocity, force = mas* accelerations; therefore doubleing both gets; P = 4*mass*a*v

I got this problem wrong because I doubled velocity and the speed of light. I knew something was fishy about it...

This is really cool. I already knew the Larmour formula before doing this problem, so it wasn't maybe the most enlightening problem, but it's comforting to know that if I ever need to re-derive it I can get pretty close just using dimensional analysis.

> I again used physical reasoning that I gained from my E&M classes, so i guess its unfair. But I intuited that doubling the velocity/acceleration doubles the number of electrons flowing or doubling the amount of current. And power is proportional to current squared, thus quadrupling the power.

I did this in a much simpler way and got the same answer- I modified the power equation P=W/t= F*d/t= (ma)(vt)/t= mav.

> Same here, only I just remembered P=F*v=mav. So although this gets the same answer, you get 4 because of a^2 but no dependence on v, whereas this answer depends on a and v. The two equations would seem to be mutually exclusive (P cannot be both proportional to a*v and a^2), so was this just pure luck that it worked out?

I did this is a simpler way and got the same answer

I got to this answer by totally neglecting the q^2/(4*pi*epsilonnot). speed*acceleration*mass make power.

I still don't see how doubling velocity affects the power, or does it?

> It doesn't have any effect. Only doubling the acceleration affects the power.
>
> I'm confused - I said P=F*v and got the right answer. How does velocity not affect power?

I think I got this with a lot less work. I 'spose it was just a lucky guess then?

I think I took too simplified of an approach to solve this problem.

That sounds dangerous. ;)

What is the mass of a black hole generally? Is it comparable to earth? I know so little about this.

> Don't black holes have almost no mass?

this was a really fun question to think about!

In gravity problems, the quantity GM/Rc^2 is dimensionless. For most objects, it is very tiny. A likely candidate criterion for a black hole is when this quantity is around 1. Therefore, the radius of the black-hole earth would be

$$R \sim \frac{GM}{c^2} \tag{5}$$

Putting in numbers,

$$R \sim \frac{7 \cdot 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \times 6 \cdot 10^{24}\,\mathrm{kg}}{10^{17}\,\mathrm{m^2\,s^{-2}}} \sim 4\,\mathrm{mm}. \tag{6}$$

(The true black-hole radius, based on general-relativity calculations, is twice this value based on dimensional analysis and a bit of guessing.)

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Roughly what is the number density of free (conduction) electrons in a copper wire?

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$$E \sim \frac{\rho R^5}{t^2}.$$

For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

is this the speed of light? how do you know that we have to use this constant

> I plugged in c for v because a black hole doesn't let light escape.

>> And yes, c is the speed of light.

I still a little confused by this problem. What is the equivalent of GM/Rc^2. The number 4 mm seems a bit small to me. Am I thinking of this in a wrong manner?

I didn't know how to go about this problem in the homework....seeing the solution clarifies it. I had a feeling that escape velocity was a good method, but I have no knowledge of black holes and assumed my hunch would be completely wrong

> Why is that quantity dimensionless? I couldn't solve this problem. I used F = GMm/R^2 but don't know how we arrived at this.

>> I figured this out after reading through reading 25 and remembering that this is a dimensionless group equal to the angle of the bended light.

I ended up going a different way around to dimensionless groups because I didn't see what/why I should set the quantity to.

Why is this true?

> Dunno. I assumed that theta (bending of light at the surface) 1, instead. it put me sooo far off.

>> 1 radian shouldn't have put you too far off. I used pi/2 and wasn't that far off. But I agree, where is this logic coming from about GM/Rc^2 being about 1?

>>> Is that from the light bending when the constant was 1, 2, or 4 depending on the model of gravity? And we assume 1 because its the most simple case?

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>>>> I used pi/2 also which I suppose is close enough to 1?

Yay I did it just like this!

> Me too!!...although I only did it because I didn't know what else to do and the only problem I could even slightly compare to it was the light bending problem. I don't know if that's useful or not...

I did this problem using the light bending, and the given 30 degree = light cant escape, and I got a very similar answer.

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

Sweet.

This is what I did:

The gravitational force equation is F = -mMG/r^2 In order to escape a black hole from radius R, a mass m must be moving with KE = 1/2 m v^2 that is equivalent to the work done by gravity as the mass moves from R to infinity. (The mass has just enough kinetic energy to come to decelerate to zero velocity at infinity)

Integrating, we set GM/R = 1/2 m v^2

Thus, R = 2 GM/v^2

In this case, v = c for light so

R = 8.873 x 10^-3 m = 10^-2 /pm 0.5 m

This seemed fishy to me, because the theory used a mass that had kinetic energy. But light doesn't have a mass... so substituting in lightspeed at the end made me uncertain.

I had no idea what the dimensionless group was until I saw the solution, and then I also saw this in reading 25 which jogged my memory that this quantity is angle of the bended light, which is why it's dimensionless.

I actually also used a different approach and got the same answer. I used the approach that we used in class when calculating how light bends around the sun. I just extended the analogy for the case when light bends 180 degrees and "can't escape"

yippie! I got one!

I used the same method but I got about 1.5 mm...

It's still within an order of magnitude. Besides, there was probably some variation in estimating division. At least your process was correct.

Used the wrong "G"

It might help to point out this is what would give the enormous density required for a black hole.

I used divide and conquer and got an answer in the same ballpark as this. Awesome

I find this result very interesting.

Having very little experience with black holes outside of a few SciFi movies (which often portray black holes as large, vast entities), this problem helped me gain a little bit more tangible intuition for similar problems.

Yeah, I was expecting some kilometers or something. The answer here really surprised me.

I used a similar approach, but came up with a different answer (probably because of my estimations). I got around 1 m and thought that was way too small–guess not!

I came up with this as well and figured it was completely wrong, but put it down anyway. Upon second glance, I trust my reasoning! This is a very cool problem.

I got it too, but did not trust my gut and completely changed my answer....,

Did you get 1 meter by initially assuming acceleration due to gravity needed to equal the speed of light? I did that at first before really thinking about it and realizing that light would still escape.

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

I think that is really cool to think about. it having to be so small.

I came up with this same answer using the same method, but I was so surprised by how small it was I thought I had made a mistake.

I'm having a hard time identifying when dimensional analysis would be useful. It didn't occur to me in this problem, or the spring-mass problem.

me too and as a result i didn't end up using dimensional analysis for several of these problems

Which is called the Schwarzschild radius.

That's what I used.

That's what I used.

i've never heard of this...

I'm so proud of myself that my answer was really close to that.

I thought this problem was a bit tricky, and kept searching for a way to figure it out using the methods we have recently learned. I guess dimensions are always a good way at looking at a problem.

I like how this problem tied in with the stuff we have been doing in class, such as the work with hydrogen.

I was having a hard time understanding what is really meant physically

Is it valid to just use the density of copper and the atomic mass instead of the Angstrom stuff?

Is this really a guess? Don't we know that it has "one free electron" in the outermost ring?

On the diagnostic I just assumed this, but then I thought about it more and thought, that's why it's such a good conductor, so I had more justification for one conduction e- per atom. Similarly, gold also has one e- in the outermost shell.

I thought it was two.

In fact, Wikipedia says copper 2+ is the most stable state, despite having 1+, 3+ and even 4+ states.

In any case, that will only make us be off by a factor of 2, which in this class is hardly worth worrying about.

I think those are oxidation states in reacting with other elements, not the number of conduction or valence electrons/atom (1) in solid copper.

Confused about this as well. I just took what wikipedia said and ran with it.

Instead of using angstroms, I just used the density of copper and divided by its molar mass to get a volume/mol. Using this I still got about 10^28 electrons per unit volume

I did the same thing. I like the size of the atom better as it doesn't require looking up numbers that I didn't already know.

yes. i'm very surprised that avagadro's number didn't come up in your equations. i thought it'd be the easiest way to get myself in roughly the correct order of magnitude.

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or

$$E \sim \frac{\rho R^5}{t^2}.$$

For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

After talking with a friend I realized this but I was definitely a bit rusty on my chemistry going into this problem

> I thought copper has 29 electrons per atom? Then I used that fact to say that the number of conductive electrons is between 0 and 29, and used that for my estimation (10^1 +- .5).
>
> > I may have cheated and used wikipedia... but anyway you just need to know how many electrons are in the outer shell.

Oops I didn't distinguish between electrons and conduction electrons...

> For some reason, I thought copper had 2, since I usually see it as "Cu 2+"
>
> > Yeah, why is it just one?

I tried using simple geometric shapes, but I never got anything close to 3A. How exactly do you get that number?

isnt a copper ion normally +2?

I thought I had this one nailed. Gotta work on my error budget calculatioins.

wow, this solution is so much simpler than I thought. I was doing a bunch of crazy calculations. If i had directly gone to using atomic volume then it wouldve made a lot more sense

> can we go over this in class... I'm unfamiliar with much of this.

I got this answer just by guessing essentially. Either I'm lucky or my estimating sense is getting better.

> Blame your gut. I mean, intuition.

This problem is much easier than I made it out to be, and for some reason, it just goes over my head.

> I got something close to the answer, but still don't really get it...

I ended up with 10^30 is that close enough?

I took the same approach of estimating how much space the atoms take up but where did you get the volume from? how did you know how thick a wire to choose?

I did something similar, but had to look up the numbers- most times I can't even estimate the numbers, because I don't know enough background info

> I agree, and sometimes if i'm going to look up the value i could just look up the answer.

It's a lot more accurate if you use the known density of copper as well as the molar mass. Though I suppose looking up these values is not the preferred method for the class.

This can already be explained by copper having 2+ as its most common oxidation state. In which case, there would be 2 free electrons per atom.

Wow, this solution looks so simple compared to the numerous calculations I did. My method was completely wong

In gravity problems, the quantity GM/Rc^2 is dimensionless. For most objects, it is very tiny. A likely candidate criterion for a black hole is when this quantity is around 1. Therefore, the radius of the black-hole earth would be

$$R \sim \frac{GM}{c^2}. \tag{5}$$

Putting in numbers,

$$R \sim \frac{7 \cdot 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}} \times 6 \cdot 10^{24}\,\mathrm{kg}}{10^{17}\,\mathrm{m^2\,s^{-2}}} \sim 4\,\mathrm{mm}. \tag{6}$$

(The true black-hole radius, based on general-relativity calculations, is twice this value based on dimensional analysis and a bit of guessing.)

Problem 5   Wire

Roughly what is the number density of free (conduction) electrons in a copper wire?

$10\,\boxed{} \pm \boxed{}\,\mathrm{m^{-3}}$ *or* $10\,\boxed{} \ldots \boxed{}\,\mathrm{m^{-3}}$

Copper has, as a guess, one conduction electron per atom. Each atom occupies roughly a 3-Angstrom cube – i.e., a volume of roughly $3 \cdot 10^{-29}\,\mathrm{m^3}$. The number density is the reciprocal of the atomic volume (since there is only one conduction electron per atom). Thus $n \sim 3 \cdot 10^{28}\,\mathrm{m^{-3}}$. (The true value is roughly 2.5 times greater, due to the atomic volume being slightly smaller than I estimated here using the 3-Angstrom rule of thumb.)

Problem 6   Yield from an atomic bomb

Geoffrey Taylor, a famous Cambridge fluid mechanic, annoyed the US government by doing the following analysis. The question he answered: 'What was the energy yield of the first atomic blast (in the New Mexico desert in 1945)?' Pictures declassified by the US government – the pictures even had a scale bar! – provided the tabulated data on the radius of the explosion at various times.

t (ms)	R (m)
3.26	59.0
4.61	67.3
15.0	106.5
62.0	185.0

Use dimensional analysis to work out the relation between radius R, time t, blast energy E, and air density ρ. Then use the data in the table to estimate the blast energy E:

$10\,\boxed{} \pm \boxed{}\,\mathrm{J}$ *or* $10\,\boxed{} \ldots \boxed{}\,\mathrm{J}$

Four quantities composed of three independent dimensions make one independent dimensionless group. The only quantities whose dimensions contain mass are E and ρ. So E and ρ appear in the group as E/ρ, whose dimensions are $L^5 T^{-2}$. Therefore the following choice is dimensionless:

$$\Pi_1 \equiv \frac{Et^2}{\rho R^5}.$$

With only one dimensionless group, the most general statement connecting those four quantities is

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

I used a pretty different method. I calculated the weight of one copper atom based on its number of protons and neutrons (29 each), then used copper's density to estimate the number of copper atoms per meter cubed of copper. My mistake was not understanding what "free" electrons were so I multiplied by 29 to find the number of electrons per meter cubed.

I also used this method. I looked up the atomic mass and density of copper and with Avogadro's number, calculated the number of copper atoms in a cubic meter. Then I multiplied that by 2 since a copper atom has 2 valence electrons.

I liked this problem as well - very helpful for understanding how to apply dimensional analysis.

I liked this question because its basic dimensional analysis but we have actual data to plug in to the equation we get, so its not really estimation.

I agree, I thought this and the first were the best problems of the set. And they were somewhat related.

This problem was one of the most difficult for me to approach that we have seen in this class. It really just left me confused, seeing the solution is a great thing and I appreciate the option and will try to remember it for the future.

Funny!

I'm really glad i got the dimensions to this problem.

not sure how you chose this...is this a common equation?

You use the dimensional analysis techniques discussed - the solutions just don't list the table (but the same information is represented in the paragraph of text above)

I used this since it was the only thing that worked, but i thought it had to be wrong because R^5 is so large an exponent!

yeah I had the same problem too just that we barely see such a high power in the problems presented in class

I worked on this problems for almost an hour trying to find a solution that didn't involve R^5.

This is where I have a complaint.

How are we supposed to know to pick 1 as the arbitrary constant? Depending on what we pick, the energy will be arbitrarily large or small.

most of the constants we deal with are on the magnitude of 1, which seems like the safest place to start anyways. If you find out later (this constant is def. possible to look up), you can easily alter your answer, which could be confusing if you picked something like 1289

I did not see how to take the jump to equating the dimensionless group to a constant, but I do see how it would be hard to pick anything but 1 since, 1 is such a nice number.

This otherwise horrible problem was so straight forward with dimensional analysis. Absolutely delightful!

I was fairly surprised by (and skeptical of) the R^5 term when I was working this out.

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For each row of data in the table, I'll estimate $\rho R^5/t^2$, using $\rho \sim 1\,\mathrm{kg\,m^{-3}}$:

Wouldn't it be more accurate to use (R2-R1)/(t2-t1) for velocity as opposed to R/t?

this is a good point since we do have the data points – but i'm sure it doesn't affect it much.

This result is very exciting. You couldn't suggest it right off the bat, but dimensional analysis spits it right out.

Yay!!! I did this the same way!!!

t (ms)	R (m)	E (10^{13} J)
3.26	59.0	6.7
4.61	67.3	6.5
15.0	106.5	6.1
62.0	185.0	5.6

The data are not perfectly consistent about the predicted blast energy E, but they hover pretty closely around $6 \cdot 10^{13}$ J.

This blast energy, expressed in more common units for such devices, is roughly 15 kilotons of TNT – in close agreement with the then-classified value of 20 kilotons. Dimensional analysis triumphs again!

I missed that this was ms instead of s. Threw me off by 10^6.

I just realized I made the same mistake.

I caught myself when I thought about what this measurement is saying. Although 10^7 J passes the "makes sense" test for me, an explosion taking 3 seconds to expand 59 m does not. (Think about it counting to 3 Mississippi.)

I wasn't really sure if at this point I should average them or what. Why are they different? Where is the energy going?

At this point, they are all the same order of magnitude and maybe the difference is just experimental error.

I originally thought it meant that as time progressed, the radius got larger, and possible energy is being dissipated.

I was confused about this too. Perhaps clarification should be offered in the problem.

I agree. I wasn't entirely sure what to do with the table either.

I took that chart as how the "shock" from the blast traveled with respect to time. At t=0, the bomb goes off. As t increases, the blast radius would increase as the "shock" traveled outward. From there, we could estimate E of the "shock" over time and distance. I took E decreasing as energy dissipated.

Similar to what they said a little lower. The order of magnitude stayed the same. I took the decrease to be roughly due to a dissipation of energy.

I really liked having 4 data points here. Seeing that there was clearly a similarity in the values of E I got for each point really helped me sense that I was on the right track.

I really liked doing this problem...it made sense, but still needed thinking.

So close...forgot to change ms to s and got 10^7 instead.

I did the same thing! I was so proud of my finding it was R^5 and I didn't even see the time scale. I wonder how many people did that.

Yeah I made that mistake too. So many units to keep track of in this problem..

Oh, ha! That explains why my answer is so off!

Nice, me too!

Just curious, what exactly is the ratio of kilotons of TNT to energy. Does TNT always have the same energy density?

Whoa! This just sounds scary, although an atomic blast always has on it's own too.

That is pretty cool.

This is really cool how close it got.... and a lot of TNT...

Triumphs over the US government?

6.055J/2.038J (Spring 2010)

Solution set 6

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Problem 1   Guessing an integral using easy cases

Use easy cases to choose the correct value of the integral

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx. \tag{1}$$

☐ $\sqrt{\pi a}$

☐ $\sqrt{\pi/a}$

> The most useful special cases here are $a \to 0$ and $a \to \infty$. When a is zero, the Gaussian becomes the flat line $y = 1$, which has infinite area. The first choice, $\sqrt{\pi a}$, goes to zero in this limit, so it cannot be right. The second choice, $\sqrt{\pi/a}$, has the correct behavior.
>
> The limit $a \to \infty$ gives the same conclusion: The first choice cannot be right, and the second one might be right.

Here it is, for the memo due Thursday.

I definitely enjoy starting with some easier problems like this one. Sometimes the week's readings get me a little confused, but problems like this ease me into the work and assure me that I am learning and do remember important facts.

> True I also like the warm up.

Oh, I think I used dimensional analysis here too.

> I did as well, it just seemed more intuitive.

I thought this was a good problem to introduce this problem set. Some people were complaining in the comments about how they don't think they could do it without choices, but in real life easy cases isn't really used to get an answer. It's much more useful for checking to see if an answer is incorrect which is very useful in real life.

> Yeah I agree with you because this question is meant to test use of easy cases. Therefore being able to rule out an answer as incorrect would be the best you can do with easy cases, as seen in this problem.
>
>> I tried doing this with easy cases but i was biased from having seen this integral multiple times in the recent past - made just using atan easier.
>
> I really like this easy cases method- It comes very easily to me, compared to the other methods we've learned. I found this pset much easier than previous ones.

can you do this with the FWHM concept you showed in class today? I guess I do not know what this graph looks like without my calculator.

in problems such as these, would anyone ever split their "answer coins" in any way but all or nothing or half and half? in other words, what would be the justification for something like 7:3?

> to clarify my comment: i feel like if you know it, it's 0:1, or if you don't, it'd be like 1:1. i don't know how you can "kinda" know it.

Hahahahahah. Wow. I had the right method, but was totally deficient about what e^0 was. Oops.

so this is useful when you have choices, but was there a way of getting an approximate answer without choices?

> We did something like this in class (I think before spring break). It dealt with creating units for x.

It might be helpful to note in the problem that this is the Gaussian too.

I approached this just substituting values for a... I used a=1 and then picked an arbitrary value of a=5... a little different but still came out to find the same answer.

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>
> The limit $a \to \infty$ gives the same conclusion: The first choice cannot be right, and the second one might be right.

How do we know this?

> You don't have to know it, but you might recognize it as the form of the Gaussian distribution (PDF).

> > I'm not familiar with the PDF. Does Gaussian just refer to the graph of the integral?

> > > The function is a form of a Gaussian function with particular parameters. You would graph the function, not the integral.

> > > > I would not have thought of this, although I think I could have figured out that it should be a line at 1 and not 0.

This is useful, I didn't think of changing a all the way to 0

> This is the only case I did. You don't really need to do both, since there are only 2 choices. If you can cancel out one, using even just one method, you know you're home free. And for me, it was pretty easy to realize the area under a straight horizontal line is infinite.

Oh I didn't even consider a = inf, a=0 just came much more naturally.

> Using a = infinity would be a second check if you used a = 0 first. You come to the same conclusion, but it's nice to see that it works twice.

what do you mean by this?

> Sorry, "Gaussian" is just jargon for the e^-something*x^2 function.

I first thought about this problem as "a increasing" and "a decreasing", and noted that a increasing should decrease the integral (because it makes the exponent more negative everywhere), and vice versa. It's essentially the same concept, but sometimes infinite and everywhere-zero integrals can be tricky.

I got the answer, but not that way. I depended too much on multiple choice, and pattern matching.

> Yeah me too

Ahh. Goodness...so simple!

wow I didn't think of this before, i thought this is a very cool approach !

That's exactly how I did it! (Though I did cheat with mathematica first...)

> I did it exactly this way too–but I had a brain fart, so when I thought to myself that as a approches 0, I somehow said that area approached 0 too. I used the easy cases method right though

This makes complete sense and I think it is a very good easy cases problem.

Doesn't the second choice go to infinity as a-> 0?

> yes, which is what we are looking for...from the line above.

that's basically the reasoning used. I guess i should have considered the a -> 0 case too so that I could have been some certain about my answer.

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> The limit $a \to \infty$ gives the same conclusion: The first choice cannot be right, and the second one might be right.

I did this a similar way, but I thought just in graphical terms. I know what exp(-x^2) looks like, and I know what adding a positive constant in front of the x^2 does, so I reasoned that the integral value had to get smaller.

I always forget to check the other case. I usually just stop when I have one easy case result. I guess I should get in the habit of always trying mins AND maxs.

I considered a->0, but not a->inf. I got the right answer anyway.

I was able to arrive at this same conclusion, but if I didn't have the two options I think I'd be stuck.

> I came to the same conclusion but I pieced together equations that I knew like e^cx and e^x^2. I didn't even think about taking the cases where a went to zero or infinity because I was so focused and so used to dealing with the x.

> > I think in general its usually easier to look at the solutions, and what is variable. Noticing that a was either on the denominator or numerator, you can immediately go back to the integral and see which one corresponds to the right behaviour of the integral.

> > > I think this example was very simple for that reason.

> > I agree. I also started to focus on the exponent rather than on a by itself. It becomes clearer that this is the thing to do later on in the pset though.

Is there a way to derive the pi^1/2 in the answers using easy cases as well?

I looked at the cases where a is 1 and x is not squared. I guessed that the answer would take a similar form and since the integral of e^(-ax) is of the form 1/a I guessed that the answer here had to have a fraction in it.

I used dimensional analysis for this problem (even though it said to use easy cases). If I assume the dimensions of x is length, then the dimensions of a have to be l^-2. The dimension of the integral is l, so the dimension of the answer has to be x^-1/2. This matches the solution obtained using easy cases.

> but since we already did this in class, I think we were trying to use different methods to check our answer here...

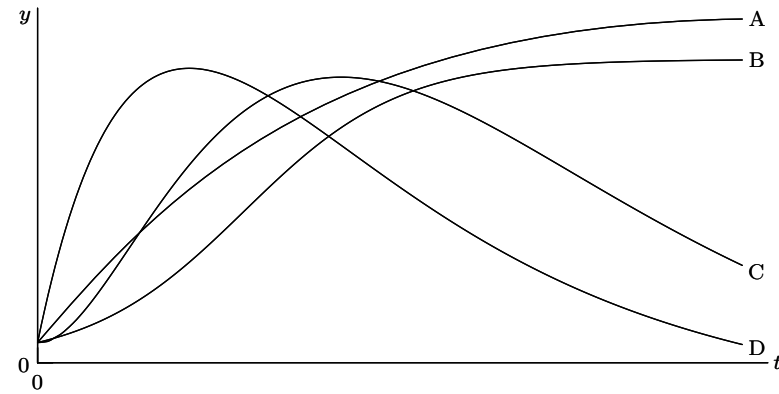would "rough graphing in my head" be considered a use of the "easy cases" method?

> I think it would be considered estimation broadly, but I don't know about calling that "easy cases"!

Problem 2   Differential-equation solution

Which sketch shows a solution of the differential equation

$$\frac{dy}{dt} = Ay(M - y),$$

where A and M are positive constants?

- [ ] Curve A
- [ ] Curve B
- [ ] Curve C
- [ ] Curve D

> Use easy cases by choosing the solution that behaves correctly in all the easy cases. Here, one easy case is small t (t ≈ 0), when y is small – in particular, small compared to M. Then the M − y term is approximately M, making the differential equation
>
> $$\frac{dy}{dt} = AMy \propto y.$$
>
> It is the equation for exponential growth (since AM is positive). Therefore, for small t, the curve should follow an exponential, which is concave upwards ('holds water'). Only curves B and C satisfy this test.
>
> In the large-t extreme case, y approaches M. Then dy/dt = 0, which makes y constant (consistent with the assumption y → M). Among curves B and C, the only curve that becomes flat is curve B.
>
> As a further piece of evidence in favor of curve B, the derivative dy/dt must always be positive. Why? For it to be negative, y would have to exceed M. But when y reaches M, then dy/dt becomes 0 and y stops changing. Therefore, y can never exceed M. Contradiction! Therefore, the derivative cannot be negative. Curve C, however, has a region of negative slope.

This is just an autonomous equation from 18.03. They are really easy to solve if you can sketch out the behavior of y' vs y.

I've never taken 18.03, so some of this class has actually been quite difficult for me.

how would it change if they were negative

I think it would depend on the signs of A and M relative to each other. For instance, if A was positive but M was negative, then dy/dt would always be negative (when y > 0). But if A was also negative, then I think dy/dt would always be positive (again when y > 0).

I guess this is a pretty standard differential equation problem. I thought about it the wrong way though - I should have seen M as a stable point (y greater, and it goes back down, y less and it goes up), and thus B would be obvious. Oh well.

I didn't quite do this, but I wound up setting the slope to zero and finding which values of y had a flat slope... in particular, y=0 was one of those values so I also came to the same conclusion that it must be concave upwards (or rather, flattish at y=0).

Are we assuming that y is also approaching 0 in this small case?

Just that y is smaller than M such that in M-y, the y can be ignored

How do we know y is small when t is small?

We know that y is small when t is small because A and M are constants. Therefore, if you let t be small, y is on the other side of the equation, and has to fluctuate accordingly.

The two cases I looked at were where dy/dt=0. looking at the equation, this happens either as y approaches 0 or y approaches M. so it has to be choice B. also, this is the logistic growth equation, which has an s shape.

I dont quite get this. Why is it proportional to y if y is really small and insignificant .

well since A and M are constants, the only variable is y. therefore it is proportional to y.

That is an interesting way of solving this. I tried to use easy cases, but it never occurred to me to use proportional reasoning as well.

When things are small it means we can cut out second order terms (hence proportional to y), but we still need some sort of independent variable to relate everything together.

agreed. if y is tiny, why is it not "porportional to zero?"

This alludes me; I don't quite understand the proportionality.

This is the same method I used to analyze it. I definitely think it's the simplest and fastest way to do it.

I made a stupid mistake here and forgot for a second that we were looking at dy/dt so I said the slope for small y was linear, which it clearly is not. I really liked this problem though.

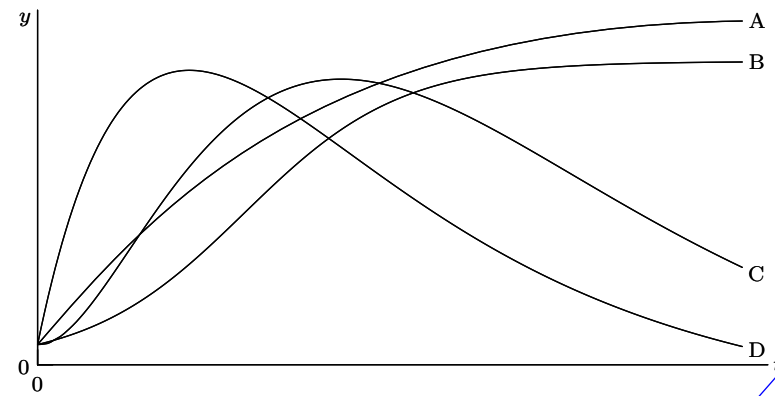I made that mistake at first also, but then I had to go back and change it.

I am not sure I even remembered this equation, but I agree now that this is a simple way to negate some options.

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I really liked this explanation. I missed the fact that at a small t the curve will show exponential growth. But it makes sense as it is rewritten here.

Didn't think of this.

I just figured that it should be a positive growth.

how about D, it also shows some exponential growth in the beginning

This is an interesting solution! However, I'm still confused as to how this is an example of the "easy cases" we've learned.

It's easy if you let M and A = 1 and look at your solution. You still arrive at the same solution but it might be a bit easier to see.

Its an example of easy cases because we are looking at smal t and large t.

I feel like easy cases is y=0 and y=M, and don't worry about limits... isn't that technically calculus? :)

please don't highlight the entire solution box in the future.

thanks.

agreed. it makes it difficult to mark other areas of the solution. That being said I agreed with the solution pretty easily.

I used similar thinking, but it was clearly faulty, since I was deciding between curves A and B.

So did I, I still don't really understand how they were choosing between B and C, I was choosing between curves C and D, oops.

How do you know what happens for large t. I basically looked at the the effects on the slope of the curve from small and large y. I still got the right answer though.

I agree. I thought that large y meant that the m term disappeared completely. I don't know why it can only approach M.

Could you explain why y approaches M in the large-t case?

Hmm i forgot about extreme cases here... i ended up looking at the zeros. I was a little off though.

Once I narrowed down to curves B and C (using a slightly different method from above), I mistakenly looked at the curve for large y and not large t. Thus, my answer was curve C based on that wrong assumption. However, could someone please explain why y approaches M for large t.

I did the same thing, apparently I've forgotten how to think about this type of differential equation.

b/c the small was made easy y comparing y to M, it holds to do the same for a large case of t. Make y–> M in order to simplify your max case into an easy case...i think...

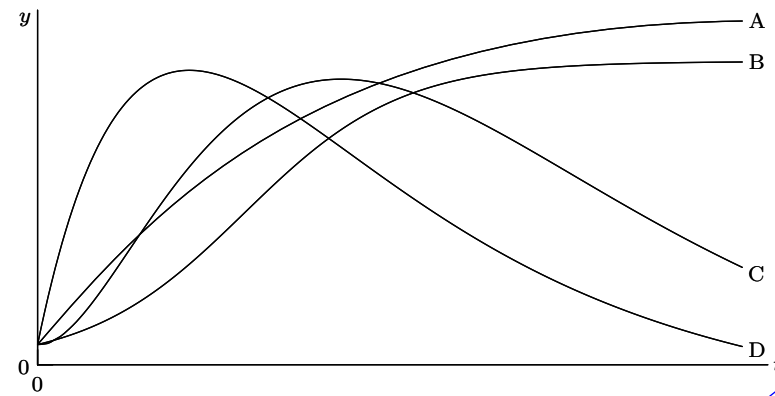Couldn't we also assume the case as a y parabola?

exactly what I was thinking!

Problem 2 Differential-equation solution

Which sketch shows a solution of the differential equation

$$\frac{dy}{dt} = Ay(M - y),$$

where A and M are positive constants?

☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

Use easy cases by choosing the solution that behaves correctly in all the easy cases. Here, one easy case is small t ($t \approx 0$), when y is small – in particular, small compared to M. Then the $M - y$ term is approximately M, making the differential equation

$$\frac{dy}{dt} = AMy \propto y.$$

It is the equation for exponential growth (since AM is positive). Therefore, for small t, the curve should follow an exponential, which is concave upwards ('holds water'). Only curves B and C satisfy this test.

In the large-t extreme case, y approaches M. Then $dy/dt = 0$, which makes y constant (consistent with the assumption $y \to M$). Among curves B and C, the only curve that becomes flat is curve B.

As a further piece of evidence in favor of curve B, the derivative dy/dt must always be positive. Why? For it to be negative, y would have to exceed M. But when y reaches M, then dy/dt becomes 0 and y stops changing. Therefore, y can never exceed M. Contradiction! Therefore, the derivative cannot be negative. Curve C, however, has a region of negative slope.

but if the slope is 0 (y=m) this could be a maximum right? In which case, C could be a solution.

This was my assumption and I also chose C, however The graph follows from the equation with y as the dependent variable. As time increases, and the derivative gets to zero, the y will no longer increase any more, which is what it would have to do in order to have a negative slope.

i hadn't even thought of this!

This is a great explanation for further proof but I would have never thought to reason this myself!

Why does it have to stop changing when it becomes 0? If there was a local maximum it would also be zero but then the slope would continue to change afterwards.

That's an interesting point. The reason is that the equation is first order. The only way for y to change is for dy/dt to be nonzero. Once dy/dt becomes zero, y won't change, and dy/dt will stay nonzero.

With a second- (or higher-) order equation, that argument doesn't work. For example, in spring motion, $dy/dt = 0$ at the extreme of the motion, but d^2y/dt^2 is not zero, and it makes dy/dt eventually nonzero, which means y can change (so the spring oscillates).

I think I'm still confused about why y can't grow larger than M.... is it because y=dy/dt is a function of y? As I was thinking about this, I realized I was thinking of the situation where dy/dt is a function of t, and I'm not sure I have any intuition for when it's a function of y.

I'm glad you answered that. It might be good to mention in the solutions that this is all due to it being a first-order system.

Also, don't you mean that once dy/dt becomes 0, y won't change, and dy/dt will remain 0? you wrote "stay nonzero"

Ah, I see. I was debating between D and C, but this makes sense.

I completely got this one wrong. I figured that the derivative was a linear term (AM)y minus a quadratic term (AM)y^2, and that there was some point where the graph would go pos–>neg. I realize I was really off after reading this explanation.

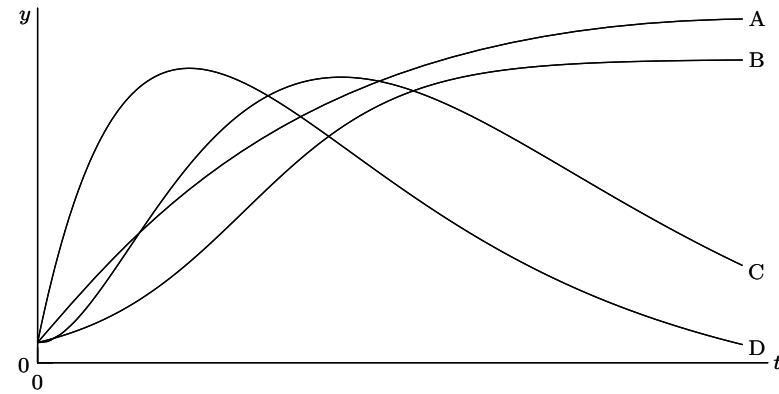Me too. I thought it was C or D, but I was pretty confident it was C.

Makes sense. I just misinterpreted the graph, by think of y as the dependent variable of the equation, instead of just the slope.

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could you explain this in class? I reasoned that if y goes to infinity then the slope should be negative after multiplying out the equation to get dy/dt=ayM-ay^2. The negative term will dominate.

if y rises above M, then yes the slope will be negative. But since all of these curves start between 0 and M, the correct curve is confined between 0 and M and is always increasing. y=M is a "sink", because for all y>0, y will tend towards M with time (from below if y<M, from above if y>M). Similarly, for y<M, y tends away from 0.

But how do we know that all of the curves start between 0 and M and are always increasing. I've forgotten a lot of my 18.03 so please bear with me.

@11:31 Thank you! I get it now....I was definitely one of the people that started thinking as t gets big.

More interestingly, we can show that every solution curve is monotonously increasing or decreasing (not strictly, since there are solutions y=0, y=M).

First, note the fact that two solutions can never cross. A rough proof of this is that if two curves were to cross, then they would have different derivatives dy/dt at the same point (t,y). But dy/dt is a function of y and t (well, just y in this case, but that's ok), so every correct solution needs to have the same derivative at (t,y) as every other solution. Moreover, if two solutions did go through the same point, they would necessarily be the same solution.

Now, here's where the fact that dy/dt=f(y) comes in useful. Since the slope doesn't depend on t, any solution curve can be translated left or right, and will still be a solution. If a solution ever increased and then decreased, it would be possible to translate it horizontally such that the new curve crossed the old curve, violating the theorem "proved" in the previous paragraph. So every solution is monotonously increasing or decreasing, or constant!

At y=0, the slope=0. The only solution that is horizontal at y=0 is B.

You won't know this is true unless you see the rest of the graph (x less than 0)...maybe all the graphs move steeply back up in the y direction.

from my interpretation of the graph, B & C look like the might have a slope of 0 at y=0

Problem 3   Fog

Fog is a low-lying cloud, perhaps 1 km tall and made up of tiny water droplets (radius $r \sim 10\,\mu m$). By estimating the terminal speed of fog droplets, ~~estimate the time that the cloud takes to settle to the ground.~~

$$10^{\boxed{} \pm \boxed{}}\,s \quad \textit{or} \quad 10^{\boxed{} \cdots \boxed{}}\,s$$

To include in the explanation box: What is the everyday consequence of this settling time?

At low Reynolds number, the drag is

$$F = 6\pi\rho_{fl}\nu\nu r, \tag{2}$$

where ρ_{fl} is the density of the fluid. The weight of the object is

$$W = \frac{4}{3}\pi r^3 \rho_{obj} g, \tag{3}$$

where ρ_{obj} is the density of the object. At the terminal speed ν, the drag and weight balance:

$$6\pi\rho_{fl}\nu\nu r \sim \frac{4}{3}\pi r^3 \rho_{obj} g. \tag{4}$$

Therefore, the terminal speed ν is

$$\nu \sim \frac{2}{9}\frac{gr^2}{\nu}\frac{\rho_{obj}}{\rho_{fl}}.$$

(This calculation neglects buoyancy, which is a small effect for water droplets falling in air.)

Calling $2/9 = 1/5$ and using $\nu \sim 10^{-5}\,m^2\,s^{-1}$ gives

$$\nu \sim \frac{1}{5} \times \frac{10\,m\,s^{-2} \times 10^{-10}\,m^2}{10^{-5}\,m^2\,s^{-1}} \times 1000 \sim 2\,cm\,s^{-1}.$$

As a check on the initial assumption, let's calculate the Reynolds number:

$$Re \sim \frac{10^{-5}\,m \times 2 \cdot 10^{-2}\,m\,s^{-1}}{10^{-5}\,m^2\,s^{-1}} \sim 0.02.$$

It is much less than 1, validating the assumption of low-Reynolds-number flow.

At $\nu \sim 2\,cm\,s^{-1}$, the droplet takes $5 \cdot 10^4\,s$ to fall 1 km. A day is roughly $10^5\,s$, so the fall time is about one-half of a day. The everyday consequence is that fog settles overnight. You go to sleep with a pea-soup fog, and by the time you wake up, it's mostly settled onto the ground – and maybe evaporated as the morning sun warms the ground.

---

this one was hard

I find I am very limited by my unfamiliarity with physics equations when solving these types of problems, but once I know which ones to use I can figure out how to estimate the solution

I had NO idea how to do this problem using the terminal speed/ reynolds number.. I took my gut feeling that when fog appears at night it is usually gone in the morning and so I figured it takes about 12 hours to settle... my intuitive answer was close but lacked any mathematical proof!

it was neat knowing the consequence before trying to approximate!

We were to use this equation for low reynolds number, which occurs when the density is low, the velocity is low, the diameter is small, and the viscosity is large. Viscosity of air has magnitude 10^-3, density has magnitude 1, and the diameter is 20*10^-6 micrometers. This gives a reynold's number of 10^-3*v, so it's fair to use this drag equation. When I first thought about it though, I didn't do it this analytically - I just thought: "oh, air isn't viscous, I'll use the other drag equation".

oh! I totally make a silly mistake, I don't know why I didn't think of this! I instead just used the terminal speed equation - but I got an answer much smaller (like 24 minutes). Drag really slows it down!

> haha, I did the EXACT same thing apparently–I also got 24 minutes. I forgot to use the Reynolds number completely, and I used the drag equation we had from before. this answer here makes much more sense.

i used drag force as F rhoAv^2 and the drop as a cube. still go the same order of magnitude though so approximating still works out

can we get an equation sheet for the important things to remember like this- I always forget them and have to go and dig them back up

can you maybe list in class when reynolds numbers are high vs low?

I should have used this equation. Instead I used F=rho*A*c_d*V^2

> Yes this formula would have been very useful!

> Yeah, I made the same mistake.

where is this equation from again?

what does the two different v's mean?

> viscosity and velocity. this came from at least two different readings

> The v with a curve on the left stands for viscosity and the v with the curve on the right stands for velocity. I agree that it is difficult to distinguish sometimes.

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$$v \sim \frac{2}{9}\frac{g r^2}{\nu}\frac{\rho_{obj}}{\rho_{fl}}.$$

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are these just equations we should know, because we've taken this class?

I'm not sure where we got this either. The formula I came up with was based on the viscous drag force equation and was different.

> Yes! That equation (Stokes drag) was the subject of reading 23 (r23... on NB) and used in the reading on the conductivity of seawater (r24... on NB), which I admit was not the most easy-to-follow reading. But hopefully the application in this problem helps make that equation more meaningful.

This equation took forever to find within the readings. Maybe we could create a class list of equations from the readings? We could use NB to comment on them and better understand their complexities and relationships.

We should have a review day and go over everything that we learned, including a recap of the equations!

I didn't think to use this equation and ended with the wrong answer.

wow... forgot gravity. There goes a factor of ten.

I forgot that there were two densities in this problem, that of the air and that of water, since both of them are fluids. Oops!

The object is air? I made this same mistake.

I think the object is water and the fluid is air.

I made the same mistake. I didn't think about air as a contributing factor. That's why my answer made no sense :(

This is something that makes a lot of sense, but I didn't realize before; at terminal velocity these balance. It makes it so much clearer and I finally understand this. Thanks.

In pset 4 you neglected this 4pi/3 term...why include it here this time?

> I wanted to get a reasonably accurate value of the settling time (how long the droplet takes to fall) because otherwise the everyday consequence doesn't work out so well.

where did the (2/9) derive from?

In the previous step, we discovered that at the terminal speed v, the drag and weight balance. We then solve for the terminal speed v, by dividing the right-hand side by everything on the left except the terminal speed v. Since the pis cancel out, 4/3 divided by 6 is 4/18, or 2/9!

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I don't think this is the same velocity that was calculated for a rain droplet, is it? Why does this have more terms?

> When i used the terminal velocity for the rain droplet instead with fog-droplet radius, it only took 2.5 hours to settle. What's wrong with this?

>> The raindrop speed was calculated assuming high Reynolds number, where the drag coefficient is approximately 1 (equivalently, where the drag force is proportional to v^2). But the Reynolds number is too small for that assumption to be valid.

I was slightly over on this, which made me underestimate the time.

Is this a necessary check?

> Checks are always a good thing!

> yes. yes. yes. It should have been done in the reading, too. Always check assumptions!

>> What is the check intended for? Fog movement doesn't seem like something that is getting turbulent.

I gave mine .5 for a sphere. It didn't work that nice

> My final answer didn't work out nicely either; it was off by a HUGE factor. Instead, i resorted to guessing the time fog takes to settle, and backsolved a speed of 10cm/s.

For this problem I used the formula for terminal velocity of a raindrop from 2 psets ago, and then changed the radius....however my answer was about an hour, guess I did something wrong

So I attacked this problem first using the everyday example. But I was steered pretty wrongly. I forgot to analyze fog at 1km and used my example form home. During which I only saw fog settling in about an hour, as it wasn't 1 km above the ground.

ah, when you said everyday consequences i for some reason thought you meant a much more dramatic conclusion

This is a pretty interesting consequence, definitely something we don't think about often!

> This is not the way I thought of it at all. Fog is one of those things that is always around you but you never really think about.

>> Do all lay lowing clouds act as fog does from this demonstration? Is it something in the property of the cloud or is it just its relative distance to the ground that makes a cloud lower itself onto the ground?

I really like how the problem had us check to see if our approximation was realistic. It's definitely just as important as the approximation itself.

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doesn't the fog settle because the temperature drops also and allows the water to condense? not just because it is falling?

Yeah, that's how my parents explained it to me when I was little. Some insight, anyone?

Does the fog also get 'burned' off by the morning sun? Or is this just something that people say.

I've never heard the word "burned" used..they probably mean evaporated.

The temperature drops, which makes the water condense out of the air (formerly it was water vapor, i.e. a gas). That condensation is the fog droplets! But that is a separate process from the settling, which is gravity pulling the droplet downward.

I just took the question to mean what happens in the morning, I too have heard that fog "burns" off...when it gets hot enough the dew point rises and the vapor evaporates

Problem 4   Hyperbolic-function sketch

Which graph is $\ln \cosh x$ (where $\cosh x = (e^x + e^{-x})/2$)?

☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

Use easy cases: $|x| \to \infty$ and $x \to 0$. In the $x \to \infty$ case, $\cosh x \approx e^x/2$, so $\ln \cosh x \approx x - \ln 2$. In the $x \to -\infty$ case, $\cosh x \approx e^{-x}/2$, so $\ln \cosh x \approx -x - \ln 2$. In other words,

$$\ln \cosh x = |x| - \ln 2 \qquad (|x| \to \infty). \tag{5}$$

This is enough information to select curve C.

But let's check that curve C is correct also in the $x \to 0$ case. There, a Taylor series for e^x gives

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{1}{2}\left[\left(1 + x + \frac{x^2}{2} + \cdots\right) + \left(1 - x + \frac{x^2}{2} - \cdots\right)\right]. \tag{6}$$

The result is $\cosh x \approx 1 + x^2/2$. For the logarithm, the Taylor series is

$$\ln(1 + z) \approx z. \tag{7}$$

So,

$$\ln \cosh x \approx \ln\left(1 + \frac{x^2}{2}\right) \approx \frac{x^2}{2}. \tag{8}$$

Thus, near the origin, $\ln \cosh x$ looks like an upward-facing parabola (concave up). Curve C passes this test.

What exactly is cosh?

* Hyperbolic cosine:

coshx= 1/2 *(e^x +e^-x)

In a circle with radius 1 x^2+y^2=1, you can plot x=cos t and y = sin t.

In the hyperbola x^2-y^2=1, you can plot x = cosh t and y = sinh t.

Hence, hyperbolic cosine and hyperbolic sine.

I did use the easy cases for this one. But when I analyzed them, I did not have enough understanding for graphing to narrow down much. In the end it came down to a guess.

Same. seeing the cosh function just threw me totally off.

When doing my approximating, I left out this - ln 2 term which prevented me from figuring out whether the answer was curve B or C. I wish I hadn't dropped what I thought was extra luggage.

To decide between curves B and C, you can use the x -> 0 easy case, which says that the curve looks like an upward-facing parabola at the origin. That test knocks out curve B.

I looked at easy cases for the derivative. As x-> infinity, f' goes to the indeterminate zero times infinity. Does this tell us anything?

I dont get the coshx notation...

do you mean x -> 0?

No this is correct, but not shown on the graph, it would be in the third quadrant. I didn't look at this case either because the graph does not show this area.

No this is correct, but not shown on the graph, it would be in the third quadrant. I didn't look at this case either because the graph does not show this area.

x -> negative infinity. I missed the - sign too.

I don't think I used easy cases to do this problem.

I used a similar approach, but I only addressed the +infinity situation, and not the -infinity situation, probably because the graph doesn't show that side. Why is it important that we look at the -infinity situation here, but not in other problems?

I just used the fact that y x at high x. I then took the derivative of the function (not nearly as hard as it looks) and found that at x=0, the slope was also zero
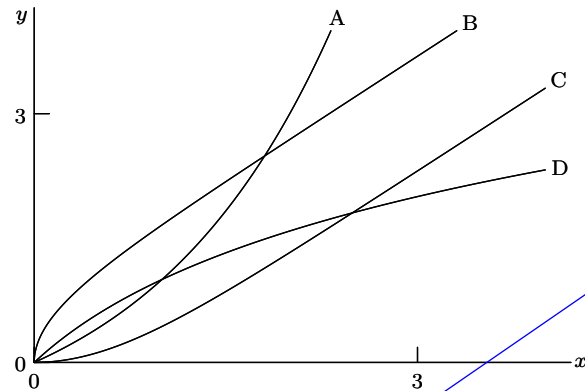
This is an interesting idea. I like the notion of looking at derivatives as easy cases, because sometimes they can be.

This is how I did it too. I first eliminated A and B by plugging in x=3 (y should be less than 3) and then by realizing that the slope should be close to 1.

I did the exact same steps...but I confused the infinity and negative infinity cases...oops...

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Thus, near the origin, $\ln \cosh x$ looks like an upward-facing parabola (concave up). Curve C passes this test.

This one was a little to math intensive for me, I had to get help on this problem. I'd never heard of cosh before really.

Interesting. I chose x=3 as my easy case, where the cosh function goes to 20 and ln coshx goes to a little over 2. C was closest so I chose C.

I wasn't completely convinced that this meant it was C. I did this same analysis and came up with the right answer, but I took this piece of information to mean that the end behaviour would have to be linear.

> I think this is indistinguishable from curve B. I used this to narrow it down to B and C, but they both look like the slope goes to 1 as x goes to infinity. I guess you could use the -ln(2) to guess where the y-intercept would be, but i didn't think that was accurate enough to guess off the graph.

>> I sort of agree. When I first read the solution, I wasn't convinced. But come to think about it, for large x, the difference between B and C is the offset. Since teh solution comes out to x - ln(2), the curve that has a negative offset is correct.

>>> when I first read the solution, I didn't see the '-ln2' ... I feel like ln[cosh(x)] = |x| narrows it down to either B or C and ln[cosh(x)] = -ln2 narrows it down to C or D.

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> This statement isn't helpful. I see how the negative y-int. as x->inf would point to C, but not how the x->-inf tells us anything, since we don't know what that part of Curve C looks like.

I should have used this method. The method I used was more complicated and had more room for failure.

Not really good w/ log manipulation. I think it threw me on this problem. Reasoning is solid though.

ooh i forgot about those

I didn't think about incorporating Taylor series into this problem. That is actually really cool.

Does this count as an easy case? I wouldn't have known how to deal with this equation even in this case.

Woah I did not even consider a Taylor series expansion, but it's nice that the math works out so nicely.
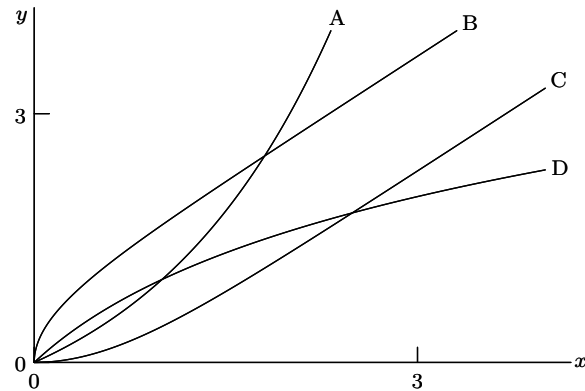
> I tried to use Taylor series at first, but my math gave me unusual terms. Good to know that I was at least heading in the right direction.

>> yeah I didn't think of this at all!

totally got this competely wrong

Problem 4   Hyperbolic-function sketch

Which graph is $\ln \cosh x$ (where $\cosh x \equiv (e^x + e^{-x})/2$)?

☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

Use easy cases: $|x| \to \infty$ and $x \to 0$. In the $x \to \infty$ case, $\cosh x \approx e^x/2$, so $\ln \cosh x \approx x - \ln 2$. In the $x \to -\infty$ case, $\cosh x \approx e^{-x}/2$, so $\ln \cosh x \approx -x - \ln 2$. In other words,

$$\ln \cosh x = |x| - \ln 2 \qquad (|x| \to \infty). \tag{5}$$

This is enough information to select curve C.

But let's check that curve C is correct also in the $x \to 0$ case. There, a Taylor series for e^x gives

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{1}{2}\left[\left(1 + x + \frac{x^2}{2} + \cdots\right) + \left(1 - x + \frac{x^2}{2} - \cdots\right)\right]. \tag{6}$$

The result is $\cosh x \approx 1 + x^2/2$. For the logarithm, the Taylor series is

$$\ln(1 + z) \approx z. \tag{7}$$

So,

$$\ln \cosh x \approx \ln\left(1 + \frac{x^2}{2}\right) \approx \frac{x^2}{2}. \tag{8}$$

Thus, near the origin, ln cosh x looks like an upward-facing parabola (concave up). Curve C passes this test.

Here's an alternate solution using easy cases:

At x = 0, coshx = 2/2 = 1. Thus, y = ln(coshx) = ln(1) = 0. All 4 curves fit that description so no elimination yet. If we take the derivative of y and see how dy/dx reacts for large x, we can eliminate curve A since curve A increases very quickly and we can see from simple calculus that this curve has a constant slope at large values of x. I chose to eliminate curve B because according to the graph, curve B has a value of y > 3 for x = 3, and we can see that ln(coshx) has a value of y < 3 for x = 3. So know we decide between curves C and D. If we take the second derivative, we see that it is = 1 > 0, which implies positive concavity. Curve D has negative concavity, so I eliminate that. Curve C has positive concavity, so I choose curve C.

> Hmmm. Cool process of elimination.

I used easy cases in a different way, by letting x=1 and getting a quick approximation. Since we have x and y =3 marked on the axis it was easy to check the curves and I arrived at the same answer.

> That's really interesting, I didn't think of it that way.

> I did the same thing, by calculating whether or not the formula would be greater than 3 when x=3. I suppose if there were no numbers labelled on the graph that would be a little harder to do

Totally forgot about this. Should have done it.

> I think using a bit of calculus is easier (and less error-prone) than using a Taylor series expansion. At least that is the case for me.

>> Could we talk about the smart use of Taylor series in class?

How do we know this is C rather than A? Isn't A also an upward-facing parabola?

> I think because we divide by 2 we expect it to be flatter than A.

>> I also went with the "lets enter a few more numbers" approach and if you take the x-ln2 approach you can enter a few numbers and see that it must be C. Especially since values for all curves are given for x=3!

I got this one right, but for a possibly shady reason. I know ln(e) =1, so I figured any function of e transforms into something roughly linear when you take the ln of it. Using the property of logs, I guessed that C was the only curve that does this.

> Curve B also has a linear portion though... You would have to decide whether the curve should be concave up or down at x=0. Also, curve B is above the y=x line, whereas curve C is below, in the limit of x -> infinity.

Problem 5   Guessing an integral

Choose the correct value of the integral

$$\int_{-\infty}^{\infty} \frac{1}{a^2 + x^2}\, dx, \tag{9}$$

where a is a positive constant.

- [ ] πa
- [ ] π/a
- [ ] $\sqrt{\pi}a$
- [ ] $\sqrt{\pi}/a$

The easiest special case is $a = \infty$. In that limit, the integrand is zero everywhere, so the integral is zero. The first and third choices are therefore incorrect.

To decide between the second and fourth choices, use the special case $a = 1$. The integral becomes

$$\int_{-\infty}^{\infty} \frac{1}{1 + x^2}\, dx$$

The integral is $\arctan x$. At ∞ it contributes $\pi/2$, and at $-\infty$ it subtracts $-\pi/2$, so the integral is π. Only the second choice, π/a, has the correct behavior when $a = 1$.

can we go over the different ways to think about this

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This was actually one of the integrals I had to memorize in school, so I knew the answer without even guessing. The explanation below is more satisfying though.

sort of, but it's only applicable when you're given four well-constructed multiple choice answers. Otherwise, dimensional analysis to give the 1/a and the a=1 trick could do it for you.

It was also the case for me that this is one integral drilled into my head in school, so I knew the definite answer without needing to do approximations. I like that there is a way to go through it if I ever forget the integral.

This one, at least, I did right. ><

I used dimensional analysis for this one too...

I decided moving a to zero and got a similar answer for the integral of 1/x^2.

I reached the same conclusion using dimensional analysis but then I got stuck figuring out if it was answer choice 2 or 4.

yeah, that's the part dimensional analysis can't do for you.

me too. I resorted to approximating an area using rectangles and triangles, but that gave me answer 4 instead of 2.

I guess I never thought to vary "a" instead of "x".

i think that's one of the points the pset was trying to get across. i varied x for the first problem, but thought about a for this one instead.

instead of using logic such as a-> to infinity, i prefer to think in terms of. as a increases/decreases... is this an okay application of "easy cases"?

I like this approach

It was really helpful for me to go over this problem in class... it helped me better understand how to use easy cases in approaching this problem.

Whoops. That seems really obvious in hindsight now.

Wow..this problem is a lot easier than I made it out to be. This was the problem that took me the most time on the problem set and I was no where close...

I thought it looked familiar to something...

I still don't quite understand a lot of these integral problems; I feel like that's a hard thing for me to grasp in this estimation course.

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This seems to illustrate that there is a limit to easy cases. integral of 1/(1+x^2) dx is certainly much easier than 1/(a^2+x^2), but it still doesn't help much if you're trying to do back of the envelope calculations and don't happen to have an integral table ready. Also, why doesn't a=0 work?

Unless you happen to have memorized this particular integral. I had used it enough times to recognize that there was something that looked like arctan of x going on here right from the beginning

yeah, I used this exact same method–the case of a approaching infinity is easy, so you can easily eliminate choices 1 and 3. I happened to recognize this integral, so I got lucky...but if you didn't remember this integral then i guess you were out of luck and had to guess between choices 2 and 4.

I agree, I couldn't choose between 2 and 4 b/c I did not realize this was arctan.

Agreed, it's still not very easy.

I did use a=0, and graphed the function. It has infinite area under it, so I think you draw the same conclusions as the first part of the solution..

You can also keep 'a' as a constant by realizing that the integral comes out to (1/a)arctan(x/a) and then plugging in infinity. $\pi/2a$ - (-$\pi/2a$) = π/a

This was tricky! Forgot my 18.01 integrals for a second there...

Bah! I didn't even think of that.

I relied on my intuition and figured that a sqrt(pi) on top just didn't seem right, so I guessed the right answer.

Had the option for a sqrt(pi/a) been present, I fear I likely would have chosen that over pi/a.

heh, i did the same...couldn't reason out why the sqrt(pi) was there.

but: sqrt(pi/a) would not have the right dimensions, so that wouldn't make sense either.

the choice is sqrt(pi)/a not sqrt(pi/a) ... so the dimensions would work even though it's wrong.

...i think.

Haha. Me too. I figured out a=1 was the best easy case, but for some reason I forgot the integral would be arctanx.

Yah, i did not recognize the arctan!

This is something that I probably should have remembered from my math but slipped my mind

I definitely didn't remember this either. I tried to use lumping to come up with a good area underneath the curve in terms of triangles, and it worked fairly well.

I was pretty stoked that I actually remembered this technique

I got suck here because I couldn't remember the integral of arctan() . Was there any way of solving this without knowing that?

I recognized this, but just reasoned that getting a sqrt(pi) out would be difficult, so I concluded 2 as well.

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The easiest special case is $a \to \infty$. In that limit, the integrand is zero everywhere, so the integral is zero. The first and third choices are therefore incorrect.

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I recognized the integral as arctan(x), but i couldn't for the life of me remember or reason the behavior of arctan(x) at either infinity, so i had to look it up.

Is there an easy way to visualize this?

I also think that the pi/2 that arctan(x) contributes is very hard to intuit. I would also love to see a way to explain/visualize this.

I couldn't figure out how I was supposed to learn something useful from the normal easy case of setting a to zero. Using 1 in this case to generate the integral of arctan really was a clever trick for this problem.

What was the correct behavior when a=1? Please state it so I can learn.

Yeah and the a isn't in the square root in any of them so it's unlikely the pi would be either

That makes sense, my reasoning for not choosing the square root was that it just looked funny for a problem so nicely written.

I couldnt pick between the two answers, but I was tempted by the "prettiness" of pi*a!

Is there some way that we could figure this out (other than Wikipedia) if we didn't recognize or memorize the integral or how to solve the arctan?

Again, I had a hard time figuring out if this was the second or fourth answer.

Problem 6   Debugging

Use special (i.e. easy) cases of n to decide which of these two C functions correctly computes the sum of the first n odd numbers:

Program A:

```
int sum_of_odds (int n) {
  int i, total = 0;
  for (i=1; i<=2*n+1; i+=2)
    total += i;
  return total;
}
```

Program B:

```
int sum_of_odds (int n) {
  int i, total = 0;
  for (i=1; i<=2*n-1; i+=2)
    total += i;
  return total;
}
```

Special cases are useful in debugging programs. The easiest cases are often $n = 0$ or $n = 1$. Let's try $n = 0$ first. In the first program, the $2n + 1$ in the loop condition means that $i = 1$ is the only case, so the total becomes 1. Whereas the sum of the first 0 odd numbers should be zero! So the first program looks suspicious.

Let's confirm that analysis using $n = 1$. The first program will have $i = 1$ and $i = 3$ in the loop, making the total $1 + 3 = 4$. The second program will have $i = 1$ in the loop, making the total 1. Since the correct answer is 1, Program A has a bug, and Program B looks good.

This one was a fun problem, even for someone who doesn't code much.

I found this one really trivial, just one case of n=1 and you can get the answer

Maybe easy, but a nice confidence builder.

I agree; this was a very easy problem, but I think it's actually a good way to teach someone who doesn't know how to code about coding. I think you can figure out a bit about the actual coding process from this problem. Very nice.

I thought this was the simplest problem on the pset, but it paralleled nicely with what we did in class

I am totally unfamiliar with programming but when a friend gave me a hint for how to solve this problem, it was definitely much simpler than I thought!

I'm glad to see something familiar to me (code).

it took me forever to find the difference (i'm not good at recognizing differences), next time could you highlight the difference in the lines?

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wouldn't this return a -1 when with n = 0, and shouldn't the answer be 0? If I am right, how are we supposed to know that the program is right if it is wrong on one of our easy cases?

You are right - that condition will return a -1, which is NOT less than i (which is 1).

With n = 0, this condition that i <= 2n-1 will not be met, so the body of the for loop is not executed. The total that is returned is just 0, since that's what total starts at.

It may be a little unfamiliar if you're not used to seeing code.

using n=0 here seems suspect here since normally you count at least one odd. And negative odd numbers wouldn't work. Nevertheless a very intuitive problem solved by simple cases

This was the logic I used, I'm pretty happy I did a problem right (even though there were only 2 options to choose)

I got this problem incorrect because I'm not familiar with the code....

The important thing about this problem is knowing that computers start indexing at zero whereas our gut intuition is to begin at 1.

That doesn't really matter here. The for loop dictates that we start iterating i at 1 anyway.

you might note how the second program behaves for i=0. counting from i=1 to -1 by +2 equates to not running at all, I assume?

I had to rerun through the numbers several times to finally get the correct results when I was doing the problem.

It's good to see I got this one right on the head. I used the same analysis as well.

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I just tested n = 0, is that enough seeing as there are only two choices and one of them did not work with n = 0, or should we always check multiple values?

I used the same test case. A simpler reasoning would be if you are counting odd numbers, you can't subtract from the lowest one, which is 1.

yup

I did this with n=2, just because, as we established in class, 0 and 1 are sometimes 'special cases' and I didn't want to risk them outputting the same answer and wasting my time.

i used n=4 for the same reason...also because I was able to better understand it with 4

Problem 7   Damped, driven spring

A damped, driven spring–mass system (e.g., in 18.03, 2.003, 2.004, 6.003, and maybe also 8.01) is described by the differential equation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 e^{i\omega t}, \tag{10}$$

where m is the mass of the object, b is the damping constant, k is the spring constant, x is the displacement of the mass, ω is the (angular) frequency of the driving force, and F_0 is the amplitude of the driving force. The solution has the form

$$x = x_0 e^{i\omega t}, \tag{11}$$

where x_0 is the (possibly complex) amplitude.

Which graph, on log–log axes, correctly shows the transfer function F_0/x_0? Don't solve the differential equation – use an approximation method to guess the answer!

Curve A

Curve B

Curve C

[In writing the solution, I realized that I made a mistake in the problem statement by asking for F_0/x_0 (input/output) instead of x_0/F_0 (output/input). Additionally, I should have used absolute value and asked about the magnitude of the transfer function $|x_0/F_0|$. I'll write the solution as if I had written the problem correctly. Apologies if you spent extra time because of those mistakes!]

Use easy cases. At low frequencies ($\omega \to 0$), the spring moves very slowly, meaning that derivatives with respect to time become tiny. Therefore, the time-derivative terms $m(d^2x/dt^2)$ and $b(dx/dt)$ become much smaller than the kx term. The remaining equation is

$$kx \approx F_0 e^{i\omega t}. \tag{12}$$

With $x = x_0 e^{i\omega t}$, the transfer function x_0/F_0 is $1/k$. This function is independent of frequency, so the curve must be flat at low frequencies. The only curve that matches this criterion is curve A.

As a check, let's try really high frequencies ($\omega \to \infty$). Then the second-derivative term $m(d^2x/dt^2)$ is the dominant term, so the differential equation simplifies to

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Using $x = x_0 e^{i\omega t}$ gives

$$-mx_0\omega^2 = F_0, \tag{14}$$

so the magnitude of the transfer function $|x_0/F_0|$ is $1/m\omega^2$. On a log–log graph, that is a -2 slope, which could be curves B or C but not curve A.

I couldn't really figure out a good way to do this problem, partly since you explicity said not to solve the differential equation- which was the only way I could think of doing it!

Yeah, I was also confused on how to approach this problem. I ended up going with my "gut" based on the limited information I remember about spring systems from 18.03.

I too was really confused on this problem (having only taken 8.01 on this list of classes). I made a guess based on low w - which sounds like it might be correct - but I don't really understand what the graphs mean in relation to the system.

8.01 would never get into things like this...

Brings back unpleasant memories of extreme cases...

I was a little confused here. What exactly does the transfer function tell you? I was able to make a reasonable guess on the homework, but I really wasn't sure what was going on in the problem.

I thought something was funny

That makes a lot more sense.

I actually followed your methods below while solving and was able to justify Curve A for low frequencies. But using the F/x, I couldn't justify the high-frequency behavior. So I went with Curve A, anyway.

me too. The F/x really threw me off in justifying every part of the graph for any answer.

I blissfully assumed it was output/input, ignored the equation, and instead just thought about a physical spring-dashpot system. When you drive it at low frequency, it behaves like just a spring, that is, with no frequency dependence. But if you try to drive it at high frequency, the dashpot absorbs all the energy and you don't get anything out.

Yikes, I solved for the original transfer function F_0/ x_0 and got the right answer (A)... I probably should have gotten (C) though since the real question is opposite, right?

I solved for the original transfer function as well, but wouldn't it only flip the graph vertically? The function should still be flat for small w (therefore A is still the best answer), correct?

I think this is why I picked C, the opposite curve of A.

I like this qualitative analysis better than all the math. This part is very intuitive.

I can't believe I never considered that option. It would have helped me greatly.

I will forever remember this demo that they did in 18.03 of the mass and spring. Makes soo much physical intuitive sense. very useful picture.

Whereas I will always remember Prof. Leeb swinging a toilet plunger back and forth in 6.131...

i agree it's usually easier to figure it out in your head this way if you know that the transfer function is getting at.

Finally 8.01 intuition paid off!

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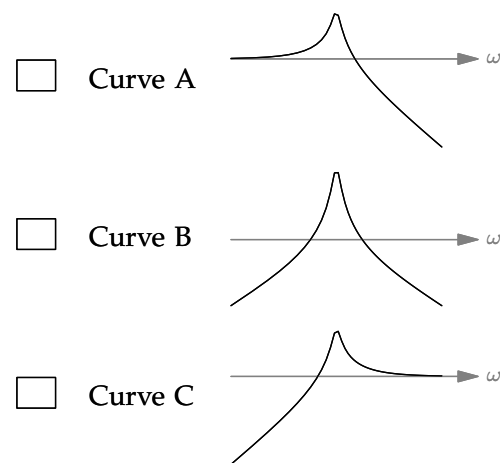
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I was sort of confused on how to start this problem, but this makes sense.

> Me too. I didn't think to use frequencies right away, but this makes so much sense.

> > I looked at this problem for a long time, and still ended up taking a pretty random guess because I had no idea how to approach this problem. I didn't think of using frequency at all. but this makes sense

cool I think I got this one. This problem looked so intimidating.

> Yeah I thought so too that it was quite intimidating. I didn't see how simple it was to disregard the time derivatives as w –> 0 which makes the problem really easy.

I think this is a good solution

I put curve A because it looked like all the second order systems we studied in 6.003, but I had no idea how to solve this problem using approximations, without doing the actual derivation.

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I like that we picked the more useful area first, but then as a double check knocked out the second two as a check

> I like how this worked out but for some reason my logic ended up with the complete opposite answer.

I didn't do this so carefully and got A, because after getting my results from the first limit, I really just wanted to check that the slope was negative in the second limit, so I didn't catch the error

I was so confused about this. since the problem was flipped I had that it was a +2 slope. I still picked the right answer because of the approximation for small omega, but I couldn't figure out why the slope was +2 instead of -2 like in the picture, which made me think it had something to do with the -w^2, even though that made no sense on a log-log graph...

What exactly does log-log mean?

> It means that log(gain) is plotted against log(frequency), the way we plotted log(drag coefficient) versus log(Reynolds number).

This problem maybe assumed too high a level of comfort with log-log plots and differential equations. But it was very well proposed and I thought the solutions were very helpful, aside from the last sentence about curve A being eliminated...isn't the answer curve A?

I got pretty lucky and guessed C on this problem just because of the definition of damping.

> I did the exact same thing haha.

Am I crazy, or does he mean it could curves A and B but not curve C? That's what it looks like to me in the graphs..

> Yeah, it must be. Curve A is definitely right.

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A damped, driven spring–mass system (e.g., in 18.03, 2.003, 2.004, 6.003, and maybe also 8.01) is described by the differential equation
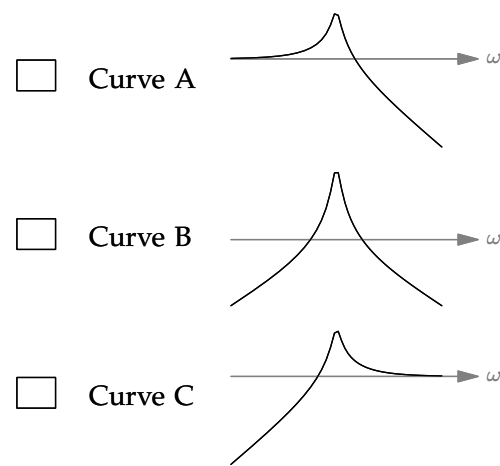
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 e^{i\omega t}, \tag{10}$$

where m is the mass of the object, b is the damping constant, k is the spring constant, x is the displacement of the mass, ω is the (angular) frequency of the driving force, and F_0 is the amplitude of the driving force. The solution has the form

$$x = x_0 e^{i\omega t}, \tag{11}$$

where x_0 is the (possibly complex) amplitude.

Which graph, on log–log axes, correctly shows the transfer function F_0/x_0? Don't solve the differential equation – use an approximation method to guess the answer!

Curve A

Curve B

Curve C

[In writing the solution, I realized that I made a mistake in the problem statement by asking for F_0/x_0 (input/output) instead of x_0/F_0 (output/input). Additionally, I should have used absolute value and asked about the magnitude of the transfer function $|x_0/F_0|$. I'll write the solution as if I had written the problem correctly. Apologies if you spent extra time because of those mistakes!]

Use easy cases. At low frequencies ($\omega \to 0$), the spring moves very slowly, meaning that derivatives with respect to time become tiny. Therefore, the time-derivative terms $m(d^2x/dt^2)$ and $b(dx/dt)$ become much smaller than the kx term. The remaining equation is

$$kx \approx F_0 e^{i\omega t}. \tag{12}$$

With $x = x_0 e^{i\omega t}$, the transfer function x_0/F_0 is $1/k$. This function is independent of frequency, so the curve must be flat at low frequencies. The only curve that matches this criterion is curve A.

As a check, let's try really high frequencies ($\omega \to \infty$). Then the second-derivative term $m(d^2x/dt^2)$ is the dominant term, so the differential equation simplifies to

$$m\frac{d^2x}{dt^2} = F_0 e^{i\omega t}. \tag{13}$$

Using $x = x_0 e^{i\omega t}$ gives

$$-mx_0\omega^2 = F_0, \tag{14}$$

so the magnitude of the transfer function $|x_0/F_0|$ is $1/m\omega^2$. On a log–log graph, that is a -2 slope, which could be curves B or C but not curve A.

The analysis here makes a lot more sense than what I tried to work out. I don't have a very good intuition into these types of problems. What was the actual answer (to the question you intended)?

i thought it was C, from what i learned in 2.004. i don't see how B could be correct.

I had no idea – so I guessed C as well because it looked familiar.

I decided it was curve C based on the physical intuition but I really don't know what is supposed to be happening in B. Is the motion just disappearing...? acting like there isn't any force at all?

B is like a band-pass filter, where only certain frequencies resonate, and high and low are damped.

So none of the solutions are correct?

so what is the actual answer to this problem? Could you also go over this one in class?

I agree with you. After reading the explanation it seems he gives argument for multiple answers. Is that what he's actually saying? Or am I just getting too caught up on the right answer and ignoring the thought process?

you mean to say it could be either A or B, but not C

I would hope so. Otherwise this question has no answer anyways he writes it.

It's A, this is a tough typo though.

Yeah, I was quite confused by this last sentence. It definitely has to be curve A, though, just based on the way second-order systems like this respond.

Why can it be two of the options?

Because only option A fits with the requirement that the function be constant for small values of omega. The second piece of evidence just confirms that it can't be option C.

so instead of working through math, i thought about it logically and realized that at small oscillations, dampening would take over and at higher oscillations, it would be easier to move which was only reflected in curve A

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Comments on page 7

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the the kinematic viscosity of air one of the numbers we should know?

It was on the table of useful constants linked from the course website.

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$$\frac{e^2/4\pi\epsilon_0}{\hbar c} \equiv \alpha \approx \frac{1}{137} \quad \text{(fine-structure constant)}.$$

Use those values to evaluate the energy of a visible-light photon. Note: The photon energy is $E = hf$, where h is Planck's constant and f is the frequency of the radiation; equivalently, $E = \hbar\omega$, where $\hbar = h/2\pi$ and ω is the angular frequency of the radiation.

$10\,\boxed{} \pm \boxed{}$ eV   ***or*** $10\,\boxed{} \cdots \boxed{}$ eV

$$E = hf = 2\pi\hbar f = 2\pi\hbar\frac{c}{\lambda},$$

where f is its frequency and λ is its wavelength. For green light, $\lambda \sim 600\,\text{nm}$, so

$$E \sim \frac{\overbrace{6}^{2\pi} \times \overbrace{200\,\text{eV}\,\text{nm}}^{\hbar c}}{\underbrace{600\,\text{nm}}_{\lambda}} \sim 2\,\text{eV}.$$

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Here is the solution set – memo due on Sunday at 10pm.

I realize I am inconsistent with my approximations especially with this pset. Sometimes, I do specific calculations and it seems to be throwing my answers wayy off. should we totally stray away from anything specific even if we are able to do the complex calculations in our head?

Personally, I thought this pset was a lot tougher than previous ones. There were several problems that I just had no idea how to address. Also, I feel like the pset problems have changed. In the beginning of the class, there were problems ranging across many topics that we solved by breaking them down into information we knew or could find out. But now all the problems are heavily focused on physics and are solved by digging through the notes to find the appropriate equation. I'm not getting anything out of this, and it took me significantly longer to do this pset than usual.

> The differences in the psets most likely have something to do with the topics we are learning. When we were learning divide and conquer it was considerably easier to be able to break down a problem. On the other hand, I agree with you about not really getting anything out of this and taking much longer on this pset. Somewhere down the line, everything did seem to change into equations.

> Wholeheartedly agree. It's cool to learn about some of the things using physics as examples, but also harder if you're not too familiar with all of it...and ends up with me losing interest or getting confused.

>> I too agree this pset was a lot tougher and does seem to be testing our knowledge of the equations we derived and not necessarily our ability to derive them through approximation. I feel like I can't "talk to my gut" for these problems since I lack a lot of intuition for the physical systems we're discussion. I understand physics is probably the most common use of approximation, but I'm really starting to get bogged down in the examples.

Totally missed these units!

> Yeah me too. My answer was ridiculously high because of it.

I found it confusing that we were given all these values here, when only the first one was actually relevant to the first problem. I spent a while trying to figure out a way to use all of them in the first problem before doing the problem the way I knew how to do it and ignoring all of the values.

> I agree. I approached it the same way that he did but I couldn't figure out how to resolve the frequency term. I thought that you were supposed to only use some combination of the given constants.

> These are very good numbers to remember, though I'm not sure I would store the fine-structure onstant in my brain. Seems just too arbitrary.

This problems make me dig through my 5.111 to remember the equations!

But you still need to memorize these values... so what's the point?

> The point is that these numbers are easier to remember than the exact values.

>> Some of these numbers are exact, they are just easier to remember in combination. For instance, I always remember c but can't necessarily remember the wavelength of something like a radio wave. But I can think of the frequency and then do the math, like we had to do here with hc.

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what exactly does that mean?

I didn't think we needed a 2 pi term if we used E=hf?

> It's not hf though, it's h-bar times f. h-bar is h/2pi.

I liked this problem, but what was the approximation methods we had to apply? It seemed mostly like algebra and plugging in numbers.

> I too am a little confused where approximation is used here (other than reducing the number things we have to memorize). I didn't really like this problem, since I was able to arrive at a reasonable answer by just blindly re-arranging the equations, but don't really have a good grasp of all the relationships used in the problem.

wow i totally missed this part, with the different h's. i cant believe it–this makes so much more sense now

> Same. The writing is hard to read though; I could only see the difference at 151%.

I feel stupid now. That was something I easily could have done in high school.

This was reminiscent of something in high school for me - what I'm confused about is what this has to do with the later material we've seen. In other words, what tools did we not have at the beginning of the term that prevented us from doing this problem?

> As I understand it, the lesson here was lumping of the constants into easy to remember values. At the beginning of the semester, we might have been tempted to solve for each constant individually and plug it into the equation.

I eventually arrived at this formula, but I think I did a bit more arithmetic first. Perhaps it would be useful to show all the steps.

These problems seemed a lot more daunting than they actually turned out to be. I thought we didnt need 2pi if using E=hf and c=f*wavelength

> that's not Plank's constant. That's "h bar" which equals h/2pi.

Idon't even remember equations like this one, but as soon as someone mentions it I can figure out what to do with it. Its frustrating that I can't just recall these simple equations.

Why did we assume and use the wavelength of green light? I was confused about the value so I just used the first one I saw while looking online.

> Visible light varies from 380 nm (purple) to 750 nm (red). He picked a number that was within the range and that made multiplication easier.

It was really helpful for me that I remembered the range of visible light.. otherwise I think I would have been totally lost at this point!

This would have been nice to include in the original problem I think - although it does test some estimation skills. I was way off...

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---

this is the piece of information i was missing! i was trying to do the problem by rearranging terms and doing dimensional analysis and wasn't getting too far.

I tried that method too, then realized that I must be missing something b/c it was taking so long!

Agreed. I knew the range of visible light, but I assumed that we weren't expected to use it and ended up wasting a lot of time trying to combine the given terms. I think the way this problem was presented made it seem like those three pieces of data were all necessary for the problem.

i mean.. he did say "visible light", and this class has sort of taught us to just go ahead and pick an easy-to-multiply value.

I agree that the "visible light" was the real clue. He doesn't often (if ever) give us information we don't need to know. So when you see things like this, you can assume it serves some purpose. You should also realize that different frequencies of light must carry different energies, so the frequency must come in to play somehow.

When you said, "Don't look anything up", what if we didn't know this value?

Well, I think you should know the wavelength of SOMETHING in the visual spectrum. As long as you pick a wavelength that's visible, you're okay, since the question asked for just a visible photon. Remembering the visual range isn't that hard.

everyone learns this in 5.11x or 3.091!

which was 3 years ago for some of us!

i took 3.091 and never had to memorize it!

A good fact to know is that visible light is in the range of 400-700 nm. I think it's a good takeaway from any class involving optics, no matter your major.

Why did you choose to use green light?

I would imagine because green light is about in the middle of the visible spectrum.

That's why I chose green.

I didn't really remember which wavelength corresponded to which color, but I remembered from high school that visible light goes from 400nm to 700nm. For lambda, I decided to choose the middle value of 550.

I forgot f = c/lamda silly me

same here. i also wouldn't have known the wavelength for visible light off the top of my head. this was a strange question.

Thats exactly how I did it! :)

This would explain why Sanjoy told us not to spend too much time on these questions. These approximations shouldn't be that overly complicated. I can't believe I forgot that equation.

Yeah, I also missed that. It completely threw off my answer. Additionally, I was unhappy that there was too much information; I couldn't remember that throwing out information was ok. Eventually I just said screw it and used just some of it, thinking I had to use all.

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Wow, I've spent the last hour trying to figure out why my calculations were so astronomically wrong when I realized I was in joules. Fail.

This explanation really helps make it clear. I guess it makes sense to use light in the middle of the visible range

This problem was pretty self explanatory

This makes perfect sense; I have no idea why I didn't do it this way.

I was way way off on this problem...I confused h and h_bar, silly.

    The difference is a factor of 2*pi, so you should be off by less than an order of magnitude.

        Yeah wow, I must have seriously messed up something - I used the same equation (forgot about the 2pi) and got like 10^9......oops.

I did it correctly, but it seems so odd to have such a large number from a single photon! also, how would we estimate the range, given that we don't know the exactly values the above constant clusters are off by?

For some reason I had a lot of trouble with this problem. I think I was just making it more complicated than I needed to.

Problem 2   Boundary-layer thickness

How thick is the boundary layer on a golf ball traveling at, say, $v \sim 40 \, \mathrm{m \, s^{-1}}$?

$10^{\boxed{}} \pm \boxed{}$ m   *or* $10^{\boxed{}} \cdots \boxed{}$ m

The thickness δ is roughly $\sqrt{vt}$, where v is the kinematic viscosity of air, and t is the time for air to travel a distance comparable to r, the radius of the golf ball. So

$$\delta \sim \sqrt{\frac{vr}{v}}, \tag{1}$$

A golf ball has a diameter of about 5 cm so $r \sim 2$ cm. The kinematic viscosity of air is $v \sim 10^{-5} \, \mathrm{m^2 \, s^{-1}}$. So

$$\delta \sim \sqrt{\frac{10^{-5} \, \mathrm{m^2 \, s^{-1}} \times 2 \cdot 10^{-2} \, \mathrm{m}}{4 \cdot 10^1 \, \mathrm{m \, s^{-1}}}} \sim 10^{-4} \, \mathrm{m} \tag{2}$$

(after neglecting a factor of 0.7). Compared to this thickness, the dimples are plenty thick enough to trip the boundary layer into turbulence and thereby reduce the drag coefficient.

Is this actually how fast a golf ball goes??

You could estimate it pretty easily. I drive about 200 meters, and the ball is usually in the air about 5 seconds. That's 40m/s right there.

I liked that this problem was in the pset, right after we did it in lecture

Sad that I missed lecture :(

Was it meant to matter if the golf ball was dimpled to begin with? I calculated it ignoring the dimpling, but now I see that that comes up in the solution notes. If the BL is disturbed by dimples, is it still the same size as the BL calculated for a smooth ball?

maybe it's just cause i'm a golfer- But I love doing calculations about golf and learning more about the physics of it. maybe you can put more sports examples in because those I always find the most interesting

how do you know it's comparable to r?

He discusses this point on R27 (last paragraph of second page). It's how he deals with the time component, by making t (v/r) where v is velocity of the ball and r is it's radius.

i was thinking a distance like the distance the golf ball was traveling. bad error...

I thought I had to incorporate Reynolds number into this..

I wonder if there was a way to set this problem up such that we could see the difference between dimples and no dimples

There's no "dimples" parameter in this equation, so we're not concerning ourselves with dimples or not. Where do you see a dependence on dimples/no dimples?

oh I know that dimples are not involved in this question. I'm saying that since we learned about why there are dimples on golf balls, it would be cool to see a problem in which we solve how much it changes.

Didn't we see this in the reading?

I re-read the readings a few times but I still don't really understand what the significance of the dimples either. How would that change this problem?

They make the flow go a bit turbulent earlier so the boundary layer stays attached till further back on the ball. That being said, I have no idea how that would change this answer! I think the layer thickness might stay the same or close to the same, it just holds on longer?

Damn, I think I switched the V's by accident (velocity and viscosity)

I wasn't sure which v was which, so I used dimensional analysis to figure it out.

I forgot the square root. Damn.

I think I did too. 10^-9?

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Oh I ended up using the wrong equation.. I think I calculated delta/r...

I did that too–i also didn't know the kinematic viscosity of air. so i directly made a ballpark estimate of reynolds number, saying 1000. then the reciprocal of the square root of that was ratio of boundary layer to radius. boundary layer to radius = 1/100, so using 2 cm, the boundary layer is around 1*10^-4 m

I ended up using this equation and then double checking with a reynolds number guess

Whoa...I totally used the wrong equation...I used delta = Re*kin. viscosity / velocity, and got a bogus answer of .35 m...I thought I was doing something wrong!

I just realized I used the wrong Re, I used the actual Re, not the boundary layer Re. If I used Re (bl) = sqrt(Re), then I got the correct answer! yay!

Darn units!!!

Me too. I used a very wrong value for this.

Ahh... I used this value as my radius... What an overestimation of the size of a golf ball!!

i think i screwed up and used the diameter for r. silly mistakes :(

I got this problem right but did it using the inverse of the Reynolds# (I got it from the readings somewhere) and the radius ...guess it worked out

Yeah that's the method I used to solve this problem. It also worked out for me.

Since the boundary layer readings were more confusing, my work on this was more guessing that knowledge, but I still think I got the right order of magnitude. Sweet!

So I also looked up this equation in wiki because at first I couldn't find it on the lecture notes and it has a value of 5 multiplied to the equation. that confused me...

Woah, I got this number even though I was using dramatically different assumptions. Pretty cool how that all works out.

hmm... off by a factor of 5. I used a similar method and similar numbers. bad math probably.

how'd you go about it?

i forgot to take the square root! i hate it when i get problems wrong for stupid mistakes like this. only 2.005 test, i missed a question because i said that 2+.5+2=2.5

Is that the limiting constraint when designing boundary layer obstructions, that they have to be bigger than the boundary layer thickness?

I believe so. Intuitively, if something is smaller than the thing it's trying to block, it won't block it.

Makes perfect sense to me.

So are there golf regulations for how deep/how many dimples the golf ball can have?

Also, would super deep dimples actually make that much difference? Like once it's reached a certain point, is there any difference having a deeper dimple?

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I'm glad you added this sentence. It seems like a good sanity/order of magnitude check

Good to see this works out again!

Yep, I totally had thunk (in the beginning of hte year) that the dimples were there just for show or tradition!

yippie! I got one :)

Feels good to get one right!

Got the right answer, I think. Pretty fascinating stuff, really. It's good to put the answer back in perspective, relating the number back to the size of the dimples - I forgot to think about that after finishing the problem.

Problem 3   Viscous versus form drag

The form drag (drag due to moving fluid aside) is

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The viscous (skin-friction) drag is

$$F_v \sim \rho v \times \text{surface area} \times \text{velocity gradient}, \tag{4}$$

where ρv is the dynamic viscosity η. The velocity gradient is v/δ, where v is the flow speed, and δ is the boundary-layer thickness.

The ratio F_{d}/F_v is dimensionless, and must therefore be a function of the only dimensionless measure of the flow, namely the Reynolds number Re. In fact, the function is a power law:

$$\frac{F_{\text{d}}}{F_v} \sim \text{Re}^n, \tag{5}$$

where n is the scaling exponent. What is n?

$$\boxed{} \; \pm \; \boxed{} \quad \textit{or} \quad \boxed{} \; \ldots \; \boxed{}$$

Using v/δ as the velocity gradient and A as the surface area, the skin-friction drag becomes

$$F_v \sim \rho v A \frac{v}{\delta}. \tag{6}$$

Therefore, the ratio of drag forces is

$$\frac{F_{\text{d}}}{F_v} \sim \frac{\rho v^2 A_{\text{cs}}}{\rho v A v / \delta}, \tag{7}$$

where A_{cs} is the cross-sectional area. For objects that are not too elongated (e.g. not a long train), the surface and cross-sectional areas are comparable. Then the areas in the numerator and denominator cancel out. Additionally, the factors of ρ and one factor of v also cancel. What's left is

$$\frac{F_{\text{d}}}{F_v} \sim \frac{v \delta}{v}. \tag{8}$$

From the reading (r27-lumping-boundary-layers), $\delta \sim r/\sqrt{\text{Re}}$, so

$$\frac{F_{\text{d}}}{F_v} \sim \frac{v r}{v} \times \text{Re}^{-1/2}. \tag{9}$$

The fraction vr/v is the Reynolds number, so

$$\frac{F_{\text{d}}}{F_v} \sim \text{Re}^{1/2}. \tag{10}$$

Thus, $n = 1/2$.

For most everyday flows, Re $\gg 1$. Thus, most of the drag is form drag rather than skin-friction drag. The exception to this rule is very long objects (freight trains), where the surface area is much greater than the cross-sectional area.

---

can you explain more what viscous drag is? I had a lot of problems with this problem just because I didn't really understand what this was.

I like these scaling problems. I think it's a cool way to look at things.

wow i forgot the "gradient" part

Wow I did this totally wrong...I think I just couldnt keep track of v versus nu, my paper was a mess of things that kinda looked like "v"

definitely the different v's made this problem take longer for me than it would have otherwise

I agree, even knowing how the equations should look I was still a little confused by the two

Also agree, I thought I had it figured out when I went back into the readings, but I kept on subst. the wrong v for one another in my rearranging...

Yeah it definitely poses a problem. But the best thing to do in this case would be to zoom into the page pretty significantly to make sure that they're different variables.

i guessed this just to make things easier, and im glad it worked out :)

I didn't realize you could make this approximation..this would have made things a lot easier!

I was stuck on this for a while as well, and made the approximation for simplicity. How much error does this introduce?

I agree. I think this assumption is crucial to solving this problem.

Yeah, this fact definitely is necessary to simplify the problem.

It took me a while to figure this out. I finally decided it was okay, after trying to determine a good ratio (such as 1/2 r) was too complicated to put back into Re.

I got down to this part with no problems, but couldn't get it to relate to the Reynolds number. This way makes sense though.

I actually canceled too many terms because I couldn't tell between the v's and the nu's. It's a messy and careless mistake, but one that could have been avoided with better typesetting.

Whoops, I think I thought this value *was* the Reynold's number, from another point in the reading. Now I'm confused why it isn't...

yeah, I did the same thing. I think its because that would give us the reynold number of the flow inside the boundary layer as opposed to for the object overall...

I think my problem was that I accidentally focused only the del and neglected velocity and viscosity. Thus my entire estimation was overly simplified to F_d/F_v   del. That would explain why my answer was n=-0.5

Likewise. Is it wrong to assume this is a Reynold's number of sorts? Doin it this way does make the next question a lot easier...

even after the note to look at boundary layers (and re-reading that part), I really didn't get this one...however, this explanation was _very_ helpful. Thank you

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Thus, $n = 1/2$.

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---

ahh, I used this for the previous problem but didn't even think to use it here!

I know. I got a lot of the canceling out through A A and the v, but I completely forgot to use the definition of d for this. It would've made this a cinch.

tricky

I remember seeing this in the reading!

I left it as Re^ -1/2. I forgot about the additional portion.

I'm just a little upset by the variables involved: nu and v, they both look exactly alike, I think at one point or another when squinting at the section notes or here, i mixed up the two and ended up with a bad equation. I wish we could've used more distinctive letters for these.

Yea, I confused these and it messed me up. That is a rather unnecessary thing to get bogged down on.

Shouldn't this be = Re^-.5 instead of *Re^-.5?

No, the delta is replaced by the r/Re^.5 so instead of dividing by Re^.5, you multiply by Re^(-.5), which is mathematically equivalent.

I understand this answer, but I would argue n = 1 is also correct because the problem did not specify which Reynolds number. Re^(1/2) is correct for Reynolds number based on the radius. However, Re^1 is correct for Reynolds number based on delta, which is also a commonly used parameter.

Yeah I got n=1 also.

Ohhh oops, I totally missed this part. Tricky to put the Reynolds number in there.

I agree, I saw this in the reading but wasn't sure when you could use the Re estimate. Why wasn't it used in the gold ball example?

That's why I still had a factor of r. I went back and used the equation I used in problem 2 which complicated things more for me.

Nice! This one made me happy to get right.

I got -1/2 for this problem.. my gut told me it was +1/2.. Like we learned in class, intuition seems to be the best bet!

I was also debating between whether it should be negative or not, but ended up leaving it at negative. Should have trusted my intuition also...

Same here, I went with intuition but am glad to see where it was right.

I got this one wrong, I somehow got n = 1 after manipulating the variables, i didn't realize delta   r/sqrt(Re)

Wasn't this very similar to something we did out in one of the memos?

Man, I missed the square root part of the problem.

I really don't know what technique/principal this problem is trying to illustrate

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Thus, $n = 1/2$.

For most everyday flows, Re $\gg 1$. Thus, most of the drag is form drag rather than skin-friction drag. The exception to this rule is very long objects (freight trains), where the surface area is much greater than the cross-sectional area.

Got something right finally

I used this idea to set the exponent, to something greater than one but I guess i didn't consider that Re is really big.

It's good to actually see this painted out for us, as it confirms what we probably already knew intuitively.

I like how these problems were inter-related. It helped me feel assured of my method as I went through the problems. However, The problem with this is that if you miss the first problem, it is difficult to move forward with the next dependent problem

Problem 4   Viscous versus form drag while walking

Use the result of **Problem 3** to estimate the ratio

$$\frac{\text{form drag}}{\text{viscous drag}} = \frac{F_d}{F_v}$$

for a person walking.

$$10^{\boxed{}} \pm \boxed{} \quad \textit{or} \quad 10^{\boxed{}} \boxed{} \dots \tag{11}$$

The ratio is roughly the square root of the Reynolds number, where

$$\text{Re} \sim \frac{\text{size} \times \text{speed}}{\text{kinematic viscosity}}. \tag{12}$$

For a person, the size is roughly $r \sim 1\,\text{m}$ (using the geometric mean of $2\,\text{m}$ for the height and $0.5\,\text{m}$ for the width). For walking, $v \sim 1.5\,\text{m s}^{-1}$. The viscosity of air is, conveniently, $1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}$, so the Reynolds number is roughly

$$\text{Re} \sim \frac{1\,\text{m} \times 1.5\,\text{m s}^{-1}}{1.5 \cdot 10^{-5}\,\text{m}^2\,\text{s}^{-1}} \sim 10^5 \tag{13}$$

The square root is $10^{2.5}$ or 300. Form drag, which is mostly independent of viscosity, is the big source of drag.

So we could have done it either way, right? That's a good way to check how accurate our approximations are.

I like this a lot - most of the approximations I feel more comfortable when there's another way to check it.

i didnt realize this problem was really just asking us to calculate the re #

Oh, I didn't even think of this. I kind of just picked a number in between, and used 1 for convenience.

Oops. That's not the way I calculated size.

oops. I was thinking in feet and knew that the "radius" of a human was approx three feet, but then I wrote it down as 3m, though this did not really affect my answer!

I got this partwrong. Ruined my entire equation.

Same here...I always forget to use kinematic viscosity, and I think I accidently used the wrong viscosity.

I've always wondered if it's kosher that so many numbers were calibrated on every day objects, like the density of water being a convenient number like 1,000, for instance. I know it all balances out, but it seems fishy, you know?

What is this based off of?

I think most people can walk 3-4 miles per hour, which is roughly 1.5 m/s.

I didn't even think of using the results from the last problem. I actually used the equations for the forces to solve it. Obviously I didn't read the entire question or just skimmed it.It clearly says to use the result from the last problem...

I did this problem correctly but my n exponent was wrong from Problem 3.

Same with me. Although, I believe my answer was just a factor of 10 off, which isn't SOO bad.

I was also a factor of 10 off. Now i see my mistake...

Awesome to see we got the same answer here, and how much easier this problem is with the information from problem 3!

I agree, these two problems work very well together explaining how you can combine these formulas to answer questions.

Dang, since I missed calculated problem 3, my answer for this question was way off. I guess this is my issue with relying on possibly wrong estimates, it carries over elsewhere.

yeah, this problem set was more frustrating for me than others because a bunch of the problems relied on correct answers from other problems, which I didn't always get.

From my mistakes in Problem 2, I accident got the inverse of the correct answer.

Yea I agree with that. Maybe there is a way in the future to have us rely on concepts from other questions but not necessarily the exact numbers.

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The square root is $10^{2.5}$ or 300. Form drag, which is mostly independent of viscosity, is the big source of drag.

It's important to note that you're assuming all the constants of proportionality (ratio of areas, drag coefficients, etc.) to cancel out to about 1.

I got the right concept, that form drag is the major source of drag (over viscous drag), but I messed up on the first part, and didn't get the factor of 1/2, so my value here was 10^5

Again, I understand this solution, but I don't know why the way I did it doesn't work. From before, Fd/Fv = v*delta/nu. Then I used delta = (v*r/nu)^1/2.

Problem 5   Rolling down the plane

Four objects, made of identical steel, roll down an 30-degree inclined plane without slipping. The large objects have three times the radius of the small objects. Which object has the greatest acceleration?

☐ a large spherical shell

☐ a large disc

☐ a small solid sphere

☐ a small ring

The goal is to find the acceleration a along the plane. It depends on g, θ (which is $30°$ here), the object's moment of inertia I, its mass m, and the rolling radius r.

$$a = f(g, \theta, I, m, r).$$

Use dimensional analysis to simplify this function of five variables. The six variables in total make three independent dimensionless groups:

$$\frac{a}{g} \quad \theta \quad \frac{I}{mr^2}.$$

Therefore,

$$\frac{a}{g} = f\left(\theta, \frac{I}{mr^2}\right). \tag{14}$$

Probably

$$\frac{a}{g} = f\left(\frac{I}{mr^2}\right)\sin\theta.$$

$$a = f\left(\frac{I}{mr^2}\right)g\sin\theta.$$

The ratio I/mr^2 is a dimensionless measure of where the mass of an object lies. The farther toward the edge and away from the rolling axis, the greater the ratio. Most importantly, I/mr^2 is independent of an object's radius; for example, a big and a small ring have the same ratio.

The bigger the ratio, the bigger the fraction of energy consumed by rolling motion compared to translational motion. Therefore, the object with the smallest I/mr^2 will have the greatest acceleration. The solid sphere (choice C) has the most mass near the rolling axis, so it will be the fastest.

I thought this problem was pretty intuitive. The most dense and aerodynamic object should have the greatest acceleration.

If they all weighed the same, would the ring be the fastest?

The size didn't end up making a difference?

I find that when I see problems I've solved before I jump right into the math. keep the fresh problems coming.

Aha...there's the variable I missed. My attempt at dimensionless analysis totally failed without it.

yeah thats the problem with dimensional analysis.

I thought we were supposed to consider drag in this problem... oops!

same! intuitively i would think that the large disc would be fastest because it would have the largest ratio of mass:drag force.

I didn't even think to use dimensional analysis here, I just went straight for Inertia and neglected g, m, theta etc. Got the right answer too

Same for me. I went from torque=I*alpha, and after making simplifications using proportionality, got down to acceleration was inversely proportional to radius ^4....I found that hard to believe, but it does give you the right answer (you also get that acc is proportional to mass, so figuring same radius amongst the small object, you know that the solid sphere must have more mass than the ring)

I did the same, a quick flip back to 2.003 gave me the right answer in a couple seconds, although it's good to see how we can use dimensional analysis here to arrive at the same conclusion.

I did the same thing and found it effective.

I also didn't think of using dimensional analysis for this problem. However, since I'm not course 2, I didn't have any notes or information from previous classes to help me, and I just had to go with my "gut".

I managed to get the correct answer without using a, g, or theta. I used dimensional analysis to find a relationship between I, m, and r and from there used reasoning. Though I didn't get the "same answer", it's nice to know my approach was right.

I thought this one was actually simpler than this. I forgot about angular acceleration...

I thought this was a reynolds number problem instead of a general dimensional analysis problem. Still I got the right answer from intuition.

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The bigger the ratio, the bigger the fraction of energy consumed by rolling motion compared to translational motion. Therefore, the object with the smallest I/mr^2 will have the greatest acceleration. The solid sphere (choice C) has the most mass near the rolling axis, so it will be the fastest.

I didn't even use these equations, I just did it by calculating which mass was greatest- which I thought would have the greatest acceleration

> Same although I considered the SA for air friction as well.

>> I agree... I was looking for some sort of relation to something we have done recently and attempted to analyze it via Re forces.

>>> I had no idea how to approach this problem via dimensional analysis (which I see now), but when I did it I used tools learned in high school physics and in 8.01.

>>>> Same here. I basically used the equations F=ma and I=mr^2.

>>> Agreed. Moment of inertia and Rotational Kinematics could also get you to the answer of this problem.

I didn't even go into all this. I just looked at what had the most concentrated mass

so you don't need to know what f is? how you know whether or not a/g is positively correlated to 1/m^2? meaning as 1/m^2increases, a/g increases? can't it be the other way around

I actually used a different set of variables here – I used the actual radius of the object and the average radius of its mass distribution, which relates to its moment of inertia.

I thought we were supposed to use cross-sectional area and look at drag resistance. That seemed in the subject of what the pset was about

> That's what I used, and I assumed that mass was a little more important than air resistance, so I got the right answer.

> This is true. The focus on drag in the beginning of the homework might have thrown several people on the wrong track.

So is I proportional to r^2? I don't see how the ratio is independent of r if there's an r^2 in the denominator.

> I think I=mr^2, so maybe they cancel out?

>> I=c*mr^2 for most simple objects (more specifically, solids of revolution about the axis of rotation), where c is a constant depending on geometry.

> See here: http://en.wikipedia.org/wiki/List_of_moments_of_inertia

>> I didn't think to consider the ratio. I just said that since the acceleration will depend on how far the mass is from the rolling axis the object with the most mass the furthest away will roll faster.

>>> I also said that the object with the most mass the furthest away will roll faster....however, it seems to me that's not how it worked? or does the small solid sphere really have more mass farther from the center than any of the others?

what is?

I've lost track of how many times I've seen experiments proving this fact, and it still seems counter-intuitive to me.

> Agreed, but because of that I tend to remember it

> I agree. I pulled the same mistake again too! I definitely don't know why it's so counter-intuitive.

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The bigger the ratio, the bigger the fraction of energy consumed by rolling motion compared to translational motion. Therefore, the object with the smallest I/mr^2 will have the greatest acceleration. The solid sphere (choice C) has the most mass near the rolling axis, so it will be the fastest.

I for some reason did not even think to apply energy here...

I think the energy balance is more clear. Assuming you know enough about the moment of inertia, you can see why one would roll faster.

I looked up moments of inertia for different objects... I is always proportional to mr^2, but there are different constants for different shapes (ring, disc, solid sphere).

I like this explanation more than the 2.003 one. it puts it more in a physical context

I identified this but somehow convinced myself the ring was better...should've thought about it a little harder.

This is basically what I did. The explanation is clear.

I notice there is no consideration for drag here. Is that simply being lumped in with acceleration?

Interesting; this makes sense when I imagine it visually, but it's pretty hard to reason intuitively unless you find all the variables for dimensionless analysis.

I actually reasoned it intuitively only and didn't do the math. The way I thought about it is: 1) you want a lot of mass so you get more force downward 2) at the same time you don't want it to have a large radius that will cause it to roll slower ie. have more radial inertia.

I did the same thing, but then I got stuck on how the ring seemedl ike it would have the least drag, but the sphere had the lowest moment of inertia.

@sat1:03 – Awesome way to think about it! Thanks

I answered this based on intuition instead of doing out the math.

I had the whole concept flipped. This explanation makes a lot of sense.

sweet, again my intuition worked.

haha, my intuition was totally wrong–i approached this from a drag perspective and said the solid disc...but this makes sense–with more mass near the center/rolling axis, more energy goes into translational motion

Glad I remembered 2.004 well enough to do this problem successfully.

I'm sure there are a lot of comments similar to mine...but I totally did this wrong...didn't even think of using dimensional analysis. I need to remember to use all the techniques!

Problem 6   Hydrogen binding energy

In lecture and readings we analyzed hydrogen (`r26-lumping-hydrogen.pdf` on NB), which is one electron bound to one proton. Using those results, one can show that the binding energy is

$$E \sim \frac{1}{2} m_e (\alpha c)^2, \tag{15}$$

where α is the fine-structure constant, c is the speed of light, and m_e is the mass of the electron.

Use the methods of **Problem 1** to calculate the binding energy in electron–volts.

$$10^{\boxed{}} \pm \boxed{}\ \text{eV} \quad \textit{or} \quad 10^{\boxed{}} \cdots \boxed{}\ \text{eV}$$

Rearranging the powers of c,

$$E \sim \frac{1}{2} \times m_e c^2 \times \alpha^2. \tag{16}$$

Because $\alpha \approx 1/137$, which is roughly $1/141$,

$$\alpha^2 \sim \frac{1}{1.41 \cdot 10^2} \sim \frac{1}{2} \cdot 10^{-4}. \tag{17}$$

Since $m_e c^2 \sim 0.5 \cdot 10^6\ \text{eV}$, the binding energy is

$$E \sim \frac{1}{2} \times \frac{1}{2} \cdot 10^6\ \text{eV} \times \frac{1}{2} \cdot 10^{-4} \sim \frac{1}{8} \cdot 10^2\ \text{eV}. \tag{18}$$

The result is 13 eV.

This problem was nice and easy once you realized that it was a simple manipulation of variables!

Haha I forgot to square alpha, no wonder my answer was high....silly mistake

　Ah crap. Same. =(

forgot the 1/2 damn

　same here. crap.

I'm not really sure I understand how this helps?

　Oh nevermind, because when you square it you get 2. It's not immediately obvious though, so you might want to make a note.

　　Ah, that is clever. It took me a little bit to get this, but it definitely makes the math easier.

　　　Another option is to make this just 10^2, and compensate for that by calling .25 * 10^6 = "few" * 10^5. You end up getting a very similar answer, and definitely within when given the bounds.

　　　　yeah, i also converted it to a few * 10^4.

　　　　　Definitely a clever trick, I did not think of this.

　　　　　　me neither, but good thing the problem didn't hinge on it.

　　　　　　　Yea, I just arbitrarily rounded while doing these calculations and got a close answer.

　　It's the fine-structure constant again!

I didn't really think to simplify the numbers this early in the stage.

I forgot to square the alpha term, which threw off my answer by a factor of 10^2. I did the problem correctly but messed up the easiest step!

I used the method explained here, but i got a major overestimate. Its probably due to me just arbitrarily picking values instead of thinking through them and figure out stuff like the 1/141.

I don't really see how this is a useful exercise. Sure, we're seeing how those 3 values from problem 1 can be used rather than the individual constants, but otherwise this is just a plug and chug problem.

　I agree, I was surprised that this problem was so simple, I feel most of the problems on these psets the hardest thing is finding the right equations to use

　　I think it's an exercise to show you how much easier the problem becomes when you know a few groups of constants.

　　　Also, it's an exercise on re-arranging variables in a way that makes it convenient and simpler.

　　　　Maybe he was just being nice? He said the pset had more problems but they were simple

　　　　　It also builds up to the next couple problems.

　　　　　　it was also good practice for quick, approximating-type math

i ignored all those 1/2s but they clearly add up - oops!

Problem 6   Hydrogen binding energy

In lecture and readings we analyzed hydrogen (`r26-lumping-hydrogen.pdf` on NB), which is one electron bound to one proton. Using those results, one can show that the binding energy is

$$E \sim \frac{1}{2}m_e(\alpha c)^2, \tag{15}$$

where α is the fine-structure constant, c is the speed of light, and m_e is the mass of the electron.

Use the methods of **Problem 1** to calculate the binding energy in electron–volts.

10 [ ] $\pm$ [ ] eV   ***or*** 10 [ ] $\cdots$ [ ] eV

Rearranging the powers of c,

$$E \sim \frac{1}{2} \times m_e c^2 \times \alpha^2. \tag{16}$$

Because $\alpha \approx 1/137$, which is roughly $1/141$,

$$\alpha^2 \sim \frac{1}{1.41 \cdot 10^2} \sim \frac{1}{2} \cdot 10^{-4}. \tag{17}$$

Since $m_e c^2 \sim 0.5 \cdot 10^6$ eV, the binding energy is

$$E \sim \frac{1}{2} \times \frac{1}{2} \cdot 10^6 \text{ eV} \times \frac{1}{2} \cdot 10^{-4} \sim \frac{1}{8} \cdot 10^2 \text{ eV}. \tag{18}$$

The result is 13 eV.

---

i ignored all those 1/2s but they clearly add up - oops!

what is this problem trying to show us?

Problem 7   Heavy nuclei

In this problem you study the innermost electron in an atom with many protons (i.e. with a heavy nucleus). So, imagine a nucleus with Z protons around which orbits just one electron. Let $E(Z)$ be the binding energy. The case $Z = 1$ (**Problem 6**) is hydrogen.

Find how $E(Z)$ depends on Z. Namely, what is the scaling exponent n in

$$E(Z) \propto Z^n \tag{19}$$

or, equivalently, in

$$\frac{E(Z)}{E(1)} = Z^n ? \tag{20}$$

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$

With Z protons pulling on one electron, the electrostatic energy contains the factor $Ze^2/4\pi\epsilon_0$. So instead of using $e^2/4\pi\epsilon_0$ as one quantity in the dimensional analysis, we should use $Ze^2/4\pi\epsilon_0$. The other quantities – a_Z, m_e, and $\hbar$ – are unchanged except for a_Z replacing a_0. The Z propagates along with the e^2 through the calculation of the radius a_Z and the energy $E(Z)$.

Since the radius a_0 has one factor of e^2 in the denominator, the a_Z picks up a factor of Z in the denominator relative to a_0. Therefore,

$$a_Z = \frac{a_0}{Z}.$$

The electrostatic binding energy is inversely proportional to the radius a_Z:

$$E(Z) \sim \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{a_Z}. \tag{21}$$

One factor of Z is directly visible, and the second factor is part of $1/a_Z$. The energy $E(Z)$ thus has a factor of Z^2:

$$E(Z) = E(1) \times Z^2.$$

Therefore, $n = 2$.

I got help to do this problem but I don't think I would have been able to do it alone..

I forgot Z was originally in the equation even after looking for it.

> Where exactly was this in the readings? I totally guessed at this for the purposes of the problem.
>
> > I couldn't find it either...and so I couldn't figure out how to incorporate Z.

Missed how this was proportional to the number of protons. Does doubling protons double the potential energy?

I thought the explanation in this paragraph was enough to explain the answer, all the stuff after this just confuses me.

what is az?

I tried to relate Z and energy to the size of the nucleus and got this completely wrong.

I'm a little confused - when did we ever find an expression for a0?

> It's an intermediary step. You can find an expression for a0 based on what was given in Prob. #1.
>
> I am still not sure how this was calculated either...
>
> > I'm also confused how this was calculated... actually I was pretty confused about this problem and the next one in general. I didn't really have any idea how to solve them. I just don't know enough about chemistry or physics to understand the problems.
> >
> > I also agree about not really understanding this step from the first problem or on this one and would like to see it explained in class.

I diddnt know this radius changed, w=or would be calculatable in these circumstances

> I feel like if you didn't think this equation, there's no way to get the answer. I tried thinking of how Z changed the radius using the equation in the notes (the one similar to the Bohr radius), but couldn't conclude any changes.
>
> > I also tried to think of this equation, but couldn't...I did, however, remember that if Z increases, there's more charge pulling the electron in, therefore the radius is smaller...i just couldnt come up with something that made sense...this seemed a little too simple for electron math

I don't think I would have guessed this step

> It makes more sense if you rearrange the equation as a_0 = Z*a_z and think about how atoms pack and electric attraction scales.
>
> I really had no idea on what to do. A friend explained it to me, and I was still clueless on how to get the exponent.

Interesting conclusion, I ended up getting close by abstracting this further by energy for each shell. I had no idea to use e^2/4pie0 for this.

Problem 7   Heavy nuclei

In this problem you study the innermost electron in an atom with many protons (i.e. with a heavy nucleus). So, imagine a nucleus with Z protons around which orbits just one electron. Let $E(Z)$ be the binding energy. The case $Z = 1$ (**Problem 6**) is hydrogen.

Find how $E(Z)$ depends on Z. Namely, what is the scaling exponent n in

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$$E(Z) = E(1) \times Z^2.$$

Therefore, $n = 2$.

I completely missed the second factor 1/az and came up with n=1, which threw off my answer to 8 as well.

Isn't this just saying that if you sub in the actual equation for a_z, you will get the answer?

I couldn't figure out the 2nd factor of Z, but after solving the next problem, I knew it had to be Z^2, so I redid this problem.

I made a mistake with the charges and thought it was proportional to one.

this was my initial guess - then I changed it, over thought it I guess

I'm not sure the way I did this was correct, but somehow I got the right answer. Perhaps I didn't interpret this solution correctly.

Problem 8   Heaviest nuclei

Consider again the system of Problem 7: a nucleus with Z protons surrounded by one electron.

When the binding energy $E(Z)$ is comparable to $m_e c^2$ – the rest energy of the electron – then the electron has enough kinetic energy to produce, out of nowhere, a positron (an anti-electron). As a result of this process, which is known as pair creation, the positron leaves the nucleus, turning one proton into a neutron. That makes the atomic number Z drop by one. The consequence is that, for large-enough Z, the nucleus is unstable! Relativity sets an upper limit for Z.

Use the results of Problem 7 to estimate this maximum Z set by relativity (feel free to ignore factors of 1/2 in $E(1)$).

$$\boxed{} \pm \boxed{} \quad \textit{or} \quad \boxed{} \ldots \boxed{}$$

To include in the explanation box: Compare your estimate with the Z for the heaviest stable nucleus (uranium).

Since the binding energy $E(Z)$ is $E_0 \times Z^2$ and $E_0 \sim m(\alpha c)^2$, the binding energy is

$$E(Z) \sim mc^2 (Z\alpha)^2. \tag{22}$$

When $Z\alpha \sim 1$, this energy is comparable to the electron's rest energy: That is when the electron becomes significantly relativistic, which permits pair creation to destabilize the nucleus. So the maximum Z is roughly α^{-1} or about 140. The heaviest stable nucleus is uranium with $Z = 92$, so the explanation for the stability of the elements looks pretty good.

I was pretty lost about this problem in general. How are people who don't have much knowledge about physics and chemistry expected to do these problems? I was searching for information for hours, and still didn't know how to do this problem.

Yeah this one got me too...

I like having related problems on these psets.

I completely misread this problem. I don't particular like it, nor did I find it easy to understand.

i didnt really understand what was going on here. i guess i need to brush up on chemistry, or physics, or something.

Same with me, i had to look up some of this stuff to get a better understanding of the problem.

This class makes me remember so much knowledge I've long-since deemed useless.

I knew this was related to alpha but for some reason I couldn't understand the exact relation. Now it all makes sense...

I couldn't figure out how to do this problem using problem 7 but my chemistry knowledge was enough for me to reason what the answer should be..

This formula (obviously) would've been very useful...I'm failing to recall, was it in one the readings somewhere or did this require a little knowledge of relativity?

it was derived from the previous two problems. at least, that's how i figured it out.

Yeah this comes directly from 6 and 7

I didn't even think of this approach... very clever.

I managed to re-write the equation into a similar form. But since my reasoning was wrong in Problem 7, my entire approach to this question was off from the start.

Don't quite understand this. Can someone please explain in more depth.

typo

I got 130. I'm surprised as to how close I was to the correct number.

would the factor of 1/2 help or hurt us here?

Problem 8   Heaviest nuclei

Consider again the system of Problem 7: a nucleus with Z protons surrounded by one electron.

When the binding energy $E(Z)$ is comparable to $m_e c^2$ – the rest energy of the electron – then the electron has enough kinetic energy to produce, out of nowhere, a positron (an anti-electron). As a result of this process, which is known as pair creation, the positron leaves the nucleus, turning one proton into a neutron. That makes the atomic number Z drop by one. The consequence is that, for large-enough Z, the nucleus is unstable! Relativity sets an upper limit for Z.

Use the results of Problem 7 to estimate this maximum Z set by relativity (feel free to ignore factors of 1/2 in $E(1)$).

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Whe $Z\alpha \sim 1$, this energy is comparable to the electron's rest energy: That is when the electron becomes significantly relativistic, which permits pair creation to destabilize the nucleus. So the maximum Z is roughly α^{-1} or about 140. The heaviest stable nucleus is uranium with $Z = 92$, so the explanation for the stability of the elements looks pretty good.

I was unsure of whether or not a value of 140 was good. If you think about it, the numbers are close but this answer would actually predict that all of the elements that we know of would have a stable nucleus.

Me too! I was conflicted. In the context of the estimations we've been doing in this class, 140 is good enough compared to an actual answer of 92, but in the context of the periodic table, it is pretty useless in predicting where the instability starts since there are less than 140 elements overall.

If we hadn't ignored that factor of 1/2 wouldn't it have been even larger? I also didn't consider this a very close approximation.

yeah I agree, the numbers seem pretty far apart...at what value would Z have been a bad approximation?

Not that I was close since I did a horribly simple arithmetic mistake in #6, but when he asks us to disregard factors so that our answers come out closer to the actual value it makes me even more skeptical/suspicious of a technique or solution.

That's a great point – can we go over this in class. What threw off the accuracy?

I think there are additional factors, besides relativistic ones. For example, space limitations, as the nucleus gets bigger.

I felt more comfortable with my guess here than i did on the previous question (my gut told me 102 +/- 10)...I ended up getting this problem ready to solve, after further contemplation on #7, but ended working backwards and solving 7 using my gut guess on 8

I like how this problem set worked off of itself

6.055J/2.038J (Spring 2010)

Solution set 8

Submit your answers and explanations online by **10pm on Wednesday, 05 May 2010**.

Open universe: Collaboration, notes, and other sources of information are **encouraged.** However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

Here is solution set 8 (this time in the right NB site – sorry about that). The memo will be due Friday at 9am.

Problem 1   Should you be worried?

Assume that 1 in 10^4 bridges in the United States are in danger of collapse unless repaired soon, and that a new test for bridge integrity has been devised. This test is 90% accurate: It always detects an unsafe bridge (no false negatives); and 10% of the time it says that a safe bridge is unsafe (10% false positives).

You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

$$10\boxed{} \pm \boxed{} \qquad or \qquad 10\boxed{} \cdots \boxed{}$$

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

Let's do it by the natural-frequencies approach. Imagine a population of 10^4 US bridges. Given the base rate of 1 in 10^4, assume that one bridge among them is actually in danger of collapse.

Now imagine applying the bridge-integrity test to all 10^4 bridges. It will spot the one unsafe bridge. But from among the nearly 10^4 safe bridges, it will also mark 10% or 10^3 bridges as unsafe. The bridge you use is among the roughly 10^3 bridges with a positive test. But only one of those bridges is actually unsafe, so $p_{\text{unsafe}} \approx 10^{-3}$. Therefore, the odds are 10^3 to 1 that the bridge is safe (or simply 10^3).

Now let's use Bayes theorem to get the same result. The odds form of Bayes theorem is

$$O(H|E) = O(H)\frac{P(E|H)}{P(E|\overline{H})}, \tag{1}$$

where H is the hypothesis that the bridge is unsafe, $O(H)$ is the odds in favor of that hypothesis being true, E is the evidence that the bridge failed the integrity test, $P(E|H)$ is the probability of a failed integrity test given that the bridge is unsafe (the false-negative rate), and $P(E|\overline{H})$ is the probability of a failed test given that the bridge is safe (the false-positive rate).

The initial odds $O(H)$ are 10^{-4} (the bridge is very probably safe). The likelihood ratio is

$$\frac{P(E|H)}{P(E|\overline{H})} = \frac{1}{0.1} = 10. \tag{2}$$

Therefore, the new odds are 10^{-3} in favor of the bridge being unsafe (or 10^3 in favor of it being safe, as computed above).

For an excellent article in the newspaper (of all places) on how to do Bayes theorem using natural frequencies, see Steven Strogatz's recent column in the *New York Times* (thanks to Sean Clarke for pointing me to it), available at **http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/**

Somehow I totally missed this when I read the problem. Woops.

That actually seems rather low

i feel like that's "okay" because there are no false negatives.

i took this to mean that for every 9 unsafe bridges, it incorrectly said 1 was unsafe. i guess it depends on what you mean by "10% of the time"

Pretty crappy test if you ask me. That's a lot of false positives. So the US would fix 1000 times more bridges than it needs. Go budget deficit.

It's probably reasonable given that you have to ensure no false negatives (seen as unsafe when it's safe), as those are unacceptable given the conditions here

As another post said, you don't want to miss anything (no false negatives). And few false negatives usually means lots of false positives. Even then, you don't have to fix everything that fails this test. There might be other, more detailed tests (e.g. taking samples of the material, doing X-rays, who knows what) that are more reliable although more expensive (hence not worth doing on all bridges).

Lots of medical tests work that way. If you are positive on the cheap but not-super-reliable test, then you go in for "further screening". As an example, if you present at the hospital with chest pains and shortness of breath, you have just tested positive on a very crude test for a heart attack – so then you are sent for further tests (which someone with more medical knowledge than I have will have to describe!).

despite what our math just told us, can you imagine the media outrage if this were ever to actually happen? sigh...

Ah, I never quite understood the meaning of odds.

or you could write this as odds=(1-punsafe)/punsafe

I feel foolish. I just could not figure out O(H). I was trying to figure out the unsafe bridges on a country-wide scale, using an approximation of 600,000 bridges. It didn't even occur to me to use a sample of 10^4 bridges.

I did it this way; in fact, I think I wrote this exact statement (almost exact) in my explanation.

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You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

10 ⬚ ± ⬚   **or**   10 ⬚ ⬚ ...

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

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Now imagine applying the bridge-integrity test to all 10^4 bridges. It will spot the one unsafe bridge. But from among the nearly 10^4 safe bridges, it will also mark 10% or 10^3 bridges as unsafe. The bridge you use is among the roughly 10^3 bridges with a positive test. But only one of those bridges is actually unsafe, so $p_{\text{unsafe}} \approx 10^{-3}$. Therefore, the odds are 10^3 to 1 that the bridge is safe (or simply 10^3).

Now let's use Bayes theorem to get the same result. The odds form of Bayes theorem is

$$O(H|E) = O(H) \frac{P(E|H)}{P(E|\overline{H})}, \tag{1}$$

where H is the hypothesis that the bridge is unsafe, $O(H)$ is the odds in favor of that hypothesis being true, E is the evidence that the bridge failed the integrity test, $P(E|H)$ is the probability of a failed integrity test given that the bridge is unsafe (the false-negative rate), and $P(E|\overline{H})$ is the probability of a failed test given that the bridge is safe (the false-positive rate).

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I really like this approach, I tried to use Bayes theorem and got lost in the probability aspect of the problem rather than trying to use logic to get through it

What were you confused about? Bayes Theorem is not that hard, but you do have to think about what you're plugging in.

I ended up setting the problem up like an asymmetric binary channel and solving it using Bayes' theorem.

I used Bayes' theorem but this is actually a lot more intuitive.

I also used Bayes' but this so clear. It really makes a lot of sense to me.

for me, bayes' theorem is not necessarily hard, but my intuition is obscured by the math.

Yeah, this intuitive reasoning actually makes much more sense to me than the Bayes rule calculations I used!

I do too. I highly recommend it for making the Bayes theorem result intuitive. Gigerenzer's studies showed a huge improvement in accuracy when the 'natural frequencies' method was explained to doctors (see the NYT article and references therein).

My analysis doesn't make any sense now, oops.

Me too. I completely overlooked this. In retrospect I definitely should not have assumed it was so simple.

wow, I must have been really tired...I did not take 10% of 10^4, I just converted it to a decimal!

I thought this method is faster and easier than the bayes theorem

Me too. I was about to try Bayes theorem then I realized how simple this way was so I did the problem this way.

this is hard to follow

where did you get only one of them is unsafe? I thought is 1 out of 10^4 not 1 out of 10^3

I wasn't sure how to put this into my calculations...

This method is so much more simpler than using Bayes theorem. Why couldn't I remember this?

maybe you should make it clear there that punsafe=1-psafe. There seems to be confusion about how to use punsafe in the formula for odds that you gave above.

It would be helpful to see bayes theorem form probability first, then change it to odds

Yea, the Bayes theorem is definitely more commonly known in probability form and that would also be consistent with the way you presented it in the reading. But it doesn't really matter, its all Bayes theorem anyway.

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I didn't think about using the Bayes theorem...

> That's unfortunate. It's useful!

> The reasoning with words is much easier to understand. I went straight to Bayes theorem because that is what we were shown in class, but failed to get the right answer.

>> That's what I used for this - took me a few tries to define the probabilities correctly

>> Same. I really wonder where I went wrong in it, I guess the number I used for P(unsafe read)

> this probability stuff isn't the most intuitive to me either. after seeing these examples, I think Bayes theorem anytime a test is used to validate something.

I think my problem with this was I was trying the traditional method.

This is exactly what I did! And I got the correct answer! Thank you 18.440...

Totally forgot about using this form...I just used the normal Bayes rule!

> Yeah using this form definitely saves a little bit of time at the end. It's important to change the odds back to the odds that the bridge is safe, not unsafe.

>> I tried Bayes' formula as well, without the odds figure. The odds figure somewhat confuses me.

for some reason the wording in these problems always throws me off, the probability that the bridge is save given it failed and the probability that the bridge failed given that it is save seems like the same thing to me

That's what I got as my answer

> Yeah, I guess I stopped at the initial odds, and didn't continue to calculate the likelihood ratio.

As a visual comment, when I'm looking at the solutions, I see this equation and immediately think that its saying the answer is 10. I t would be nice if you could clearly display what the answer is.

> Yeah, I agree. I often find it annoying to wade through all this description when I just want to see what the answer is to compare to mine. It would be much better to display the answer more clearly.

>> Good point. For the next pset, I'll make the answer the final displayed equation in the solution (or maybe with a gray background, in case there's discussion after the answer, so that the answer still stands out).

where is the "likilihood ratio" in the notes? I didn't see it.

> Reading 28, page 153

This was my answer but it took me a while because I kept thinking there was more to it.

> These are just the initial odds though

I still don't understand odds very well. Probability makes a lot more sense to use for an answer. Why did you ask for the odds?

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You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

$$10^{\boxed{} \pm \boxed{}} \quad \textit{or} \quad 10^{\boxed{} \boxed{} \cdots}$$

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

Let's do it by the natural-frequencies approach. Imagine a population of 10^4 US bridges. Given the base rate of 1 in 10^4, assume that one bridge among them is actually in danger of collapse.

Now imagine applying the bridge-integrity test to all 10^4 bridges. It will spot the one unsafe bridge. But from among the nearly 10^4 safe bridges, it will also mark 10% or 10^3 bridges as unsafe. The bridge you use is among the roughly 10^3 bridges with a positive test. But only one of those bridges is actually unsafe, so $p_{\text{unsafe}} \approx 10^{-3}$. Therefore, the odds are 10^3 to 1 that the bridge is safe (or simply 10^3).

Now let's use Bayes theorem to get the same result. The odds form of Bayes theorem is

$$O(H|E) = O(H) \frac{P(E|H)}{P(E|\overline{H})}, \tag{1}$$

where H is the hypothesis that the bridge is unsafe, $O(H)$ is the odds in favor of that hypothesis being true, E is the evidence that the bridge failed the integrity test, $P(E|H)$ is the probability of a failed integrity test given that the bridge is unsafe (the false-negative rate), and $P(E|\overline{H})$ is the probability of a failed test given that the bridge is safe (the false-positive rate).

The initial odds $O(H)$ are 10^{-4} (the bridge is very probably safe). The likelihood ratio is

$$\frac{P(E|H)}{P(E|\overline{H})} = \frac{1}{0.1} = 10. \tag{2}$$

Therefore, the new odds are 10^{-3} in favor of the bridge being unsafe (or 10^3 in favor of it being safe, as computed above).

For an excellent article in the newspaper (of all places) on how to do Bayes theorem using natural frequencies, see Steven Strogatz's recent column in the *New York Times* (thanks to Sean Clarke for pointing me to it), available at **http://opinionator.blogs.nytimes.com/2010/04/25/chances-are/**

This should actually be 1/999.9 exactly, right?

Hooray! I did this correctly! I still feel like the odds fraction is a slightly less intuitive way of using Bayes' formula, especially for those who already have probability.

I agree with you, but odds just seems to be the ratio between probabilities of the bridge being safe over bridge being unsafe

Maybe not less intuitive, but the Bayes thm just seems most systematic and like it is covering all the bases.

Agreed, knowing how to do Bayes gave me a lot of confidence in the answer, not just the ability to solve the problem.

I answered in terms of the odds of the bridge being safe (10^3) instead of being unsafe... Same thing.. wrong reading of the question.. oops

This pretty cool hope this works - I got that answer, but and still impressed at how unlikely it is that the bridge is bad!

haha. Sounds like an article we should all read.

can this be a hyperlink?

This is supposed to be a book, so probably not.

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Yeah, and its not too much trouble to just copy and paste the link.

I made it a hyperlink in the PDF file itself (so if you download the file and view it on your computer). But when NB turns that file into images, I think the hyperlink gets lost. But it's something that I think Sacha mentioned as on his 'to do' list (if not, I'll suggest it).

COMMENTS ON PAGE 3

Problem 2   Reusing plausible-range combinations

In lecture, we saw that if the width of an object has a plausible range $w = 1 \ldots 10$ m and the length has a plausible range $l = 1 \ldots 10$ m, then the area $A = lw$ has the range $2 \ldots 50$ m^2.

If instead the plausible ranges are $w = 2 \ldots 20$ m and $l = 5 \ldots 50$ m, what is the plausible range for the area?

$$\boxed{} \pm \boxed{} \text{ m}^2 \quad \textbf{\textit{or}} \quad \boxed{} \ldots \boxed{} \text{ m}^2$$

Multiplying everything (the top and bottom) by a constant changes the midpoint of the range and the lower and upper endpoints, but does not change the width itself (the ratio of the upper to lower endpoints). For example, a factor of 25 uncertainty is still a factor of 25, just around a new midpoint. In fancy words, the width of a range is invariant to changes of scale.

From lecture, we are given that $1 \ldots 10 \times 1 \ldots 10 \approx 2 \ldots 50$. Now multiply the lower and upper endpoints of the first range by 2; and multiply the lower and upper endpoints of the second range by 5. Those changes multiply the lower and upper endpoints of the product by 2×5. So,

$$2 \ldots 20 \times 5 \ldots 50 \approx 20 \ldots 500. \tag{3}$$

Problem 3   Singing a logarithm

Estimate 1.5^{40} using the singing-logarithms method from lecture (a copy of the handout is on the course website).

$$10^{\boxed{}} \pm \boxed{} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \ldots \boxed{}$$

1.5 is 3/2, which is 7 semitones (a perfect fifth). Each semitone is a factor of $2^{1/12}$ which is also $10^{1/40}$ (40 semitones make a factor of 10). Therefore, $1.5 \approx 10^{7/40}$ and

$$1.5^{40} \approx \left(10^{7/40}\right)^{40} = 10^7. \tag{4}$$

The true value is just above $1.1 \cdot 10^7$.

This description of how to calculate the plausible range makes a lot more sense than the formula i used from the notes

I missed the "factor of" part some how from the readings and was confused for a while about how 2...20 had a range of 25. Then I realized it was a factor of 25. doh.

> Yeah, I didn't get this at first, (where the 25 came from) but I still got the right answer–now its really clear to me, though after reading this solution

i didn't think of it this way. i just used the formula in the readings, although i noticed the trend between the two examples.

> Same here, I used the formula in the readings.
>
> > I mainly used a Symmetry argument for this problem. If the initial range was 2-50 for a value between 1-100, then I figured 20-500 made sense for 10-1000.
>
> It really helps to think in log space here, or in factors like 2x or 10x.
>
> > I didn't even think of log space, but it makes a ton of sense when you mention it.
> >
> > > i just tried to think about factoring out the right numbers so that we are left w/ quantities we know about.

I feel like the key to this problem was realized the ranges were identical so you already knew what 'r' was and could go from there easily

I got it correct! :)

how did you choose 2 and 5? not arbitrarily right

I used the method in the notes to find it. I dont really understand this method...

> i agree. I also used the method from the notes. Calculating the plausible range using this is not very intuitive to me. I don't really get how the width of the range is invariant to changes of scale.
>
> > Yeah I agree. Although I used the method and arrived at the correct answer, it is not very intuitive for me either.
> >
> > > Also agreed, the lecture notes, were a great reference and thoroughly explained, step-by-step, so I relied on the readings.

I ended up using the longer explanation used in the lecture notes. I don't remember where we used this method.

> So did I. I used the formulas in the notes that give values for midpoint and range.

I did this by noticing the pattern. I guess it's an estimation technique of sorts, but I'm a little hazy on this. I'll probably ask you about this after class tomorrow.

> I did the entire analysis out. I guess I wasn't completely confident that you could carry factors through like this. I guess this will definitely just make my life easier in the future. It's nice to double check though.
>
> > I also did the entire analysis, but I used the factors to see how it carried through the problem. I think it was more useful to see how it worked, than to be told that something works.

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---

wow, never thought about doing this problem with patterns from previous questions....what if you changed the endpoints by a lot – meaning a change in the width itself? would you have to recalculate, or could you add and subtract?

> If you double all the widths (on a logarithmic scale), then you'll double the output's width. That's all on a log scale, so that means squaring the actual widths. For example, if 1..10 x 1..10 makes 2..50 (a width of 25), then 1..100 x 1..100 (doubling each logarithmic width, from 1 to 2 factors of 10) will make 4..2500 (a width of 25^2).

i understand the solution for this part but at the same time, it could probably be written out a bit more to be explicit (separate lines, use brackets to show what to multiply/why)

It is important to remember to multiply correctly in this problem, multiplying lower bounds together and upper bounds together.

So where does doing the logarithum procedure come into play? can you explain this solution in further detail?

For some reason I entered this in completely wrong but I got the same answer using the method from the reading I didn't notice the invariant but that would have made this problem a little quicker.

ooh i think i got that! or something like it?

I did this the hard way and got a different result. This way is much easier!

This solution makes sense.

yes! finally got one right

> Same here- sometimes I have the right idea, but do the calculation wrong, but definitely got this right!

> > I did something way more complicated although this was my initial inclination

> > > I used a different method to solve this problem, not the one shown in the lecture notes. Instead of finding the geometric means and then the midpoint, and then the range, I used a pattern in the example, multiplying the first term by 2 and the end term by 1/2, and got the same answer.

At least for me, this concept was pretty tough to understand so I resorted to using simple logarithms (not singing logarithms)

even with my notes and the handout i struggled with this problem.

Although, I don't think I'm ever going to use it outside this class, I think this is a really neat trick.

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---

i didn't understand this in the handout at all. it just seems like a really complicated process that you need a handout for. why not just use a calculator?

Sometimes you just don't have a calculator handy. Personally i thought this was a really cool method that I hadn't seen before, and it was cool to connect music to math.

I agree even though it might be a little round about, the method is definitely cool and something I had never thought about before

I still also don't know how to do this without memorizing a few of the intervals, which I don't know, since I haven't studied music much.

I understand we might not always have a calculator at hand, but I'm definitely more likely to have a calculator than I am to have a copy of that handout, and this method seems useless without the handout. Why are we doing this?

It does seem complicated to me, but someone explained it as A^B = B*P1 = C*40+D*P2 then A^B = 10^C * E^D where P1 is the semitones for A and P2 is the semitones for E. That made it easier.

i think the handout would be easier to understand if the steps for each situation were listed out more in an outline form instead of just written in paragraph form. if there's anything latex's taught me, it's the power of white-space.

This explanation definitely helped clear a bit of it up - honestly the handout was the only way I would've gotten through this problem at all.

Yeah, not having studied music at all; I still don't understand this method...I couldn't get it from the handout and couldn't find any other information on it online...

I don't think you need music theory to solve it. Since I never learned anything about music, I just ignored the music terminology and focused on the patterns themselves.

I try and try to understand the singing logarithms and I still cannot get them. the worst part is I play the violin. any suggestions on understanding them?

This makes more sense, i approximate 1.5 as 1 and tried to do it but got very confused

I really enjoyed this problem. It's an interesting way to estimate exponents.

I agree–it's really cool how we can estimate exponents like this. I got the right answer, but obviously i feel like in an everyday situation i'd have no possible way of applying this method because I probably wouldn't have the chart with me

I liked getting practice with this singing logarithms method - it's quite useful. Will we be allowed to use the logarithms sheet during the final?

Yeah i really liked this problem. It was simply but useful for understanding the basics of using the algorithm.

Well I'm sure we aren't expected to memorize the whole table!

I was shown this method after doing it out with 3^2 = 10 and 2^10=10^3. Need more practice here and with singing logs in general.

can we have a white paper that explains how to use the semitones handout?

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$$1.5^{40} \approx \left(10^{7/40}\right)^{40} = 10^7. \tag{4}$$

The true value is just above $1.1 \cdot 10^7$.

I was very confused by the singing a logarithm handout. This explanation helped a lot though.

> The handout could use a lot more explaining. I'm actually sit a bit confused on how to do this w/out looking at the sheet.

>> Yes I agree, this explanation makes a lot more sense than the handout.

>>> yeah I agree...I'm still really confused about the singing logarithms even though I got the right answer by following the examples.

I really liked this problem, the "singing logarithms sheet" was extremely helpful. It's just gonna be hard to memorize for the final exam

> Will we have the table of singing logarithms for the final? I feel like that's a LOT of information to memorize if we don't.

>> Yeah agreed. If we are given the handout then singing logarithms won't be that bad. I can't imagine we wont get it if it's on the final.

I got something similar to this.

> I was off by a factor of 10 for some reason. I'll have to go back and check what I did.

This was just hard until I looked into semitones. Then the conversions weren't difficult.

> I missed this class but was able to find a friend to explain it - not that bad.

> I had a little trouble understanding the theory of semitones at first but was able to get the answer using the chart in the document on the website.

this is such a cool trick! and as a plus, I was able to wow my roommate for a bit, as she thought I just had a .pdf open from another class.

I did this one right!

I did this one wrong, but my answer wasn't really off by all that much.

I thought this one was really simple but I liked it a lot! It made me feel like i learned something useful from a simple knowledge of music

Problem 4   Estimating a mass

You are trying to estimate the mass of an object. Suppose that your plausible range for its density is $1 \ldots 5\,\mathrm{g\,cm^{-3}}$ and for its volume is $1 \ldots 5\,\mathrm{cm^3}$. What is (roughly) your plausible range for its mass?

$$\boxed{} \pm \boxed{}\ \mathrm{g} \quad \textit{or} \quad \boxed{} \ldots \boxed{}\ \mathrm{g}$$

Each range is a factor of 5 in width. In semitones,

$$5 = \underbrace{\frac{5}{4}}_{4\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} = 28 \text{ semitones.} \tag{5}$$

Because 40 semitones make a factor of 10, the 28 semitones means a base-10 logarithm of 0.7. Add the squares of (logarithmic) widths to get the new (logarithmic) width squared:

$$0.7^2 + 0.7^2 \approx 1. \tag{6}$$

So, the new plausible range has a width of 1 decade (a factor of 10). The range is centered at 5 g:

$$m = \rho V \sim \underbrace{\sqrt{5}}_{1\ldots5}\ \mathrm{g\,cm^{-3}} \times \underbrace{\sqrt{5}}_{1\ldots5}\ \mathrm{cm^3} = 5\,\mathrm{g}. \tag{7}$$

So the plausible range is $1.7 \ldots 15\,\mathrm{g}$. (A full calculation, without using the semitones approximation, gives $1.6 \ldots 15.6\,\mathrm{g}$.)

Problem 5   Which is the wider range?

Suppose that your knowledge of the quantities a, b, and c is given by these plausible ranges:

$$\begin{aligned}
a &= 1 \ldots 10 \\
b &= 1 \ldots 10 \\
c &= 1 \ldots 10.
\end{aligned} \tag{8}$$

Which quantity – abc or a^2b – has the wider plausible range? (The 'width' is the ratio of the upper to lower endpoints; so a, b, and c are each a factor of 10 wide.)

☐ abc

☐ a^2b

☐ Both quantities have the same width.

Both choices have b in them, so ignore it and instead compare ac versus a^2. When computing ac there is a chance that an overestimate in a will compensate an underestimate in c (and vice versa). However, when computing a^2, any error in estimating a is magnified – a factor of 2 error in a becomes a factor of 4 error in a^2. So, a^2 has a wider plausible range than ac. Numerically,

$$\begin{aligned}
ac &= 2 \ldots 50. \\
a^2 &= 1 \ldots 100.
\end{aligned} \tag{9}$$

it does still seem weird to try and do this in semitones

> agreed, that never occurred to me. is that necessary?

>> There was no indication to use this method in the problem and I didn't think it was related to the semitones thing. Seems weird to use it here for no reason. I used the same formula for plausible range from the other reading.

>> Same, I wish this problem would have mentioned that method.

> I'm also a bit surprised that this problem used semitones. I tried to reverse calculate a range in a fashion similar to the paper area method.

>> You don't have to do it that way and can use the formulas from the reading. It was just more fun for me (one of the benefits of teaching after all) to not use a calculator and thereby keep my singing-log muscles in training.

I think I guesstimated a lot to do this problem in stead of doing it step by step like the solution.

Nice, I did this one somewhat differently but had the same estimated answer. I still used singing logs, but estimated range = 5^sqrt(2) =5^1.41 = 10^1.41 log(5) = 10^1.41 * 28/40 = 10^1.

whats the point of doing it this way?

This was the step I missed. Coo'

> i actually didn't spot this as a semitones problem at all.

>> I agree. This problem was kinda tricky about it. But it makes you wonder if you should be using the semitone estimation more often.

>> Where do the 2s come from?

>>> The twos come from our (arbitrary) breakdown of 5 into 5/4*4 = 5/4*2*2, which then happens to be easy to calculate in terms of semitones.

I did not even see this as a way to approach this problem. I ended up just using the range equations.

I think I will have to go through this problem and try it again.

where did 40 come from?

> You can find this information on the approximating logarithms chart...

So if it weren't 1, we'd have to take the square root to get the width, right? (which might be hard)

> i don't follow this part of the solution

my computer must have really bad resolution, because that looks like a minus sign, but that would make no sense.

> Try zooming in NB, it's a double tilde for approximately.

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I found the same new logarithmic width, but got values for the endpoints closer to those of the full calculation.

Based on hidden step of 5*1/sqrt(10) and 5*sqrt(10), or is there an easier way just knowing the width of the range and the midpoint?

> I should have written out that step. For me, sqrt(10)=3 is so automatic, as is 5/3=1.7, that I didn't even think of it as a step – but that's a reason to explain it so that it'll become automatic for everyone else.

i used the same method here as i did in question 1. it seems more intuitive to me.

> I also did the same method as i did in question 1. It might be a few more steps, but I still think I got the answer faster than if I had tried the log approach.
>
> > You were still taking logs though, just not using the semitones method, right?
> >
> > > I also used the method from problem 1. It makes a lot more intuitive sense to me and seems less contrived than singing logarithms (though I still think the singing logarithms are an incredibly useful and interesting tool).
>
> I did also! dont know where these semitones came into the picture here

What is the full calculation- the method from the notes?

> yes - that is what I did and I got the 1.6 to 15.6 g, I found the method from the notes much easier to use than the semitones!
>
> > agreed...the application of semitones here took a couple of minutes to understand how it broke down properly. I like seeing these things applied elsewhere though!

if you had wanted us to use the semitones method, perhaps you should have said so. I naturally just assumed this was another range-calculation and i didn't even have a feeling that it was supposed to be done another way

> I don't mind what method you use. My solutions are just one approach. One of the purposes of making the solution sets a reading memo is that I hope others will suggest alternative solutions, and everyone can benefit.
>
> I did it by singing logarithms (which are just logarithms to the base 10^1/40) because it was more fun than using a calculator to do the logarithms.

I think I got this; and I used the non-semi-tone-method with the mus.

> I solved it without using semitones, but I ended up with a much smaller plausible range. However the semitones method seems easier to use.

I got here without using semitones. They seem a lot more complicated here than following the method given in class to find these.

I arrived to this answer intuitively based on our discussion in class about the inherent difficulty in eyeballing area versus the greater ease in estimating two lengths.

I can't believe I also thought to do this, I feel so smart! :)

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$$\boxed{}\pm\boxed{}\;\mathrm{g}\quad\textit{or}\quad\boxed{}\ldots\boxed{}\;\mathrm{g}$$

Each range is a factor of 5 in width. In semitones,

$$5 = \underbrace{\frac{5}{4}}_{4\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} \times \underbrace{2}_{12\text{ semitones}} = 28 \text{ semitones.} \tag{5}$$

Because 40 semitones make a factor of 10, the 28 semitones means a base-10 logarithm of 0.7. Add the squares of (logarithmic) widths to get the new (logarithmic) width squared:

$$0.7^2 + 0.7^2 \approx 1. \tag{6}$$

So, the new plausible range has a width of 1 decade (a factor of 10). The range is centered at $5\,\mathrm{g}$:

$$m = \rho V \sim \underbrace{\sqrt{5}}_{1\ldots5}\,\mathrm{g\,cm^{-3}} \times \underbrace{\sqrt{5}}_{1\ldots5}\,\mathrm{cm^3} = 5\,\mathrm{g}. \tag{7}$$

So the plausible range is $1.7\ldots15\,\mathrm{g}$. (A full calculation, without using the semitones approximation, gives $1.6\ldots15.6\,\mathrm{g}$.)

Problem 5   Which is the wider range?

Suppose that your knowledge of the quantities a, b, and c is given by these plausible ranges:

$$a = 1\ldots10$$
$$b = 1\ldots10 \tag{8}$$
$$c = 1\ldots10.$$

Which quantity – abc or a^2b – has the wider plausible range? (The 'width' is the ratio of the upper to lower endpoints; so a, b, and c are each a factor of 10 wide.)

☐ abc

☐ a^2b

☐ Both quantities have the same width.

Both choices have b in them, so ignore it and instead compare ac versus a^2. When computing ac there is a chance that an overestimate in a will compensate an underestimate in c (and vice versa). However, when computing a^2, any error in estimating a is magnified – a factor of 2 error in a becomes a factor of 4 error in a^2. So, a^2 has a wider plausible range than ac. Numerically,

$$ac = 2\ldots50.$$
$$a^2 = 1\ldots100. \tag{9}$$

Oh this is extremely clever. I can see how this would be a problem if you thought you could eliminate a factor of a from both equations too.

Invariant! Which I forgot...

oo, i get it now, i thought the opposite. since in ac, there are 2 variables, and you will get a wider range, since is more "varied"

See, this is what i felt when i saw a "squared". I thought "oh, that must have a larger error because it's getting squared". But then i reasoned out that since a and c have the same uncertainty, and the uncertainty is the only thing taken into finding plausible ranges, that they must be equal. How do you account for this using the formulas you gave?

Exactly my question–could we see how this plays out in the math? I don't know how to distinguish the square of one term from the product of two terms in the range calculation.

This problem brings out an assumption that I did not make explicit enough in the reading. Namely, the formula for combining the plausible ranges assumes that the errors are uncorrelated. Said differently, information about the accuracy of one factor does not tell you anything about the accuracy of the other factor's estimate.

That assumption is fine when you are multiplying say number of people in the US * cars per person. But that assumption is not true when you are talking about a*a. There, any error in a becomes twice (on a log scale) that error in a^2. So, if the range for a is x..y, then the range for a^2 is x^2..y^2.

Exactly what I thought! I don't see how this problem used an approximation method

Hahahaha I got exactly the opposite! That was silly, I should have thought that through more. I just assumed that not having another variable w/error would reduce the width – but of course, they balance each other out!

I actually didnt think about the compensating for underestimates, or errors...I just thought that if a is either really small or really large, the range becomes huge–either 1^2 or 10^2, the two ends of the spectrum. And if we include c, c doesnt have to be the same as a (if a is 1 or 10, c can be something in between), which narrows the range

Wow...I didn't even think of canceling out the b. I just tried to include it in the formulas. Probably why I didn't get the right answer...

I like this explanation. I never really thought of this problem until this hw assignment.

I also like this explanation but I find it difficult to resolve the fact that a and c also have more diversity than a^2. This seems to suggest that they should have a wider potential range.

wow this never occurred to me, i just took the fact that they have the same ranges at face value!

This is so much easier to say than what I said. I'm actually kind of confused by my answer now that I go back, even though I got this right. Glad I have a simpler explanation now.

I chose "both" for this problem, because all values are equal. Thinking about in terms of how an error will be magnified makes a lot more sense though!

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$$\begin{aligned} ac &= 2\ldots 50.\\ a^2 &= 1\ldots 100. \end{aligned} \tag{9}$$

Ah, this is quite insightful. For some reason my intuition said they would be the same width because each of the values for a and c were equally probable. That doesn't make much sense now. Cool problem!

Yeah, I spent a while reasoning about this in different ways too... this was one of my favorite (or at least most insightful) problems on this homework

I kept trying to reason my way into thinking that a^2 would have the wider range, but was never quite able to get to it.

Now I realize that my thinking was quite close to the solution here.

I did not think of that. a^2 will magnify an error in an approximation while (ac) can have its factors canel out its approximation errors.

Yeah, I definitely thought both quantities would have the same width, since you're multiplying 3 values that all have the same range. I didn't realize we were supposed to think about any potential error in the ranges.

I had the right answer and the right intuition, but I think putting my explanation into writing was definitely not as concise or understandable as it is here. Well put.

Oh I get it now, I also failed to think in terms of probability for this problem. Tricky!

Ah this makes so much sense. Great explanation. Its actually is really intuitive.

That does make a lot of sense. I used a dummy variable to represent the range 1...10, but I didn't realize that in doing so I neglected to include information about the errors present in the measurements.

I put the same and then explained that ac was probably better for some reason. Good explanation.

I thought this intuitively, but figured the math wouldn't work out that way, considering they were all the same. This analysis definitely makes sense though.

this is what i guessed too. the fewer variables you have the greater chance that no errors will cancel out.

I disagree with this answer. Sure, there' sa chance error in c will cancel with error in a, but isn't there also a chance that the error in c is not only in the same direction, but actually of greater magnitude (both overestimate, but c does so by 10% more) in which case ac would have a wider range than a^2...

I liked this problem - it made sense to me.

this makes sense now but when I was doing it I was totally convinced it was the other way around

If we said this without any math behind it, is that ok?

I hope so! I definitely had the same reasoning, just a much poorer explanation.

I understand why the plausible range should be larger with a^2, but I don't know how he was able to get those ranges.

i mean, the way this class works, it doesn't matter whether we got a certain problem perfectly right or not, right? you understood what was happening and by getting to see the solutions, you get to further understand it. that's it

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I thought that aˆ2 was smaller, since we only picked 1 random number rather than 2. I realize now why that's wrong.

yeah exactly! that's the same thing I thought too!!

I didn't understand why the ac factor didn't affect the range the same way as aa did. But now the explanation makes sense because the c gives a chance to cover up for mistakes in estimating the a

I knew there was probably a reason for the a²b term having a greater range, but I didn't know what. This makes so much more sense.

Agreed, it seemed to easy to be the same but I still couldn't understand exactly what the difference would be.

I thought it seemed to easy to blame the square term for making a larger range and somehow reasoned out (incorrectly) that they were the same. funny how that works out

I was originally thinking of this, but then got confused by the reading. I should have trusted my intuition...

I agree. I tried to use the range method from the reading to no success.

Thing that got me was that aˆ2 could be smaller than ac. But once I realized this would increase the range, it made sense.

but c could also be overestimated? this is assuming someone is estimating - i assumed numbers were chosen randomly from the range

i see what you're saying here, but i came up with a reason that made perfect sense to me, but gave the wrong answer. i said that -abc depends on 3 terms, while a^2*b depends on 2. (or 2 terms and 1 term, if you cancel out the b), so there's a bigger chance that something will go wrong with the -abc. why is this logic incorrect?

It's because more (independent) terms means more chance for errors to cancel. There's no guarantee that they will cancel, but it's at least possible. Whereas if everything depends on one quantity a, then in figuring out a*a, the errors have no chance of cancelling.

Problem 6 Golf-ball dimples

Why do golf balls have dimples?

☐ The dimples make the main airflow around the ball become turbulent.

☐ The dimples stabilize the flight.

☐ The dimples are there by tradition but have no physical justification.

☐ The dimples make the airflow turbulent in the thin boundary layer adjacent to the ball.

Let's first calculate the Reynolds number (a good first instinct in understanding a fluid flow). Let's say that the golf ball is hit at $30\,\mathrm{m\,s^{-1}}$ (70 mph!). It's diameter is a few centimeters, say $3\,\mathrm{cm}$. Using $\nu \sim 10^{-5}\,\mathrm{m^2\,s^{-1}}$ for the viscosity of air, the Reynolds number is

$$\mathrm{Re} \sim \frac{30\,\mathrm{m\,s^{-1}} \times 3\cdot 10^{-2}\,\mathrm{m}}{10^{-5}\,\mathrm{m^2\,s^{-1}}} \sim 10^5. \qquad (10)$$

That means the Reynolds number in the boundary layer is roughly $\sqrt{\mathrm{Re}} \sim 10^{2.5} \sim 300$. This is not high enough for turbulence, so the boundary layer is laminar.

A laminar boundary layer separates easily on the back of the golf ball, creating a large turbulent, low-pressure region behind the ball – that means lots of drag. If only the boundary layer could be made turbulent! Then the boundary layer would stick to the golf ball farther along the back side, and the drag would be lower. That's just what the dimples do: They trip the boundary layer into turbulence at a lower Reynolds number than is otherwise required (Choice D).

This problem didn't really test my knowledge since it was taken directly from the reading. Although, I don't think I would have been able to re-derive this on my own.

I liked that it was here- we saw it on the diagnostic test when we had no knowledge of the subject and now we can actually reason it!

I explained this using theory but I guess working with numbers creates a better explanation.

Why don't airplanes and rockets, and bullets, and such have dimples on them? Is the boundary layer airflow already somehow turbulent? Or is the shape too complex, or some other reason.

I didn't even bother calculating the Reynolds number- didn't we go over this in a previous pset?

yeah that's what I thought, so I just put D as my answer.

I think this question was more about understanding physical concepts, instead of the usual "find the approximation".

Yea I feel like we went over this before. Maybe it was on the 1st exam?

yeah it was on the diagnostic... a lot of the pset problems have been

same here. i also hate thermo so taht doesn't help either.

I understand this and know how to get the solution to this problem but I still don't understand intuitively why a turbulent boundary layer wants to stick to the surface of the golf ball.

I'm not really sure, but I think maybe it has to do with the vorticies that turbulent flow produces in comparison to laminar flow. When it's laminar, the flow is less likely to change direction, so going over a shape would not "bend" the flow downwards - it would want to continue more straight. However, if turbulent - the flow is much more likely to be pulled closer to the surface as there is no sense to the flow. this is just my guess though, I don't know for sure.

I still dont understand how a turbulent boundary layer sticks better and helps

This make perfect sense especially after the pset last week.

it was straight from lecture!

I just remembered the fact from lecture, not so much math as shown here.

It's interesting to see applications of the boundary layer. I didn't understand it very well when we first learned about it.

I think choice A is pretty similar to D. D is the better answer, but I don't see how A is wrong

The main flow is already turbulent, and that's true with or without the dimples (because the main-flow Reynolds number is so large for any actual golf-ball stroke). So, the dimples don't change anything about the main flow (directly). But they do make the boundary layer turbulent (which indirectly changes the main flow because the bdy layer stays attached farther back).

Problem 6   Golf-ball dimples

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☐   The dimples are there by tradition but have no physical justification.

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---

this was easy if you remembered the explanation from the reading. no calculations required! also mythbusters told me this...

> I believe it's also stated in the reading that by creating turbulence in the boundary layer, there would be less drag..etc

>> I agree - my statement was something along the lines of "Cause you told me so"

got it right! However, I didn't do any calculations. I just reasoned that tradition wouldn't sell, the flight path isn't stabilized, from my experience, and then guessed btwn a and d.

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Problem 7   Singing logarithms to combine plausible ranges

You are trying to esimate the plausible range for the volume of an object. You have assigned the length, width, and height the plausible ranges

$$l = 1 \dots 10\,\text{m}$$
$$w = 1 \dots 10\,\text{m} \tag{11}$$
$$h = 1 \dots 10\,\text{m}.$$

In other words, each range is a factor of 10 wide (the 'width' is the ratio of the upper to lower endpoints). Convince yourself that the plausible range for the volume $V = lwh$ is a factor of $10^{\sqrt{3}}$ wide and is centered on $10^{1.5}\,\text{m}^3$.

Each factor in lwh is centered at $10^{0.5}\,\text{m}$ (the geometric mean of the lower and upper endpoints). Therefore lwh is centered on
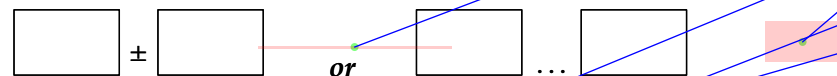
$$\left(10^{0.5}\,\text{m}\right)^3 = 10^{1.5}\,\text{m}^3. \tag{12}$$

To compute the width of the range for lwh, note that each factor in lwh is 1 factor of 10 in width. For plausible ranges, add the squares of the (logarithmic) widths to get the square of the final (logarithmic) width:

$$1^2 + 1^2 + 1^2 = 3. \tag{13}$$

So the plausible range for V is $\sqrt{3}$ wide (in its base-10 logarithm); in other words, the range has width $10^{\sqrt{3}}$.

For the answer box, use $\sqrt{3} \approx 1.7$ or $\sqrt{3} \approx 1.73$ and the singing-logarithm method from lecture (a copy of the handout is on the course website) to estimate $10^{\sqrt{3}}$.

$$\boxed{} \;\pm\; \boxed{} \quad \textbf{\textit{or}} \quad \boxed{} \;\dots\; \boxed{}$$

I'll first use $\sqrt{3} = 1.7$. Then

$$10^{\sqrt{3}} = 10^{1.7} = 10 \times 10^{0.7}. \tag{14}$$

But $10^{0.7}$ is 28 semitones (40 semitones is a factor of 10).

$$28\ \text{semitones} = \underbrace{\frac{5}{4}}_{4\ \text{semitones}} \times \underbrace{2}_{12\ \text{semitones}} \times \underbrace{2}_{12\ \text{semitones}} = 5. \tag{15}$$

So, $10^{1.7} \approx 50$.

For fun, let's correct that estimation slightly by using $\sqrt{3} = 1.73$. The extra factor is $10^{0.03}$. Since 0.03 is roughly 1/40, $10^{0.03}$ is roughly 1 semitone. Based on the observation that 1.25 is 4 semitones, 1 semitone is given by

$$1.25^{1/4} = (1 + 0.25)^{1/4} \approx 1 + \frac{0.25}{4} \approx 1.06. \tag{16}$$

So, we should raise the earlier estimate of $10^{1.7}$ by 6%, which gives 53. [An exact calculation gives $10^{\sqrt{3}} \approx 53.96$.]

I made this problem harder than it could have been. I essentially did two problems: estimating the plausible range of l*w and then estimating the plausible range of (l*w)*h.

> me Too!!

> Yeah, for some reason I didn't think to add.

I definitely did not see this trick for this problem, pretty cool

> I don't quite get where this came from.

> This is pretty cool, I tried to do it in a much more complicated way.

This one was kinda fun to do

> I did so much more work than what was shown here.

I'm not sure where my confusion is on this, because I follow my notes and I think I'm doing it right and then I get it wrong.

I thought this was the same kind of question. After doing the others, I knew I needed to estimate using semitones again.

this is the trick i was missing :(

I broke this up differently. I noticed that 1.7 was about 70/40

confused about this. seems that ur examples make sense if i stare at them, but singing logs didn't click for me like the other stuff. i guess practice makes perfect...

> Perfect practice makes perfect...

This method was really fun to work through! I don't really understand the practicality of it yet (access to a singing logarithms table implies that you would also have access to a calculator, maybe on the Internet) but it was surprisingly accurate and very interesting to learn.

> Yeah i agree. Its interesting to see the application and a different way of thinking about calculating, but I don't really see its applicability in everyday life.

>> Yeah agreed. I had a lot of fun working through the problem and even got the right answer, but I don't know when else I'll use this method.

> With a bit of practice, the singing-log table will become part of your mental toolbox. I never have the table in front of me. For example, I wrote all the solutions that used singing logarithms without the table. You need just a few ideas to derive the whole table, or any item in it as needed. First, 2^1/12 = 10^1/40 [which is from 2^10 = 10^3]. Then you need two more numbers: 7 semitones = factor of 3/2 (perfect fifth); 4 semitones = factor of 5/4 (major third). Everything else can be figured out from those.

> For the book itself, I'm going to make the singing logarithm handout into an actual section with explanations, and we'll build up the table step by step using the preceding information (like what wedid in lecture).

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So, we should raise the earlier estimate of $10^{1.7}$ by 6%, which gives 53. [An exact calculation gives $10^{\sqrt{3}} \approx 53.96$.]

This is pretty incredible accuracy for such a goofy method.

I can't believe I got this one right. I thought I had it wrong for sure

surprisingly close approximation though

Problem 8   Perfume

If the diffusion constant (in air) for small perfume molecules is $10^{-6}\,\mathrm{m^2\,s^{-1}}$, estimate the time for perfume molecules to diffuse across the lecture room.

$$10\;\boxed{}\;\pm\;\boxed{}\;\mathrm{s}\quad\textit{or}\quad10\;\boxed{}\cdots\boxed{}\;\mathrm{s}$$

To include in the explanation box: Now try the experiment, at least mentally. How long does it actually take to smell the perfume from across the room? Explain the large discrepancy between the theoretical estimate and the experimental value.

The dimensions of a diffusion constant D are L^2T^{-1}, so the diffusion time is given by $\tau \sim x^2/D$, where x is a length. The lecture room is perhaps $10\,\mathrm{m}$ deep (and maybe $15\,\mathrm{m}$ wide). It doesn't matter exactly which length I use, so I'll use the one that is simpler to square: $x \sim 10\,\mathrm{m}$. Then

$$\tau \sim \frac{10\,\mathrm{m^2}}{10^{-6}\,\mathrm{m^2\,s^{-1}}} \sim 10^8\,\mathrm{s} \sim 3\,\mathrm{years}. \tag{17}$$

That time does not agree with experiment! In reality, it takes perhaps a minute to notice that someone has opened a bottle of aromatic stuff. The discrepancy is that the molecules must travel not just by diffusion; in fact, the unavoidable air currents in the room transport the molecules much farther and faster than diffusion can.

I actually found this problem somewhat out of place on this pset - dimensionless groups were some time ago.

> yeah, although we talked about diffusion when we talked about random walks more recently.

>> Yea, the formula I found for this was from the probabilistic random walks reading

Did this mean given a few molecules released at an instant or is the bottle left open? What's the role of concentration or flux here?

> the final answer was soooooooooooo many orders of magnitude off that i don't think those factors really matter much here.

I am glad this was included in the problem, or I would have been really confused, b/c the approximating was not that difficult, nor was the math!

This was so interesting, especially because of the huge discrepancy between the estimated value and real life.

Forgot to square the length, otherwise would have gotten the same answer...

> This problem was really simple, but I didn't see it. I didn't trust my gut and confused myself by trying to find the velocity of air (which doesn't make sense).

> I took this into account by looking at the surface area of a sphere created by the perfume. This has the same factor, x^2, as the way solved here.

I totally missed this. i thought the room was a lot larger!

> i did too. i said the room was 100m long... i did math backwards and said that 10m was 3ft instead of 30 ft. oooops.

Should this be 100m^2 since x   10m?

this is exactly what I got, at least now I know why the numbers didn't make sense. It just shows that problems in real life are more complicated than you think they are... except for the easy cases.

> I agree–i got this answer too, and at first i thought to myself, doesnt this just show us that our estimations can be completely off? then I thought–well in a good estimation we're not just using one method–here, if we just apply a little common sense and relate it back to our daily experiences, we add a lot of insight to our estimation

I just thought my answer was off because I missed a step, when it was actually correct. The opposite explanation makes sense

> same... i got a few years and thought it couldnt be right since it was so far from the convection result

>> I remember you talking about this in class, so I was vaguely confident about my answer, but it definitely seems off intuitively.

>>> Yeah so I got this number and wrote it as my answer. I completely forgot to convert units to check if the answer was realistic. I don't know how I would have felt if it was multiple years, but I guess it would have worked out.

Problem 8   Perfume

If the diffusion constant (in air) for small perfume molecules is $10^{-6}\,\mathrm{m^2\,s^{-1}}$, estimate the time for perfume molecules to diffuse across the lecture room.

$$10\ \boxed{}\ \pm\ \boxed{}\ \mathrm{s} \qquad \textbf{\textit{or}} \qquad 10\ \boxed{}\ \ldots\ \boxed{}\ \mathrm{s}$$

To include in the explanation box: Now try the experiment, at least mentally. How long does it actually take to smell the perfume from across the room? Explain the large discrepancy between the theoretical estimate and the experimental value.

> The dimensions of a diffusion constant D are $\mathrm{L^2T^{-1}}$, so the diffusion time is given by $\tau \sim x^2/D$, where x is a length. The lecture room is perhaps 10 m deep (and maybe 15 m wide). It doesn't matter exactly which length I use, so I'll use the one that is simpler to square: $x \sim 10\,\mathrm{m}$. Then
>
> $$\tau \sim \frac{10\,\mathrm{m^2}}{10^{-6}\,\mathrm{m^2\,s^{-1}}} \sim 10^8\,\mathrm{s} \sim 3\,\mathrm{years}. \tag{17}$$
>
> That time does not agree with experiment! In reality, it takes perhaps a minute to notice that someone has opened a bottle of aromatic stuff. The discrepancy is that the molecules must travel not just by diffusion; in fact, the unavoidable air currents in the room transport the molecules much farther and faster than diffusion can.

This was an interesting problem, I was a little worried when I got 3 years, but the fact that you need air currents, etc... helps reassure that our estimation was off

its pretty amazing how large the discrepancy is, epically when I feel like the air int he class room isnt' really moving

This was a really surprising fact. I didn't realize that small air currents can move particles that much faster compared to if they weren't present.

> Lots of air currents. Don't know what we'd do without them.
>
> Hmmm. Makes sense that the air currents would be making the difference. Now I kind of wish people didn't use this perfume example to explain diffusion all the time.

Are there any accessable environments that do not have significant air currents that this sort of test could be performed?

I got a crazy long answer too! So, if there wasn't air currents in the room, it would really take THAT long for perfume odor to disperse? And wouldn't the odor go away in that time?

is there a way to account for that?

> My mass transfer education is minimal, but maybe you could find an effective diffusion constant given convective mass transfer.
>
> > Air flow due to pressure and temperature gradients might also be considered in analyzing the air flow.

it's sort of like the difference between conduction and forced convection.

> Yeah i was really surprised at how small of a role diffusion plays in this scenario

if it were a sealed room, would it be true?

> Very likely, because any temperature differences between the top and bottom of the room would produce convection (air currents). It's really hard to get rid of convection.

Problem 9 **Teacup spindown**

You stir your afternoon tea to mix the milk (and sugar if you have a sweet tooth). Once you remove the stirring spoon, the rotation starts to slow. In this problem you'll estimate the spindown time τ: the time for the angular velocity of the tea to drop by a significant fraction.

To estimate τ, consider first a physicist's idea of a teacup: a cylinder with height l and diameter l, filled with a water-like liquid. Tea near the edge of the teacup – and near the base, but for simplicity we'll neglect the effect of the base – is slowed by the presence of the edge.
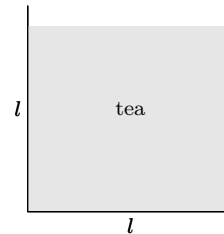
Because of the no-slip boundary condition, the edge creates a velocity gradient. Because of the tea's viscosity, the velocity gradient produces a force along the edge. This force tries to accelerate each piece of the edge along the direction of the tea's motion. The piece in return exerts an equal and opposite force on the tea. That is how the edge slows the rotation. Now analyze this model quantitatively using the following steps. Keep the results in symbolic form until the final step (**Step e**) when you get a numerical value for τ.

a. Convince yourself that the spindown time τ is given by

$$\tau \sim \frac{\rho l^5 \omega}{\sigma l^3} = \frac{\rho l^2 \omega}{\sigma}, \tag{18}$$

where ρ is the density of tea, σ is the viscous stress (the viscous tangential force per unit area), and ω is the initial angular velocity. *Hint:* Consider the torque on and the angular momentum of the rotating blob of tea. In addition, drop all dimensionless constants like π and 2 by invoking the Estimation Theorem 1 = 2.

If the tea is spinning at angular velocity ω, then it has angular momentum $L = I\omega$, where I is the moment of inertia. The moment of inertia is given by mass times a squared distance from the origin:

$$I \sim \underbrace{\rho l^3}_{m} \times l^2 = \rho l^5. \tag{19}$$

Not all of the mass is at a distance l from the center, but the twiddle accounts for the omitted dimensionless constant. With that I, the angular momentum is

$$L \sim \rho l^5 \omega. \tag{20}$$

The viscous stress produces a torque that reduces this angular momentum. The viscous torque is

$$\text{viscous torque} \sim \underbrace{\text{viscous stress}}_{\sigma} \times \underbrace{\text{area}}_{l^2} \times \underbrace{\text{lever arm}}_{l} \sim \sigma l^3. \tag{21}$$

Because torque is dL/dt, it has dimensions of L/t. So a time is given by L/torque:

$$\tau \sim \frac{\rho l^5 \omega}{\sigma l^3}. \tag{22}$$

b. Now estimate the viscous stress σ by using the idea that

$$\text{viscous stress} \sim \rho \nu \times \text{velocity gradient}. \tag{23}$$

The velocity gradient is determined by the thickness of the region over which the the edge significantly affects the flow; this region is the boundary layer. Let δ be its thickness (you'll find δ

I like this problem very much. There is no sure procedure to solve the problem that jumps to mind immediately. But just playing with dimensions and intuition gets you a decent solution.

I got this problem wrong... solution is still hard to follow.. Could we maybe go over this in class?

I also got it wrong. I can follow the solution, but the class might benefit from an in-person explanation.

Agreed, I thought this problem was pretty difficult, and this solution doesn't make much sense either. I think this problem was a little too heavy on physics, particularly for those of us who don't use 8.01 knowledge everyday.

I would similarly appreciate a comment or two in class in relation to this problem.

I will discuss it, hopefully with a related demonstration that just occurred to me (details to be worked out in time, hopefully!).

where were these in the first place?

For example, in the moment of inertia of a spinning cylinder of water.

How did I forget this??

this is were I messed up!

Yeah, I never attempted this one, wouldn't have thunk of this!

well i definitely didn't get this on my own, but i believe you.

Yeah I agree, I definitely did not know how to even start this question, let alone work through it and finish.

in **Step d**). In terms of δ, estimate the velocity gradient near the edge. Then estimate the viscous stress σ.

The velocity gradient is

$$\frac{\Delta v}{\Delta x} \sim \frac{\omega l}{\delta}. \tag{24}$$

Therefore the viscous stress is

$$\sigma \sim \rho \nu \frac{\omega l}{\delta}. \tag{25}$$

c. Insert your expression for the viscous stress σ into the earlier estimate for the spindown time τ. Your new expression for τ should contain only the boundary-layer thickness δ, the cup's size l, and the viscosity ν.

After substituting,

$$\tau \sim \frac{\rho l^5 \omega}{l^3 \times \rho \nu \omega l/\delta} \sim \frac{l\delta}{\nu}. \tag{26}$$

[The information in the problem statement is sufficient to arrive at this result, because $l\delta/\nu$ is the only way to make a time from l, δ, and ν.]

d. Now estimate the boundary-layer thickness δ using your knowledge of random walks. The boundary layer is a result of momentum diffusion – and ν is the momentum-diffusion coefficient. In a given time t, how far can momentum diffuse? This distance is δ. Estimate a reasonable t for the rotating blob of tea. [*Hint:* After rotating by 1 radian, the fluid is moving in a significantly different direction than before, so the momentum fluxes no longer add.] Use that time to estimate δ.

A reasonable time is the time to rotate 1 radian, namely $t \sim 1/\omega$. In that time, the diffusion distance δ is

$$\delta \sim \sqrt{\nu t} \sim \sqrt{\nu/\omega}. \tag{27}$$

e. Now put it all together. For a typical teacup stirred with a typical stirring motion, what is the predicted spindown time τ? [Tea is roughly water, and $\nu_{water} \sim 10^{-6}\,\mathrm{m^2\,s^{-1}}$.]

$$10^{\boxed{}} \pm \boxed{}\ \mathrm{s} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \dots \boxed{}\ \mathrm{s}$$

Substituting for δ in the expression for the spindown time τ gives

$$\tau \sim \frac{l\delta}{\nu} \sim \frac{l}{\sqrt{\nu\omega}}. \tag{28}$$

Now put in numbers. My nearby teacup is a few inches across, so $l \sim 10\,\mathrm{cm}$. When I stir the tea, it rotates at a frequency $f \sim$ few Hz, so $\omega = 2\pi f \sim 20\,\mathrm{s^{-1}}$. The result is

$$\tau \sim \frac{0.1\,\mathrm{m}}{\sqrt{10^{-6}\,\mathrm{m^2\,s^{-1}} \times 20\,\mathrm{s^{-1}}}} \sim 20\,\mathrm{s}. \tag{29}$$

The velocity gradient formula is what I lacked to get this problem right

I actually did this correctly! I was just trying something although I didn't know if it was reasonable to make the velocity gradient into omega*L.

i didn't get this step. i guess i didn't understand what it was asking for.

Ooops, I guessed it as omega*delta instead. That threw my final answer of significantly, but I realized this in my submission online

This was the furthest I could get on the problem.

For some reason, I always forget to consider dimensional analysis in approximating. If I had remembered this, I could have finished the problem.

I still think random walks was a pointless unit where I learned nothing.

I found this part really confusing. In particular, I was unsure how to apply random walks to analyze the momentum diffusion. I tried taking slices, etc. but eventually gave up and set t 1s.

Definitely the hardest question on the test. Even reading the question took a few goes. But this explanation does walk you through it well. As long as you don't get discouraged by the fact it's sooo long.

THis question really confused me. The hint not helping much at all

this is the piece that I was missing to completely solve this problem. Now that makes sense.

I only thought to do this because the problem statement said to drop all constants like pi, and i immediately thought of reducing w.

Oh interesting, this never crossed my mind. I went straight to t=1sec.

Ya, this estimate was elusive to me. I ended up going with smaller t, and still wound up with a huge huge number for my time constant answer (thousands of seconds)...

huh?

It took me a while to convince myself that it made sense to have omega in the denominator, here. Basically it means that the faster you spin it, the faster it slows to a fixed percentage of its initial momentum – not to a momentum constant for all cases!

ahhh i messed plugging stuff in here!

this seems like a really long time to me. i guessed it would only be 1 or 2 seconds.

I got 100,000 years

I actually got this right, but only because I got a lot of help on this one. It was really hard.

in **Step d**). In terms of δ, estimate the velocity gradient near the edge. Then estimate the viscous stress σ.

> The velocity gradient is
>
> $$\frac{\Delta v}{\Delta x} \sim \frac{\omega l}{\delta}. \tag{24}$$
>
> Therefore the viscous stress is
>
> $$\sigma \sim \rho v \frac{\omega l}{\delta}. \tag{25}$$

c. Insert your expression for the viscous stress σ into the earlier estimate for the spindown time τ. Your new expression for τ should contain only the boundary-layer thickness δ, the cup's size l, and the viscosity v.

> After substituting,
>
> $$\tau \sim \frac{\rho l^5 \omega}{l^3 \times \rho v \omega l/\delta} \sim \frac{l\delta}{v}. \tag{26}$$
>
> [The information in the problem statement is sufficient to arrive at this result, because $l\delta/v$ is the only way to make a time from l, δ, and v.]

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> A reasonable time is the time to rotate 1 radian, namely $t \sim 1/\omega$. In that time, the diffusion distance δ is
>
> $$\delta \sim \sqrt{vt} \sim \sqrt{v/\omega}. \tag{27}$$

e. Now put it all together. For a typical teacup stirred with a typical stirring motion, what is the predicted spindown time τ? [Tea is roughly water, and $v_{\text{water}} \sim 10^{-6}\,\text{m}^2\,\text{s}^{-1}$.]

$$10^{\boxed{}} \pm \boxed{}\ \text{s} \quad \textbf{\textit{or}} \quad 10^{\boxed{}} \cdots \boxed{}\ \text{s}$$

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> Now put in numbers. My nearby teacup is a few inches across, so $l \sim 10\,\text{cm}$. When I stir the tea, it rotates at a frequency $f \sim$ few Hz, so $\omega = 2\pi f \sim 20\,\text{s}^{-1}$. The result is
>
> $$\tau \sim \frac{0.1\,\text{m}}{\sqrt{10^{-6}\,\text{m}^2\,\text{s}^{-1} \times 20\,\text{s}^{-1}}} \sim 20\,\text{s}. \tag{29}$$

i think the most frustrating part of this problem for me was that this is about the answer i would get if i just guessed off the top of my head. i don't think alllllllllllllllllll the work that was just done made the answer tooooooo much better.

> And now you also know why (physically) it takes about that much time.

wow i think i was way off!

To include in the explanation box: Estimate τ_{exp} experimentally by stirring tea. Compare the experimental time with the predicted time.

I just tried it, and $\tau_{\text{experimental}}$ (the time for the rotation to slow significantly) was around 10 s. Not bad!

This was the problem I didnt manage to do from the problem set, the step by step was fantastic.

I wasn't able to do this problem either. But you're right, the solution step-by-step does a great job of explaining how to go about the problem!

I had assumed it was until the water pretty much stood still...

I made this assumption as well - I wasn't sure exactly what a "significant fraction" meant. My experimental result was closer to 1-2min.

I think "slow significantly" roughly means until its speed is reduced by a factor of $1/e$, (i.e. about a factor of 1/3).

Hmm, I got significantly lower for my tested result, maybe because I used water in a glass and it appeared to stop earlier than it actually did?

I also got lower.

Yeah I tested it and got around 5 seconds. The answer I got using this problem was 10. I guess it comes down to how you define when the rotation significantly slows down.

This pset felt like it had just the right amount of computation. As a result, I enjoyed it the most out of all the psets we had this semester.

6.055J/2.038J (Spring 2010)

Solution set 9

*This homework is **not for turning in** (by MIT rules, no assignment may fall due after May 7). But I hope that you enjoy thinking about the problems.*

Here are the solutions to the last set of problems (all optional). I've put it on NB in case you want to discuss the solutions (but no memo is due).