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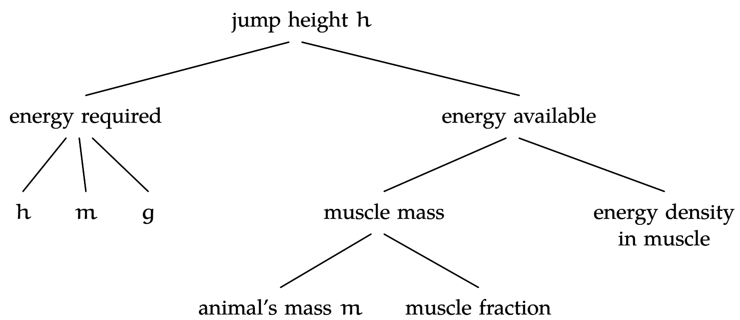

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jump height $h$
  energy required
    $h$
    $m$
    $g$
  energy available
    muscle mass
      animal's mass $m$
      muscle fraction
    energy density|in muscle
```

These 10 lines – simple to understand, write, and change – expand into 34 lines of tedious, error-prone code in the boxes language. And they expand into 1732 lines of PostScript code! As Bertrand Russell said, "a good notation has a subtlety and suggestiveness which makes it almost seem like a live teacher" (quoted in [23, Chapter 8]).

2.1 Diagrams as abstractions

Diagrams are themselves a powerful kind of abstraction. Diagrams are an abstraction because they force one to discard irrelevant details, reducing

This explains abstraction a lot better. Before it seemed like you were creating things to make life easier, which doesn't help estimation from my point of view. Now it is clear that you have to think about problems differently sometimes.

I am having trouble with replying in the notes section. When I hit "notes" and want to reply, I go to the options and the drop down menu comes out nicely. Then, when I hit reply, the box comes out just as expected. However, when I try to click anywhere inside the box (in case I want to fix a typo or anything) the same drop down menu comes up again. I just realized a lot of my previous nb posts were never posted because I didn't realize what as going on...

Also, it happens within this box as well.

This example is a lot easier to follow and I think it illustrates the concept of an abstraction more easily than the UNIX example. Maybe if you gave this section first and followed it with the UNIX example it would work better.

It seems that abstraction would be much harder for a problem in three dimensions.

It seems to me that this version of abstraction is easier to understand than the previous example given in Wednesday's reading.

So, is this kind of a correlation between visual diagrams and memorization. I wonder if that helps with divide and conquer or when you have to remember certain constants/figures for abstraction

I am a bit confused as to why this is a downward schedule. What is a downward schedule? Are you thinking of time as cyclical?

seems like a pretty interesting research topic...course 9 perhaps? wonder if other primates have intuitive estimation skills (moving from tree to tree, for example)

is this referring to something like the Rorschach test?

like a previous commenter, I am wondering what you mean by considering the Navier-Strokes equation as symbolic information...are we supposed to look at it as a picture and remember it ie. photographic memory-like?

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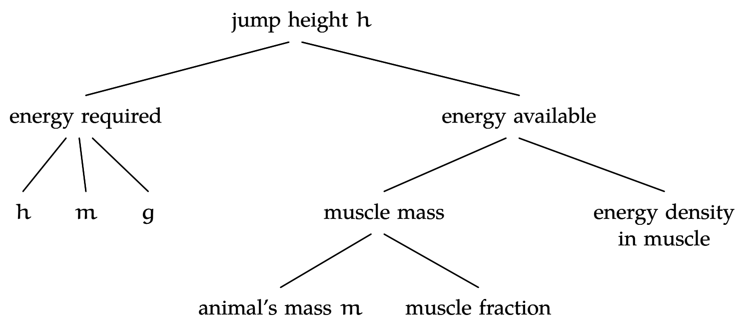

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was the question of the constant slope ever resolved? I agree that this is a confusing diagram, I don't think walking up a mountain and resting along the way corresponds to a straight like as depicted.

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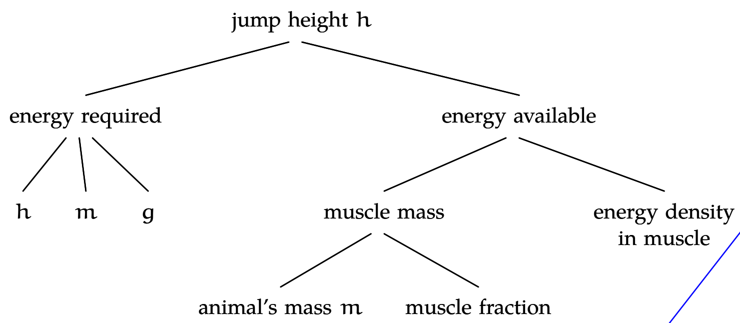

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Read Section 2.1 and do the memo by Thursday at 10pm. Hopefully a change of pace for those weary of programming!

I agree that this was much more accessible than the previous chapter - programming does limit your audience, while everyone can relate to and understand the mountain problem at the end of the chapter.

While I did not find the previous section(s) inaccessible - despite not having tons of programming experience - I found this section to be extremely clean and compact, and good for demonstrating abstraction of data and graphical representation. I don't really think the complaints about the graphs are valid, either, as they serve their purposes without being overly complicated.

To me this is the key to abstraction, look at the big picture and don't stress over the details.

does this mean that tree diagrams are a form of abstraction?

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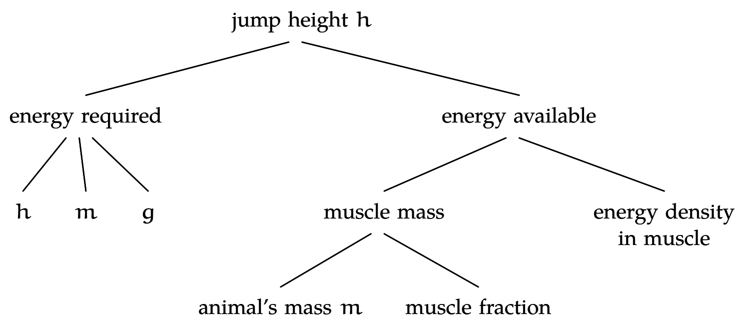

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This is how I usually think about abstraction. Today I was a bit confused when abstraction=repeatability. For me its about discarding details and broadening definitions.

I agree–and this is also how you explained this in the very first lecture as a way of organizing your desk. Just throwing some useless stuff away.

Is this the idea behind discarding certain details (for instance, in the MIT budget example, we considered the wages to be by far the most significant contributor)? How should we judge which factors are irrelevant besides general intuition?

I'm also confused as to when we should consider a factor irrelevant. It seems that by using certain abstractions we may be tempted to add even more "irrelevant" details to the problem.

I think there's a difference between neglecting factors and discarding details. Abstraction is the latter. When we talked about wages, it was abstraction when we discarded the detail that people's wages differ. It doesn't seem like abstraction to assume that the budget is made up solely of wages.

This definition doesn't really line up with the re-using definition we have been talking about. Re-using seems like it would be more modulating than abstraction.

I agree. I'm confused why we are considering separating tasks and discarding details as the same procedure.

I don't think abstraction throws useless stuff away. I think abstractions just sums up the unnecessary details into one compact thing. I used this example before but its kind of like a circuit element. Instead of using mu*A*N^2/length we simply use L for inductance. Abstraction would be using the L instead of the big long equation.

That's the same way I think about abstraction. My personal example is writing MATLAB and excel functions formulaically so that the values can be easily changed instead of always having to retype the equation or calculation.

I look at abstraction as a way to take the information we know to solve a larger problem. I try to throw out small details by changing the information I know.

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I feel that this should read "our brain's visual perceptual hardware"

They

I also feel like using diagrams can leave out a lot of necessary details and confuse the readers and create misunderstanding.

too bad. seems like the symbolic-processing is a lot more prevalent at MIT

I really like this description of diagrams- I feel like it really ties back to the example in class where we were much better able to understand powers of 2 through a box (1ˆ2 + a top and side layer= 2ˆ2 + a top and side layer= 3ˆ2 and so on).

This is a lot easier to understand description of abstraction as well. For some reason the ones in the past section didn't work as well for me.

Is this true for everyone, despite what type of learners they are? It strikes me this would vary between auditory/oral/written learners, etc.

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I feel like this would be more powerful if you supported the point with an example demonstrating it.

It's interesting because we talked about this earlier this week in 6.UAT. We were shown a concept in signal processing first with equations and math, and then graphically. Everyone voted and overwhelmingly agreed that they understood the graphical representation more clearly and preferred it.

We were then asked again to describe the difference of how we felt between the two methods. Overwhelmingly, people described the mathematical explanation as more rigorous, complete, etc.. However, Prof. Freeman then said that in this case, we were wrong, the graphical explanation actually was more rigorous, it described more aspects of the signal than did the math, and from it we could extract more information!

It seems that people attribute rigor to mathematical explanations more easily and give it more credibility, even though people tend to perceive and understand graphically explanations more quickly and easily.

A famous example along the same lines is Feynman diagrams. They are used throughout almost every field of theoretical physics now, but they were first invented by Richard Feynman (MIT Class of 1939) to calculate otherwise almost impossible integrals in quantum electrodynamics (roughly, quantum mechanics plus electromagnetism and special relativity).

The previous methods involved horrendous symbolic calculations; but Feynman made it so easy. One of the fellow winners of the Nobel Prize is reputed to have said, "Feyman made it so that any fool can do quantum electrodynamics."

I think it's more than just human evolution. Within our own lifes, we are taught at a young age to learn with pictures, and we don't learn until much later how to formally express our ideas in words.

This might be a good place to include your example of the power of diagrams in class; with the area of a square equaling the sum of sequential odd numbers.

Interesting.

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I think showing an example of solving the following problem about the hiker without a diagram would be more effective at proving that one method is easier than this description is. Going into the history of how we think seems kind of random although it is interesting.

I agree that it's a bit of a digression, but it's definitely interesting and sort of frames the argument for why diagrams are powerful abstraction tools.

I thought I was reading my psych or linguistics book for a minute there.

Do you think the human capacity for signal processing will catch up with the human capacity for visual processing?

I don't think so. A lot of signal processing still uses vision and so visual processing would be strengthened even as signal processing is strengthened. This may not be absolutely the case, but to support the distance between the two, there is actually a term in neuroscience - the Pictorial Superiority Effect or PSE. PSE specificlally relates to memory but has the same gist as what is described here in the text. Check out Brain Rules by John Medina

(10ˆ5 vs 100,000. Nice touch...)

I like this section. Examples for why things work better tend to help people understand what's going on and why they should care.

This explanation is a really interesting way to think about why visual learning is more effective....although I'm unsure how valid an argument it is

A perfect example of this is the exmple in class of the sum of odd integers and the square

I can't remember if this was discussed when diagrams were first introduced, but I think it would be more appealing to see it the first time we discuss diagrams, because it is definitely the more intuitive rendering of knowledge for a lot of people.

I am a little confused about how sequential analysis leads to symbol processing. I would have thought that symbol processing is more related to visual processing than it would be to language. A little more explanation about this distinction would be helpful for me.

couldn't you also say then that there were limitations to diagrams that required the use of symbols- yes diagrams make things easier but most difficult problems can't be solved using them?

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I love how much random information we learn in this class.

I agree a million times.

use: "(even longer among vertebrates)"

Not exactly sure what you're getting at with the line about "general perceptual processing". Do you mean that because at some point our ancestors could hear a predator rustling in the branches, we're better at solving problems today? Is this a reference to our "gut feelings"? Or is this just telling us we've been thinking critically in some way for a very long time (I'm not sure that's even true...)?

We went over this idea in class a few lectures back. Perceptual refers more or less to visual or organized in a way conducive to that.

I think what he's referring to is like the example we did in class where we have memorized the squares of numbers but we don't necessarily know HOW we know them. Then we looked at that box that started at 1 box and added boxes to the top and side to get the squares (ex. 1 box + a top and side row = 4 boxes (2ˆ2), then plus a top and side row= 9 boxes (3ˆ2), etc.)

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It would be fun if this diagram was actually included in the reading with a short explanation afterwards of what the picture is - or are we doing this in class?

Does it have copyright restrictions? That's the only reason I could see for not including it.

http://www.bbc.co.uk/radio4/reith2003/images/lecture3_dog1.gif I think that's the one.

Alas, almost everything has copyright restrictions now. I could probably get permission to include the diagram, if I were publishing the book like normal; but I am planning on publishing it (in print and online) using the Creative Commons NonCommercial ShareAlike license, and I doubt any commercial publisher would allow "their" photograph to be licensed that way.

Since the terrible Supreme Court judgment in Eldred v. Ashcroft, which allowed Congress to extend copyright terms as much as it wants, this problem is only getting worse.

In this particular case, there might be a way out. I know Richard Gregory, and it's possible he still has the copyright to the picture, in which I think he'll be happy to give me the necessary license. I will ask.

Meanwhile, I'll show it in class one day (maybe Friday).

I think there might also be other versions that are not copyrighted? I've seen the image though and it is rather striking that once you've seen it once...you cannot go back to the original blotchy view!

I looked at the link several posts earlier, but I did not see any sort of figure. Is this the right picture? Is there anything special that we should look for?

Where is the picture? I want to see it!

I agree. Without the picture this is just confusing...

"...not to spoil the surprise." Clever technique to peak interests of readers.

I'd like to see this...or you should come up with an example that you _can_ print in your book.

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Don't images / diagrams decay over time? where as language or equations must be more enduring to be structurally sound?

I think his point is that you will remember the hidden picture if you see it again. He isn't referring to the physical decay of the picture over time.

also, in terms of equations, if you haven't used it for a long time, you will tend to forget about it.

Yeah, I think he is talking about the first impressions and the impact on the brain.

Perhaps it would be better if you used a different visual then, one that is more well-known.

ooh that makes sense

This paragraph may be a little unnecessary. Going directly from the previous to the next may prove to be more fluid and natural. Also, there will be no chance for people to complain about not understanding the Navier-Stokes equation.

what is the connection here? how does the picture connect to Navier-Stokes equation? the transition isn't so clear to me

The arguement could me made that the primary reason that this is harder to understand is that the meaning of these symbols is defined by someone else. If we had a deep understanding of the symbols, perhaps the equation would be understood almost as quickly

I think presenting the equation is distracting. It draws the eye, placing too much emphasis on a minor detail for a broader explanation. For your explanation, it's not important that people know what Navier-Stokes is. Therefore I think you should omit the actual equation.

well, no, I sort of disagree with this. It would've been nice to have a picture alongside the equation just to point out that it is easy to remember photos but difficult to remember equations, but it reminded me of how I often have trouble remembering equations while I usually remember photos I've seen. Then again, I guess I don't have to recall them as often.

This seems a little random, what does it have to do with the topic?

what is the difference in learning symbols and pictures? do they use different parts of the brain?

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Even with math (say linear algebra), it is easier to deal with R2 space because you can simply draw things out easily as opposed to R3 or RN

jumpy. either cut this line or make it flow from the previous & into the next sentence better.

Maybe have the equivalent diagram that would make it easier to understand, to compare the two medias?

Like another reader, I've never seen this equation before. Can someone give a brief explanation?

According to Wikipedia, it is a fluid dynamics model type equation. I too would have liked some sort of BRIEF phrase describing it (or perhaps some other more easily conceivable example), though I'm against removing it. The point is to illustrate some confusing concept that required many presentations, and that is one such equation.

As mentioned, its the fluid flow equation for things that have pressure and velocity (like your arteries). I also would vote to keep it, although I know the equation its also a nice demo of many symbols which, despite having been seen before (or never) nonetheless look confusing and easily forgotten

It is used in fluid dynamics for describing fluid flow. It takes into account the various forces which are at work and allows you to full characterize (describe) the flow of the fluid. However, it is a fairly complex equation and often can only be solved analytically in a a few cases.

It does not matter what the equation is - it's the idea that it has a lot of symbols that are harder to process than pictures. The fact that you don't get it probably just proves his point.

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Would it be more useful to use an example that more people in the class would be familiar with? I for one havec no idea what this equation represents, so I can't really relate to the example.

Now that I've read the whole chapter, this example actually seems entirely unnecessary - I would actually vote for scrapping it entirely.

Agreed that it seems unnecessary. Said by someone who has seen this equation a lot. And for me, learning it didn't require many presentations but many uses (ie I didn't learn it from reading and lecture but from the problem sets and examples)

The equation just serves as a counter example to the diagram argument. The more confusing the better in my opinion. The point is to convey the complexity of symbolic information.

I think he should at least say what this equation is used for. I've never seen it before.

Navier-Stokes equation is used to describe fluid flow.The wikipedia page has a pretty good description:http://en.wikipedia.org/wiki/Navier-stokes

I think he was just trying to say that, generally speaking, it can be harder for us to memorize symbolic information like formulas as opposed to visual information. But I do agree that, by not knowing this formula, I lost some of the meaning of this example.

Just mentioning that it's "the Navier-Stokes equation to describe fluid flow" in the text would seem to alleviate this problem.

There have been a lot of examples that were a bit harder to get through and were completely unfamiliar, but I think that's one of the appeals of this class. These are things that I would probably never learn in my course, but a small does of it is fairly interesting.

The author isn't teaching this equation to us! The point is to show the reader an example of how visual perception differs from symbolic perception. All in all, I think the examples definitely prove the point, no matter what the readers background is.

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I have to say as someone course 2 I can't help feeling a bit of glee: "Ha, after all this UNIX and course 6 material, finally something we course 2 people can understand and those course 6 people can't! Hahaha!" That said I'm not sure learning an equation is a visual thing for most people, so maybe this isn't the best example.

I think the author's point in including this seemingly obscure equation was to show that our mental hardware is more devoted to looking at diagrams rather than symbolic information–in this paragraph he made the comparison of Richard Gregory's splotch picture to this Navier-Stokes equation

you must be course 6. the course 2 people had no freaking clue what the unix stuff was about.

lol, so true!

i think there is even more mental hardware devoted to cognitive reasoning than visual processing.

Is this referring to the parts of the brain like V1 and somatosensory processing areas?

Not sure what the author means either, but is it necessary to include this? How about just starting from "A good problem-solving ..." I think the author already makes a valid point in the previous example about how the brain's visual perception is very unique and powerful.

probably but more generally just that we have a lot of infrastructure in our brain set up for visual processing

I'm wondering the same thing. What is this referring to; can we get a note? Not necessarily in the reading, but just for our knowledge. I'd like to know if this is referring to a specific part of the brain, like V1-V4, or the prefrontal cortex, etc.

So I just think that making diagrams are a waste of time. Maybe they are useful for teaching others that don't understand the question, but I don't see how making one yourself will let you learn better.

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I could imagine that this is not universally true. Are there some problems where I diagram might mislead you or not be the optimal problem solving strategy?

Although a diagram may not be the most efficient way to solve every problem I think by definition a good diagram should not mislead or confuse you

There probably are, but the author just says here that "a good problem-solving strategy" is to use diagrams. The purpose is just to make things simpler, since in a diagram you only include necessary info. So if in a problem you had a simple, easy to see situation that needed lots of calculation, a diagram wouldn't help.

I think most of us already do this, although I sometimes make the diagram in my head.

I agree, we did this a lot in 8.01/8.02

Its like block diagram representation in any signal processing or control class like 6.003 or 2.004. We don't necessarily know the implementation of the blocks but its the first step to figure out the solution.

While a lot of us already do this I think it's beneficial to explicitly state it since some may not realize they are doing this. Additionally for anyone that may also be interested in teaching of any kind, stating this is a good reminder that visual learning is pretty powerful.

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is a tree an abstraction then? what other types of diagrams are we talking about?

I think the whole point of this section is that a tree IS an abstraction...a visual one

But it's also important that there are more possible visual abstractions than trees, such as graphs, charts, etc. that can help one understand a problem or result.

So any pictorial diagram can work, as long as it simplifies the problem (enough to generalize a situation without losing "key" details)?

I would say a tree is a good abstraction. If you've ever taken 6.041 or an equivalent, the probability trees are incredibly helpful when trying to look at scenarios.

Not sure whether it technically is or isn't. I would say that it's important for it to be a reusable unit for efficiency in our approach, like the number of poeple in the US.

I also thought this was intuitively yes, and then realized I had no good explanation why. The graph at the end was very nice in explaining this.

This is a great problem - it's appeared on interviews multiple times for me.

I agree. I remember I first encountered this problem reading Marilyn vos Savant and found the solution fascinating. I've also found that several companies use it as well in interviews.

I also found this problem and the solution below interesting, but I felt like I wasn't that interested in it because it was merely a mental exercise. Would it be possible to have an example that had more of a real world application?

In regards to the real world application, I think the fact that this question appears in interviews serves as an indicator of its usefulness. It tells the interviewer how you think, whether you can think this way. Personally, this class is teaching me different ways to think about problems and this was one of the many interesting examples we've encountered.

i really like this problem also. is there any way we could have numbers associated with it at the end to give us something more to imagine (the different paces, etc.)?

I think it's interesting how on interviews a lot of companies don't let you write/draw diagrams in these interviews. It's like, anybody can figure it out if you draw it, but they want to know who doesn't need the visual, which seems a little odd to me.

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Why is it mentioned that you sleep for 24 hours at the top? Wouldn't it be the same if you just hiked back after reaching the top in 24 hours?

Since it seems a lot of people have already heard this example or think it is intuitive, it might serve well to have a different problem that is more difficult or has more distractions that could confuse people for the diagrams- I would say in this example it's clear that we only care about the distance on the path and the time so there aren't really any distractions in the problem. In other problems, sometimes the hardest part can be determining the relevant information so a question that requires that would be a good fit here.

I disagree. I think this is a great example and works really well here

I also agree; if anything, it reminds me of the word problems we used to do in elementary school math that I had a lot of difficulty with, and is great evidence for me that trying to process this via language rather than visually is much more difficult.

I'm trying to edit this to "I disagree" but I seem to be having trouble editting...

I disagree, I think this example is fine and even if we know the answer, seeing how what we thought in our heads translated into graphs was useful. at least here the problem doesn't detract from the concept

I also disagree – I've certainly heard this question and its solution before, but I think it's much easier to think about in a diagram format.

wouldn't you just rest as different times this way you would be on a different schedule.

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I found this to be a pretty quick problem to solve intuitively based on a simple continuity argument (which is, I guess, demonstrated by the diagrams). I feel like it would be more helpful to either in place of this example, or after, give a problem that can't be rationalized in our head because it contains several possible interdependent situations, or some other complication.

I agree.

Well, not necessarily. Examples are meant to be super simple cases, the concepts of which can be applied to harder and more realistic problems. Here, I'd rather have a nice and easy to understand example with basic diagrams to prove this point rather than one that will lose me along the way (like UNIX code!). Plus, we're MIT students, so we don't count.

Haha, I think we count, but it's nice to have something that makes sense and can be explained to demonstrate what we're studying

I agree, the simpler problem let me focus on what we were actually trying to learn rather than trying to understand the example so i could understand the concept and getting lost.

It is also nice to have all the information that is needed in front of you so you don't need to try to delete information in your mind.

I like this! Often times questions contain useless words/data that aren't needed and usually end up confusing me

I agree. This problem was so much easier using abstraction. I would like to see another example of this in class.

I take back my last comment. This is a good example of when a diagram is useful.

wouldn't it make more sense to thing about the details you do care about and then ask yourself if you really do care about them?

When I first thought about this problem, a visual did come to mind, but it was not a graph, like you presented. It was a very literal mental image of me walking up the mountain.

since it's a 24 hour schedule. Don't you have to pass where yourself at some point?

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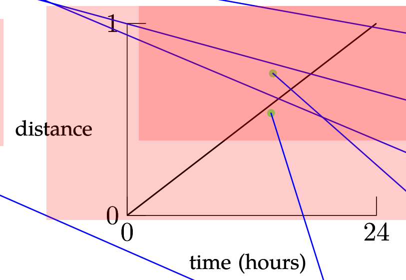
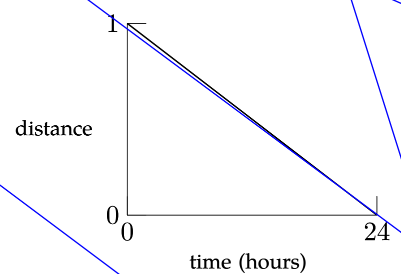
this is assuming a constant speed taken throughout the climb yes? if yes, what would happen if you were to vary the speed going up and down the mountain and how would the graphical representation look in this instance?

the question is: Must $u(t) = d(t)$ for some value of t? Or can you choose $u(t)$ and $d(t)$ to avoid the equality for all values of t?

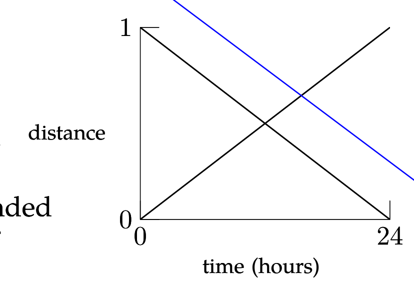
With these abstractions, the question is cleaner, but it is not yet easy enough to answer. A diagrammatic representation makes the answer more obvious. Here is a diagram illustrating an upward schedule. Distance is measured from the bottom of the mountain (0) to the top of the mountain (1). According to the indicated schedule, you walked fast (the initial slope), rested (the flat part), and then walked to the top.



Here is a diagram illustrating a downward schedule. On this schedule, you rested (initial flat line), walked fast, then walked slowly to the bottom.



And this diagram shows the upward and downward schedules on the same diagram. Something interesting happens: The curves intersect! The intersection point gives the time and location where the upward and downward schedules landed on the same point at the same time of day (but on separate days).



Furthermore, the diagram shows that this pattern is general. No matter what schedules you choose, the upward and downward paths must cross. So the answer to the question is 'Yes, there is always a point that you reached the same time on the upward and downward journeys'– an answer hard to reach without abstracting away all the unessential details to make a diagram.

Comments on page 3

it would be nice to have figure numbers so that you can reference the figures easier in the text. (this could be my own bias from reading papers a lot recently)

I think you should specify what you mean by t here. It's not absolute time, since the events happen on different days; rather, it's time since the start of that particular day.

Are formulas also abstractions?

I like how you turned the problem question into a mathematical concept. It helps to understand what we are trying to do witht the diagrams

I suppose if you really wanted to convince people you could show multiple graphs, like exponential movement or something else that wasn't simply linear. I know the result is the same, but it sometimes helps to see multiple graphical displays!

So the diagram represents the abstraction in this case, or is it the abstraction?

At the risk of being even more convoluted... I think it's an abstraction of an abstraction.

So, to answer your question. Both?

It does seem like we're dealing with two types of abstractions: a problem solving abstraction (the diagram) and the data abstractions themselves. So I agree, sounds like both are correct.

i didn't know that was a word.

It is a word, but this seems less like a diagram (a simplified drawing of something like a tree or a schematic) and more like a plot. I think "graphical representation" is perhaps a better term. It's very similar to finding the intersection of two linear equations graphically rather than solving for it. We wouldn't call that diagramming.

I felt kinda dopey when I realized that the answer to people's requests that the graphs should be non-linear was so simple: the linearity of the path is an *abstraction* of a pace that is probably full of curves. We abstract so we can simplify and solve.

I think these drawings are missing the flat parts that you mentioned in the reading

Yeah I agree. the description describes a very non linear walking schedule. First walking fast indicates a larger slope than the walk after the rest. and the resting part is missing.

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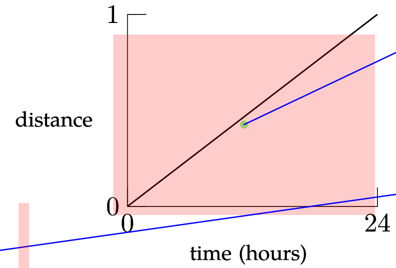

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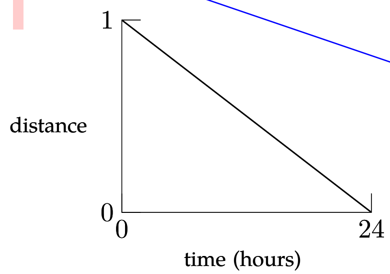

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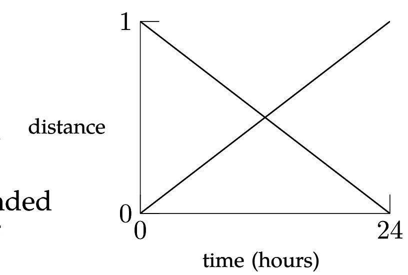

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Maybe I'm missing something, but the graphs don't seem to correspond to the position curves described in the text. Not that it really matters, since the point is to show the intersection is hard to find without plotting them. Still, it's a bit of a detractor.

Well, the graphs are only showing the hour of the day, not the absolute time. It's really just time MOD 24 hours, which helps explain why they have to intersect.

This is not the best diagram for this problem, it is not intuitive from looking at the diagram and reading the problem how they are connected.

While I agree that it is certainly not immediately intuitive, all graphs require a moment's concentration to get your bearings. To decrease this time and considering the topic is on visual learning, a picture of a mountain on the y axis and a graphic of light and night on the x might help the reader jump to understanding an instant sooner. However, then this work would look too much like a pretty textbook, which detracts from its excellent rational and spartan grounding.

there aren't flat lines in the graphs...there should be if you're going to say there are...

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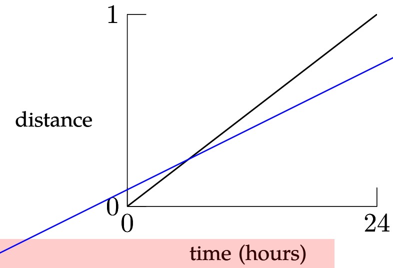

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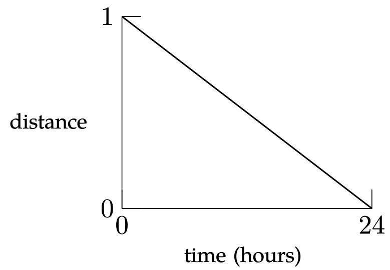

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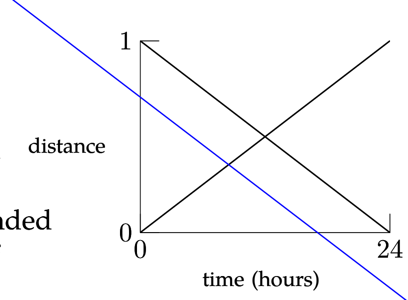

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I'm not sure where the flat part of the diagram is? My guess is that you had a diagram with three parts: an upward part, a flat part, and a downward part, and then later decided to remove it and provide these diagrams instead without changing the text description.

I was also wondering about the "flat part of the diagram". However, it doesn't appear to be necessary to show it, since it is 24 hours and won't shift the schedule of d(t) relative to u(t). Thus, it's ok to just omit that part and overlap d(t) and u(t) exactly on top of each other since their time of day corresponds.

Yeah, this graph is sort of mod(24 hours), but without the flat part.

The 24-hour wait at the top seems like an oversimplification. Doesn't this still hold for any offset because time is mod(24) as someone pointed out? I pictured sliding the descending line left or right. I guess there's a trivial solution where you're only at the top and bottom at the same time of day

You could say that figuring out that the 24 hour long rest is arbitrary is part of separating the problem details from into the abstraction.

The flat part would be in the overall schedule, but in this overlay of graphs, we're only concerned with the time of day when you're at any given position, and the question says that you start going up and going down at the exact same time of day.

These diagrams don't really seem to correlate with what he is saying, since the slope appears to be constant. I understand that it doesn't matter, but it does make it confusing to say one thing and draw another. Either the text should be rewritten to say "constant speed" or the diagrams should be redrawn to match, including the level parts.

Now I remember what happened here. I wanted to make a diagram that corresponded to the text, i.e. with a flat part, but was in a rush to make a diagram, so I just put on in there that was "approximately right", with the intention of fixing it later. But I never did. Sorry about causing confusion with that.

but there's still no change in slope here, regardless of whether or not there's a flat portion

This makes sense, but the graph doesn't totally make that clear to me. Maybe there's just too many comment lines on the page that're distracting me.

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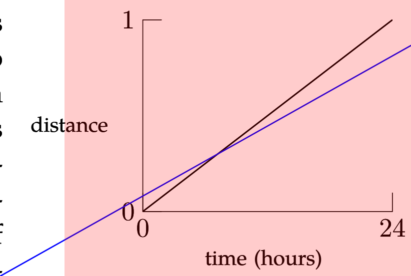

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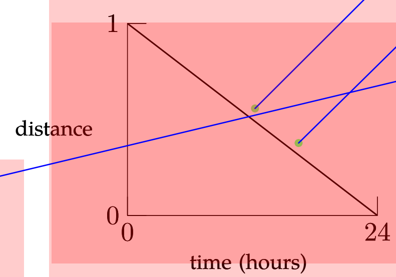

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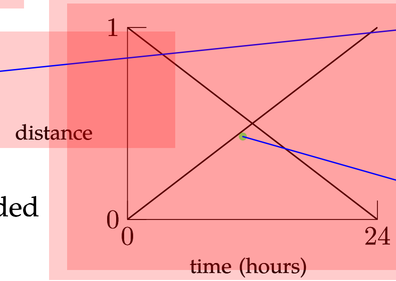

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I don't see the flat line that represents resting...?

Me either I had the same question..

Same situation as with the first graph. Check the comments left on that one, it's sort of a typo.

This simplifies it nicely.

As the problem gets more complex, would it be possible to simplify it once to several diagrams, and then simplify those again to one or two diagrams?

If that's the case, then wouldn't it be easier to create a more simplified diagram?

The formatting of this page seems odd with random white space. It was rather confusing (particularly with the lack of proper diagrams a few paragraphs earlier). Would simply numbering the figures/diagrams, and referencing them in the text, make it easier to read?

i think latex takes care of all the formatting. i'm not sure how much control the author has over it.

It's probably because the diagram is connected to that paragraph, so although the words stop the paragraph continues until the end of the diagram.

I didn't think that you would be at the same point at the same time at first. And I wouldn't be able to express an argument any better than this last diagram.

I agree, the problem was very difficult to think about in my head but with these diagrams it is almost impossible to doubt the fact that you would be at the same place at the same time at some point.

While the graphs may not exactly match the description from the reading, the message trying to be sent seems really clear here.

Agreed, I really liked this example a lot.

Yea it's definitely very clear from this example how much easier it is to understand visuals quickly.

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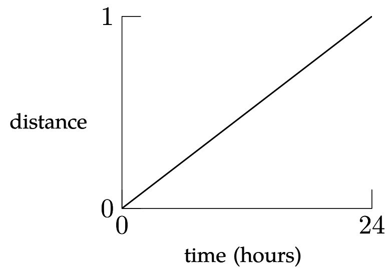

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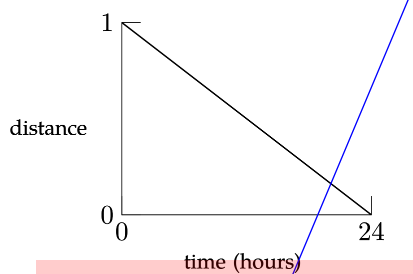

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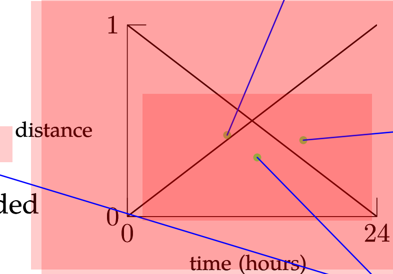

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Usually I understand graphical arguments better than others, but for some reason these graphs don't convince me that there is always a point you reach at the same time going up and going down. Is there another proof of this?

Think of it as two people (this is how I originally solved it back in high school) one going up, one coming down. Will there be one point in time when they are on the same location on the mountain? Since they have to pass each other, the answer is yes. There is no difference here, except that you have to think of one person going both up and down.

Does the two people walking toward each other count as abstraction? Because in my head I'm picturing two things moving at each other.

I think there are other ways to describe this solution to this problem, but ultimately it reduces to this graph. The graph serves as the simplest form of the solution. I also believe that the curve does not necessarily have to be linear either on the way up or down, but either way the curves will intersect and you will be able to find a point at which you are the same distance at the same time of day.

The description of two people walking in opposite directions (given by the earlier commenter) really helped drive the point home, and ultimately made the graphs understandable for me. I think a description like this in the text might help solidify the abstraction presented by the graphs.

The main thing to notice is that the person has to walk down the mountain in the same time that he hiked up. That's what made the light bulb go off for me.

I think most people already solve problems visually like this in their head instead of the abstract way, but overall I think it is a great philosophy. A picture is always easier to understand than an abstract concept.

I'd say that when I think about a problem like this in my head I'm definitely using abstraction, even if i don't realize it.

Agreeing with previous readers, the diagrams should be changed, and just to reiterate your point (visually with diagrams!) you might consider including intersecting graphs with totally different schedules. Thus, one could look at three graphs and realize that yes, the paths will always intersect.

this quickly resolves any doubt or attempt to find a way to not be at the same place at the same time of day

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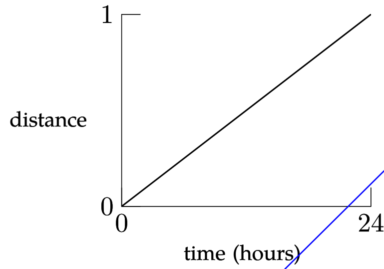

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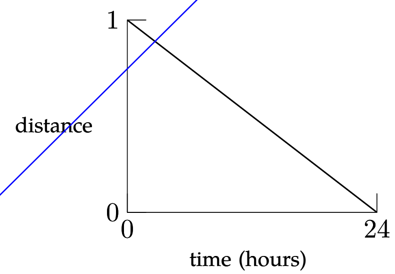

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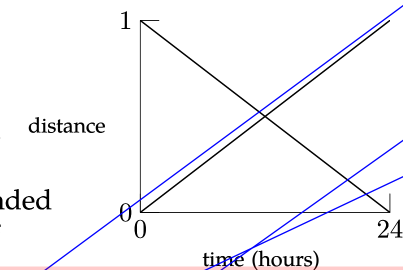

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I still am not completely convinced. It seems to me that you should be able to walk in such a way that you won't be in the same spot at the same time of day. Like what if you walked fast on the way up and slow on the way down? Or maybe I am not fully understanding this diagram or the problem statement.

I had this same problem, but I think it's the specification that both up and down trips are over an entire 24-hour period, so even if you walk fast part of the time, the curves will still cross, even if it's just at some resting point.

If the diagrams don't help, another way to think about it is to imagine two people. One at the top one at the bottom, either walks at an arbitrary speed at one point they will pass each other provided they each reach their destination within 24 hours.

Thinking about the problem in terms of two people is really helpful..I think it would be worth including in the explanation of the diagram, otherwise it is somewhat difficult to understand why the solution is right.

I agree, that finished clearing it up for me.

Maybe include another possible set of diagrams (different slopes or flat parts in the middle of one or something) to really show the generality.

yeah I agree

I thought so

This is a good example to demonstrate the advantages of using methods that utilize visual processing. Could you give an example that is best solved with "perceptual processing"?

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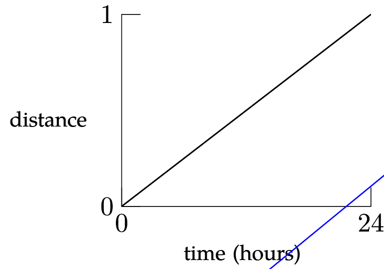

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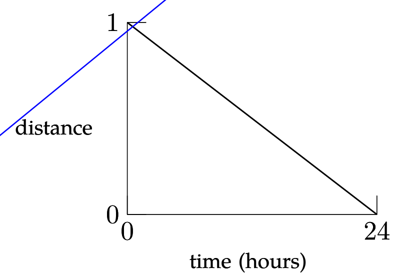

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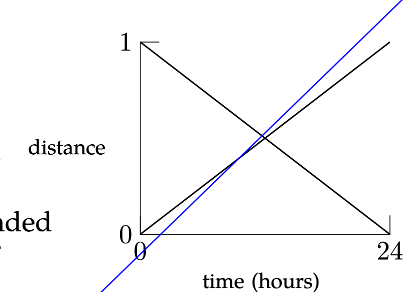

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This is cool, my initial reaction (and as others have mentioned), is yes - my mind was thinking about the midway point, if you are hiking at the same speed, etc...you would be halfway at the same time. However, without the diagram and the abstraction, I'm not sure I would have realized quickly that it has to intersect once always (even if pace is different). I was thinking through other situations, and its definitely not easy without this method! This was a surprising for me and quite a useful method and example!!!

I agree, I still haven't been really sure of what exactly abstraction was and how it was useful, but this example sums it up really well. I wouldn't have been able to think through the problem as quickly without thinking about it this way.

I agree with this and think that once the graphs are updated it'll be even more definitive. This abstraction showed how useful it is to take the time to ignore the extraneous parts of a problem and that sometimes too much information can be detrimental

I think it would be interesting to illustrate a way of doing this problem without diagrams, just to compare the relative difficulties.

Yeah I agree, being able to see the advantage it brings would be pretty useful.

I don't know how you would... But maybe that's your point.

My initial reaction to reading the problem didn't think of this intersecting curves approach. Initially it seemed a little more thought intensive, but the explanation of the diagram helped simplify it greatly. The fact that this diagram approach seemed so simple seems like enough of a demonstration to show the benefits of using diagrams. Initial reactions from reading the text might be enough to serve as the "other" approach without diagrams.

I always thought graphing out a problem or making a picture added to the work. In this case, though, I may not have gotten the right answer independent of the diagrams.

this seems to be the only time you actually connect diagrams to abstraction

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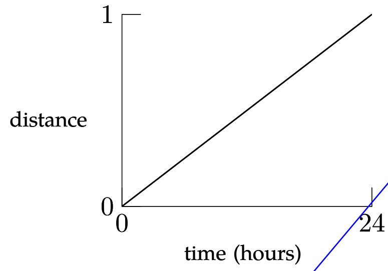

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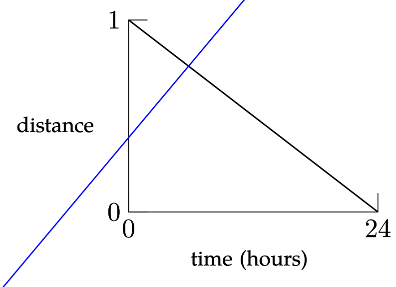

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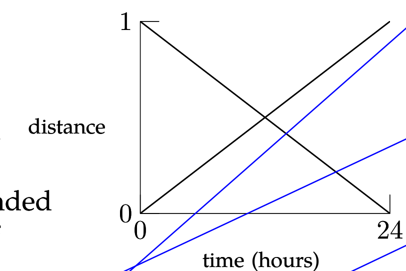

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This problem is pretty intriguing to me (I've never seen it before). I've been trying to think of ways in which the paths wouldn't cross but I haven't been able to, it's pretty cool.

Yes, but I probably wouldn't have been able to come to that conclusion without the diagrams. Good example!

Without using this method of abstraction and drawing diagrams, this problem would have taken me much longer to do, and would have gotten a lot more complex

really? i thought the answer was apparent right away. although, i agree with others that another set of "drastic" plots would drive that point home.

I think having the diagram just makes the problem very simple to understand. Some may find this intuitive from the start, but the diagram essentially makes the problem easy to understand for people who didn't see it from the beginning.

We should have an example using Venn Diagrams as well

I really liked this explanation because without it I would have come up with myriad hypothetical alternatives where it didn't hold true and maintained that I was right.

good summary

I thought the answer 'Yes' was intuitive. After I read the problem, it just seemed like 'This has to be yes, right'. I don't need the need for this example, but I do agree with an above comment, I'd like to see the diagram.

I really liked this example- I actually understand what you are talking about now

straightforward and easy to understand reading.

This final point would be more clear if you used graphs that were nonlinear as well (start out fast, slow down, etc)

This reading was very easy to follow.

Except for the mismatch between the diagrams and the text, I think that this section is overall very clear and well written. I honestly have no comments (that would not almost be verbatim to what was already said) except the one I made above.

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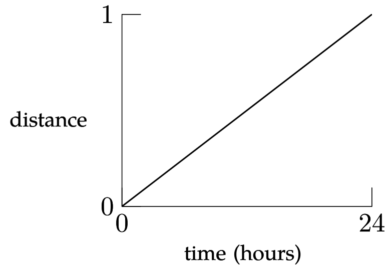

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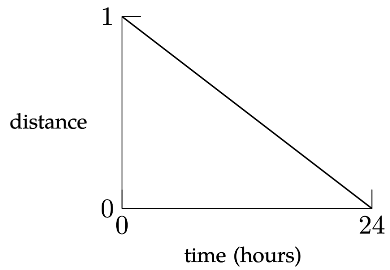

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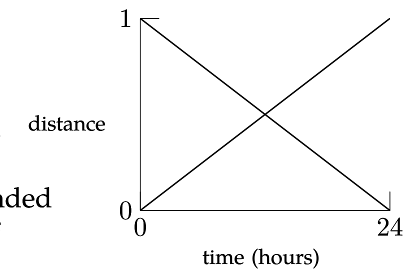

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looking back, I think i might have been doing a lot of proof reading around the time of this (& the previous) reading