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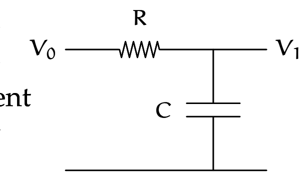
2.3.1 RC circuits



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are input/output signals always going to be so readily recognizable?

I think a tree would be useful here to show all the factors that can affect resistance to daily temp variations

I agree, by the time you reach the end of this section you realize that these two situations parallel nicely.

Now we can do the last 2 problems of the pset! woohoo!

I enjoyed this chapter because I understood all of it after taking 2.005 and 6.002, which is cool!

It seems like this chapter combines abstraction with divide and conquer. It seems like a really practical application based on the example.

WHy is this a transfer function? Is B a Laplace transform?

Thanks for incorporating course 2 material (ie. 2.005) and course 6 material. I think it makes it easier for both majors in the class to understand.

Are these approximations taken from your book of constants? Or like your usual general estimates?

2.3 Low-pass filters

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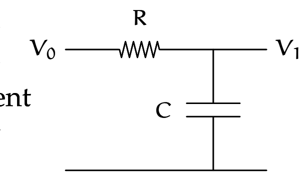

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Read Section 2.3 (memo due Tuesday at 10pm). Lots to discuss about it in class, I am sure!

In a section like this, I didn't really see that many points to make. I think most of the other students just made comments about good sentences or summaries. I also thought the section was well done, so I don't have much to say.

Something telling what ontological bonds you are referring to would help here as well as something giving background will help.

Ontology deals with questions concerning what entities exist, and how such entities can be grouped, related within a hierarchy, and subdivided according to similarities and differences. I agree that a definition would be helpful, but I think the definition is implied in the phrase the follows afterwards.

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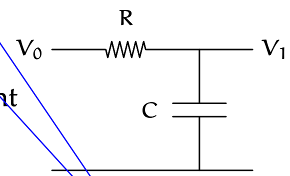

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Which ones exactly? This sentence made me stop reading and try and figure out which bonds you are referring to..circuits in general? Abstraction?

yeah, using more common vocabulary would make this better. I spent more time trying to figure out what "ontological" meant than necessary.

Agreed, I'm a big fan of new and interesting words, but some like this one just seem misplaced.

I'm in complete agreement with these comments - I did a reading of the sentence, stopped, then went back and read it again. Using that word unnecessarily disrupts the flow.

Is this really that important? I think people are starting to worry too much about grammar and vocabulary

I think it is important because it means that we are having a harder time reading the book. Not to mention, one of the points of us reading is this is to 'provide another set of eyes' and proof read as well...see the how to do reading memos page.

I think this is actually a good place to use the word, the words in between the dashes are another way to say what he means by snipping the ontological bonds. Although I've never heard the word ontological before, I had a good idea of what it meant from the context, I think this is well done.

I think it'd be better to include the words in quotes first, then use the word ontological. It allows the reader to already know the meaning of the word without having to stop focusing on the material.

yeah, the last response hit the spot, reversing the order of the sentence may create less pause and confusion.

those? this doesn't make sense.

Referring perhaps to the bonds of thinking about the study of circuits

definitely going to have to look that one up

2.3 Low-pass filters

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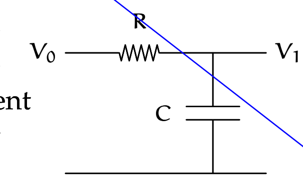

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I understand what you are saying, but I feel that its rather confusingly worded and took me twice reading it to understand

I agree with this completely. I feel like you could condense this whole paragraph down into a much simpler sentence, something to the effect of: "The next example comes from the study of circuits, however we will see that the core idea (the abstraction) will be useful in understanding diverse natural phenomena"

I really like the rephrased comment and greatly agree that the sentence in the reading is overtly confusing. Ontological makes the reading sound sophisticated but is very different from the other vocabulary used in the reading. I personally think it detracts more than it adds

I agree. I got confused from the use of the word those, not really sure what it's referring to. Overall, the paragraph brings up an important point about abstraction.

I think it's the combination of the hyphens, quotes, and parentheses, which really break up the flow of the sentence.

From whence this quote–the dictionary? The quotation marks seem unnecessary unless it's a critical citation.

I agree with the other comments about the confusing sentence, but I also think that after reading this entire paragraph I am almost entirely lost on what you are actually trying to say, besides the idea that you introducing the examples you are about to discuss.

I agree. I think this paragraph does not do a very good job of preparing the reader for what is coming next - it feels wordy in a convoluted way, but doesn't really set up the idea of a low-pass filter, as is the section title.

I disagree. He is explaining that he will use the idea that came from low pass filters and apply it to other things like temperature fluctuations. It motivates the discussion of low pass filters by telling you how it is an example of abstraction.

comma

2.3 Low-pass filters

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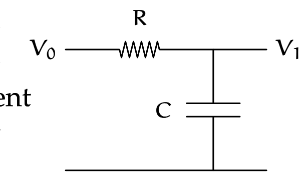

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After reading this section, I just have to say that I'm glad I'm 6-3, and not 6-1 or 6-2. I don't know if it's the material or the way it was presented, but I pretty confused throughout this whole section.

now think how the course 2 people feel...

Now think how the non course 2 and 6 people feel...

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I'm getting a little annoyed by this I'm not course 6, I dont get it thing...not everyone in course 6 knows how to code [i'm just as much at a loss for that unix stuff as a below average person in an athena cluster...more than joe-shmo on the street, but not enough to get 98% of what he's said in this book.]

my point, please stop insinuating that your knowledge gaps are totally the fault of your major. It's really about your interests...half the course 2 people I know are better at unix than I am (and I'm course 6-1)

besides everyone should remember this from 8.02!

as far as this section goes, it's not just your coursework background. I know exactly what he's talking about and it was still a little hard to follow

Exactly. These are all just basic engineering concepts (OK, maybe not the UNIX). But it's still way more accessible than if he were using, say, drag equations from aero/astro or thermo from chemE.

I agree about the 6-3 part, but I think one of the problems with this is that "new" material from a completely different subject is being taught in just 2 pages.

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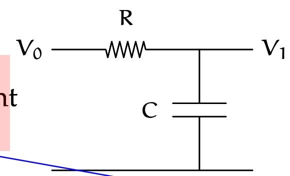

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Don't capacitors and inductors make a circuit nonlinear?

This is what I thought too - he goes on to state that inductors and capacitors make the circuit have time dependent behavior and if I remember correctly an RC circuit with time is definitely not linear.

No, ideal resistors, capacitors, and inductors are part of linear circuits. Here linear means that when a linear combination of signals ax1(t) + bx2(t) is applied to the circuit F(x), the output is aF(x1)+bF(x2). Informally, what this means is that the *values of the electronic components*, (i.e. the resistance, cap, etc) dont change with voltage or current in the circuit.

Exactly, even if the voltage over tie, for example, is not linear, the *circuit* is still linear because its response to a sum of inputs is the same as the sum of its responses to those inputs.

I feel like it would be clarifying here to state what exactly it means when a circuit element has time depended behavior. It might be helpful to give a brief description of what each component does and how each varies with time. A resistor isn't time variant because the resistance is a fixed quantity, but it takes time for charge to be stored in a capacitor because the electrons have to be deposited.

I think its ok. He assumes basic knowledge

This sentence doesn't sound terribly grammatical, it might be the infinitive phrase being used as the subject. Perhaps a gerund prhase such as "Getting time-depending behavior..."

what's your definition of interesting here?

Interesting here means that the circuit's response changes with time. It's tough to do anything very useful with just resistors and voltage sources.

Exactly - resistors can provide us only with a linear response to voltage changes, whereas capacitors and inductors introduce some more interesting behavior as it takes time for them to respond to voltage changes.

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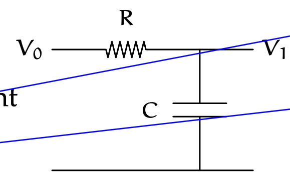

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While you may assume that the reader understands what these elements are, it may be useful to either have a little note box clearly defining them - it may be nice for those who do know they are, but would like to be reminded. And from reading below and the comments, also including the governing equations for circuits and their elements will be useful when introducing (2.11)

side bars rock!

I agree that a reminder of their governing equations could help.

I think resistors are perfectly interesting, but perhaps this is like an electrical engineer saying levers are perfectly interesting.

and/or, since RLC circuits are good too.

Having an EE background I understand what this means, but a different audience might not catch that RC actually means resistors and capacitors.

Agreed. It would be very useful if you included a diagram of an RC circuit here.

I agree that it should say "an RC circuit, made up of a resistor and a capacitor," but isn’t there a diagram right there?

write as V(t)

I think V0 here is constant, that’s why he didn’t write it as V(t)

I don’t know the scope of your book, but maybe replace with "freshmen E&M class" to make it non-MIT friendly.

This equation is also covered a lot more in depth in 8.03 than in 8.02.

Unlike 8.03 though, 8.02 is a GIR.

Not that this is beyond my understanding, but everyone keeps expecting me to remember my GIRs. I wish someone had told me I’d need them after freshman year.

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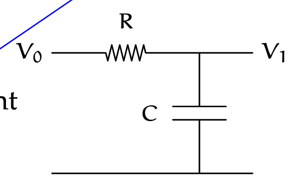

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It's been a while since I've done this. It would help me a lot if you could justify this equation with a few arguments or convince me that it's correct. The dependence on the product RC is also not obvious. I guess this is what happens when your circuit E/M mostly was learned in high school...

I agree that equations might help us remember where this comes from (although that understanding isn't crucial to the rest of this analysis, it seems). The RC dependence is clear because the only time 'R' or 'C' appears is when the term 'R*C' appears. I found that to be well explained.

I'm not sure that the math needs to be explained - the chapter is about abstraction, and making RC = tau is very clear to me without all the mathematics that derived this differential equation. I think we're perfectly fine without it; it allows us to focus on the abstraction without getting bogged down in details.

i agree with this...skim the ee stuff, stick to the abstraction.

It would be really helpful if there was an explanation for these equations and this section in general. Since it has been a couple of years since I took 18.03 and 8.02, a lot of this material in this section is unfamiliar, and is therefore difficult to follow.

This seems to be a prime example of the conflict between not having enough explanation and being confusingly explicit. No matter what, someone is going to complain that it's either of the above. This example works fine using only the explanations given and vague memories of fulfilled GIRs. As mentioned before, see the tau substitution.

Maybe it would be useful to add an appendices for each of these sections. I think it's fine to use a birds eye view of things to ease clutter, but providing appendices would allow people reading the book to still follow in more detail.

kinda makes sense since they force each other. Although it seems a little strange cause shouldn't it just act like a capacitor if the resistor is really small

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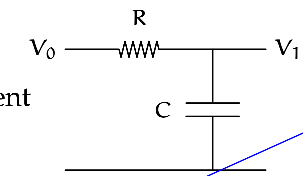

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can you explain why you multiply the R and C together? what is the physical meaning of this product?

The sentence before explains it: it's a product that happens in the equation that has no importance as individual units, only as an abstraction.

It falls out of the math. The temperature analogy is more intuitive to explain. The time constant determines how quickly the temperature of a particular room, object etc, approaches that of a temperature it's exposed to. The higher this product the longer it takes. This makes sense - an ocean with a lot of capacity and resistance, takes a long time to change temperature.

This part is very clear to me. It is pretty easy to follow so far.

I agree this section was pretty easy to follow and useful, as it's been a while since I've dealt with circuits and needed a quick refresher.

i like this description–that it doesn't matter what R and C are, and that we don't need to know the detail to know what tau does

I agree, although this is a method I have often used for solving complex equations.

I agree, it seems like this is almost the opposite of an abstraction, that we are grouping together terms in order to ignore the confusion multiple variables could cause.

This explanation is a great example of abstraction, especially for those of us that have long forgotten the meaning of these equations.

What is this triple bar sign? Does it mean something like "from now on, let tau be defined as rc"?

Very clear abstraction! Some of the previous readings seemed to make them less clear.

Why is RC considered a time constant? Not familiar with circuits...

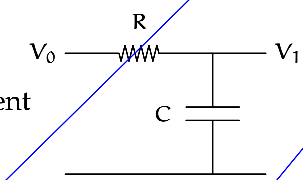
There wasn't much of a transition from defining t=RC to calling it a time constant. For the less EE savvy, is this confusing?

Yes, I think it should be brought up that it's a constant or a quantity with physical meaning, then that it is a TIME constant after the behavior is described.

2.3 Low-pass filters

The next example is an analysis that originated in the study of circuits (Section 2.3.1). After those ontological bonds are snipped – once the subject is "considered independently of its original associations" – the core idea (the abstraction) will be useful in understanding diverse natural phenomena including temperature fluctuations (Section 2.3.2).

2.3.1 RC circuits



Linear circuits are composed of resistors, capacitors, and inductors. Resistors are the only time-independent circuit element. To get time-dependent behavior – in other words, to get any interesting behavior – requires inductors or capacitors. Here, as one of the simplest and most widely applicable circuits, we will analyze the behavior of an RC circuit.

The input signal is the voltage V_0, a function of time t. The input signal passes through the RC system and produces the output signal $V_1(t)$. The differential equation that describes the relation between V_0 and V_1 is (from 8.02)

$$\frac{dV_1}{dt} + \frac{V_1}{RC} = \frac{V_0}{RC}. \quad (2.11)$$

This equation contains R and C only as the product RC. Therefore, it doesn't matter what R and C individually are; only their product RC matters. Let's make an abstraction and define a quantity τ as $\tau \equiv RC$.

This time constant has a physical meaning. To see what it is, give the system the simplest nontrivial input: V_0, the input voltage, has been zero since forever; it suddenly becomes a constant V at $t = 0$; and it remains at that value forever ($t > 0$). What is the output voltage V_1? Until $t = 0$, the output is also zero. By inspection, you can check that the solution for $t \geqslant 0$ is

$$V_1 = V(1 - e^{-t/\tau}). \quad (2.12)$$

In other words, the output voltage exponentially approaches the input voltage. The rate of approach is determined by the time constant τ. In particular, after one time constant, the gap between the output and input

My EE experience is limited, but is it possible for R or C to vary with voltage (ie the resistor becomes less resistive at high voltages)? Therefore, is it possible for tau to be dependent on V0 (or t, if V0 is a function)?

No, we are dealing with simple, fixed resistors and capacitors, which have a given resistance, R and capacitance, C respectively.

So... a step function? I think calling it by name might make this more familiar. Edit: it is actually called by name later... but I think it would be helpful to do so here (and perhaps still include the definition).

I felt this discription would be more familiar visually than saying step function, because it literally draws at the voltage, 0 and then to a constant in words as you read along.

Whoh, how did you get this again? Was this just a lucky guess from diff eq. intuition? Or is there some greater concept I'm missing?

i would appreciate an explanation also

All they did was take 2.11 and integrate it. They kept the dV1 term on the left and put dt and everything else on the right.

need to review 18.03. ew.

You just put dV to one side and dt to the other side of the equation, and then integrate. Very little 18.03 involved. (plus it is not that bad)

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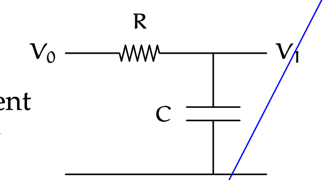

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Another great explanation; this should be done for the majority of equations.

Agreed! This explanation makes the whole equation seem less intimidating and makes me feel like I don't need to search back into my 18.03/8.02/ old textbooks to understand what's going on.

I agree– the abstraction made with tau and the result here are very clear

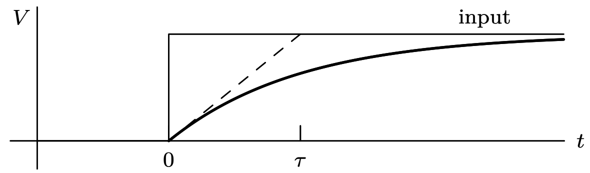
I agree that the explanation of the equation is clear and very helpful, but I'm still unclear as to how we arrived at it. I guess if the point of the example is to show how abstraction is useful to understand the equation, then it doesn't really matter.

Definitely a good explanation. I wasn't really going to try to understand this equation on my own. Kinda glad they did that for me.

I agree this is a very helpful method, although I can't honestly say it's the first time I've ever seen it.

This is an easy example to explain, I think attempting to explain some previous equations would do more harm than help. Although I do agree I enjoyed seeing this step by step.

voltages shrinks by a factor of e. Alternatively, if the rate of approach remained its initial value, in one time constant the output would match the input (dotted line).



The actual inputs provided by the world are more complex than a step function. But many interesting real-world inputs are oscillatory (and it turns out that any input can be constructed by adding oscillatory inputs). So let's analyze the effect of an oscillatory input $V_0(t) = Ae^{i\omega t}$, where A is a (possibly complex) constant called the amplitude, and ω is the angular frequency of the oscillations. That complex-exponential notation really means that the voltage is the real part of $Ae^{i\omega t}$, but the 'real part' notation gets distracting if it is repeated in every equation, so traditionally it is omitted.

The RC system is linear – it is described by a linear differential equation – so the output will also oscillate with the same frequency ω. Therefore, write the output in the form $Be^{i\omega t}$, where B is a (possibly complex) constant. Then substitute V_0 and V_1 into the differential equation

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$$Bi\omega + \frac{B}{\tau} = \frac{A}{\tau}, \tag{2.14}$$

or

$$B = \frac{A}{1 + i\omega\tau}. \tag{2.15}$$

This equation – a so-called transfer function – contains many generalizable points. First, $\omega\tau$ is a dimensionless quantity. Second, when $\omega\tau$ is small and is therefore negligible compared to the 1 in the denominator, then $B \approx A$. In other words, the output almost exactly tracks the input.

Third, when $\omega\tau$ is large, then the 1 in the denominator is negligible, so

is this the definition of a time constant?

i dont understand how this relates? why do we care about the dotted line?

I think the dotted line represents the output if the initial rate stayed constant, so I think it's there just to point out that voltage doesn't change at a constant rate and it actually decreases as afunction of time

Sorry I shouldn't have said "decreases as a function of time" but rather changes as one since it doesn't necessarily decrease.

Its just another interpretation of how tau relates to the transient response of the circuit

Yeah–I don't think it's relevant to solving the problem, but it might help some people understand the situation better and how tau fits in (part of an exponential rather than a linear model)

I agree that it gives real meaning to tau. "the time at which the voltage difference shrinks by a factor of e" is useful for determining tau but not necessarily intuitive, and doesn't offer much insight. I personally had never noticed the relation indicated by the dotted line, and it was definitely a cool thing to learn.

It was a new concept to me and a useful way of thinking about time constants. Is it only true for exponentials?

I'm a little confused by the graph itself. It would have been useful to have a line that represented what the output would be if the rate of approach remained its initial value.

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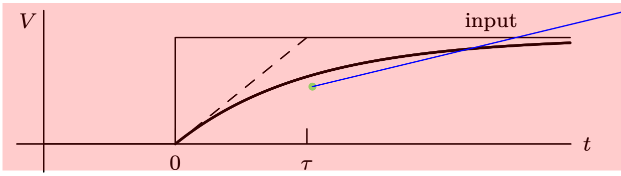

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You might consider marking the point on the V axis that the two lines converge upon.

Agreed, labeling it Vo would be useful for visualization

Vo=0... do you mean labeling it just 'V'? V1–>V as t–>+inf

Why is the time axis offset by so much from the 0 point? We aren't going to be considering negative time.

Otherwise the step rise from 0 to V falls right on the y axis and is then hard to distinguish from the y axis.

At least on my screen, it is still hard to discern a difference in the lines and if the voltage traces at t<0 are drawn on the X-axis.

I also found the fact that Tau is related to the initial rate, and the factor it decays by very interesting. Could you explain in the text the significance and reason for this relationship.

I think I should make the input and output graphs in black and the axix in gray (or not show it at all, since the t<0 region shows where the x axis is).

At this point, I'm wondering why this example is useful for this class. What was the point that was trying to be proven through this example?

I believe this example was more of a set up for the later example. In previous readings, examples have been given without any background knowledge, which confused many people. This reading actually takes its time to provide some sort of base when trying to understand the thermal problem. I found it very helpful.

Some example of really cool things with oscillatory inputs would be nice

Grammatically, this seems like an odd sentence

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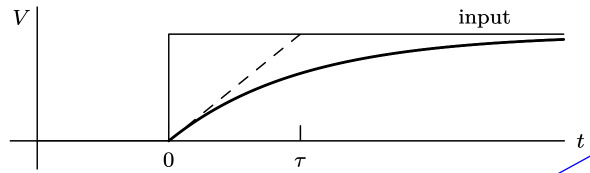

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I'd like an example or two?

I agree that another example would help, maybe one that is more EE focused.

I also agree. In all of my classes, we look at the step function because it's the easiest to learn from. It'd be nice to see a real world example of a sine wave, exponential, etc.

The example of daily temperature variations shows up in the next section, but there's no way to know that when you're reading this section. So it needs a mention right here.

Might be worth it to throw "Fourier" in here somewhere for people who might have seen it before.

I agree, it would have been helpful to see the Fourier here so we see the real part even if it is omitted.

But then people who don't know about Fourier stuff are going to complain that you put it in without explaining it. Dilemmas, eh? :P

But as written, I was somewhat confused about what it meant until I thought of the Fourier example

a. whats with the little side note things? b. don't complicate it unnecessarily

I think that the side note would be useful if it were actually put on the side...not in-line with the rest of the text

So (comma) let's...

This class really sneakily teaches you a lot of things you don't expect to learn. Not complaining, just observing.

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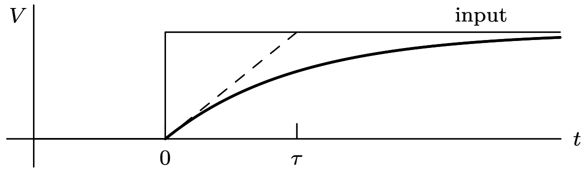

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What is "real part"? Is this a physics notation?

I think it's referring to the real part of a complex number (without the 'i' part)–since they said A is possibly a complex number

Here Ae^(iwt) = Asin(wt) + A*i*cos(wt), so the real part is A sin(wt).

I think it would be helpful to actually right exactly what real part you are referring to. Since for some this may be very new and abstract information, the more explicit you can make it the better

That's true when A is real. But if A is complex, e.g. x + yi, then the real part of Ae^(iwt) is x*sin(wt)-y*cos(wt).

Alternatively, if A is Ce^(i*phi), where C and phi (the phase) are real, then A*e^(iwt) is Ce^(i(wt+phi)), so the real part of A*e^(iwt) is C*sin(wt+phi).

After the confusion earlier since time-dependence was mentioned, this sentence really helped clear up in what way an RC circuit is linear. Maybe the fact that it is linear due to a linear diff eq in that paragraph near the beginning would help?

even with a oscillatory input?

You should probably have V_1(t) = Be^(iwt) written explicitly somewhere, and bpossibly show at least some of the algebra of substituting the 2 equations into 2.13

It could go right after "in the form", just as "V1 = Be..."

add tau in here to make it more coherent.

For me, the whether or not tau is in here doesn't make a difference since i know that tau is just RC. But it would help if there was a little more math shown–how we got to these expressions

Something (like RC=tau) needs to be different here, otherwise this is the same as eq. 2.11.

I agree that these are the same equation now and it would be nice to differentiate them

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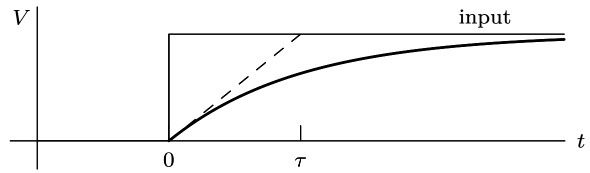

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I would like to see these math steps in a little more detailed manner. I am slightly confused what you are using for V1 and I would appreciate seeing the intermediate step before the common factor is removed.

I too was a little confused at first, but he actually defines what they are in the paragraph above. V1=Be^(iwt) and V0=Ae^(iwt).

Can we see the math for this simplification?

Also, this could be nice for the dimensional analysis leading to the quantity being unitless, as mentioned below by someone else.

Agreed - in this case, I think the math would be useful, and it doesn't take too much to put the values into the differential equation.

i'm not sure if you mean replacing V1 = Be^iwt and V0 = Ae^iwt and dV/dt = Biwte^iwt and then taking out the e^iwt but maybe just having a secondary step in the above equation which shows this replacement would be helpful. we can always go back and figure it out in our heads but it'd be nice just to see it anyway.

definitely, 18.03 was awhile ago for most of us.

Why not convert this to the conventional a + ib form, instead of having i in the denominator?

This form is much more applicable to the problem even if mathematically it isn't as pretty

why is this the transfer function? it does not seem to go along with what I have learned about them

Is it called a transfer function because it contains many generalizable points or is it that this particular function contains these points and there is another definition of a transfer function?

its a standard term. According to wikipedia: A transfer function (also known as the network function) is a mathematical representation, in terms of spatial or temporal frequency, of the relation between the input and output of a (linear time-invariant) system. " So to answer your question, yes

This is exactly what we're doing in 2.004 right now..

I like this way of breaking down the problem but how would this compare to the abstractions we've been doing, it seems like we're going in the opposite direction.

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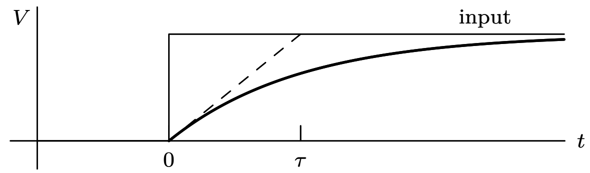

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It might be useful to show the unit analysis here

Agreed.

Also agreed - not immediately clear that it's dimensionless.

angular frequency has units of 1/t and a time constant has units of t. Multiply and its dimensionless!

is there "credit" given for the checkbox "I agree" or do I really have to enter Agreed everywhere?

Neither gets credit! There's always a question from someone to answer, so you shouldn't ever feel like there's nothing to contribute.

Given a choice between clicking the "I agree" box or just typing in "I agree" as a comment, click the "I agree" checkbox. Then NB can tell me what comments are most pressing. It's not [yet] smart enough to parse the text, and uses only the explicit "I agree" button.

I agree that it should be clear that it's frequency*time and therefore unitless, but it's not that clear how R*C gives units of time.

Again, a great summary sentence!

Weird spacing. Seems like it should be one big paragraph since sentence transitions are "first, second, third."

I always find it useful when "large" is defined... If you go by the magnitudes of (1+iwt) vs. (iwt), then wt 10 gives you within 10%, I think.

Almost! It would be true without the "i". So, 1+wt is within 10% of wt, if wt 10. But with the "i", the mangitudes are within 0.5%.

Oops, I'm not sure what I did wrong. I tried to compare the magnitudes of (1+iwt) and (iwt).

Redoing it I get: |1+iwt|=sqrt(1+(wt)^2), and |iwt|=wt. At wt = 10, |1+iwt| 10.05, so there's your 0.5%.

Turns out (solving the algebra), you only need wt=2.2 (|1+iwt|=2.4) to get the magnitudes within 10%.

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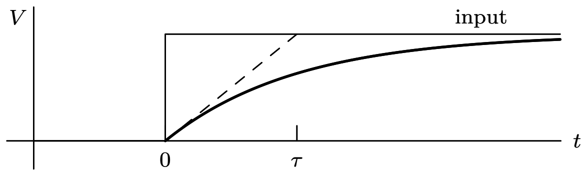

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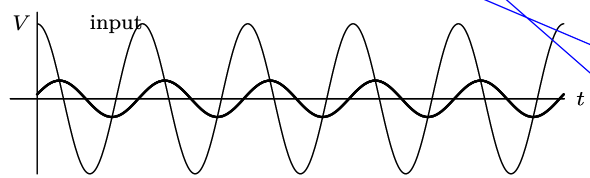
This equation – a so-called transfer function – contains many generalizable points. First, $\omega\tau$ is a dimensionless quantity. Second, when $\omega\tau$ is small and is therefore negligible compared to the 1 in the denominator, then $B \approx A$. In other words, the output almost exactly tracks the input.

Third, when $\omega\tau$ is large, then the 1 in the denominator is negligible, so

One great thing would be to summarize paragraphs or smaller sections with boxes similar to the way you summarize full chapters (or parts of chapters) with the little boxes. It makes it easier to get the exact point across; the first time through I missed a key point in the previous paragraph, and I think summary boxes (or bold type or something) may make the reader be attracted to the point,

$$B \approx \frac{A}{i\omega\tau}. \tag{2.16}$$

In this limit, the output variation (the amplitude B) is shrunk by a factor of $\omega\tau$ in comparison to the input variation (the amplitude A). Furthermore, because of the i in the denominator, the output oscillations are delayed by $90°$ relative to the input oscillations (where $360°$ is a full period). Why $90°$? In the complex plane, dividing by i is equivalent to rotating clockwise by $90°$. As an example of this delay, if $\omega\tau \gg 1$ and the input voltage oscillates with a period of 4 hr, then the output voltage peaks roughly 1 hr after the input peaks. Here is an example with $\omega\tau = 4$:



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So I don't fully understand how this explanation (how the differential equation works) describes how a low pass filter works. I feel that the last paragraph accurately explains what a low pass filter does, but I dont understand from the previous explanation how it actually does this. It seems like one minute we are talking about the waveform and the differential equation and then we jump to a low pass filter.

Good Explanation.

I like this explanation. The page prior to this was important in understand how abstraction fits in but it's nice to get an explanation of some of the fuzzy points even though they may not be as relevant.

why does this happen? what leads to the 90 degree delay

you're going to explain this but not the equations or math?

Whenever we get to the symmetry part of the course, there are some nice examples in the 18.04 book that use symmetry to calculate long expressions of complex arithmetic (in particular powers of binomials)

Is this delay the interesting part that we figured out earlier, so we can apply it to more real life examples?

How exactly did you come to the conclusion that the delay is roughly 1 hour? Is it due to dividing by 90 (which is 1/4 of 360, so 1/4 of the input's 4 hour period) or is it due to wt=4 (so input's 4 hour period divided by wt is 1 hour)?

@ writer-of-comment: Your word choice is confusing. But the general idea is that the 1 hour is a quarter of the 4 hour period. Thus at that time, the max output should occur then.

I believe this is because 90/360 = 1/4 of a period. 1/4 of the 4-hour period gives us 1 hour after the input peaks for the output peaks.

Yeah, it's because it's a quarter of the 4 hour period–the graph right under it shows what he's talking about

yup 90/360 is 1/4th and 1 hour is 1/4th of 4. Would you ming showing us why it lags by 90 degrees.

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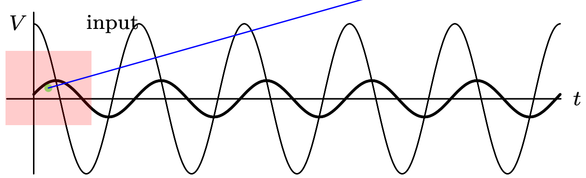
In this limit, the output variation (the amplitude B) is shrunk by a factor of $\omega\tau$ in comparison to the input variation (the amplitude A). Furthermore, because of the i in the denominator, the output oscillations are delayed by 90° relative to the input oscillations (where 360° is a full period). Why 90°? In the complex plane, dividing by i is equivalent to rotating clockwise by 90°. As an example of this delay, if $\omega\tau \gg 1$ and the input voltage oscillates with a period of 4 hr, then the output voltage peaks roughly 1 hr after the input peaks. Here is an example with $\omega\tau = 4$:



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Is the output the bold curve in the graph? Why is it's amplitude smaller?

Its explained a little higher up, tat the amplitude by is shrunk by a factor of wt compared to the input amplitude

Again, it might help to label the V access

access? do you mean axis? in which case, it is labeled.

I would like to see a few labels on both axes to establish relative magnitudes.

If you just want relative magnitudes then can't those be deduced from the unlabeled graph? You can tell by looking that the taller fn is more than twice the amplitude, etc.

it'd be nice to label the two curves w/ a key or something, just for emphasis

I also agree, I think I get it but there's no need not to have labels.

I usually find your summary paragraphs helpful, but this one is a bit confusing. It kinda just goes over my head.

While I know that this is true, I'm not entirely this is an adequate follow up from the previous discussion. You prove that for omega*tau >>1 that the output is a dampened version of the input, but I can see a reasonable question being "so what if omega is really small (low frequency) but tau is just really big?" All I'm saying is perhaps you should add a line or two clarifying further that the "low frequencies" are not relative to normal frequency range but to the selected tau (that a large tau means only extremely low frequencies pass through and the rest are considered "high-frequency")

I'm not really sure that I follow how what we've done so far has demonstrated this.

I understand, but I think there's a missing bit that the limits of wt essentially mean limits of frequency–wt being small means low frequency, and wt large means high frequency. It could be confusing that there is still the factor of the time constant in there, but it just scales the exact values of "low" and "high" frequency. Later, in the thermal example, almost all frequencies are too high because it is the tau in that case that is usually high.

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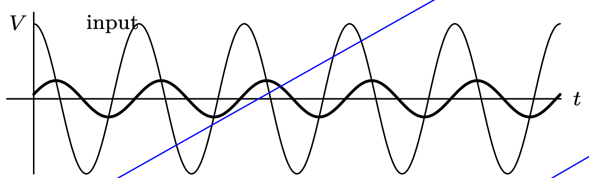

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Great description of low- pass filters, simple and to the point.

Agreed, this may be the first time so far where enough background information was given so that lack of knowledge doesn't bog down the later example.

I agree... readers could actually skim, or not even understand much of what was talked about previously but understand its application here by this simple paragraph.

This is probably the best explanation of a low pass filter I have ever had

Was a circuit explanation of low pass filters really necessary? I took 6.003 and we learned about lowpass filters without using circuits- I was lost through the entire article until this paragraph

I don't understand the applications of this example.

It would be nice also if we can relate the low pass filter back to the beginning of the class where we talked about cd's and frequency. Low pass filters in electronic music to me is very interesting.

is the word "abstract[ed]" used here the same way as we've been talking about "abstraction" in general?

yeah, I think so. Because in the next section we go on to use this abstraction in another setting.

Yes it is, abstraction was defined in prevoius sections of Chapter 2, and this one is really just examples in different contexts.

It would be useful if throughout the text the information was put into an abstraction perspective.

Perhaps I'm just already used to seeing RC filters as examples of low pass filters, but I didn't quite follow how all this circuit analysis led to an abstraction away from the circuit origin.

I don't see how it is abstracted away from its origins

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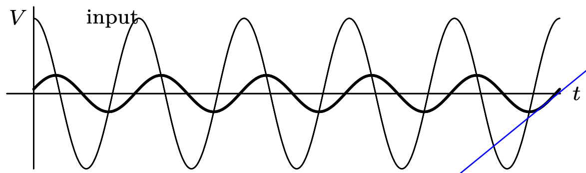

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I thought it was really interesting that heat flow is so closely connected with RC circuits. But I would have liked a short explanation of why these two concepts are so closely connected. I feel like, in this section, I learned how to reappropriate the RC circuit equations for use as heat flow equations, but I didn't know why I was allowed to do so.

I agree.

I guess looking back, not knowing what resistors and capacitors do physically/qualitatively makes the thermal analog not as clear. A brief explanation as to how capacitors store charge and how resistors dampen current flow might make the transisition from circuits to thermodynamics more clear.

This idea is pretty interesting to me. Having seen a lot of stuff related to RC circuits through course 6 classes I'm always amazed at how many things they can be used to model.

This might make the section too lengthy; it's to serve a purpose for the section; if someone is then further interested on capacitors, they should look it up. But that's just my point of view; I like these chapters short.

It would be nice if we could go over the "pool temperature" exercise in class. I somehow don't quite get how to solve it correctly.

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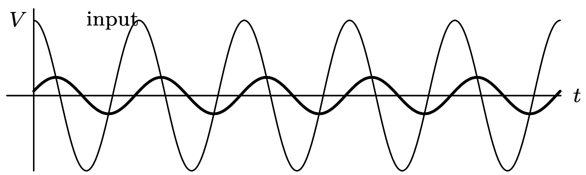

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Where can we see an example of a high-pass filter? And how do we know that this is a low pass filter?

Heat flow can be understood as a low pass filter because the time constants of heat flow systems are usually high. That is, it takes a relatively long time for the system to respond to changes in temperature. And if the temperature is changing very rapidly (high frequency), then the system will respond minimally (the high frequency fluctuations will be filtered out).

Yup. It makes more sense if you've actually seen a high pass and low pass filter work, because you can figure out what they do to the frequency, and then apply the concepts in real life. I think it would help a little if there was a demo involved with this section, but that is hard to do on paper!

"Rapidly" and "slowly" are relative terms. Looking at the value of w*t will give a clue to whether we are in the high or low frequency regime. If w*t is large, then the frequency is relatively high for this time constant. If w*t is low, then the frequency is relatively low.

Heat transfer behaves like a low pass filter because it can be modeled by the same RC circuit as before (heat capacity is like capacitance, and thermal resistance is like electrical resistance). Any system that can be modeled in such a way will act as a low pass filter, regardless of the time constant. The value of the time constant only affects what counts as a "high" or "low" frequency.

Thanks for the clarification I would've had the same question.

that makes complete sense and i've never thought of it before, not in 2.003 or 2.005 or 8.02

how is it a model for heat flow?

this makes sense because the heat flow through materials has resistance as well as the material holding energy

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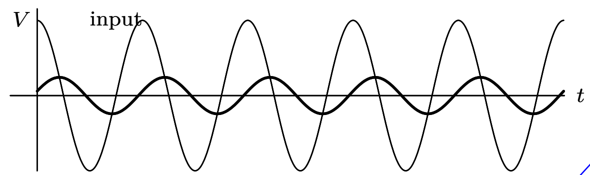

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Interesting point! I never really thought of the connection before, but it makes sense after reading this paragraph.

I agree. I really liked this paragraph and the interesting insight it brought up.

This is a really cool comparison. I never thought about this before, and the way you connected the more difficult low-pass filter with the everyday cup of coffee makes both concepts easier to understand.

If anyone is interested in this, 18.303 goes into it a little more in depth.

i dont think you're referring to this example, which is extremely simple, but everyone course 2 here has taken thermo.

I guess this is why in Coffee shops they warm the cups and jars on the surface above the machine, so that they delay the time constant and the milk and coffee is kept cold for longer periods of time.

Perhaps this example could be placed before the RC circuits as it is easier to relate to.

I think it's best how it is, since this one is easier to just relate to the RC circuit. Also, logically it makes sense because here we see how the abstraction of resistance and capacitance actually apply to other seemingly unrelated situations–like a cup of tea cooling down–which is pretty crazy

this was a very good explanation relating the electrical circuit model to the thermal model

perhaps i'm being too nit-picky here, but i believe that, technically, heat cannot be "stored" because it is defined to be heat transfer.

thermal energy...?

This is a good description of the components of the system in terms of their electrical analogies.

do you really need to say "so-called"? we get it.

Perhaps he is defining a difference between the term thermal capacitance as a way of comparing the behavior with an EE capacitor and the actual physical behavior of the process in question

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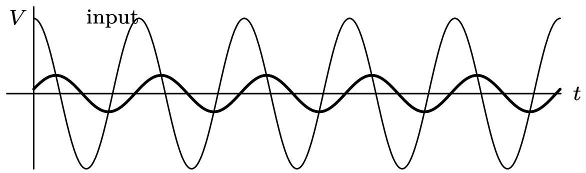

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but this isn't what the abstraction was in the previous section. in the low pass filters we used tau as the abstraction instead.

I agree with that except for the "instead". Tau was one of the abstractions, and resistance and capacitance are two further abstractions.

This makes sense to me.

I never thought of the analysis for heat before, very interesting

thats really cool! haven't taken any thermo class since I'm 6-1, but the analagy with RC really resonates.

I like the last sentence in this paragraph - it summarizes everything nicely and drives home the purpose of the previous section.

Can you show this mathematically?

If R_t is the thermal resistance and C_t is the thermal capacitance, their product R_tC_t is, by analogy with the RC circuit, a thermal time constant τ. To measure it, heat up a mug of tea and watch how the temperature falls toward room temperature. The time for the temperature gap to fall by a factor of e is the time constant τ. In my extensive experience of neglecting cups of tea, in 0.5 hr an enjoyably hot cup of tea becomes lukewarm. To give concrete temperatures to it, 'enjoyably warm' is perhaps 130 °F, room temperature is 70 °F, and lukewarm is perhaps 85 °F. The temperature gap between the tea and the room started at 60 °F and fell to 15 °F – a factor of 4 decrease. It might have required 0.3 hr to have fallen by a factor of e (roughly 2.72). This time is the time constant.

How does the teacup respond to daily temperature variations? In this system, the input signal is the room's temperature; it varies with a frequency of $f = 1\,\text{day}^{-1}$. The output signal is the tea's temperature. The dimensionless parameter $\omega\tau$ is, using $\omega = 2\pi f$, given by

$$\underbrace{2\pi f}_{\omega}\,\tau = 2\pi \times \underbrace{1\,\text{day}^{-1}}_{f} \times \underbrace{0.3\,\text{hr}}_{\tau} \times \frac{1\,\text{day}}{24\,\text{hr}}, \qquad (2.17)$$

or approximately 0.1. In other words, the system is driven slowly (ω is not large enough to make $\omega\tau$ near 1), so slowly that the inside temperature almost exactly follows the outside temperature.

A situation showing the opposite extreme of behavior is the response of a house to daily temperature variations. House walls are thicker than teacup walls. Because thermal resistance, like electrical resistance, is proportional to length, the house walls give the house a large thermal resistance. However, the larger surface area of the house compared to the teacup more than compensates for the wall thickness, giving the house a smaller overall thermal resistance. Compared to the teacup, the house has a much, much higher mass and much higher thermal capacitance. The resulting time constant R_tC_t is much longer for the house than for the teacup. One study of houses in Greece quotes 86 hr or roughly 4 days as the thermal time constant. That time constant must be for a well insulated house.

In Cape Town, South Africa, where the weather is mostly warm and houses are often not heated even in the winter, the badly insulated house in which I lived had a thermal time constant of around 0.5 day. The dimensionless parameter $\omega\tau$ is then

Comments on page 4

This is a very neat way of looking at thermal transfers. However, I am not sure how the diagram for a low-pass filter translates to this cup of tea. It's just non-obvious how a the thermal capacitor or thermal resistor should be connected with thermal wires and where the temperatures should be sampled.

I'm not really sure but I think that these two things are juxtaposed due to the time constant similarity

Oh, that's a really clever analog! I took thermo last term, but we never approached it in this manner.

Really? In 2.005 we did, and I just realized now that my thought process in solving those problems was abstracting things – treating physical objects as resisters and capacitors to analyze them. Neat that this chapter helped me explicitly realize that now.

its amazing how many things you can model with resistance and capacitance. I think we did something in 2.004 where we modeled springs and dampers similarly... yay cross-disciplinary connections :)

Yes! One thing I've really liked about this class is, for these kinds of problems, it seems to either explain what i've been doing in my head or give me some new wayt o approach problem!

Could this alternatively be calculated using Newton's Law of Cooling?

This is a very clear paragraph, the time constant example is explained well here

I agree. This paragraph strikes a nice balance of technical and understandable.

I particularly liked the clear explanation of how one might go about estimating tau for a real system.

temperature estimation of lukewarm...?

oh, it's answered below

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Isn't this not true, since F is not on an absolute scale from 0?

I agree. I don't really understand how you could make this calculation accurate. If you change units (to Kelvin, let's say) you would definitely not get a factor of 4. Farenheit is a pretty arbitrary unit....

That flaw in the Fahrenheit scale doesn't matter here since it's just the delta T that matters. The same gap between room temp. and tea temp. might have gone from 55C-21C = 34C when hot to 30C - 21C = 9C when it cooled, or also about a factor of 4 (with the differences just scaled by the 9/5 F/C ratio)

Is there a quality factor Q for this, as there is for an RLC circuit?

LRC circuits are second order circuits whereas RC circuits are 1st order circuits. Only seconds order circuits have Q factors.

Wouldn't this also depend on surface area and other variables that aren't used to calculate time constant?

Is this saying we can find tau by letting exp(-t/tau) = 2.72?

Close, I think it's that exp(-t/tau) = 1/2.72. If you solve that equation by taking ln of both sides, you get t=tau.

It would have been helpful to see this quick comparison when tau was initially introduced.

why does the concept of time constant come from an RC circuit? I think if you switched the order of these examples it would make more sense

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Why is the frequency day^-1? Are we just considering the period to be 1 day? And ignoring day-to-day changes? (a summer day being much different than a winter day)

I think that's the frequency of the driving function, the daily fluctuation of temperature. However, the RC circuit does not change the overall frequency (see previous page), so the overall frequency is the frequency of the driving function.

I believe the assumption is that each day's temperature fluctuation is, more or less, the same. So the room's temperature at a given time of day is the same as it is +-24 hours.

I agree with the above comment - I think the idea is the temperature at 9am today will be the same as the temperature at 9am tomorrow, giving you the 1-day period, or the 1-day^-1 frequency.

but frequency in this context is a little weird.

Just think of it as 1/period as mentioned above

This is also confusing because it doesn't specify that we're just leaving a teacup in a room for a time of a day or so AFTER it has come to equilibrium.

Are the sub-brackets necessary?

I think so.

Agreed, I think they are necessary. I probably would have been confused without them, or I would have had to continually scroll back up to where the equation was initially written. I don't think they detract at all from the text or equation.

I would definitely say yes, they make the point clear for us and the analog to RC circuits easy to understand.

If they are necessary, the only formatting thing that seems to bug me (and it may just involve being OCD about alignment) is that the f sub-bracket dropped below the others

I am finicky about alignment as well. I wrote a few TeX functions (macros) to handle that; I'll insert the magic code and make the alignment work out.

This whole section seemed clear and easy to follow. It might be kind of nice to relate it to Fourier's law of conduction.

Could this be explained quantitatively by any chance?

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so what are the possible factors that affect the thermal resistance? length, surface area, and what else?

here we're looking at thickness of the walls and surface area–the thicker the wall, more thermal resistance. but at the same time, teh greater the surface area, the more heat is able to escape, so smaller thermal resistance

area, material

Are we just ignoring the fact that a teacup has no top? Would it be fair to assess the comparable thermal resistance between a teacup and a house with no roof?

Good point for a further refinement, though we aren't directly comparing the two physical situations, just their overall time constants (abstraction). It's more that in the first example we've neglected convection off the top of the liquid, which *is* a significant route of heat transfer from a teacup.

edit to my earlier note: We didn't really neglect convection but we lumped it into our abstraction, since we didn't consider modes of heat transfer, just our empirical experience of the time constant as our tea cooled.

http://en.wikipedia.org/wiki/R-value_%28insulation%29 Thermal conductivity is covered here.

although the house has more surface area and will lose more heat to convection - conduction is a much more efficient heat dispenser and the resistance is greater in the house (due to it's material properties)

never thought of that

I don't entirely understand what is compensating for what here. But besides this, I really like this example, it really makes the point of the article clear.

So if the house has a smaller thermal resistance, shouldn't it have a SMALLER time constant?

So I didn't do UPOP, but some of my friend's did, so this question goes out to you guys. Does a house really have smaller thermal resistance than a cup of tea?

this is how they explain heat transfer in 005- with resistances

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This paragraph helped me a lot in understanding how the house compares both to the tea cup and a circuit.

I think I may be getting the results of resistance and capacitance confused here in terms of the time constant. There is a lower thermal resistance (smaller R) but it is balanced by a significantly larger C, so the overall time constant is greater, right?

so, just to clarify, thermal capacitance is related to the size/volume of the object?

not quite; heat capacity has the units of J/kg-K, so if there is more mass then there is more thermal capacitance. you can have a larger volume but less thermal capacitance if the materials are different.

specific heat capacity is J/kg-K, which you multiply by mass to get heat capacity of, e.g., your brick house.

So the air mass is the capacitance and the walls the resistors to temperature difference, right? So this can be applied to anything that stores energy and looses it over time.

The walls are both resistance and a contributor to the capacitance. I suspect most of the thermal capacitance (often called thermal mass) is in the walls and other solid objects, rather than in the air.

When we study random walks, we'll be able to estimate the relative contributions. The paper I was reading about time constants of Greek houses said that furniture made a significant contribution to the time constant. I think it was 30 hours (out of the total of 90 hours).

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I'm a little confused on how a house can have a 4 day time constant when each day the outside temperature rises and falls...thus making temperature fluctuations inside of the house a daily thing

Ah, don't confuse the time constant, which is an intrinsic property of the house, with the input or driving frequency omega (or with 1/omega). They can be totally different.

Because the system is linear, the driving frequency is also the output frequency, and neither need be the same as 1/tau. Rather, the time constant helps you figure out the output amplitude given the input amplitude (e.g. if the input frequency is much higher than 1/tau, then the output amplitude is small compared to the input amplitude).

If it's anything like mine, your Cambridge/Somerville apartment probably has a time constant closer to South African than Greek. You can tell because the delay is short between when it gets cold or hot outside and your apartment follows suit.

this was also confusing me but the response helped. However, I wonder how you test the time constant of a house if you don't have conditions where you can heat it up and then let it drop back down (like the tea cup) without it being greatly affected by outside conditions

why Greece?

Greece gets quite hot in the summers and thus it is necessary to have well insulated houses to avoid getting baked in the summers. Also, older house construction methods in the Mediterranian region were designed to be well insulated (narrow streets, houses built with thick clay/stone walls, etc)

Does the temperature of the room affect the time constant?

In a linear system, the time constant is independent of the temperature (just as the RC time constant is independent of the voltages). No system is completely linear, but both the room and the RC circuit are very good approximations.

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Just wondering, does humidity play a role? Warm air with more moisture having a longer or shorter time constant?

That is a very interesting question. I don't know the answer but I suspect that humidity leads to greater thermal transfer and thus reduces the thermal time constant. Heat is transfered through phonons (vibrations of molecules) and higher humidity means higher water vapor and thus more efficient heat transfer.

But humid air has a higher specific heat, though the effect is small.

It might be nice to give the dimensionless time quantity for the houses in Greece. Not that it's hard to find, but it'd just be more convenient to have direct comparisons between the teacup, the house in Greece, and the house in South Africa.

This sentence is hanging out, and read funny. I think something more similar to, "That must be a well insulated house!"

I agree - I was confused as to how you were able to assert that the time constant must be for a well insulated house. Is there a reason homes in Greece are "well insulated"? I like the inclusion of the South African homes later, but this sentence really just sticks out awkwardly

These facts are pretty interesting.

Why were you living in South Africa?

I was one of the founding faculty of the African Institute for Mathematical Sciences (http://www.aims.ac.za) in Cape Town, and I taught several of the courses.

And, almost all my better half's family is still in South Africa.

$$\underbrace{2\pi f}_{\omega}\,\tau = 2\pi \times \underbrace{1\,\text{day}^{-1}}_{f} \times \underbrace{0.5\,\text{day}}_{\tau}, \tag{2.18}$$

or approximately 3. In the (South African) winter, the outside temperature varied between 45 °F and 75 °F. This 30 °F outside variation gets shrunk by a factor of 3, giving an inside variation of 10 °F. This variation occurred around the average outside temperature of 60 °F, so the inside temperature varied between 55 °F and 65 °F. Furthermore, if the coldest outside temperature is at midnight, the coldest inside temperature is delayed by almost 6 hr (the one-quarter-period delay). Indeed, the house did feel coldest early in the morning, just as I was getting up – as predicted by this simple model of heat flow that is based on a circuit-analysis abstraction.

I think that while the Unix example in the previous section was more interesting to the course 6 audience, this example is more interesting and understandable for the course 2 audience.

I really like this example. I did not think I would ever be able to understand the pool changing temperature in the diagnostic and problem set, but now i do.

Same here. The house temperature variations is a really good example of using extraction to solve a problem.

I would've thought the coldest temperature is near dawn, since the outside air had all night to cool.

this makes sense to me i just get a little confused when it cycles to a new day

So this really is just a universal truth?

I'm sure there are varations and this is a massive oversimplification, but it does make sense by experience, no?

Agreed - I'm sure it can't be true for everything, but that does sound about right for my house, and it's probably just a general rule of thumb to follow.

A little intuition tells us that 1/4 period is reasonable. More than that would mean that your house would still be cooling off later in the morning even after the temperature outside had begun to warm back up for a while.

Previously this was said to apply when omega*tau>>1...but I wouldn't consider 3>>1, so perhaps this should be clarified or the restriction relaxed to just >1?

slightly confused. 0.5 is the period which is 12 hours, right? so isn't 1/4 of the period 3 hours, not six? (nevermind i got it, the time constant is 0.5 but the period is 1]

Finally someone explains to me why my dorm room is so cold when I wake up even though it is warm when I go to bed at the coldest point in the night.

This is also a little biased by presumably being covered in blankets for most of the night...

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Your blowing my mind Professor Sanjoy.

Agreed, its awesome seeing how these simple abstractions and comparisons let you analyze real world situations quickly.

Is it just a coincidence that the RC abstraction could apply to heat flow, or could we find similar abstractions throughout the physical sciences?

The RC abstraction can be applied to _many_ different systems...think back to any physics class you've taken that used /exp/ ever...same concept.

I would imagine it would apply to anything which can be modeled as a value flowing through a system. So it probably extends to fluid dynamics, and I'd guess air flow as well. Of course, all of these would only be rough approximations. Fortunately, that's what we're looking for in most cases.

I like the example used in this section. While following the math and circuitry may have been a little tedious at first, its application to simple heat flow within the home was very unique and easy to follow!

I agree. I have seen circuits as models for various systems and it really emphasizes abstraction.

Just checking, but abstraction is reusing principles of something we know on something new. But all this approximation seems like a stretch. Is this not so?

I really liked the variety of examples in this section. Something more typically analytical and "science-y," an everyday application, and a broadened application that helps explain a larger aspect of life.

I very much liked the examples included here. The EE example was perhaps a little too in depth, but the thermal examples made it much clearer.

I think that the variety of examples not only helps to ensure that more people can relate to at least one example and gain a better understanding, but it provides some room to compare and contrast so that we can pick out the similarities between the examples and see what information is applicable to different situations and what was specific to one example.

I happened to also like the thermal example as opposed to the circuit one. ME vs EE for you.

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Good section, with the right amounts of explanation and interesting information.